

Electromagnetic radiation of variable frequency

Wojciech Tadeusz Chyla, PhD

Self-review of research activities

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PART I

DIPLOMAS, INSTITUTIONS, DIDACTICS, MISCELLANEOUS

1. Diplomas and education certificates

- **1990 – 1993** United States Particle Accelerator School, certificates of completion.
 - * **1993** Harvard University, Cambridge, USA.
 - * **1992** Stanford University, Stanford, USA.
 - * **1991** University of Illinois, Urbana-Champaign, USA.
 - * **1990** Harvard University, Cambridge, USA.
 - **1992** – Philosophy Doctor, Physics, University of North Texas (1987 – 1992), Denton, USA.
 - * Title of the doctoral dissertation: *Expulsion of carriers from the double-barrier quantum well and investigation of its spectral and transport consequences.*
 - * Nostrification of the doctoral degree: Department of Physics, Warsaw University, 1994.
 - **1987** – Master of Arts in Physics, University of Southern California (1985–1987), Los Angeles, CA.
 - **1974** – Master of Science in Chemistry, Department of Chemistry, Warsaw University (1969-1974).
 - * Title of thesis: *Calculation of the inductive energy of interaction in the He-H⁺ molecule.*
 - **1970** – English Language College, „Methodists” (1966-1970), Warsaw – Certificate of completion.
-

2. Employment in academic institutions and didactic experience

- **Academic institutions**
 - * **1994 – 1998** Adjunct, Department of Mathematics and Science, Pedagogical University (presently Warmia and Mazury University), Olsztyn.
Responsibilities. Lectures and examinations in general and modern physics for full-time and part-time students in the Department of Mathematics and Science. Teaching laboratory classes and calculation practice classes in general physics. Participation in the licentiate examination committee.
 - * **1987 – 1991** Instructor (Teaching fellow), Teaching assistant, Department of Physics, University of North Texas, USA.
Responsibilities. Instructor (Teaching fellow): Lectures in mechanics and thermodynamics (the Resnick & Halliday calculus-based undergraduate lecture course for science majors), midterms and final examinations. Teaching assistant: Teaching undergraduate laboratory courses for science majors (mechanics, thermodynamics, electricity and magnetism, optics) and teaching open laboratory courses in general physics for non-science majors.
 - * **1985 – 1987** Teaching assistant, Department of Physics, University of Southern California, USA. *Responsibilities.* Teaching undergraduate laboratory courses in mechanics for science majors. Grading of homework and examinations of undergraduate science majors; checking and scoring of homework of graduate physics students.
 - * **1974 – 1978** Assistant, Institute of Physics, Polish Academy of Sciences, Warsaw.
Responsibilities. Individual research and numerical analytical work for other members of the Section for Solid State Theory and Microwave Spectroscopy.
- **Other research and education activities**
 - * **2014 – present** Senior specialist, Central Statistical Office.
 - * **2013 – 2013** Laboratory worker, Radwag, Radom, Poland.
 - * **1998 – 2013** Applied Science Enterprise, Warsaw, founder.
Activities: Scientific research and publications, consultancy, didactics (private tutoring and teaching preparatory courses in physics for the national maturity examination and the International Baccalaureate examination).
 - * **2008 – 2011** Principal specialist, Central Office of Measures, Warsaw.
 - * **1992 – 1993** Teacher (substitutions), Denton Independent School District, TX, USA.

- * **1992 – 1993** Consultant, Medical College Admission Test, American Institute for Research, Washington, DC, USA.
- * **1992 – 1993** Consultant, American College Testing Program, Iowa City, IA, USA.
- * **1992 – 1993** Technical translator, SCITRAN, Santa Barbara, CA, USA.
- * **1983 – 1985** Lecturer of preparatory courses in physics and chemistry at Warsaw University.
- * **1983 – 1984** Lecturer of preparatory courses in physics at the Society for Dissemination of Science, Warsaw.
- * **1981 – 1983** No permanent employment. Private tutor in mathematics, physics and chemistry.
- * **1981 – 1981** Laboratory worker, Institute of Experimental Biology PAN, Warsaw.
- * **1978 – 1980** Consultant and numerical analyst, the Metal Structures Group, Warsaw Technical University.
- * **1978 – 1980** Consultant, Section of Crystallography, Warsaw University.

3. Miscellaneous

- **Bibliographic data**

- * Number of articles – 33
- * Number of articles published in Philadelphia List journals with IF – 15
- * Number of articles published in MNI SW-recognized national journals – 13
- * Number of articles published in other scientific and technical journals – 4
- * Number of articles published in conference materials – 1
- * Number of presentations at conferences and seminars – 12
- * Number of citations – 13 (WoS), 14 (JCR), 42 (Google Scholar)
- * H-index – 3 (JCR), 4 (Google Scholar)
- * Total Impact Factor – 14.9
- * Total MNI SW score for published articles – 305

- **Awards**

- * **1993, 1992, 1991, 1990** – Stipends from Fermilab for participation in the US Particle Accelerator School, USA.
- * **1991** – Industrial Sponsorship Award (American Physical Society, Texas Section), USA.
- * **1991** – L. Miller Scholarship (University of North Texas), USA.
- * **1985** – Idalene Merrill Fellowship (University of Southern California), USA.
- * **1969 – 1974** Nine awards from the President of the Warsaw University.

- **Membership in scientific organizations**

- * **1989 – 1993** Member of the American Physical Society, USA.
- * **1989 – present**, member of $\Sigma\Pi\Sigma$ (Physics Honor Society), USA.

- **Organizational work**

- * **2013** – Delegate to the 63rd General Assembly CECIP (European Association of Manufacturers of Weighing Instruments), Rotterdam, Netherlands.
- * **2010** – Secretary of the TC1 OIML (International Organization of Legal Metrology) Technical Committee session, Warsaw.

- **Editorial work**

- * **2008 – 2011** Scientific and technical editing of *Metrologia – Biuletyn GUM* (quarterly).

- **Reviews of articles for scientific journals**

- * Journal of Electromagnetic Waves and Applications
- * International Journal of Astronomy and Astrophysics
- * British Journal of Applied Science and Technology
- * Physical Review and Research International
- * Journal of Applied Physical Science International
- * Physical Science International Journal

- **Reviews of monographs published in Polish journals**
 - * T. J. Quinn, "From artefacts to atoms. The BIPM and the search for ultimate measurement standards", Oxford University Press, Oxford, 2012.
 - * R. Nater, A. Reichmuth, M. Borys, P. Zervos, „Dictionary of weighing terms. A guide to the terminology of weighing”, Springer, Dordrecht, 2009.
- **Analyses, expertises**
 - * **2009** – W. T. Chyla, Analysis of the British government document "A strategy for the National Measurement System 2010 – 2014", performed for the British National Measurement Office, DIUS, May 2009.
 - * **2009** – W. T. Chyla, Analysis of the Proposal for a Decision of the European Parliament and the Council on the European Metrology Research Program (EMRP), performed for the Central Office of Measures.
 - * **2010** – W. T. Chyla, Analysis of the proposition for the „Law on Measures”, performed for the Central Office of Measures.
 - * **2011** – W. T. Chyla, Analysis of the document „EURAMET Strategy 2020”, performed for the Central Office of Measures.
 - * **2011** – W. T. Chyla, Analysis of the CCU-CIPM project of the redefinition of the SI base units, performed for the Central Office of Measures.
- **Translations**
 - * See 'The cumulative list of translations', PART V, section 3.
- **Participation in workshops**
 - * **2013** – United States Pharmacopeia, Understanding USP requirements for balances, Basel, Switzerland.
 - * **2012** – QS Zurich, Manager of the EFS projects, Warsaw.
 - * **2009** – GUM, Measurement uncertainty calculations in practice, Warsaw.
 - * **2009** – IMS, Management systems in research and calibration laboratories according to the PN-EN ISO/IEC 17025:2005 standard, Warsaw.
 - * **2009** – Science Academy, Effective processing of information, Warsaw.
 - * **2009** – OPI, European Program "Innovative Economy", Warsaw.
 - * **2009** – ALTKOM, Handling of difficult situations – transformation of organizations, Warsaw.
 - * **2008** – GUM, Selected topics in general metrology, Warsaw.
 - * **1975** – Institute of Mathematics PAN, Programming in Fortran, Warsaw.



PART II

DISCUSSION OF THE SINGLE-TOPIC CYCLE OF PUBLICATIONS

Electromagnetic radiation of variable frequency

1. Theoretical fundamentals of propagation of variable-frequency radiation (VF rays)

Frequency (or wavelength) of the electromagnetic radiation plays the key role in the classical physical optics. On the other hand, in the classical geometrical optics, frequency of the light ray is an irrelevant constant parameter. However, there are some phenomena (e.g., reflection from a mirror in motion, propagation in a strong gravitational field or quantum interactions), where frequency of the light ray varies along its path. This is an intermediate situation between physical optics and classical geometrical optics that is not covered by the Fermat principle.

The classical physical optics can be derived from the principle of least action. The laws of the classical geometrical optics can be derived from the Fermat principle. Since the classical geometrical optics is a special case of the classical physical optics, the Fermat principle should not be considered an independent physical principle, but it ought to be derived from the principle of least action. It turns out that this is possible indeed; the derivation gives a more general form of the variation principle for geometrical optics that allows for changes of frequency of the light ray along its path. The new variation principle for geometrical optics comprises the classical Fermat principle as a special case. The simultaneous solution of both these problems is presented in¹ [1], {9}.

1.1. The origins of the research project.

1.1.1. Unification of the Fermat principle and the principle of least action.

The first variation principle in physics was the Fermat principle (the principle of least time, PLT) formulated in the 17th century (1662), which states that a light ray travels between two points in the configuration space along a trajectory that minimizes the propagation time. The initial goal of the Fermat principle was to form a theoretical basis for the experimental laws of geometrical optics. However, from the contemporary point of view, the historical meaning of the Fermat principle is that it paved the way for the Hamilton variation principle (the principle of least action, PLA), which provides a unifying formalism for the established fundamental laws of theoretical physics and a paradigm for construction of new theories, especially in the area of quantum fields and particles.

Although the laws of classical physics, quantum field theories and a variety of phenomenological theories can be derived from the principle of least action, the classical geometrical optics does not refer to the Hamilton principle and physicists still use the Fermat principle, which is considered as a fundamental principle independent of the principle of least action. The initial goal of this work was the derivation of the Fermat principle from the principle of least action, so that the number of elementary building blocks of physics is decreased by one, in accordance with the Ockham's postulate of simplicity of the natural philosophy laws.

The belief in existence of a connection between both variation principles (PLT and PLA) results from the sense of formal aesthetics in physics, which manifests in the strive for unification of fundamental physical notions and theories, and in the search for symmetries in the physical description of the universe. Such motivation is always important, but it is usually insufficient, because it is so general that it can hardly ever be specialized to a specific research undertaking.

A clue that indicates the *direct* relationship between the Fermat principle and the principle of least action is the existence of the indirect relationship between the PLA and the classical geometrical optics. Namely, since the PLA can be used to drive Maxwell equations, which comprise classical physical optics, which in the short-wave limit turns into the classical geometrical optics, there should also be a *direct* relationship between both variation principles (PLT and PLA) from which geometrical optics can be derived.

The search for the relationship between the PLT and the PLA cannot be performed at the level of Maxwell equations, because these equations impose restrictions that uniquely determine paths of light

¹ The number in the square bracket denotes the number of the article cited in the given part of this self-review and the number in the curly bracket denotes the same article in the final cumulative list of publications.

rays and do not allow for variation of the paths (any variation, i.e., a change of the path of the light ray would have violated the set of four Maxwell equations). Therefore the search for the relationship between the PLT and the PLA must be performed at the higher level of generality; the PLT must be derived directly from the PLA.

1.1.2. Incorporation of the VF radiation into the geometrical optics domain.

In the classical geometrical optics, based on the Fermat principle, frequency of light is an insignificant constant parameter. Frequency serves only for establishing phenomenological properties of a given medium (refractive index), which is used when changing parameterization of the path of the light ray from the temporal one to the spatial one. The possible dependence of the trajectory of the light ray on its frequency cannot manifest also where geometrical optics is derived from physical optics by taking the limit of infinite frequency of the electromagnetic radiation (the short-wave limit of physical optics). As long as frequency of the light ray is constant along its path, the classical geometrical optics is valid and both routes that lead to this theory (the Fermat principle or Maxwell equations) are correct.

However, there are some interactions, unknown a few hundred years ago, which can change frequency of the light ray along its path of propagation. For example, the light ray that is reflected from a mirror in motion changes its frequency discontinuously at the reflection point (the special relativity effect); the ray propagating near a large mass changes its frequency continuously due to the time dilation in a strong gravitational field (the general relativity effect); and a coherent Raman scattering changes the frequency due to a quantum interaction. Let the light rays that are subject to such interactions be called “rays of variable frequency” (VF rays). By definition, such rays are monochromatic at every point of the trajectory (one Fourier component at every point), but the numerical value of frequency can vary along the path due to interactions encountered along that path.

Since the electromagnetic radiation of variable frequency does exist, the domain of geometrical optics should be broadened to include those rays. The need for taking the existence of VF rays into account implies the necessity of replacing the PLT with the new variation principle (New Extremum Principle, NEP for short) that is “sensitive” to the changes of frequency of the light ray along its path. The physical phenomena responsible for the change in frequency can be different, but whatever the physical origin of this effect is, this additional degree of freedom of light rays must be taken into account when deriving the NEP from the PLA.

1.2. Derivation of the new extremum principle (NEP) for VF radiation from the principle of least action (PLA).

In this part of the discussion we consider the physical reasoning and interpretation of important aspects of the formalism that has been used to derive the new variation principle for geometrical optics from the principle of least action. On the other hand, the mathematical tools that are used in this derivation are standard, without any inventions of my own.

1.2.1. The variation principle for the light ray in the context of path integrals and the Schwinger principle.

The variation principle is closely associated with the concept of Feynman path integrals and the Schwinger’s equivalent formulation of the principle of least action.

According to the Feynman’s concept, all trajectories that connect the initial state A with the final state B of a classical or quantum system are equally probable, i.e., the modulus of the amplitude of the transition probability of $A \rightarrow B$ is the same for all trajectories that connect A and B, but the (complex) phase factor of the transition amplitude depends exponentially on the action integral for each individual trajectory. The total amplitude of the transition probability from the initial to the final state is the sum (integral) of transition probabilities of all trajectories that connect the two states. Since the unit of the action (the Planck constant) is very small, the (complex) phase factors, which depend on the trajectory, oscillate very fast (and change the sign) even for very small changes in the trajectory, so that the contributions from practically all trajectories cancel each other, except for the so called stationary trajectory, for which the action integral takes on the extremum (minimum) value, $\delta_{\text{path}} S_{\text{field}} = 0$. Since the stationary trajectory, together with the group of trajectories (paths) that differ infinitesimally, give the only non-vanishing contribution to the total amplitude of the probability of transition from A to B, this trajectory is interpreted as the observable (or macroscopic) path of evolution of the system from the state A to the state B.

The variation of the trajectory of the path of the light ray is equivalent to the Feynman's concept of path integrals, except for the fact that variation of the path of the light ray occurs in a 3-dimensional configuration space, and not in an arbitrary abstract space of field states.

The concept of path integrals is equivalent to the Schwinger's formulation of the principle of least action, which also can be written as the variation $\delta_{\text{path}} S_{\text{field}} = 0$. The difference between the two approaches is that the Schwinger's picture assumes variation of trajectories in the operator space, rather than variation of trajectories in the space of field states. Consequently, the variation of the action in the Schwinger's picture requires calculation of the expectation values for variations of the operators in a fixed state of the field, whereas we are going to calculate the expectation values of fixed operators over the varying trajectories in the space of states of the electromagnetic field, or simply speaking, over different trajectories of the light ray in the configuration space. Therefore, although both pictures (Feynman's and Schwinger's) of the principle of least action are equivalent, our variation approach to the geometrical optics is closer (or rather analogous) to the Feynman concept.

1.2.2. Construction of the Lagrangian density, Lagrangian, action integral and the principle of least action (PLA) for VF light rays in vacuum.

The first, key step of any formalism based on the PLA is construction of the Lagrangian (L) or, in the case of fields – the Lagrangian density (\mathcal{L}). All the features that constitute the minimum set of constraints that uniquely define the system must be built into the Lagrangian, and it cannot comprise any additional features that do not represent the actual physical situation.

