Continuous measurement of position and momentum of a quantum particle

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Doctoral thesis summary

As a result of the impressive development of experimental techniques in the last decades, it is now possible to observe and control single quantum objects such as atoms or ions. Such a level of precision was unthinkable at the birth of quantum mechanics a hundred years ago. Meanwhile, quantum theory stands out from other fields of physics, among other things, in that measurement becomes a fundamental issue.

The concept of projective measurement developed by the fathers of quantum mechanics is one of the basic elements of the theory. At the same time, it is insufficient to describe the vast majority of experimental situations. During measurement, a quantum system interacts with the measuring device. A detailed description of the dynamics of the latter is usually neither desirable nor technically feasible. Therefore, it becomes necessary to describe the object of measurement within open system theory. The analysis of physically admissible mappings of such systems leads to the formalism of the positive operator valued measure (POVM) and the currently accepted, generalized notion of quantum measurement.

Describing the change of the quantum state induced by a measurement is particularly important when we are interested in measuring the same system repeatedly. Due to the indeterministic nature of quantum mechanics, each measurement causes a random perturbation of the particle. This leads to noisy trajectories, different in each realization. This kind of dynamics is described by stochastic differential equations.

The main goal of this work is to propose a new model for continuous measurement of position and momentum of a quantum particle. This model meets the requirements of the above-mentioned modern theory of quantum measurement. At the same time, it is quite close to the original projective theory. As a result of the measurement, the particle at random times experiences perturbations and its wave function is projected onto a state localized in position and momentum space. A natural choice is a Gaussian state. We assume a discrete set of possible outcomes of these measure-

ments. This corresponds to a grid of points in phase space where the "detectors" are located.

Formally, the dynamics of the system is described by the stochastic Schrödinger equation, whose deterministic counterpart is the Gorini-Kossakowski-Sudarshan-Lindblad (GKSL) equation for the density matrix. In numerical calculations, we use the Monte-Carlo Wavefunction (MCWF) formalism, which is a method for solving the GKSL equation. This allows for obtaining single quantum trajectories in which continuous non-linear evolution is interrupted by jumps, i.e. particle detections. We interpret these trajectories as individual realizations of particle dynamics. Taking the detections as the only physical signal available to the observer, we base all further analysis on them.

In the variant proposed here, the model has two key parameters: measurement intensity and detector grid density. Depending on these quantities, we identify a number of dynamic regimes: We investigate under what conditions the observed trajectories statistically coincide with classical motion. In our simulations we also observe the Zeno effect. We supplement the numerical results here with analytical calculations for the simplified scenarios of one and two detectors. This allows an estimation of the particle motion delay caused by frequent observation. We also identify a regime in which the average position of the particle is a steplike function, moving from one measurement point to another.

In addition to determining the average trajectories, we also examine their dispersion. We calculate the dispersion of detections in position and momentum space, showing how these quantities scale with time. In the dense-grid approximation, we calculate the diffusion constant analytically and present effective classical stochastic equations that reproduce the observed dynamics. In the opposite scenario of sparsely located detectors, we present an alternative to MCWF numerical method that allows to exactly calculate the dynamics of statistical variables.

Our calculations are for a free particle and one in an external harmonic potential. In the latter case, apart from analogous calculations concerning average trajectories and dispersion, we study the change in energy of the system. We present an analytical formula for the energy distribution in the initial state and show how the ensemble of trajectories thermalizes over time.

The model proposed by us has a number of features that distinguish it from models of continuous measurement studied so far in the literature. We consider a simultaneous measurement of position and momentum, resulting in a system with a definitive quantum state. In contrast, most other models consider only one weakly measured dynamic variable – this means that the action of the measurement operator M is not the same as measuring twice (M^2). The use of projective measurement operators is not only a reference to the "classical" theory of quantum measurement. Our description leads to a model whose dynamics can be studied within generalized renewal theory - a class of standard stochastic processes. This fact is used many times in this work and allows to derive a number of analytical results. Finally, in the paper we make a detailed comparison of our model with the "filter" approach popular in similar studies. On the basis of numerical simulations, we indicate significant differences in the scaling of the above-mentioned dispersion functions between the two approaches.

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