Construction of the Lagrangian reminds the job of a baker. Having taken the same main ingredients (flour and water) and some appropriate additives (yeast, sourdough, salt) and having avoided any harmful additives (e.g., a glue), the baker can apply a variety of baking recipes to produce the whole spectrum of bakery products. However, at a given moment he is producing a single intended product; the success of this kind of job depends on professional qualifications of the baker, who is supposed to choose the right ingredients and apply recipes appropriately. Similarly, we are constructing the Lagrangian for the electromagnetic field in such a way to get the new extremum principle for geometrical optics, and we are not going to use the method that leads to Maxwell's equations (we are going to bake bread, rather than the birthday cake). Here are the characteristic features of our construction.

- *Invariance of the Lagrangian density with respect to gauge transformation.*

The Lagrangian density of the electromagnetic field of the light ray can be expressed in terms of the 4-potential A_μ (which assures Lorentz covariance) or in terms of fields \mathbf{E} and \mathbf{B} (which assures gauge invariance). Since geometrical optics is evidently gauge-independent and it is not Lorentz-invariant (in particular, variation of the trajectory in the 3-dimensional configuration space is not a Lorentz-invariant operation), we use the other option, i.e., $\mathcal{L}_{\text{field}} = \mathcal{L}_{\text{field}}(\mathbf{E}, \mathbf{B})$. Potentials A_μ are implicitly comprised in the fields.

- *Invariance of the Lagrangian density with respect to time reversal.*

Since the Lagrangian must be a rotational scalar, it must depend on fields via E^2 , B^2 and cannot comprise the term $\mathbf{E} \cdot \mathbf{B}$, because this is a pseudoscalar that changes sign under time reversal; a mix of terms that are symmetric and antisymmetric with respect to time reversal would have violated reversibility of light rays in geometrical optics. The Lagrangian must be a function of squares of field strengths, because the electromagnetic field has to be locally linear (nonlinear phenomena are beyond the scope of geometrical optics). Therefore, the most general form of the Lagrangian density that satisfies these requirements is $\mathcal{L}_{\text{field}} = \text{const}_1 E^2 + \text{const}_2 B^2$.

- *Constants in the Lagrangian density must remain undetermined.*

In principle, the values of constants const_1 and const_2 can be determined by comparing coefficients in field equations (that can be derived from the PLA by variation of potentials) with experimentally determined coefficients in Maxwell equations. However, such a procedure would be inconsistent with the idea of varying the *path* of the light ray, because one cannot simultaneously vary the trajectory of the light ray and satisfy Maxwell equations that uniquely determine that path (one can, of course, use a single field equation, because only the complete set of four field equations constitute the constraints that exclude the variation approach). This is why both these constants in the Lagrangian density are considered as (fixed) unknown parameters.

- *To include the VF rays, we have to use the QED.*

In the classical electrodynamics, the amplitude and frequency constitute two independent degrees of freedom of the electromagnetic wave. This is not so in quantum electrodynamics (QED), where the normalization coefficients and the expected values of field operators are explicitly dependent on frequency (this has been actually known a half century before formulation of the QED, i.e., since the explanation of the photoelectric effect). Therefore, in order to expose frequency hidden in the classical amplitudes, we have to consider the electromagnetic field of the light ray as a quantum entity, i.e., we have to use the QED. In practice, it means that the squares of the electric field and the magnetic field in the Lagrangian density must be replaced with the expectation values of the corresponding field operators, computed on the very state of the field that represents the light ray.

- *The VF ray is represented as a spatial sequence of monochromatic field states having the same occupation number.*

Since frequency can change along a trajectory of the light ray, the whole path (either virtual or real) is divided into small sections, where frequency and the wave vector can be considered constant; we compute the expectation values of field operators in every of these constituent volumes. We assume that the field in every segment of the trajectory is monochromatic (a single Fourier component) and its occupation number is fixed; frequency in consecutive segments can be different, but the excitation number of the photon field is the same in all segments, because absorption and emission of photons is beyond the scope of geometrical optics. Such a representation of the field state does not restrict the generality of the formalism; if the light ray were represented by a coherent state or an arbitrary linear combination of Fock states, rather than a single basis state, the notation would have been complicated, but the formalism and the final result would remain the same.

- *The gravitational field in geometrical optics must be static.*

The next aspect of the physical situation is the presence of the gravitational field in the area where the light ray propagates. We assume that the gravitational field is static, because otherwise the 4-dimensional spacetime could not be split into the 3-dimensional configuration space and time; this is necessary to keep valid the basic notions of geometrical optics, such as a trajectory of the light ray in the configuration space and the propagation time.

- *The expectation values of field operators are computed locally.*

We require that the formalism (and consequently the final result) be valid in strong (albeit static) gravitational field, i.e., in the globally curved spacetime; at the same time, we want to apply quantum electrodynamics. This is possible due to the equivalence principle, which says that even though the spacetime is globally curved, all definitions and relations between physical objects (fields, states, operators, expectation values, etc.) in *small* volumes are the same as in the flat spacetime. Therefore, we can use the QED formalism in each of the small volumes which enclose the whole path of the light ray, just as in the flat spacetime.

- *We use standard measures of time and length calibrated in local procedures.*

The important implication of the equivalence principle in the gravitational field is that the quantities under consideration must be measured by mutually motionless inertial observers, with the use of *standard* clocks and measures of length, rather than with measuring instruments calibrated in some other way (e.g., *coordinate* clocks and measures of length). This is just what was expected, because all measurable quantities in physics and relations between them (physical laws) are defined in the (locally) flat spacetime; the quantities are measured with standard measures of time and space, because calibration of measuring instruments must be performed locally, without referring to some other points and gravity there.

- *Transition from the Lagrangian density to the principle of least action (PLA) for geometrical optics in vacuum.*

The Lagrangian of the electromagnetic field associated with the light ray is the Lagrangian density integrated over the whole volume. The action (the action integral) is obtained by integrating the Lagrangian over time. The contributions to the action integral come only from the areas where the (virtual or real) light ray is passing through, because these are the only regions where the expectation values of the electromagnetic field operators are non-zero. Therefore, the action associated with the light ray is the functional of the path of the light ray. Variation of such an action functional with respect to the

path of the light ray, $\delta_{\text{path}} \iint \mathcal{L}_{\text{field}} d^3V dt = 0$, constitutes the principle of least action (PLA) for geometrical optics. The trajectory that minimizes the action is the real (i.e., the macroscopic, observed) path of the light ray.

1.2.3. Derivation of the variation principle for geometrical optics (NEP) from the principle of least action (PLA) for VF light rays in vacuum.

The Lagrangian, the action integral and the principle of least action (PLA) for geometrical optics contain the expectation values of the electromagnetic field operators computed on the field states, which represent the light ray that propagates along an arbitrary trajectory. Calculation of the expectation values of the field operators is a standard algebraic task that requires no comment. Having substituted the expectation values of field operators into the PLA, we perform some purely algebraic transformations and get the sought implication $\text{PLA} \Rightarrow \text{NEP}$:

$$\delta_{\text{path}} S_{\text{field}} = 0 \Rightarrow \delta_{\text{path}} \int \omega_{\text{local}} dt_{\text{local}} = 0, \quad (1)$$

where the subscript “local” reminds that time and frequency must be measured by local, mutually motionless observers equipped with standard measures of time (and/or length²).

1.2.4. Construction of the Lagrangian density, Lagrangian, the action integral and the principle of least action (PLA) for VF light rays in a material medium.

We assume that the medium where the light ray propagates is refractive, but scattering and absorption of light are not in the scope of geometrical optics. Construction of the Lagrangian density, Lagrangian, the action principle and the PLA in presence of a material medium is nearly the same as it is in vacuum. The only significant difference is the necessity of taking into account the interaction between the light ray and the charged particles of the medium.

- *Lagrangian density of the electromagnetic field that interacts with the medium.*

The Lagrangian density of the whole system consists of three parts:

$\mathcal{L}_{\text{total}} = \mathcal{L}_{\text{particles}} + \mathcal{L}_{\text{interaction}} + \mathcal{L}_{\text{field}}$. The first term is responsible for motion and gravitational interaction between particles; since it does not comprise any dynamical variables of the electromagnetic field, it plays no role in propagation of the light ray and so it does not enter further considerations. The third component is the Lagrangian density of the free (non-interacting) electromagnetic field, and it has already been considered in detail in the case of the light ray that propagates in vacuum. The difference between the light ray in vacuum or in a material medium is exclusively due to the Lagrangian density of interaction between charged particles of the medium and the electromagnetic field, $\mathcal{L}_{\text{interaction}} = j_{\mu} A^{\mu}$; the four-current j_{μ} represents motion of the electrically charged particles of the medium. Although the

interaction term comprises the four-potential A^{μ} explicitly, its scalar product with the 4-current preserves invariance with respect to the gauge transformation.

- *The interaction between particles of the medium and the field of the light ray can be singled out from the Lagrangian density for the electromagnetic interaction $\mathcal{L}_{\text{interaction}}$.*

Since the temporal and the longitudinal component of the four-potential A^{μ} are not independent dynamic variables of the electromagnetic field and together they represent the electrostatic interaction between charged particles of the medium, we can transfer those two terms (i.e., the Coulomb interaction) to $\mathcal{L}_{\text{particles}}$ without losing the generality of our considerations concerning propagation of the light ray. The interaction of the light ray with the medium is fully represented by the scalar product of two transversal components of the 4-potential with the current density 3-vector, $\mathcal{L}_{\text{int}} = \mathbf{j} \cdot \mathbf{A}^{\perp}$, where the current density \mathbf{j} is the sum of contributions from all (free and bound) charged particles of the medium. The analogous operation is used also in formalisms based on the Hamiltonian.

² According to the 1983 definition of the meter (Resolution 1 of the 17th CGPM) the unit of length and the unit of time are inseparably related via the definitional value of the speed of light in vacuum, c .

- *Variation of the path of the light ray implies variation of the current that is induced by the ray.*

There is a fundamental *physical* difference between the meaning of the current density \mathbf{j} in the principle of least action, where the *path* is varied in order to derive the variation principle for geometrical optics,

$$\delta_{\text{path}} S_{\text{total}} = 0, \text{ and where the } \textit{potentials} \text{ are varied to derive Maxwell equations, } \delta_{A_\mu} S_{\text{total}} = 0. \text{ In order}$$

to derive Maxwell equations, one varies components of the four-potential, but the four-current is considered as a fixed source of the 4-potential, so it is not varied. On the other hand, when we are varying the path of the light ray, the current density is no longer a fixed source of the field; just the opposite: the current in the medium is the *response* of charged particles of the medium to the electromagnetic field of the light ray, so that the current is subject to variation together with the path of the light ray. In practice, it means that before we can perform variation of the path, we have to determine how the charged particles of the medium react to the electromagnetic field.

- *Elimination of the response current of the medium from the interaction Lagrangian density.*

The relation between the field and the current is given by one of the field equations that can be obtained by variation of potentials; the use of one such equation does not impose constraints that would preclude variation of the path of the light ray (only the full set of four Maxwell equations excludes variation of the path of the ray). Once we know the relation between the current and the field, it can be used to eliminate the current from the interaction Lagrangian; in this way the troublesome relation between variation of the path of the light ray and the associated change in the response current can be eliminated. The resultant Lagrangian density is comprised exclusively of field quantities. As a side remark, we can note that if the same trick is performed during derivation of the field equations, we would have got only a few algebraic identities instead of Maxwell equations; the physical reason for that is evident: if the current is considered as a fixed source of the 4-potential and that source is eliminated, then one should not be surprised that the formalism produces only algebraic identities, instead of physically meaningful results. Variation of the potential and variation of the path constitute two physically different concepts.

- *The transition from the Lagrangian density of interaction to the principle of least action (PLA) for geometrical optics in a material medium.*

Just as in the case of vacuum, the field quantities in the Lagrangian density are replaced with the expectation values of the corresponding field operators. By integrating the Lagrangian density over the whole space one gets the interaction Lagrangian. Integration of the interaction Lagrangian over time gives the action (the action integral) for the light ray, which propagates in a material medium. Those parts of the Lagrangian, which do not comprise the dynamic variables of the field can be neglected, because the variation over the trajectory of the light ray identically nullifies those terms. The action

$$S_{\text{field}} = \iint (\mathcal{L}_{\text{field}} + \mathcal{L}_{\text{int}}) d^3V dt \text{ is the functional of the trajectory of the light ray. Variation of the action}$$

integral $\delta_{\text{path}} S_{\text{field}} = 0$ with respect to the path of the light ray constitutes the principle of least action (PLA) for geometrical optics in a material medium. The trajectory that minimizes the action is the real path of the light ray.

1.2.5. Derivation of the variation principle for geometrical optics (NEP) from the principle of least action (PLA) for VF light rays in a material medium.

The expectation values of field operators on field states associated with the light ray in a medium are computed similarly as in the case of the free field. Having substituted the computed expectation values into the PLA and having reduced the constituent terms, we get the implication, which has the same form as in the vacuum case, $\text{PLA} \Rightarrow \text{NEP}$:

$$\delta_{\text{path}} S_{\text{field}} = 0 \Rightarrow \delta_{\text{path}} \int \omega_{\text{local}} dt_{\text{local}} = 0, \quad (2)$$

where the subscript “local” reminds that time and frequency must be measured by local, mutually motionless observers equipped with standard measures of time (clocks). Although no fundamental constant (speed of light in vacuum c , the electromagnetic coupling constant e_0 , the Planck constant h , and the gravitational constant G) appears explicitly in the variation principle for geometrical optics of VF rays, the actual trajectories of light rays depend on those constants, as they are (implicitly) comprised in the time of propagation, in frequency of the light ray and in the refractive index (c and G are comprised in the spacetime metric; c , e_0 and h affect the structure and physical properties of the medium that consists of charged particles).

1.2.6. The option of spatial parametrization of the trajectory of the light ray.

In practice, it is more convenient to parametrize the trajectory of the light ray with the spatial variable, rather than with the time. The transition to the new parametrization can be performed just as in the case of the classical Fermat principle, i.e., with the use of the refractive index n_{local} , so that one gets

$$\delta_{\text{path}} \int \omega_{\text{local}} dt_{\text{local}} = 0 \Rightarrow \delta_{\text{path}} \int \omega_{\text{local}} n_{\text{local}} dl_{\text{local}} = 0. \quad (3)$$

The refractive index, just as the other quantities, is measured by mutually motionless local observers with the measuring instruments calibrated with the use of local standard procedures. The price for the change of the parametrization from the temporal to the spatial one is the necessity to use the phenomenological properties of the medium, n_{local} , whereas the NEP in its primary form is free from phenomenology.

1.3. Discussion of the new extremum principle (NEP) for the VF radiation.

Both problems of the classical geometrical optics, pointed out in the earlier discussion of the origin of this research undertaking, have been solved. Firstly, the new variation principle for geometrical optics, the NEP, has been derived from the principle of least action, PLA, so that the status of the Fermat principle as the independent entity has been reduced to the status of a corollary from the Hamilton principle. Secondly, the domain of geometrical optics has been broadened to include light rays of variable frequency (VF rays).

1.3.1. Compatibility of the NEP with other variation principles for light rays.

When a phenomenon, such as propagation of light rays or any other effect, is viewed in a new way, we need to verify the compatibility of the newly derived law with the previously established laws that refer to the same phenomenon. The verification must be performed in the common domain of validity of both laws.

- *The case of light rays of constant frequency.*

In absence of interactions that affect frequency of light rays, ω_{local} becomes a constant factor, which can be factored out and cancelled. Then, the new variation principle reduces to the Fermat principle. Therefore we have the following implication: PLA \Rightarrow NEP \Rightarrow PLT.

- *The case of light rays in vacuum in the strong gravitational field.*

Light rays propagate in vacuum along null geodesics that are determined by the variational principle, $\delta_{\text{path}} \int dt_{\text{coordinate}} = 0$, where time is measured with coordinate clocks. In the discussed paper, we have shown that in absence of the refractive medium and in absence of other than gravity interactions that affect frequency of light rays, the new extremum principle (NEP) reduces to the extremum principle for null geodesics, i.e.,

$$\delta_{\text{path}} \int \omega_{\text{local}} dt_{\text{local}} = 0 \Rightarrow \delta_{\text{path}} \int dt_{\text{coordinate}} = 0. \quad (4)$$

The proof requires that time and frequency, which are measured with standard clocks or coordinate clocks ought to be carefully distinguished, both in the emission point and in further points of the path of the light ray, as explained in detail in the discussed paper.

- *Other situations.*

The Fermat principle and the principle of null geodesics are the only classical variation principles that address the problem of propagation of light rays, and no other variation principle needs to be checked against the NEP. The domain of the NEP is wider than the domain of the two variation principles. For example, where frequency of the light ray changes due to a quantum effect or where the gravitational shift of frequency is accompanied by refraction in the material medium, the NEP is the only variation principle that can handle such situations.

1.3.2. Geometrical optics of VF rays is not a short-wave limit of physical optics.

The derivation of the variation principle for geometrical optics, the NEP, did not require any assumptions about the physical nature of the effect responsible for the change of frequency (we have mentioned the reflection from the mirror in motion, strong gravity or the coherent Raman scattering, but only as a few important examples). Therefore, the domain of the NEP includes all VF rays, irrespective of the physical effect responsible for the change of frequency of the light ray. Since physical optics does not cover many kinds of interactions that can affect frequency (gravity, quantum effects), thus the NEP (which is able to deal with such situations) cannot be considered as a short-wave limit of physical optics; such a

relation between physical optics and geometrical optics is valid only in a restricted domain of constant frequency light rays.

1.3.3. *Alternative methods of solving the variation principle NEP.*

- *First integration, next variation with respect to the parameters.*

From the algebraic point of view, there are two methods of solving the NEP in specific physical situations. First, one can express the path of the light ray as a certain function of one or more parameters (e.g., by expanding the path into a linear combination of a complete set of functions); next, one does the integration, and finally performs variation with respect to the expansion parameters. This route leads to as many algebraic equations, as many unknown expansion parameters were used. Having solved the set of algebraic equations, we get the values of the variation parameters and the path of the light ray. This route has been used to derive the general law of reflection of the light ray from an object in motion covered with the refractive material [2], {10}.

- *First variation, next integration of the partial differential equations.*

The second method of solving the variation problem, the NEP, is to use the Euler-Lagrange equations that yield a set of partial differential equations; these can be solved to get the path of the light ray. This route is used in the paper on propagation of light rays that are subject to strong gravity and refraction (simultaneous gravitational and refractive lensing) [3], {11}.

2. Application of the new extremum principle (NEP) to reflection of VF rays

From the new extremum principle for geometrical optics, we can derive the law of reflection of light rays that are subject to the shift of frequency in the point of reflection. In that case, the reflection angle is dependent not only on the angle of incidence, but also on the frequency shift and dispersion of the refractive index. This problem is considered in [2], {10}.

2.1. The problem of reflection of radiation with a change of frequency.

The classical law of reflection of light rays, which is derived from the Fermat principle, states that the angle of reflection is equal to the angle of incidence and neither the frequency of the light ray nor presence of the refractive medium is relevant.

There are some physical phenomena, however, which can change frequency of the light ray in the reflection point. For example, the frequency changes when the ray is reflected from an object in motion, from a layer of molecules where coherent Raman scattering occurs, or where a beam of UV or X-rays bounces of the surface of a crystal with partial dissipation of energy in the skin layer of the sample. Moreover, reflection of the ray from an object can occur in vacuum or in a refractive medium in which the body is immersed. There is no reason to assume that these circumstances are physically irrelevant.

With this motivation, we use the variation principle NEP to derive the new law of reflection for light rays, which takes into account that the frequency in the reflection point can change and that the reflecting surface can be covered with a refractive material.

2.2. The general law of reflection from a mirror covered with refractive material.

Reflection of the light ray is a local phenomenon, where gravity does not play any role. In that case the variation principle NEP takes on the form

$$\delta_{\text{path}} \int_A^B \omega(t) dt = \delta_{\text{path}} \int_A^B \omega(l) n_{\omega}(l) dl = 0, \quad (5)$$

where A and B are the fixed initial and final point of the varied trajectory, respectively, ω is the local value of the angular frequency of the ray, t is time, l is the running parameter of the path, and n_{ω} is the local value of the refractive index at the indicated frequency. Since the light ray in a uniform medium runs along a straight line, and the direction and frequency can change only in the reflection point, we split the integral into 2 sections (before and after reflection), perform the elementary integration and get the result that depends exclusively on the position of the reflection point (the only variation parameter). Next we perform variation of the resultant expression with respect to the parameter.

The procedure produces the new reflection law for light rays that are subject to a change of frequency in the reflection point. The law can be expressed in two equivalent forms

$$\sin \alpha_r = (n_{\omega_i} / n_{\omega_r}) (\omega_i / \omega_r) \sin \alpha_i \Leftrightarrow \sin \alpha_r = (n_{\omega_i} / n_{\omega_r}) \frac{1}{1 \pm \Delta\omega / \omega_i} \sin \alpha_i, \quad (6)$$

where α_i is the angle of incidence, α_r is the angle of reflection, ω_i is the angular frequency of the incident radiation, $\omega_r = \omega_i \pm \Delta\omega$ is the angular frequency of the reflected ray, $\Delta\omega > 0$ is the modulus of the frequency shift, and n is the refractive index of the material that covers the mirror, where the index identifies the frequency of the ray.

The generality of this law manifests also in the fact that there are no restrictions as to the physical nature of the phenomenon, which is responsible for the shift of frequency in the reflection point. In practice, it means that the shift of frequency must be computed theoretically or determined experimentally for every kind of reflection individually.

2.3. The law of reflection of radiation from a mirror in motion covered with refractive material.

An important example of reflection of the light ray with simultaneous change of frequency is reflection of the ray from the mirror covered with a refractive material, which is in motion with respect to the reference Lorentz observer. Since the law of reflection from a moving mirror can be derived by referring exclusively to special relativity, we can compare the result obtained from purely relativistic considerations with the general law of reflection that has been derived from the NEP, so that the new law of reflection can be verified in this specific case.

For this purpose, we write down the 4-wavevectors of the incident and the reflected ray (in the quantum interpretation, when multiplied by \hbar , these would be the energy-momentum 4-vectors of the corresponding photons) in the proper frame of the mirror and in another inertial frame that moves with respect to the first one. Both inertial frames are related by the Lorentz boost transformation. By performing such a transformation, we obtain relations between components of the 4-vectors, from which we immediately get the relation

$$\omega_r n_{\omega_r} \sin \alpha_r = \omega_i n_{\omega_i} \sin \alpha_i, \quad (7)$$

which is exactly the same as the general law of reflection.

From the relations between components of the 4-vectors one can compute also the frequency after reflection

$$\omega_r = \omega_i \frac{1 - \beta^2 \pm 2\beta n_{\omega_i} \cos \alpha_i}{1 - \beta^2} \quad (8)$$

and the final form of the reflection law in this case

$$\sin \alpha_r = \left(\frac{n_{\omega_i}}{n_{\omega_r}} \right) \frac{1 - \beta^2}{1 + \beta^2 \pm 2\beta n_{\omega_i} \cos \alpha_i} \sin \alpha_i, \quad (9)$$

where β is the speed of the mirror in units of c . These relations are cited here, because – much to my surprise – I was not able to find any source, which would give the formula for the change in frequency and the law of reflection from a mirror in motion that is covered with the refractive material. If the mirror is not covered with such a material (reflection of the ray from the mirror in vacuum, $n_{\omega} \equiv 1$), the above two formulas for the frequency shift and the reflection angle reduce to the form that can be found in many sources.

2.4. Areas of application of the law of reflection with a shift of frequency.

The direct result of this work is a new, general law of reflection of light rays, which takes into account the shift of frequency at the reflection point and possible presence of the refractive medium in the area of reflection. The law explains, how the angle of reflection depends on the angle of incidence, the frequency shift in the reflection point, the refractive index n of the material that covers the reflecting surface and dispersion of n . The law is general, because there are no restrictions on the physical cause of the shift of frequency in the reflection point. The new law of reflection has been positively verified by comparing it with the relativistic law of reflection from a mirror in motion; we have derived the explicit form of the equations for the reflection angle and the frequency shift of the light ray that is reflected from a mirror in motion, which is covered with a refractive material.

The most important application of the new law is in perfecting radar and lidar systems. In the case of a distributed radar system (i.e., such a system, where the sources of incident beams and detectors are situated at different locations), the formula that links the speed of the object, the frequency shift, the angle of incidence and the angle of reflection with refractive properties of the medium in which the object propagates, can be instrumental in positioning and tracking the object. That relation covers also the specific case, where the object of interest is covered with a material that absorbs the radar beam and

its refractive index is highly dispersive at radar frequencies. The radiation reflected from inner layers of such an object has a significantly decreased intensity; also the frequency and direction of reflection are very much changed, compared with reflection from an object of the same speed and direction of motion, which does not have such a coating. Detection and analysis of motion of such objects is complicated due to the fact that the surface coating can consist of many layers positioned at tilt with respect to each other (composite materials), which enhances absorption and generates many very weak secondary beams of various frequencies, reflected in different directions.

Also in ordinary radar systems, where the source of the searching beam and the detector are practically at the same point (the angle of incidence and the angle of reflection are zero), the refractive index of the material that covers the object, as well as the medium in which the object propagates, are of crucial importance for the correct interpretation of data. Namely, the relative shift of frequency at the normal incidence is

$$\Delta\omega/\omega \approx \pm 2\beta n, \quad (10)$$

i.e., the shift is a function of not only the relative speed of the object, β , but also the refractive index of the ambient medium, n . If the radar system is calibrated in dry air, where $n \approx 1$, it may provide inaccurate results when used in presence of dust, fog, snow, ice, tropical torrents and where the beam is reflected from composite materials; in these situations, the speed, height and distance will be overestimated. This effect lowers safety of flight at low altitudes (e.g., on approach to the touchdown in difficult meteorological conditions) and handicaps the self-guided flying objects that use radar for detecting short-distance obstacles, especially in flight at low altitudes in a mountainous area, covered with snow or ice, and also in flatlands covered with a thick layer of snow, ice or permafrost. Similar effects can occur in the lidar technology.

One can think of purely scientific applications of the new law of reflection, e.g., the coherent Raman scattering in a thin layer, where frequency can be determined from the angle of reflection of the Stokes and anti-Stokes components, instead of the direct measurement of frequency of the reflected light. In the case of X-rays or the gamma beam reflected from the surface of a crystal with partial dissipation of energy in the reflection point, measurement of the reflection angle can be easier and more precise than the direct measurement of frequency of the reflected radiation.

3. Application of the new extremum principle (NEP) to propagation of VF rays in the general relativistic environment

The trajectory of the light ray is determined by the refractive index of the medium and the gravitational field. In the situation where neither of the two factors can be considered a small perturbation, a general theory of propagation of light rays affected by gravity and refraction has to be formulated. This problem is considered in [3], {11}.

3.1. Propagation of light rays in the gravitational field in presence of the refractive medium.

Light rays in a uniform medium propagate along straight lines. In optically non-uniform media (e.g., at the interface between different materials) the light ray is deflected due to refraction. These facts are known for thousands of years, but they were quantified by Snell only about 400 years ago (1621).

Deflection of light rays can also be caused by the strong gravitational field. This effect had been predicted first by Einstein about 100 years ago, and it was confirmed observationally by Eddington in 1919. When the light ray from a distant star passes close to the surface of the Sun (i.e., it is subjected to strong gravity), the image of the star is shifted with respect to the other stars, which constitutes the proof of interaction of the light ray with the gravitational field of the Sun. Since such observations can be performed only during the eclipse, there has been no significant progress in this field for about 60 years. In 1979 Walsh *et al.* observed multiplication of the image of a distant luminous object due to gravitational lensing for the first time; comparison of spectra of the constituent images proved that all of them come from a common, single luminous source. A few years later, gravitational lensing was observed by monitoring brightness of astronomical objects (microlensing). Since then, the number of identified lensed objects and microlensing events is fast increasing, which is due to creation of a few large research programs devoted exclusively to this effect, e.g., OGLE, MACHO, EROS, MOA, μ FUN and PLANET. Along with the increase in the number of observational records, the gravitational lensing data became the source of information about stellar objects, galaxies and provided insights into the universe in the cosmological scale.

Interaction of light rays with the refractive medium and the gravitational field used to be considered as two independent effects. In classical optics, gravity plays no role (except for technical reasons, such as deformation of optical instruments due to their own weight) and gravitational lensing – by definition – does not require refraction being taken into account. On the other hand, it is known that the gravitational field in vacuum can be considered as a kind of a refractive medium (the “refractive index” can be constructed from the metric of the field), and the optical formalism can be transformed so that it reminds the gravitational one (the “metric” for the medium can be constructed from its refractive index). In spite of all this, specialization of physicists and differences in research methods used in classical optics and in general relativity effectively prevented combining the two effects into a single theory.

Nevertheless, Nature is independent of the way scientists work and does not seek our approval for simultaneous gravitational and refractive lensing. Such a phenomenon can occur where a massive object is embedded into an expanded gaseous envelope, where lensing is due to a gaseous cloud, where the lensing galaxy is rich in the interstellar gas and has a gaseous halo, or where gravitational lensing occurs at cosmological distances in the early universe rich in the gaseous medium.

The theory of simultaneous gravitational and refractive lensing can be derived from the new variation principle (NEP) for light rays of variable frequency (VF rays). That theory has the form of a set of partial differential equations; it is non-perturbative, i.e., it covers the case of a nonlinear superposition of both effects, which is important where neither of the two effects can be treated as a small perturbation.

The theory of combined gravitational and refractive lensing gives (in principle) the possibility of finding partial contributions of gravity and refraction to the total lensing, based on observational data: since gravitational lensing is achromatic and refractive lensing depends on dispersion of the refractive index, this theory can be used to analyze data registered simultaneously at many different frequencies and gives the (theoretical) possibility to separate both effects. It has to be noted that it would be a new way of getting insight into the gravitational field and the non-luminous refractive medium; the disadvantage of this approach, unavoidable in the case of observational data (contrary to the use of experimental data) is the dependence of the results on the assumed model of the phenomenon, which is sometimes briefly commented by saying that “paper is patient”.

3.2. Equations for the path of the light ray subject to gravity and refraction.

The starting point for finding the solution to the problem of combined gravitational and refractive lensing is the new variation principle, the NEP, which determines trajectories of light rays subject to interactions that change their frequency. The physical nature of such interaction can be arbitrary; in particular, it covers the case of the gravitational shift of frequency. The variation principle NEP states that the real path of the light ray minimizes the functional

$$\delta_{\text{path}} \int \omega_{\text{local}} dt_{\text{local}} = \delta_{\text{path}} \int \omega_{\text{local}} n_{\text{local}} dl_{\text{local}} = 0, \quad (11)$$

where all quantities (time t , frequency ω , length l and the refractive index n) are measured by mutually motionless local Lorentz observers equipped with clocks and measures of length calibrated with the use of local standard procedures.

Having expressed the ω_{local} and l_{local} in the integrand as functions of the local value of the spacetime metric $g_{\mu\nu}$, one gets the variation principle that determines the curve in the 3-dimensional configuration space,

$$\delta_{\text{path}} \int L d\lambda = 0, \quad (12)$$

where λ is the world line parameter. The Lagrangian L is a function of the local value of the refractive index and the spacetime metric, $L = \sqrt{\gamma_{ij} \dot{x}^i \dot{y}^j}$, where $\gamma_{ij} \equiv -n^2(g_{ij}/g_{00})$ is the effective metric in the 3-dimensional configuration space shaped by the gravitational field and refraction of the material medium that fills the space.

From the mathematical point of view, this is a typical problem of the stationary value of the integral functional, which can be solved by using the standard Euler-Lagrange formalism. The procedure yields three partial differential equations that determine a curve on a 3-dimensional manifold

$$\begin{cases} \ddot{x}^1 + \Gamma_{ij}^1 \dot{x}^i \dot{x}^j = 0 \\ \ddot{x}^2 + \Gamma_{ij}^2 \dot{x}^i \dot{x}^j = 0 \\ \ddot{x}^3 + \Gamma_{ij}^3 \dot{x}^i \dot{x}^j = 0 \end{cases} \quad (13)$$

where Γ_{ij}^k are connection coefficients, which are defined in the paper discussed here in terms of the metric of the gravitational field and the refractive index of the material medium that fills the configuration space; the coefficients Γ_{ij}^k were calculated and cited in the Annex to the discussed paper in two specific cases of high symmetry.

To solve the set of three differential equations, one needs to know the initial conditions; in our case these are coordinates of the point where the trajectory begins and its initial direction, i.e., $x^i(0)$ and $\dot{x}^i(0)$, $i = 1, 2, 3$. By solving this set of differential equations with the given initial conditions one gets the sought trajectory.

3.3. The analytical approach in the case of spherical symmetry of gravity and refraction.

In the case of an arbitrary symmetry of gravity and refraction, the corresponding differential equations are too complicated to be solved analytically and one has to resort to numerical methods. However, there are important cases of high symmetry, where these equations can be significantly simplified, or even can be solved analytically. Such cases are considered in the discussed paper to illustrate the proposed theory.

At the first stage, we assume the Schwarzschild metric and arbitrary refraction. This allows for simplification of the set of differential equations, so that the number of 27 initial connection coefficients reduces to only three different values; nevertheless, the set of differential equations is still too complicated to be solved analytically.

If we also assume the spherical symmetry of the refractive index, we can compute the connection coefficients and the set of three differential equations reduces to two (the third one is satisfied identically). When the trajectory of the light ray is parametrized with one of the spatial variables (instead of the world line parameter), the set of two partial differential equations reduces to one ordinary differential equation.

Having assumed a specific model for the spherical distribution of the refractive medium, one can solve the differential equation analytically, using the small angle approximation and the iterative method. This procedure has been carried out in the discussed paper and the final result was given in the form of the analytical expression for the trajectory of the light ray and its deflection from the original direction. In absence of refraction, the analytical result reduces to the well known formula for purely gravitational deflection of light rays.

3.4. General remarks on lensing due to gravity and refraction.

The fast and continuous progress in observational astronomy allows registration of ever more perfect, sharper and richer in details images of macrolensing, as well as the microlensing brightness curves. The distinguishability of ever subtle features in the observational material creates the need for development of such analytical methods, which can account also for other than gravity effects that affect the lensing phenomenon.

The theory of lensing due to combined action of gravity and refraction has been derived from the new variation principle, the NEP, for geometrical optics of light rays of variable frequency, VF. The core of this theory is the set of 3 partial differential equations, which determine “geodesics” in the 3-dimensional configuration space shaped by static gravity and the refractive matter. Equations of this theory account for a nonlinear superposition of both effects (gravity and refraction) and work also in the situation where neither of the two effects can be considered as a small perturbation.

To solve the set of equations that describe simultaneous gravitational and refractive lensing, one needs to know the metric of the gravitational field and refractive properties of the medium. In the case of an arbitrary field, the numerical solution is the only option. In the case of high symmetry of both gravity and refraction of the medium, the problem of deflection of light rays can be solved analytically in the small angle approximation. In the limiting case of negligible refraction, we reproduced the known formula for the gravitational deflection due to the spherically symmetric mass.

The observational data represent the combined lensing due to gravity and refraction. The theory of this type of lensing provides (at least theoretically) the possibility of splitting the total lensing into gravitational lensing (that is achromatic) and the refractive lensing (which depends on frequency). The necessary precondition for taking advantage of this possibility is to monitor the lensing phenomena (“events”) at many different frequencies. If the two contributions to the total lensing can be separated with the use of this method in practice (the reverse lensing problem), that would constitute a new source of information on the non-luminous gaseous matter in the universe.

4. Index of refraction of the relativistic dispersive medium

The refractive index is measured in the proper frame of the medium and tabularized as $n_0(\omega_0)$, $n_0(f_0)$ or $n_0(\lambda_0)$, as a function of angular frequency ω_0 , frequency f_0 or wavelength λ_0 ; it plays a crucial role in many branches of optics, such as geometrical optics, image formation and resonant phenomena. The problem of Lorentz transformation of the refractive index from the rest frame of the medium to another inertial coordinate system is the subject of the paper [4], {4}.

4.1. Relativistic medium and the relativistic index of refraction.

If motion of a medium is handled fully consistently with principles of special relativity, i.e., without the low velocity approximation, the medium can be referred to as a relativistic medium. The relativistic index of refraction $n(\omega)$ is the refractive index of the relativistic medium.

The problem of the relativistic index of refraction has been evoked by combined gravitational and refractive lensing, because very high speeds can occur in astrophysical and cosmological situations and the low speed approximation may often be insufficient. The quantity $n(\omega)$ can be expressed in terms of the refractive index of the medium at rest $n_0(\omega_0)$, dispersion of n_0 and velocity (speed β and direction of motion determined by the angle ε) of the medium in a given reference frame. Once we know how n depends on the velocity of the medium (its speed and direction of motion), we can trace the light ray in a single reference frame even if the velocity of the medium is not uniform; this happens, e.g., where the medium rotates or it is subject to turbulences, so that the local values of β and ε are not the same everywhere. In principle, it is possible to use the rest frame value of n_0 in all calculations, but it would require performing the Lorentz transformation to the rest frame of the medium at every point where the local value of the velocity is different.

Motion of the medium, in which the wave propagates, affects not only the phase-normal velocity of the electromagnetic wave (i.e., the refractive index $n(\omega)$), but also the group velocity of the wave and its frequency.

4.2. Lorentz transformation in a relativistic dispersive medium

If a certain quantity is a tensor, its transformation properties are known. Where a given quantity is not a tensor, it is not known *a priori* how it transforms to another reference frame. Transformation properties of such a quantity must be derived from transformation relations of an arbitrary tensor quantity that comprises the quantity of interest.

The explicit formula for the refractive index in a relativistic medium $n(\omega)$ can be derived from the Lorentz transformation of any tensor comprising the refractive index, because the proper frame of the medium and the reference frame in which that medium is in motion are related by the Lorentz transformation. The relation between the refractive index in a motionless medium $n_0(\omega_0)$ and the index of refraction in a relativistic medium $n(\omega)$ is purely kinematic, because the speed and direction of motion of the uniform medium results from the choice of a (local) inertial frame, and not from physical interactions.

The refractive index is a part of the classical Minkowski frequency-wave number 4-vector; in the quantum picture (i.e., when the 4-vector is multiplied by \hbar) the 4-vector represents the 4-momentum of the photon. Since both 4-vectors are Lorentz tensors of the first order, we can use the Lorentz transformation equations and derive the explicit expression for the refractive index of the relativistic medium

$$n(\omega; \beta, \varepsilon) = \frac{(n_0^2 - 1) \beta \cos \varepsilon - \sqrt{1 - \beta^2} \sqrt{(n_0^2 - 1)(1 - \beta^2 \cos^2 \varepsilon) + 1 - \beta^2}}{(n_0^2 - 1) \beta^2 \cos^2 \varepsilon - 1 + \beta^2}, \quad (14)$$

where ε is the angle between the 3-vector \mathbf{k} and the velocity of the medium $\mathbf{V} \equiv c\boldsymbol{\beta}$ in a given inertial frame

$$\cos \varepsilon = (\beta_x \sin \theta \cos \varphi + \beta_y \sin \theta \sin \varphi + \beta_z \cos \theta) / \beta. \quad (15)$$

The Lorentz transformation, which connects the rest frame of the medium with the rest frame of the source of radiation, which has been used to derive $n(\omega, \boldsymbol{\beta}, \varepsilon)$, gives 4 relations between 4 components of the 4-momentum of the photon in both reference frames. From these four relations one can compute four unknown quantities; one of those is the relativistic index of refraction n . Two other relations determine the direction of the wave vector (\mathbf{k} or \mathbf{k}_0) in both reference frames, i.e., having known any two angular coordinates from among the four angular coordinates $\theta_0, \varphi_0, \theta, \varphi$, one can compute the other two coordinates.

The fourth relation that results from the Lorentz transformation gives the formula for the kinematic shift of (angular) frequency of the radiation

$$\omega_0 = \omega \gamma (1 - n \beta \cos \varepsilon), \quad (16)$$

where

$$\gamma \equiv (1 - \beta^2)^{-1/2}. \quad (17)$$

The value of the relativistic index of refraction $n(\omega)$ can be very much different from the value of the index of refraction of the motionless medium, $n_0(\omega_0)$. The particularly rapid change of n occurs in the case of the light wave that propagates at the angle $\pi/2 \leq \varepsilon \leq \pi$ with respect to \mathbf{V} , because $n(\omega)$ has a singularity at the threshold velocity of the medium $\beta_s = [1 + (n_0^2 - 1) \cos^2 \varepsilon]^{-1/2}$, i.e., n tends to $\pm \infty$ on approach to the singularity from either side. In the case of velocities larger than β_s , the value of $n(\omega, \boldsymbol{\beta}, \varepsilon)$ of a uniform medium becomes negative; it means that the negative value of n occurs not only in specially designed, non-uniform engineered materials, but also in a uniform medium at speeds $\boldsymbol{\beta}$ above the threshold value β_s .

Depending on the direction of propagation of light rays with respect to the velocity of the medium, one can observe the redshift or blueshift of frequency. Where the source emits the electromagnetic wave in the direction of its motion and its velocity approaches the threshold value β_s , a luminous shock wave is formed, which manifests as a high frequency burst and a surge in amplitude.

4.3. Areas of application of the relativistic index of refraction

The relativistic index of refraction $n(\omega)$ can be useful in all situations, where the source of radiation, the refractive medium and the observer are in relative motion, especially where (1) speed of the medium is large (i.e., $\boldsymbol{\beta} \rightarrow \beta_s$), so that the first order approximation in terms of $\boldsymbol{\beta}$ is insufficient; (2) the system is highly sensitive to tiny changes in frequency, e.g., the slow light effect in the oscillating medium of electromagnetically induced transparency; or (3) in situations that require utmost accuracy, e.g., in high precision positioning and tracking of objects with the use of radar, lidar and GNSS (Global Navigation Satellite System), which are very sensitive to changes in frequency and the speed of propagation of the electromagnetic waves. Relativistic corrections may have some meaning also in other areas that require high accuracy, e.g., in the Laser Doppler Vibrometry (LDV).

5. Group velocity of electromagnetic waves in a relativistic dispersive medium

The speed of electromagnetic pulses in a refractive medium in motion is determined by the group velocity of the wave packet in a relativistic medium. The group velocity of electromagnetic waves is a function of the relativistic index of refraction $n(\omega)$ and its dispersion. This problem is considered in [5], [3].

5.1. Formulation of the problem and the way to solve it.

The group velocity of electromagnetic waves in vacuum or in a non-dispersive medium (i.e., the medium where the phase velocity of light is frequency-independent) does not depend on frequency and it is the same as the phase velocity

$$V_g = V_{ph} = c/n. \quad (18)$$

In a dispersive medium, the group velocity of the wave packet can be computed from the refractive index n of that medium

$$V_g \equiv \frac{d\omega}{dk} = \frac{c}{n + \omega(dn/d\omega)}. \quad (19)$$

If the medium is motionless and the refractive index and its dispersion are known, calculation of the group velocity is straightforward. Where the medium is in motion, the relativistic index of refraction has to be used. If velocity of the medium is large or the required accuracy is high, the approximate value of the refractive index (up to the first order in velocity) is insufficient and the exact expression for the relativistic index of refraction n must be used.

5.2. Group velocity of electromagnetic waves in a relativistic medium.

Having used the already derived exact expression for the relativistic index of refraction n , one can compute the group velocity of the wave packet that propagates in an arbitrary direction relative to the velocity of the refractive medium.

The direct calculation gives

$$V_g(\omega; \beta, \varepsilon) = c \left[\frac{n_0^2(1+\Delta_0)(1-\beta^2 \cos^2 \varepsilon) \beta \cos \varepsilon - \beta^3 \cos \varepsilon \sin^2 \varepsilon + \sqrt{1-\beta^2} \sqrt{n_0^2(1-\beta^2 \cos^2 \varepsilon) - \beta^2 \sin^2 \varepsilon}}{n_0^2(1+\Delta_0)(1-\beta^2 \cos^2 \varepsilon) - \beta^2 \sin^2 \varepsilon + \beta \cos \varepsilon \sqrt{1-\beta^2} \sqrt{n_0^2(1-\beta^2 \cos^2 \varepsilon) - \beta^2 \sin^2 \varepsilon}} \right], \quad (20)$$

where Δ_0 is the parameter that characterizes dispersion of the refractive index.

5.3. Areas of application of the formula for the group velocity in a relativistic medium.

The exact expression for the group velocity of electromagnetic waves in a dispersive relativistic medium is useful where velocity of the medium is high or high accuracy is required.

High speeds of the medium occur in astrophysics, where gaseous clouds (atomic, molecular or ionized) can flow at high velocities. The expression for the group velocity can be useful in the spectral analysis of the observed delays of signals emitted by pulsars.

Low speeds of the medium may have significant consequences, where a given physical phenomenon is very sensitive to the value of frequency. One important example is the slow light phenomenon in the situation of the electromagnetically induced transparency, where the group velocity of the electromagnetic wave packet is many orders of magnitude less than the speed of light in vacuum.

It is commonly assumed that the slow light effect requires very high dispersion of the refractive index. It is interesting to note that the formula for the group velocity in a relativistic medium shows that the value of V_g can be very low also in the case of a non-dispersive medium ($n_0 = \text{const.}$), provided that the medium oscillates with speeds on the order of c at right angles ($\varepsilon \approx \pm \pi/2$) with respect to the light pulse; in these conditions we have $V_g \rightarrow c \cos \varepsilon$.

Motion of the medium with respect to the source and/or the observer affects also the satellite navigation GNSS (e.g. GPS, GLONASS, GALILEO). Since accuracy of positioning depends strongly on even tiny changes in frequency and the time of propagation of the pulses emitted by satellites, the highest-accuracy algorithms for the defense use ought to take into account the atmospheric effect in positioning and tracking of objects, i.e., the motion of the atmosphere with respect to the satellite and the observer. A similar phenomenon occurs in the radar tracking systems and in the lidar technology.

6. Refraction-dependent kinematic shift of spectral lines in astrophysics and cosmology

The shift of spectral lines of astronomical objects can result from expansion of the Universe in the cosmological scale, due to gravitational interactions and due to motion of the luminous object relative to the terrestrial observer (orbital motion of the Earth, orbital motion of the Solar System around the center of the Galaxy, relative motion of galaxies, etc.). It is silently assumed that the shift of spectral lines occurs in vacuum, which is well justified in most, but not in all situations. The refraction-dependent kinematic shift of spectral lines in the case of type Ia supernovae (Ia SNs) is discussed in [6], {1}.

6.1. Refraction-dependent kinematic shift of spectral lines.

The Doppler shift of spectral lines in vacuum depends only on the relative velocity of the source of radiation and the observer. However, if the luminous source is embedded in a moving refractive medium, the lineshifts registered by the external observer depend both on the relative velocity of the medium and its relativistic index of refraction.

Let us assume that motion of the refractive medium is the only source of the frequency shift. If the source of radiation S is embedded in the medium that flows with velocity β_S towards the observer T, motionless with respect to the source S, the frequency of the electromagnetic wave emitted by S can be measured in 3 different inertial reference frames. If frequency of the radiation in the rest frame of S is denoted as f_S , the frequency of the same wave in the rest frame of the medium is

$$f_P = f_S \frac{1-n_S \beta_S}{\sqrt{1-\beta_S^2}} = f_S \frac{\sqrt{1-\beta_S^2}}{1+n_0 \beta_S}, \quad (21)$$

where n_S is the relativistic index of refraction of the medium in motion and n_0 is the refractive index of the same medium at rest; frequency of the same radiation in the rest frame of the external observer T is

$$f_T = f_P \sqrt{\frac{1+\beta_S}{1-\beta_S}} = f_S \frac{1+\beta_S}{1+n_0 \beta_S}. \quad (22)$$

Therefore, although the inertial frames associated with S and T are mutually motionless, motion of the medium makes the registered frequency f_T different from the emitted frequency f_S . The frequency shift caused by motion of the refractive medium is

$$z_m = \frac{\lambda_T - \lambda_S}{\lambda_S} = \frac{f_S - f_T}{f_T} = \frac{\beta_S(n_0 - 1)}{1 + \beta_S}, \quad (23)$$

where λ denotes the wavelength in the reference frame indicated by the subscript S or T, respectively, and the index 'm' means that the *medium* (in motion) is responsible for this effect. Where $n_0 \rightarrow 1$ or $\beta_S \rightarrow 0$ (i.e., vacuum or the situation where the medium is motionless with respect to the source of radiation or the observer) the frequency shift reduces to the value given by the Doppler formula in vacuum. Let us note that the frequency shift is independent of the thickness of the layer of the refractive medium and does not depend on changes of the refractive index along the trajectory of the light ray, because changes in $n(r)$ affect the wavelength but not the frequency. Therefore, this is a local effect, which depends on the physical situation at the point of emission of the radiation.

The refraction-dependent shift of frequency is different for different spectral lines; it results directly from dispersion of the refractive index $n_0(\omega)$ and allows to qualitatively distinguish this effect from the frequency shift caused by the cosmological expansion, gravity or a regular Doppler effect in vacuum, which are all achromatic; it allows to distinguish this effect also from spectra of binary sources or colliding galaxies, which consist of a double series of spectral lines, with two different values of the frequency shift z .

The sign of the frequency shift z_m depends on the kind of the medium. In a non-ionized medium ($n_0 > 1$), the observer T registers lower frequency, i.e., $f_T < f_S$ (redshift); in an ionized medium ($0 < n_0 < 1$), the observer T registers higher frequency, i.e., $f_T > f_S$ (blueshift). The sign of the frequency shift depends also on the direction of the wave vector with respect to the velocity of the medium; if it is opposite ($-1 < \beta_S < 0$), the sign of z_m also changes.

6.2. Shift of spectral lines due to combined action of a few effects.

The refraction-dependent kinematic shift of spectral lines is a local effect, which is independent of the cosmological redshift due to expansion of the universe. When light travels in vacuum from distant parts of the universe toward the terrestrial observer, it is subject to redshift dependent on the cosmological scale factor at the instant of emission of the radiation (a_S) and at the time of observation (a_{obs})

$$f_{\text{obs}} = f_S \frac{1+\beta_S}{1+n_0 \beta_S} \cdot \frac{a_S}{a_{\text{obs}}}, \quad (24)$$

Having denoted the cosmological redshift z_c as

$$z_c = \frac{a_{\text{obs}}}{a_s} - 1 \quad (25)$$

one can express the total shift of frequency in the form

$$(z_{\text{obs}} + 1) = (z_c + 1)(z_m + 1). \quad (26)$$

Other independent phenomena, such as gravity (z_g) or a regular Doppler effect in vacuum (z_D), can be accounted for in a similar way

$$(z_{\text{obs}} + 1) = (z_c + 1)(z_m + 1)(z_D + 1)(z_g + 1), \quad (27)$$

where the cosmological redshift is the dominating factor for distant galaxies and other luminous objects.

6.3. Magnitude of the refraction-dependent shift of spectral lines in the case of Ia supernovae.

During explosion of the type Ia supernova (Ia SN) temperature reaches the value of 10^{10} K (1 MeV), so that the electron-positron plasma is being generated; the equilibrium density of such plasma is on the order of $10^{36} \div 10^{37}$ particles per m^3 . So hot and dense plasma is opaque and does not show any atomic spectral lines. However, when the e^-e^+ plasma ball spreads with speeds on the order of $\beta_s = 0,1$ its density and temperature gradually decrease. The electron plasma becomes transparent for the visible, near infrared (IR) and ultraviolet (UV) light when density of plasma drops by $10 \div 11$ orders of magnitude, i.e., to about 10^{26} particles per m^3 . Further drop in temperature and density of plasma, down to the levels determined by ionization of atoms, implies $n_0 \rightarrow 1$ and the contribution of plasma to the shift of spectral lines decays; in other words, refraction-dependent shift of spectral lines of Ia SNs occurs near the maximum of brightness of these objects.

The index of refraction of plasma n_0 depends mainly on its density N , mass m of its constituent particles and frequency f

$$n_0 = \sqrt{1 - Ne_0^2 / (4\pi^2 \varepsilon_0 m f^2)} = \sqrt{1 - f_{\text{plasma}}^2 / f^2}, \quad (28)$$

Assuming that $N = 10^{26} \text{ m}^{-3}$ and $\beta_s = 0,1$, one can estimate the refraction-dependent frequency shift z_m . In the near UV ($f = 1 \times 10^{15}$ Hz) the contribution of refraction is small $z_m = -0.00037$; in the visible band ($f = 5 \times 10^{14}$ Hz) it increases to $z_m = -0.0015$; in the near IR ($f = 1 \times 10^{14}$ Hz) it equals $z_m = -0.051$ and increases with $z_m \sim f^{-2}$ for even lower IR frequencies (but simultaneously intensity of such spectral lines decreases due to lower transparency of plasma for longer waves). In all of these situations plasma causes the blueshift.

The computed values can be compared to the observed values of the redshift z_{obs} of type Ia SNs. The ‘nearby’ objects of this type, which were used for calibration as standard candles, are characterized by z_{obs} in the range of $0.01 \div 0.1$; the principal conclusions on accelerated expansion of the universe and existence of dark matter that presumably fuels this process were based on observations of Ia SNs with z_{obs} in the range $0.1 \div 1$, whereas the largest values of z_{obs} of such objects observed with the Hubble telescope exceed the value of 1.7. The uncertainties of redshifts are relatively small ($\Delta z_{\text{obs}} = 0.001 \div 0.01$), especially when compared with the uncertainty of brightness of these objects. However, some of the objects identified as Ia SNs have z_{obs} that diverge significantly from what is expected based on theoretical predictions (the blueshift on the order of $0.1 \div 0.2$ relative to the expected values); some of these objects were included into the analysis and some were excluded based on their probable darkening by the galactic dust.

Having compared the magnitude of the refraction-dependent kinematic shift of frequency with the redshift of Ia SNs, one can conclude that the contribution of this effect to z_{obs} is very small in the UV range, in the visible band it is comparable to the uncertainty of measurement, but in the infrared band it can be very important. Inclusion of this effect into the analysis of data on the Hubble diagram requires prior verification of existence and parameters of the plasma ball produced by the SN explosion; this is possible by observation of the 1 MeV peak that corresponds to the annihilation of the electron-positron pair and the simultaneous observation of the spectral line in the microwave range 2×10^{11} Hz (in the rest frame of the positronium) that corresponds to the transition $1^1\text{S}_0 \rightarrow 1^3\text{S}_0$.

The refraction-dependent shift of spectral lines can be searched for in spectra of Cefeids, in spectra of young stars and accretion binary systems, in active galactic nuclei and objects of the ‘young universe’ that interact with the gaseous medium. Since the value of n fast increases at very high relativistic speeds, the refractive medium can significantly heighten the magnitude of the kinematic shift of spectral lines. Refraction can affect recession velocities and distances of some of such objects.

7. Generation of the collimated gamma beam of ultra-high intensity

The ultra-high intensity, collimated gamma beams can have direct uses or can be used for pumping gamma lasers. However, the problem of generation of such beams is still unsolved. What is even more important, there is no idea (no physical phenomenon) that could give hope for solving this problem in future. This problem is considered in [7], {8}.

7.1. The lack of high-intensity sources of collimated gamma radiation.

Construction of gamma lasers (grasers) had been considered since the early 1970s, but there has been no progress in this field, because no kind of gamma sources are known, whose intensity could be sufficient to pump lasers in that range of frequency. There is neither a way to generate high-intensity gamma beams nor a method to focus such beams. The phenomenon of spontaneous thermalization of the laser beam of low frequency (e.g., in the visible or infrared range of frequencies) and ultra-high intensity, if confirmed experimentally, can be used to generate white collimated gamma beams of intensity over 20 orders of magnitude higher than it is currently possible.

7.2. Spontaneous conversion of frequency of the photon field of ultra-high intensity.

A strong photon field can be in the form of a beam (e.g., a laser beam in the visible range or a pencil of synchrotron radiation) or a quasi-isotropic field (concentration of many photon beams in a small active volume, as in devices designed for controlled inertial fusion).

The idea of spontaneous thermalization comes from the observation that the photon field of narrow spectrum is extremely unstable from the thermodynamic point of view (low entropy). According to the principles of thermodynamics and statistical physics, such fields should undergo spontaneous thermalization, i.e., they ought to drift towards the maximum entropy state. The necessary condition for thermalization, full or partial, is the existence of material entities that can mediate interaction between photons. The noble gas atoms (He, Ne) or other material particles that interact with the strong photon field and generate higher harmonics (spectrum components that are multiples of the initial frequency) can play a role of such a mediating entity; this method, however, is unable to produce collimated gamma beams of high intensity.

The idea of spontaneous thermalization of the photon field in vacuum results from the effect of self-interaction of the field of extreme intensity, which is mediated by the virtual electron-positron field. The spectral instability of the photon field can be illustrated quantitatively in the case of a quasi-spherical field and a photon beam. In the spherical geometry, the threshold intensity of photons in the visible range (2 eV) that is sufficient for self-interaction and spontaneous thermalization is $I \geq 2.4 \times 10^{28} \text{ Wm}^{-2}$; the corresponding value for the laser beam is $I \geq 9.6 \times 10^{28} \text{ Wm}^{-2}$. The threshold energy density of the photon field in the visible range is $\varepsilon \approx 3.2 \times 10^{20} \text{ Jm}^{-3}$, and the estimate of the diameter of the volume necessary for the interaction is at least $\Delta L \approx 3 \times 10^{-5} \div 3 \times 10^{-2} \text{ m}$. The minimum power of the laser beam that satisfies such requirements is $P \approx 3.6 \times 10^{18} \div 3.6 \times 10^{21} \text{ W}$, depending on the estimate of the minimum interaction volume. These parameters are more than 4 orders of magnitude above the presently attainable technical possibilities.

In the case of the quasi-spherical symmetry, the equilibrium temperature of the fully thermalized photon field is $T = 8.1 \times 10^8 \text{ K}$, the maximum of the spectral intensity is $i = 3.1 \times 10^8 \text{ W m}^{-2} \text{ Hz}^{-1}$, and energy of photons at the maximum intensity is $\varepsilon = 0.2 \text{ MeV}$. In the case of a single photon beam, the parameters of thermodynamic equilibrium of the field depend on the beam divergence; assuming that the solid angle of divergence is 10^{-5} steradian, temperature of the fully thermalized photon beam is $T = 3.2 \times 10^{10} \text{ K}$, and the maximum of spectral intensity $i = 3.1 \times 10^7 \text{ W m}^{-2} \text{ Hz}^{-1}$ occurs at the photon energy $\varepsilon = 7.8 \text{ MeV}$.

The spectral intensity of the fully thermalized photon field is enormous, over 20 orders of magnitude higher than in the case of the strongest (controllable) source of gamma radiation, such as the nuclear reactor; therefore, full thermalization of the photon beam is not necessary for practical applications.

Before full thermalization occurs, there can be sub-threshold phenomena, such as spontaneous generation of higher harmonics of the initial frequency (in vacuum), partial scattering (including backscattering on vacuum fluctuations), self-focusing of the beam and decrease in the speed of propagation of the pulse. These are the phenomena analogous to those that occur in the interaction of the photon beam with real massive particles; particles of the virtual field substitute for the real massive

particles. If any of these phenomena is observed, that would be a strong indication of sub-threshold thermalization taking place.

7.3. Areas of application of ultra-intense, collimated gamma beams.

The enormous intensity, collimation, easy aiming in an arbitrary direction, the possibility of switching the device on and off by turning the power on and off, the possibility of generation of a series of pulses, the speed of propagation and the power of the thermalized gamma beam makes possible the direct use of such beams as the “energy delivery system” for defense purposes. Such devices would be able to operate from the space, including the geostationary orbit.

Presently, there is no other concept of a defense against a massive attack of multi-warhead ICBMs. Different variants of the antimissile defense that are known as the “missile against a missile” scheme, which comes from 1960s, their cost and unreliability even in the case of single missiles and warheads, do not require a comment. Moreover, such systems imply the necessity of making an irreversible decision of global consequences in a matter of few minutes. The present laser defense systems, including chemical lasers, which use infrared and visible frequencies, are effective only at a short or tactical range, because of the physical limitation due to scattering and divergence of the beam. The minimum cross section of the beam is proportional to the square of the wavelength; the wavelength of gamma radiation is at least 5 orders of magnitude shorter than the wavelength of the visible radiation, so the range of effective action of the gamma beam can be, at least theoretically, 10 orders of magnitude longer (in vacuum, i.e., in space).

In the case of the gamma beam, it is not possible to counteract by spinning the target or covering it with a mirror surface, because there are no mirrors in the gamma range and the penetrating ability of gamma radiation is incomparably larger than that of the low frequency beam. Besides, the gamma-beam defense systems can operate from space, without direct infringing the territory of another state that could have irreversible consequences.

The other possible application of white gamma beams is in construction of gamma lasers. Such beams can be used as the pump field for gamma lasers, even if only a small fraction of the total power is transformed into the gamma radiation.

Furthermore, thermalized gamma beams can be used to facilitate ignition in the fusion pellet, to induce transmutation of chemical elements, or to investigate photon-photon collisions in devices known as “photon-photon colliders”, because the probability of interaction increases fast with energy of photons. Therefore, the collimated gamma beams can open those areas of science, technology and national defense that are beyond the reach of contemporary technologies.

8. The list of publications related to the VF radiation

- **Articles on *Electromagnetic radiation of variable frequency* published in Philadelphia List journals with IF:**

- [1] W. T. Chyla, *I. Geometrical optics of variable frequency light rays: Theoretical basis*. Canadian Journal of Physics, vol. 78, pp. 721-745 (2000). {9}
- [2] W. T. Chyla, *II. Geometrical optics of variable frequency light rays in special relativistic and nonrelativistic situations: The general law of reflection*. Canadian Journal of Physics, vol. 78, pp. 747-754 (2000). {10}
- [3] W. T. Chyla, *III. Geometrical optics of variable frequency light rays in the general relativistic regime: Combined gravitational and refractive lensing*. Canadian Journal of Physics, vol. 78, pp. 755-767 (2000). {11}
- [4] W. T. Chyla, *Refraction in a relativistic medium*. Optik – International Journal of Light and Electron Optics, vol. 124, pp. 1477-1479 (2013). {4}
- [5] W. T. Chyla, *Group velocity of the electromagnetic wave packet in a relativistic dispersive medium*. Journal of Electromagnetic Waves and Applications, vol. 27, pp. 938-943 (2013). {3}
- [6] W. T. Chyla, *Refraction-dependent kinematic shift of spectral lines*. European Physical Journal Plus, vol. 130, paper 78, pp. 1-6 (2015). {1}
- [7] W. T. Chyla, *On generation of collimated high-power gamma beams*. Laser and Particle Beams, vol. 24, pp. 143-156 (2006). {8}

The contextual literature is not cited in this self-review, because:

- (1) The notion of optics of variable frequency (VF) radiation, the theoretical basis and examples of application of this branch of optics constitute the individual achievement, which has no precursors. *The VF radiation, whose frequency can change along the path, should not be mistakenly taken for the classical notion of radiation emitted by a source of variable frequency of emission.*
- (2) The historical and physical aspects of these research topics are briefly presented in this self-review.
- (3) The literature of a broader context is discussed and cited in the original articles.
- (4) Mixing of one's own works with papers of other authors hampers evaluation of the actual individual research contribution.



PART III

A NOTE ON PUBLICATIONS IN THE FIELD OF METROLOGY

1. Evolution of the international metric system of units SI

The international metric system of units was introduced by virtue of the Metric Convention, signed as a diplomatic treaty in Paris on May 20, 1875, by the leading economic, political, and military powers of that time (except for the Great Britain which joined the Convention in 1884). The treaty has been based on two artifacts, the platinum-iridium primary standard of the kilogram and the primary standard of the meter made of the same alloy. The earlier (1870) proposal of Maxwell to base the international system of units on atomic standards could not be realized at that time for technical reasons.

In 1960, the 11th General Conference on Weights and Measures (CGPM) introduced the first definitional standard of length (the meter) defined in terms of the wavelength of radiation emitted by the krypton lamp. The atomic standard of the unit of length was not perfect either, because the wavelength of the emitted electromagnetic wave is always somewhat diffused. The same problem concerns the unit of time, the second, defined by the 13th CGPM in 1967/8 in terms of the hyperfine transition in ¹³³Cs.

The 17th CGPM in 1983 defined the first SI base unit by fixing the numerical value of a fundamental physical constant: the unit of length has been defined as the length of the path travelled by light with the fixed speed c in vacuum in a specified period of time. The definition of the SI base unit by fixing the numerical value of a fundamental physical constant is abstract and implies the necessity to distinguish between the definition of the unit and its practical realization (e.g., in the case of the meter, the interferometric method is used).

The unit of mass is still defined as the mass of the definitional platinum-iridium artifact of 1889. Although the mass of this artifact is constant by definition, it is well known, based on comparisons with other standard artifacts, that its mass is subject to the slow drift, e.g., due to chemical reactions on the surface or nuclear reactions in the whole volume; besides, the primary standard of mass is hardly ever used for calibrations, because it is so precious (as it is the world's only primary artifact) – whereas the standards, also those of the highest order, should be readily available for calibrations. This is why redefinition of the kilogram was proposed at the end of the 20th century. It is proposed to define the kilogram by fixing the numerical value of the Planck constant; the unit of mass is going to be realized with the watt balance. The competitive idea was to define the unit of mass by fixing the numerical value of the Avogadro constant and realize it as a silicon sphere that comprises a known number of atoms.

In the first years of the 21st century, it has been realized that the redefinition of the kilogram is not sufficient to satisfy all measurement needs at the highest metrological level; it is necessary to redefine simultaneously four SI base units: the kilogram, the ampere, the kelvin and the mole by fixing the numerical values of four physical constants (the Planck constant, the elementary charge, the Boltzmann constant and the Avogadro number, respectively). The goal of this program is not only to eliminate the last SI definitional artifact; it is equally important to redefine the electric unit, the ampere, because the contemporary electric measurements are traceable to quantum standards (based on the Josephson effect and the quantum Hall effect), but they are not traceable to the outdated, classical definition of the SI ampere. To make the electric measurements traceable to the SI units and to use the quantum standards as primary standards of the SI, one needs to fix the numerical values of e and h by the virtue of definitions.

The modernization of the SI, known as the New SI, has been initially intended for 2007, but it could not be realized due to discrepancies between results provided by the watt balance project and the Avogadro project. The problem turned out to be the consequence of insufficiently accurate determination of the isotopic content of the silicon spheres. At the 24th CGPM in 2011, the scheme of the future shape of the New SI system of units was adopted in Resolution 1, but technical requirements are still not satisfied; besides, it is easily noticeable that certain professional circles organize the opposition against realization of these plans.

The New SI program is pragmatic; it concentrates exclusively on the redefinition of 4 out of 7 SI base units and the practical realization (*mise en pratique*) of these 4 units, i.e., tasks that can be realized in the foreseeable future. Nevertheless, it is worth considering the possible final form of the system of units, irrespective of our current technical capabilities; it gives a new perspective on the contemporary system of units and the direction of its further evolution. This problem is considered in [1], {7}.

Definitions of base units ought to be independent of any references to material objects, not only artifacts (that are subject to physical processes and chemical reactions that cause their drift), but they should also be independent of any material constants of macroscopic objects (such as the triple point of water that defines the unit of temperature, the kelvin) and properties of atomic entities, as they are chosen arbitrarily and they cannot be determined exactly, e.g., due to the uncertainty principle, not to mention purity, isotopic content of the standard substance, the influence of temperature and pressure, as well as internal and external interactions that can alter properties of the definitional standard.

Definitions of base units (which have to be distinguished from realizations of these units) should also be independent of any phenomena and physical theories, because definitions should not depend on the current state of knowledge and should not change together with the scientific and technological progress.

These two postulates will be satisfied when *all* base units of the international system of units are defined by fixing the numerical values of the *fundamental* physical constants. In this approach to defining the base units, all references to material properties (macroscopic or microscopic), physical phenomena (classical or quantum) or physical theories (established or novel) are shifted from the level of the definition of the unit to the level of realization of the unit (i.e., to its *mise en pratique*); in that situation, the technological progress and better understanding of the theoretical background of physical phenomena will result in improvements of the *mise en pratique*, but the definition itself will stay intact.

Every definitionally fixed value of a fundamental physical constant constitutes one constraint, which is imposed on the set of base units. The number of constraints must be equal to the number of degrees of freedom of the system, so that the system of units is not underconstrained or overconstrained. Therefore, the number of base units must be equal to the number of fundamental physical constants that define the units.

There are many physical constants and many views on which constants are truly fundamental. I believe that fundamental physical constants constitute references for the primary (indefinable) physical notions and make them measurable. Therefore, the number of independent fundamental physical constants must be equal to the number of (indefinable) primary notions in physics. These are time, length, mass and quantized charges, i.e., three kinds of fundamental physical interactions (electromagnetic, weak and strong). There is a total of 6 notions and 6 corresponding fundamental constants (c , h , G and 3 coupling constants of quantum interactions), whose numerical values can be fixed by definition; therefore, there should be 6 base units of the system of units: the unit of time (the second), length (the meter), mass (the kilogram), the elementary electric charge (the coulomb), the elementary weak charge (the weinberg?) and the strong charge (the name?). Let us notice that the elementary charge cannot be identified with the charge of a certain chosen elementary particle, because the charge of a specific particle belongs to the category of microscopic material constants. Measurement of the charge of a certain elementary particle is a kind of a *mise en pratique* of the elementary charge and the result may depend on the energy scale of the measurement and our theoretical interpretational abilities; this is due to the internal structure of particles (hadrons) and vacuum polarization that occurs also in the case of point particles, such as leptons. The practical illustration of the dependence of the effective charge of a particle on the energy of measurement is the difference between the “dressed charge” and the “bare charge” of particles; this pertains all three kinds of charges.

Since physical dimensions of fundamental constants c , h and G are combinations of three base units, we cannot ascribe one constant to one fundamental notion and one unit; units of time, length and mass must be defined simultaneously by specifying the numerical values of those three fundamental constants at the same time. This is not just an accidental feature of the three constants and the associated units, but it is a consequence of the physical reality, which is easiest to see in general relativity, where space, time and mass are closely related. When the unified theory of the electromagnetic, weak and strong interactions (quantum chromodynamics) reaches its final form, the three units corresponding to the three quantum interactions ought to be defined simultaneously by ascribing the definitional numerical values to the coupling constants of these three kinds of interaction. As for now, weak and strong interactions are beyond the domain of metrology.

The derived units can be defined as combinations of base units. With this respect, the proposed ‘final’ system of units, the international system SI, the New SI and other (historical) systems of units basically do not differ.

The units that are convenient in practice (the mole, the kelvin, the candela) would stay in the ‘final’ system of units, but they would not have the status of base units. For example, the mole, the SI base unit of amount of matter is very convenient and useful in chemistry, but it could be – in principle – eliminated, because it is just a conventional number of microscopic particles. Similarly, the unit of thermodynamic temperature, the kelvin, could be replaced with the unit of energy; the unit of luminous intensity, the candela, is also the superfluous but convenient in practice SI base unit.

Due to the continuing process of introducing the principles of metrology to chemistry, biology, and medicine, there is a need for defining new units (e.g., katal, gray) and developing the corresponding standards. Such units are introduced by pragmatic motivations and they ought to exist in all systems of units, including the “final” one. Since such units are of entirely different character, compared with the base units, I call them “descriptive units”; they require the operational definition or a special comment that explains how they ought to be understood.

The accuracy of measurement of a fundamental constant (before its value is established by virtue of the definition) determines accuracy with which the unit of the fundamental physical quantity associated with that constant can be realized, i.e., it determines the accuracy of its primary standard. Since measurement of the gravitational constant G with the required relative uncertainty on the order of 10^{-8} is not possible yet, the implementation of the “final” system of units (FC SI – Fundamental Constants SI) is a matter of further future.

The system of units FC SI, which has been outlined here, allows to infer that the current proposal of the reform, known as the New SI, is not only the act of getting rid of the last arbitrary definitional artifact (the kilogram) and the response to the needs of practical metrology (the lack of traceability of electric measurements to the SI ampere), but it is a large leap toward formulation of the final form of the international system of units, where definitions and definitional values of base units will constitute the unchangeable element of the system.

To appreciate the crucial importance of the idea of introducing invariable definitions and definitional values of base units, it is sufficient to realize that the next in line for the redefinition is the unit of time, the second (the definitional hyperfine transition in ^{133}Cs will be replaced by a spectral line in the visible range, which will improve the uncertainty in measurements of time by about 2 orders of magnitude). The change of the definition of the second will cause a change in the values of the meter, the ampere and the candela in the SI system, as well as all but the mole base units of the New SI. The change in the value of the second, and (consequently) the other units associated with the second, will be very small (on the order of $10^{-15} \div 10^{-16}$, depending on the choice of the definitional spectral line), but the point is that the very principle of keeping the definitional values of base units invariable will be violated. If the second, the meter and the kilogram were defined by fixing the values of 3 fundamental physical constants (c , h and G), this problem would have never occurred; the change of the standard spectral line (i.e., the choice of a different reference frequency) that is used in the metrological practice would have the character of a correction to the *mise en pratique* of the unit of time, introduced by the International Committee for Weights and Measures (CIPM) and would require the decision of the General Conference on Weights and Measures (CGPM).

2. The new definition of the mole

In 2010 the Consultative Committee for Units (CCU) published on the BIPM (the International Bureau of Weights and Measures) webpage a proposal for new definitions of the kilogram, the kelvin, the ampere and the mole and proposed reformulation of definitions of the remaining three SI base units of the international system of units, i.e. the second, the meter and the candela (the program that is called the New SI).

The mole is to be redefined by fixing the numerical value of the Avogadro constant N_A , i.e., the number of particles in one mole; this will eliminate the relationship between the mole and the kilogram at the level of the definition. This idea is known for half a century; it has many supporters and few, but influential opponents. Yet, the proposed formulation of the new definition contains a few imperfections that come from the present definition of the mole. This problem is considered in [2], {6}.

Both the present and the new definition of the mole contains a list of particles whose macroscopic number can be expressed in moles. The list is highly incomplete and the listed particles and kinds of particles are chosen arbitrarily; and most of all, such a list in the definition of the mole is entirely unnecessary.

The new definition of the mole uses the notion of the “elementary object”, although the concept of the mole applies mostly to complex entities (atoms, ions, molecules), which cannot be called elementary. The specific character of chemistry requires that the term “particles of the same kind” ought to be used instead of the term “elementary object”; the “particles of the same kind” must be interpreted in accordance with a given physical or chemical situation and cannot be decreed in the definition. For example, molecules of different isotopic content, tautomers, optical isomers or molecules of different conformations can be considered as being of the same kind (e.g., from the point of view of most chemical reactions) or different (from the viewpoint of the molecular mass, spectrum or biological activity). Particles are of the same or of a different kind, depending on criteria adopted in a specific physicochemical situation. The analysis and the criticism of the new definition of the mole, promoted by the CCU and the BIPM, have been presented in the BIPM journal.

3. Formulation of the new definitions of the New SI base units

Redefinitions of the four base units and new formulations of the remaining three base units of the New SI have the same structure. On the one hand, this is an advantage, but on the other hand the imperfections committed in the new definitions are multiplied. This problem is considered in [3], {5}.

The new definitions of the base units use the derived units, which is logically unacceptable, use the conditional phrases (which is not advisable and not necessary), and in some cases the unit and its symbol are not sufficiently distinguished. The analysis of these problems led to the proposition of correcting the structure of the new definitions.

4. Electric metrology of nanostructures and the necessity to redefine the ampere

The current definition of the SI ampere, which refers to the interaction between two parallel conductors with the electric current, was worked out in mid 1940's and adopted by the CGPM in 1948. It is based on Maxwell equations and worked very well in practice as long as quantum effects were negligible in electric metrology. At that time, quantum electrodynamics was at the early stage of development.

For the past 30 years, however, electric measurements of high accuracy are traceable not to the official definition of the ampere, but to quantum standards whose accuracy and stability are a couple orders of magnitude better than those of the classical standard: the primary standard of voltage (based on the Josephson effect) and the quantum standard of resistance (based on the quantum Hall effect) realize the corresponding units with the relative uncertainty on the order of 10^{-9} . The price, which electric metrology pays for so high accuracy of measurements traceable to quantum standards is the lack of the SI traceability. This situation has been formally endorsed by Resolution 2 of the 19th CGPM in 1991 which adopted the conventional values of the Josephson constant ($K_J = 2e_0/h$) and the von Klitzing constant ($R_K = h/e_0^2$).

The redefinition of the ampere by fixing the value of the elementary (electric) charge e_0 and redefinition of the kilogram by fixing the value of the Planck constant h will make possible computation of exact values of the Josephson constant and the von Klitzing constant, so that the measurements traceable to quantum standards will also be traceable to the electric unit of the international system of units. The practical difference between measurements based on the classical definition of the ampere and on the modernized SI (known as the New SI) depends on the size of the measured system, because the role of quantum effects increases with the decreasing size of the objects. This problem is considered in [4], {2}.

The logical consistency requires that if the classical definition of the ampere is used in a given situation, one ought to use the classical interpretation of the raw data. In the considered paper, I compute the systematic errors that could have resulted from the consequently classical approach, when measuring the current density, density of the electric charge, voltage and the electric field in layered structures. The Casimir effect is considered as a representative example. Electric measurements in macroscopic structures (the size of the structure being on the order of 1 mm) are not sensitive to quantum effects and the way the ampere is defined (the differences are negligible, on the order of $10^{-8} \div 10^{-10}$), but the systematic error increases fast with the decreasing size of the sample and in the case of layered nanostructures the application of the classical definition of the ampere and the consistently classical interpretation of the raw measurement data gives final results that are a couple orders of magnitude off

the true value. Although we knew that this is to be expected, the quantitative illustration of this problem constitutes the argument for prompt ridding of the dichotomy in electric measurements, where metrological practice based on quantum standards and physical knowledge is inconsistent with the official definition of the electric unit.

5. The origins and the present day of the NIST with references to metrology in Poland

The largest and the most vigorous National Metrology Institute (NMI) in the world is the National Institute of Standards and Technology (NIST), established in 1901 (until 1988 known as the National Bureau of Standards, NBS). The NIST is the research institute financed from the federal budget, whose responsibility is to work for the benefit of the American industry and its strategic goal is the broadly understood wellbeing of the nation. Responsibilities of the NMIs of the other economically developed countries are similar to those of the NIST, although the scales of their activities are different.

In the situation of continued attempts to carry out the reform of the Central Office of Measures in Poland (GUM), which has been ineffective for years, it seems worth acquainting oneself with the structure and functioning of the NIST and note the structural and functional peculiarities of its Polish counterpart. This problem is considered in [5], {24}.

In that article, I discuss the origins, the formal status, finances, structural organization, activities and functioning of the NIST, referring also to the corresponding characteristics of the GUM. I note those features of the GUM that make this institution (the state office) unique in the global scale, compared with other National Metrology Institutes.

6. Metrology in modern global economy

Metrology is a horizontal science, which integrates advanced scientific knowledge and economy. The important role of metrology in trade and industry, and in other areas of responsibility of the government, is reflected in legislation; metrology is granted a special, privileged (compared to other sciences) legal, organizational and institutional status, which is regulated in a separate act of the Parliament. This is discussed in [6], {23}.

In this article, I consider historical roots of modern metrology, organization and development of the world metrological infrastructure, the significance of scientific, industrial and legal metrology in the situation of a global economy, and in particular the increasing role of metrology in reducing technical and administrative barriers to trade and in the market supervision.

This paper describes also the developments that shaped the institutional structure of metrology in Poland after regaining sovereignty in 1918. The political situation of that time determined the status of the central metrological institution in Poland, which is a state administration office, rather than a research institute working for the benefit of the national economy (contrary to what is practiced in technologically leading countries); after 1945 that status has been maintained, as it was compatible with idea of the state control over all economic and scientific activities in the country. In this context, the article discusses the recurring but failing efforts to adjust the institutional organization of metrology in Poland to the standards observed in highly developed countries.

7. The Avogadro Project as the element of the program of redefinitions of the SI base units

The primary standard of mass in the form of the platinum-iridium cylinder that has been maintained in the safe at Sèvres since 1889 is presently the only definitional artifact of the base unit of the international system of units SI. The definition of the kilogram will be changed when the absolute measurement of mass (i.e., weighing without comparing masses) with relative uncertainty of $\sim 10^{-8}$ is possible.

There are two concepts of the redefinition of the kilogram that exist for nearly half a century: (1) the definition based on fixing the Avogadro constant (the Avogadro number), realized by means of the monocrystal of known number of atoms; and (2) the definition based on fixing the Planck constant, realized by means of the watt balance. Both methods are entirely different from the physical point of view, but their compatibility can be verified due to the relation between the Planck constant and the Avogadro constant.

Both methods are technologically very demanding; it turns out that the watt balance method has been realized in the NIST much more accurately and reliably than the silicon crystal project. In the meantime, it turned out that the contemporary metrology, and especially the traceability of electric measurements to the SI ampere, requires redefinition of not only the kilogram, but also the ampere, as well as the kelvin and the mole.

Except for a few opponents of the reform, there is a common opinion, expressed in Resolution 1 of the CGPM in 2011, that the redefinition of the kilogram will be based on fixing of the Planck constant with the watt balance as a way to realize the unit. On the other hand, the two silicon monocrystal spheres created in the Avogadro project (IAC), which initially were intended for the redefinition of the kilogram, will actually be realizations of the definition of the mole. This problem is considered in [7], {25}.

The discussed review article presents the context of introducing the notion of the mole, its present definition, the origin of the Avogadro project, the measurement principle, the technical aspect of the realization of the Avogadro project, its outcome and perspectives. Referring to the proposed formulation of the new definition of the mole, I note that it needs to be corrected.

8. Organization and the role of the contemporary metrology

Soon after the Metric Convention had been signed in 1875, the technologically leading countries founded first National Metrology Institutes (NMI); first Regional Metrology Organizations (RMO) have been established about a quarter of a century ago. The current fast expansion of the global metrological structure is caused by globalization of production and trade at the unprecedented scale; it results with ever increasing responsibilities of metrology organizations. The scientific and organizational role of the International Bureau of Weights and Measures at Sèvres (BIPM) also changes. These problems are considered in [8, 9, 10], {26, 27, 28}.

These articles present organization and responsibilities of contemporary metrology, its economic and social importance, as well as the international cooperation in science, industry and trade. Special attention is devoted to the position of the Polish NMI, i.e., the Central Office of Measures (GUM).

9. Works of the International Bureau of Weights and Measures

The International Bureau of Weights and Measures (BIPM) has been established by the virtue of the Metric Convention as a scientific institute responsible for maintaining the definitional primary standards of the kilogram and the meter and conducting research associated with units of the international metric system.

Since the redefinition of the meter in 1960, the responsibility of the BIPM as the depositary of the definitional standards of mass and length has decreased by half, and after the redefinition of the kilogram in the coming years – it will vanish entirely. On the other hand, the scientific and technological progress enhances importance of the BIPM research in the area of new measurement methods and standards, and increases the role of the BIPM as the world metrology center, which has the expert knowledge and standards from which many countries derive their traceability in ever increasing number of measurement areas.

The BIPM director prepares a detailed annual report on activities of the Bureau and publishes a brief review of the scientific work of the BIPM in the international journal *Metrologia*. That information is worth transferring to the metrology community in Poland. This problem is considered in [11, 12, 13], {30, 31, 32}.

Those articles present general issues concerning the BIPM and research works carried out in all the Departments (formerly Sections) of the International Bureau of Weights and Measures.

10. Reform of the international system of units

The international system of units was introduced by Metric Convention of 1875; the decimal, metric system of units has been gradually developed, and since 1960 it is called *Système International d'Unités* (SI). This contribution presents physical motivation for the comprehensive reform of the SI (known as the New SI), which supposedly will be concluded in the nearest years; according to this reform, the kilogram, the ampere, the kelvin and the mole will be redefined by fixing the numerical values of the Planck constant, the elementary charge, the Boltzmann constant and the Avogadro number. Advantages and disadvantages of this reform are discussed in [14], {33}.

11. The list of publications in the field of metrology

- **Articles published in Philadelphia List journals with IF:**

- [1] W. T. Chyla, *Evolution of the international metric system of units SI*. Acta Physica Polonica A, vol. 120, pp. 998-1011 (2011). {7}
- [2] W. T. Chyla, *On the proposed redefinition of the mole*. Metrologia, vol. 49, pp. L11-L13 (2012). {6}
- [3] W. T. Chyla, *On the structure of the New SI definitions of base units*. Metrologia, vol. 49, pp. L17-L19 (2012). {5}
- [4] W. T. Chyla, *The problem of scale in electric metrology of nanostructures in the context of the New SI redefinition of the base electric unit*. Acta Physica Polonica A, vol. 125, no. 1, pp. 3-12 (2013). {2}

- **Articles published in the MNiSW-recognized national journals:**

- [5] W. T. Chyla, *The origins and the present of the NIST with references to the situation of metrology in Poland (Geneza i współczesność NIST z odniesieniami do realiów funkcjonowania metrologii w Polsce)*. Nauka, vol. 2, pp. 93-114 (2012). {24}
* Reprint in Zarządzanie Jakością, vol. 1/2013 (31), pp. 88-102 (2013).
- [6] W. T. Chyla, *Metrology in modern global economy (Metrologia we współczesnej gospodarce globalnej)*. Nauka, vol. 4, pp. 139-169 (2014). {23}
- [7] W. T. Chyla, *Avogadro project (IAC) and redefinition of the mole (Projekt Avogadro (IAC) i redefinicja jednostki liczności materii)*. Wiadomości Chemiczne, vol. 66, no. 7-8, pp. 767-787 (2012). {25}
- [8] W. T. Chyla, *Origins, organization and priorities of modern metrology (Geneza, organizacja i priorytety współczesnej metrologii)*. Zarządzanie Jakością, vol. 1/2013 (31), special edition Metrologia 2013, pp. 30-39 (2013). {26}
- [9] W. T. Chyla, *National metrology institutes in Europe and in the world (Krajowe instytuty metrologiczne w Europie i na świecie)*. Zarządzanie Jakością, vol. 1/2013 (31), special edition Metrologia 2013, pp. 62-73 (2013). {27}
- [10] W. T. Chyla, *Developments in the contemporary scientific metrology (Kierunki zmian we współczesnej metrologii naukowej)*. PAR – Pomiar, Automatyka, Robotyka, vol. 10/2013, pp. 144-149 (2013). {28}

- **Articles published in the national technical journal:**

- [11] W. T. Chyla, *Discussion of the 2010 BIPM director's annual report (Omówienie sprawozdania Dyrektora Międzynarodowego Biura Miar z działalności BIPM w roku 2010)*. Metrologia – Biuletyn Głównego Urzędu Miar, vol. 6, no. 1, March 2011, pp. 23-38. {30}
- [12] W. T. Chyla, *Discussion of the 2009 BIPM director's annual report (Omówienie raportu Dyrektora Międzynarodowego Biura Miar z działalności BIPM w roku 2009)*. Metrologia – Biuletyn Głównego Urzędu Miar, vol. 1(16), March 2010, pp. 19-32. {31}
- [13] W. T. Chyla, *Discussion of the 2008 BIPM annual report (Omówienie raportu Międzynarodowego Biura Miar za rok 2008)*. Metrologia – Biuletyn Informacyjny GUM, no. 1(12), March 2009, pp. 3-12. {32}

- **Article published in conference materials:**

- [14] W. T. Chyla, *Reform of the international system of units SI (Reforma międzynarodowego układu jednostek miar SI)*. Metrologia w systemach zarządzania – 7. Praktyczne aspekty metrologii, 23-25 września 2013 r., Kielce-Ameliówka, pp. 31-42. {33}



PART IV

A NOTE ON PUBLICATIONS IN OTHER FIELDS

1. The electrostatic effect in layered nanostructures and its spectral manifestation

Application of semiconductors in the electronics industry started the tendency to miniaturize the electronic elements and final products of this sector of industry, which continues to date. The second breakthrough occurred in 1970s, when it turned out that bulk electronic semiconductor elements can be replaced with layered structures a few nanometers thick (2-dimensional structures). During one decade, the technology of producing thin layers with the single atom precision has been mastered and the method of calculation of their optical and electric properties has been discovered.

In theoretical investigations, nanostructures are represented by the potential function that consists of square potential barriers and potential wells, whose heights or depths result from the edge energies of conduction bands and forbidden bands of bulk materials from which the particular layers of the nanostructure were formed. The charge carriers (electrons and holes) in such systems behave as particles localized in potential wells (in quasi-stationary states) and move by tunneling through potential barriers.

In the next decade, i.e., in 1980s, the technology of layered nanostructures has been developed to such a level that it was possible to apply these elements in devices designed for commercial purposes (high frequency generators, modulators, amplifiers, infrared detectors, switches and lasers). Simultaneously, investigation of nanostructures in which the other two dimensions are of atomic size (1-dimensional quantum wires and 0-dimensional quantum dots) has been launched. The decrease in the size of each dimension from the macroscopic (e.g., 10^{-3} m) to the atomic scale ($10^{-9} \div 10^{-7}$ m) gives the possibility of miniaturization of electronic elements by a few orders of magnitude (not to mention the proportional reduction of costs). On the other hand, the significance of diffusion, defects or impurities in nanostructures is higher than it is in bulk materials, which imposes ever more stringent requirements on technology of production.

From the viewpoint of applications, the two most important physical features of nanostructures are the possibility to create population inversion of quasi-stationary states localized in a thin layer, which makes possible laser action, and the negative differential resistance that is important in electronics. Both these features result from quantization of the energy states of electrons (or holes) in thin layers (size quantization).

In order to compute optical and electric properties of a nanostructure, one has to assume a certain model of the potential function that reflects its physical structure. The square potential model proposed by Esaki and Tsu in 1970s assumes that the edges of potential barriers and potential wells correspond to the energies of the valence band, forbidden band and the conduction band of the bulk material from which the constituent thin layers are formed; the model has been successfully used in layered nanostructures. However, the thinner the layers (and this is the permanent trend in layered structures), the more subtle effects ought to be taken into account in modeling of the potential function, because those effects begin to play a relatively more important role in the observed phenomena. This problem is considered in [1], [12].

The subject of the analysis is the electrostatic effect that results from quantization of the stationary states in thin layers; it is the source of a significant correction to the square potential model of Esaki and Tsu. Namely, the density (number) of electrons that can be accommodated on discrete energy levels in the thin layer of the semiconductor is smaller (sometimes by a few orders of magnitude) than the electron density in the corresponding bulk (3-dimensional) semiconductor. This means that a fraction of electrons from the thin layer must relocate to the semiconductor in which the nanostructure is embedded. Consequently, the positive electric charge of atomic cores in the thin layer is not neutralized with the negative charge of electrons and the layer gains a net positive charge, which together with the charge of the electrons relocated to the bulk area modifies the square potential model. The electrostatic correction affects both the optical properties of nanostructures and the current resonances that occur when external voltage is applied to the nanostructure.

The electrostatic effect has been investigated in the case of the double barrier quantum well system (DBQW), i.e., the structure that consists of two thin layers of high energy of the conduction band (the double barrier potential) with one thin layer in between (the potential well) made of the same

semiconductor in which the whole system is embedded (the ABABA structure, where A and B symbolize 2 different materials); to fix attention, we assume that both semiconductors are n-type.

The thermodynamic equilibrium of the electron gas in the nanostructure occurs when the Fermi energy in all layers is the same. Since there is an explicit relation between the Fermi energy and density of carriers (at a given temperature), we begin with calculation of the density of carriers localized in the thin layer (in the potential well) relative to the density of carriers in the bulk semiconductor, in which the nanostructure is embedded. The density of carriers in the thin layer depends very strongly on parameters of the nanostructure (width of the potential well), density of carriers in the external bulk semiconductor and temperature. Since parameters of nanostructures vary in a very wide range of values, calculations were performed for the potential well of widths from 10 Å to 150 Å, electron densities from 10^{17} cm^{-3} to 10^{19} cm^{-3} and temperatures from 50 K to 300 K (calculations were actually performed in a wider range of values of these parameters, but they were neglected, because they did not provide any new insights).

Next, the Poisson law was used to compute the electrostatic correction to the square potential model. Calculations took into account that the distribution of the electric charge across the thin layer is not uniform, because the wave function of the discrete stationary states of the electrons in the potential well is not uniform (the charge density is distributed uniformly in planes parallel to the thin layers, of course). The Poisson equation was solved for each layer with the following boundary conditions: the electrostatic correction is continuous at the interfaces, the nanostructure as a whole is electrically neutral and the electrostatic correction must vanish at infinity.

Having known the electrostatic correction to the potential, one can use the perturbation theory to compute the corrections to the energies of the stationary states of electrons localized in the potential well of the nanostructure and calculate the shifts of spectra of such structures. All calculations were performed numerically with the Fermi-Dirac distribution function; just for a comparison, the partly analytical calculations in the non-degenerate electron gas approximation (with the Maxwell-Boltzmann distribution function) were also performed.

The calculations were completed and presented in the assumed wide range of parameters. The results cover the Fermi energy, density of electrons localized in the nanostructure, energy of stationary states, the value of the electrostatic correction for individual localized states and the shift of spectral lines. The results were shown in the form of graphs and tables to facilitate recognition of how properties of the nanostructure depend on parameters of the nanostructure. The specific (i.e., not systematic) features that can be seen in the graphs pertain mostly to the ground state and the first excited state; that results from the difference in symmetry of both states (the first one is symmetric and the other is anti-symmetric with respect to the center of the nanostructure).

The numerical results and their physical interpretation are discussed in detail in the original work. The general conclusion is that the electrostatic correction to the potential of the nanostructure is the more important, the thinner the layer and the higher density of carriers. The temperature dependence is more complicated, but in general the higher the temperature, the smoother the dependence on the other two parameters.

The electrostatic correction to the energy of stationary levels is significant (it may be as high as 300 cm^{-1} , i.e. $\sim 0.03 \text{ eV}$ or $\sim 10 \%$ of the energy of the stationary state); however, since energy shifts of stationary states between which the optical transition (absorption or emission) occurs are similar in magnitude, the shifts of spectral lines are smaller and do not exceed a couple percent of the transition energy. Nevertheless, the effect of this magnitude is very important, because nanostructures are used to generate laser radiation of a very narrow frequency half-width; in other words, the electrostatic shift of emission lines is much larger than the half-width of the laser radiation.

The discrete change in the nanostructure laser frequency can be achieved by varying the width of thin layers and the doping level. This work shows that even small changes in the geometry of nanostructures (thickness of layers), doping and temperature can significantly shift the frequency of laser radiation. These effects allow for tuning the laser frequency by applying tension, strain or changing temperature. The model presented in the paper allows to investigate effectively these effects.

2. Manifestation of the electrostatic effect in current resonances in layered nanostructures

Voltage applied to a layered nanostructure induces the electric current of resonant character, i.e., there are current peaks at certain values of voltage, which decay fast with the increasing voltage (negative differential resistance). From the physical point of view, the current resonances occur where the average energy of electrons (or holes), which tunnel through the nanostructure in the direction imposed by the external voltage, overlaps with the energy of the quasi-stationary states in the nanostructure. The resonant conductivity is important, e.g., in construction of amplifiers and switches.

The resonant voltage (i.e., the voltage that corresponds to current resonances) depends on energies of quasi-stationary states in a given nanostructure. Therefore, the calculated resonant voltage and resonant currents depend directly on the potential model that is assumed to represent the nanostructure. The numerical results are very sensitive to any changes in the assumed model, because the probability of tunneling of a particle through the potential barrier depends very strongly (exponentially) on parameters of the barrier. This is why physical phenomena that affect the potential model of the nanostructure, such as the electrostatic effect, are so important for resonant currents. This problem is considered in [2], {13}.

We have analyzed the magnitude of the electrostatic correction in current resonances of the DBQW nanostructure (the potential well between two potential barriers embedded in the bulk semiconductor). The electrostatic correction to the square potential model, which results from depletion of carriers (electrons) from the potential well (compared with the carrier density in the external bulk material) has been derived and discussed in the previous paper. The only significant difference is that the external voltage applied to such a structure exhausts practically all carriers from the quasi-stationary states in the potential well to the external area of lower potential. This is easily understood, because the inflow of carriers to the potential well occurs due to tunneling through a potential barrier that is higher than the potential barrier through which the carriers escape from the potential well, down the external potential gradient. Secondly, electrons can be captured in the quasi-stationary state if and only if the energy of the electron corresponds to the energy of the quasi-stationary state (otherwise the capture could occur only due to other, less efficient mechanisms that require participation of photons and phonons); on the other hand, the escape is easier, because right behind the potential barrier there is a continuum of energy states in the bulk material that is available to the electron. The conclusion is that the electric charge of the layer that constitutes the potential well is at the maximum, independent of the external voltage and uniformly distributed in the layer. Consequently, the electrostatic correction to the square potential model of the layered nanostructure in presence of the external voltage is somewhat higher than the correction in the stationary case (with no external voltage) that has been discussed in the former paper; e.g., the magnitude of the correction is -0.035 eV at the carrier density of 10^{18} cm^{-3} and temperature 300K.

To investigate how the electrostatic correction affects the current resonances, we assume that parameters of the nanostructure (DBQW) are fixed (height and width of potential barriers, width of the potential well, the dielectric constant and the effective mass of carriers) and their assumed values are typical. We assume that the nanostructure is embedded in a semiconductor of a spherical and parabolic conductivity band. The tunneling current density in such a nanostructure has been calculated at different values of the applied external voltage, for a wide range of electron densities (from 10^{17} cm^{-3} to 10^{19} cm^{-3}) and temperatures (from 100 K to 300 K).

The key quantity in calculations of the tunneling current in the nanostructure is the probability of tunneling of particles of a given energy through the nanostructure. In order to compute this quantity, we use the classical approach of Esaki and Tsu. First, the Schrödinger equation is solved with the assumed potential function, where the incident particles (partly reflected and partly transmitted through the nanostructure) are represented by the wave function that consists of plane waves. Next, we compute the probability current density for the plane wave that represents particles incident on the nanostructure and the probability current density of the transmitted particles. The ratio of the two quantities gives the probability of transition through the nanostructure of carriers of a certain energy. For example, the probability of transition of the carrier (the electron) of thermal energy $E = 0.025 \text{ eV}$, through the nanostructure represented by the assumed potential model, computed as a function of the external voltage, gives three sharp peaks that correspond to 3 quasi-stationary states that facilitate tunneling in this nanostructure. These three peaks are the precursors of the 3 current resonances that occur in the nanostructure.

The current density in the direction down the external voltage is computed by integrating the probability of tunneling of carriers, with their charge, velocity, density of states and the Fermi-Dirac distribution function. Since electrons are fermions, we add a factor which accounts for the fact that tunneling is possible if and only if the final state (on the opposite side of the nanostructure) is unoccupied; this factor is more important at high density of carriers, because occupation of electron states is higher in that case and the number of available unoccupied states is lower. This effect is more important at low voltage, i.e., in the case of the first resonance.

The expression for the current density in the opposite direction is analogous. The return current is usually neglected, as it is rather small, but we take it into consideration, because its role increases at high density of carriers and low voltage (so, it is most important for the first resonance). The net tunneling current density is the difference between the two currents.

The tunneling current density as the function of the external voltage applied to the nanostructure (DBQW) has been computed numerically both for the square potential model and with the electrostatic correction taken into account. The calculations were performed at carrier densities 10^{17} cm^{-3} , 10^{18} cm^{-3} and 10^{19} cm^{-3} and temperatures 100 K, 200 K and 300 K.

In the case of lower carrier densities (10^{17} cm^{-3}) we can see three sharp resonant peaks that correspond to three quasi-stationary states that facilitate tunneling through the nanostructure. The electrostatic correction increases the current resonances, because it lowers the potential barriers and makes tunneling easier. Since the correction lowers energies of stationary states, the maxima of current resonances occur at lower resonant voltages. The increase in temperature heightens the resonant current peaks (higher thermal energy facilitates tunneling), but simultaneously significantly broadens the peaks. The resonant voltages are lower at higher temperatures, because carriers of higher thermal energy tunnel more efficiently. The thermal effect is qualitatively the same irrespective of whether the calculations are performed with or without the electrostatic correction; the differences are of quantitative character.

At a higher density of carriers (10^{18} cm^{-3}), the electrostatic correction affects the two higher resonances in a similar way: heightens the resonant currents and lowers the resonant voltage. The largest electrostatic correction is observed in the case of the first resonance: the correction lowers the voltage of the first peak by half; the first resonance diffuses at higher temperatures.

When the carrier density increases to 10^{19} cm^{-3} , the electrostatic correction entirely changes the current resonances. The resonant voltage and maxima of current resonances are greatly changed and the first peak disappears altogether. The disappearance of the first resonance results from the fact that the electrostatic correction (which is negative) shifts the first stationary state below the bottom of the energy band (below the base of the “upstream” potential barrier) so that it becomes useless in the tunneling process. In calculations without the electrostatic correction, the first current resonance does exist, but it is very weak and hardly visible, because the difference between the tunneling current in both directions (down and up the voltage) is ever smaller at high densities of carriers and at small voltages.

The results of these calculations obtained for the model nanostructure (DBQW) shows that the electrostatic correction significantly affects the parameters of current resonances in the whole range of carrier densities and temperatures. In the case of heavily doped semiconductors, the electrostatic effect produces a qualitative change, as it makes the first current resonance disappear.

3. The hyperbolic potential of the heavy quark interaction

The strong interaction between quarks can be described in terms of the interaction potential. Such an approach gives particularly good results (i.e., the spectrum of the potential is in agreement with masses of the corresponding resonant particles) in the case of heavy quarks (t, b and c), for which the relativistic effects are less significant. The inter-quark potential is introduced either as a postulate or it results from the assumed phenomenological model for strong interactions. There are over a dozen of such models; they contain the fitting parameters, sometimes only one, but usually more. This problem is considered in [3], [14].

The proposed model of the inter-quark interaction assumes that the strong interaction is carried out by gluons “dressed” in a cloud of virtual particles that provide mass to the gluons. Pairs of virtual particles surrounding the exchange gluon are induced by the strong (color) charge of that particle. This model results from the fact that the difference in mass of “dressed” (constitutive) and “bare” quarks is ascribed to the massive virtual field induced by the strong charge of quarks.

Emission or absorption of the exchange gluon that mediates the inter-quark interaction must occur together with its virtual field, because otherwise that process would correspond to production of new particles, i.e., the decay of quarkonium. Therefore, the gluon field that mediates the strong interaction and keeps the quarks in the bound state can be considered as the real massive field.

The simultaneity and temporal ordering of events in the non-relativistic situation are well defined; thus the inter-quark interaction in this model is mediated by 1 gluon at a time (it may be one component of the gluon field or a linear combination of all components of the field). Thus the non-relativistic approach reduces the interaction mediated by the 8-component tensor gluon field to the interaction mediated by one real, massive scalar field. Consequently, the Lagrangian of the field has a very simple structure.

The Euler-Lagrange equation applied to such a Lagrangian has two linearly independent solutions, two hyperbolic functions. The linear combination of these solutions produces the Yukawa-type potential, which is rejected, because it is attractive for two particles of the same kind and repulsive for the particle-antiparticle pair (we need the binding potential for the quark-antiquark system to ensure the confinement of quarks in hadrons). One of the two solutions of the Euler-Lagrange equation is rejected, because its series expansion does not contain the linear term for small interquark distances; practice shows that potential models with the linear term produce spectra that are in better agreement with masses of excited states of quarkonia. Therefore, the second solution has been adopted as the interquark potential for the assumed model of the interaction. The potential ensures confinement of quarks, but it does not account for the asymptotic freedom of quarks.

The hyperbolic potential comprises only one fitting parameter interpreted as the mass of the constitutive gluon dressed in the massive virtual field. The lower limit for the mass of this fitting parameter can be estimated from the fact that the bound state, i.e., resonances qq and $\bar{q}\bar{q}$ do not exist, and the upper limit for the mass results from the mass of the virtual field induced by the strong charge of quarks.

4. The hopping model of electric conductivity in semiconductors

The electric conductivity in non-crystalline materials and in crystals built of large organic molecules, especially the aromatic compounds and other molecules with conjugated double bonds that host delocalized electrons, is handled differently than electric conductivity in typical inorganic semiconductors, where the classical band theory works fine. This problem is considered in [4], {15}.

The hopping model of electric conductivity, which is presented in the discussed paper, assumes that carriers (electrons or holes) travel randomly by hopping from one localized state to another state localized on a neighboring molecule. The hopping is understood as tunneling through potential barriers that separate the localized states positioned on the molecules. Having known the shape of the intermolecular potential or having assumed a simple model for it (e.g., in the form of the square potential barrier) one can compute the probability of transition of the carrier between the neighboring localized states and the effective time of transition of the carrier between those states as a function of energy of the carrier. From this we can get the average velocity of the carrier of a given energy in absence of the external voltage.

The hopping motion of carriers between localized states in the molecular crystal produces, on average, the uniform motion of carriers whose velocity depends on their energy; the carriers behave as if they were free particles. The lattice effect is taken into account by introducing the notion of the effective mass of the electron, just as in the band theory of semiconductors. We can compute the velocity of the hopping motion, the effective mass of carriers and the density of states as functions of the wave vector or energy.

The external voltage applied to the sample changes the shape of the potential barrier that separates the localized states, which affects the probability of tunneling of carriers in different directions and changes the effective velocity of the charge carriers (the electrons) in different directions. By integrating the drift velocity with the previously computed density of states and the Fermi-Dirac distribution function, we get the expression for the current density in this model.

5. Simultaneous gravitational and refractive lensing

The theory of simultaneous gravitational and refractive lensing has been discussed earlier in this self-review; it can be illustrated with a numerical example. This problem is considered in [5], {29}.

Assuming a model of a point microlensing mass embedded in a gaseous envelope, we get the superposition of the achromatic gravitational lensing and chromatic refraction. In this example, the mass of the lensing object is the dominant parameter of lensing and the choice of refractive parameters is quite arbitrary. For now, one does not have much of a choice, but to use the trial and error method of modeling.

6. Issues in pharmacology

The science of pharmacology has evolved from the art of medicine and these two areas of knowledge are closely intertwined. A short account of this relationship and the process of narrowing the gap between the scientific pharmacology and the traditional medicine is the subject of the paper [6], {16}.

Biologically active substances and other materials used in pharmacology and medicine are characterized with the use of physical and chemical methods that are precisely described in written standards that are called pharmacopeias. Most of technologically developed countries used to have their own pharmacopeias, but due to the globalization process the national pharmacopeias are being harmonized; for example, the present Polish Pharmacopoeia is actually the exact translation of the European Pharmacopoeia. The US Pharmacopeia and National Formulary plays a special role in this system, as it is recognized in most countries in the world; other pharmacopeias of international significance are the European Pharmacopoeia, developed under the auspices of the Council of Europe, the International Pharmacopoeia prepared by the World Health Organization (WHO), British Pharmacopoeia, as well as the Japanese, Chinese and Russian pharmacopeias, which are also recognized in many other countries. The subject of contemporary pharmacopeias, including the US Pharmacopeia, and the problem of their harmonization is discussed in [7, 8], {17, 18}.

One of the most important types of measurement in pharmacology is weighing. The environmental factors that affect accurate weighing in pharmaceutical laboratories and metrological parameters of balances are discussed in articles [9, 10], {19, 20}. Regulations concerning significant figures and the rules of rounding numbers in simple laboratory calculations are presented in [11], {21}.

International standards, which regulate weighing in quality assurance systems that are in force in pharmacology and in the pharmaceutical industry, are the subject of paper [12], {22}.

7. List of publications in other areas

- **Articles published in Philadelphia List journals with IF³:**

- [1] W. T. Chyla, W. D. Deering, *Expulsion of carriers from the double-barrier quantum well and investigation of its spectral consequences*. Applied Physics A, vol. 55, pp. 218-225 (1992). {12}
- [2] W. T. Chyla, W. D. Deering, *Electrostatic correction to the square-potential model of nanostructures significantly affects current resonances*. Applied Physics A, vol. 58, pp. 91-98 (1994). {13}
- [3] W. T. Chyla, *The hyperbolic potential of the heavy-quark interaction*. Il Nuovo Cimento A, vol. 105, pp. 619-623 (1992). {14}
- [4] W. T. Chyla, *The semi-classical hopping model of the electrical conductivity of semiconductors*. Journal of Physics and Chemistry of Solids, vol. 41, pp. 797-799 (1980). {15}

- **Article published in a new international journal:**

- [5] W. T. Chyla, *Simultaneous gravitational and refractive lensing*. International Journal of Astronomy and Astrophysics, vol. 2, pp. 76-80 (2012). {29}

- **Articles published in a Philadelphia List journal without IF, recognized by MNiSW:**

- [6] W. T. Chyla, *Medical pharmacology from antiquity to the present (Farmakologia medyczna od starożytności do współczesności)*. Farmacja Polska, vol. 70(5), pp. 258-266 (2014). {16}
- [7] W. T. Chyla, *Contemporary pharmacopeias and their harmonization (Farmakopee współczesne i ich harmonizacja)*. Farmacja Polska, vol. 70(3), pp. 134-136 (2014). {17}
- [8] W. T. Chyla, *American pharmacopeia – The US Pharmacopeia and National Formulary (Farmakopea amerykańska – the US Pharmacopeia and National Formulary)*. Farmacja Polska, vol. 70(4), pp. 179-182 (2014). {18}
- [9] W. T. Chyla, *Environmental factors that affect accurate weighing in pharmaceutical laboratories (Warunki środowiskowe wpływające na dokładne ważenie w laboratoriach farmaceutycznych)*. Farmacja Polska, vol. 70(8), pp. 428-435 (2014). {19}
- [10] W. T. Chyla, *Metrological parameters of non-automatic balances used in pharmaceutical laboratories (Parametry metrologiczne wag nieautomatycznych stosowanych w laboratoriach farmaceutycznych)*. Farmacja Polska, vol. 70(11), pp. 651-661 (2014). {20}
- [11] W. T. Chyla, *Significant figures and rounding rules according to FDA (Cyfry znaczące oraz zaokrąglanie liczb według FDA)*. Farmacja Polska, vol. 71(3), pp. 194-199 (2015). {21}
- [12] W. T. Chyla, *International regulations concerning weighing in pharmacology and in the pharmaceutical industry in the context of quality assurance systems (Międzynarodowe dokumenty regulujące ważenie w farmacji i w przemyśle farmaceutycznym w kontekście systemów zarządzania jakością)*. Farmacja Polska, vol. 71(4), pp. 246-255 (2015). {22}



³ Articles [1] – [2] were published after PhD, but they are based on the doctoral dissertation. Articles [3] – [4] were published before PhD, but their subjects are entirely different from the doctoral dissertation.

PART V

CUMULATIVE LIST OF ARTICLES, PRESENTATIONS AND TRANSLATIONS

1. The cumulative list of articles published in scientific and technical journals⁴

- Articles published in Philadelphia List journals with IF:

- {1} W. T. Chyla, *Refraction-dependent kinematic shift of spectral lines*. **European Physical Journal Plus**, vol. 130, paper 78, pp. 1-6 (2015). [II-6]
- {2} W. T. Chyla, *The problem of scale in electric metrology of nanostructures in the context of the New SI redefinition of the base electric unit*. *Acta Physica Polonica A*, vol. 125, nr 1, pp. 3-12 (2014). [III-4]
- {3} W. T. Chyla, *Group velocity of the electromagnetic wave packet in a relativistic dispersive medium*. **Journal of Electromagnetic Waves and Applications**, vol. 27, pp. 938-943 (2013). [II-5]
- {4} W. T. Chyla, *Refraction in a relativistic medium*. **Optik – International Journal of Light and Electron Optics**, vol. 124, pp. 1477-1479 (2013). [II-4]
- {5} W. T. Chyla, *On the structure of the New SI definitions of base units*. *Metrologia*, vol. 49, pp. L17-L19 (2012). [III-3]
- {6} W. T. Chyla, *On the proposed redefinition of the mole*. *Metrologia*, vol. 49, pp. L11-L13 (2012). [III-2]
- {7} W. T. Chyla, *Evolution of the international metric system of units SI*. *Acta Physica Polonica A*, vol. 120, pp. 998-1011 (2011). [III-1]
- {8} W. T. Chyla, *On generation of collimated high-power gamma beams*. **Laser and Particle Beams**, vol. 24, pp. 143-156 (2006). [II-7]
- {9} W. T. Chyla, *I. Geometrical optics of variable frequency light rays: Theoretical basis*. **Canadian Journal of Physics**, vol. 78, pp. 721-745 (2000). [II-1]
- {10} W. T. Chyla, *II. Geometrical optics of variable frequency light rays in special relativistic and nonrelativistic situations: The general law of reflection*. **Canadian Journal of Physics**, vol. 78, pp. 747-754 (2000). [II-2]
- {11} W. T. Chyla, *III. Geometrical optics of variable frequency light rays in the general relativistic regime: Combined gravitational and refractive lensing*. **Canadian Journal of Physics**, vol. 78, pp. 755-767 (2000). [II-3]
- {12} W. T. Chyla and W. D. Deering, *Expulsion of carriers from the double-barrier quantum well and investigation of its spectral consequences*. *Applied Physics A*, vol. 55, pp. 218-225 (1992). [IV-1]
- {13} W. T. Chyla and W. D. Deering, *Electrostatic correction to the square potential model of nanostructures significantly affects current resonances*. *Applied Physics A*, vol. 58, pp. 91-98 (1994). [IV-2]
- {14} W. T. Chyla, *The hyperbolic potential of the heavy quark interaction*. *Il Nuovo Cimento A*, vol. 105, pp. 619-623 (1992). [IV-3]
- {15} W. T. Chyla, *The semi-classical hopping model of the electrical conductivity in semiconductors*. *Journal of Physics and Chemistry of Solids*, vol. 41, pp. 797-799 (1980). [IV-4]

⁴ Articles {1}–{11} were published after PhD and are unrelated to the doctoral topic; articles {12}–{13} were published after PhD, but they are based on the doctoral dissertation; articles {14}–{15} were published before PhD and are unrelated to the doctoral topic. The rearmost symbol in square brackets gives the number of the part (roman numeral) and the number of the note in that part (Arabic number) where a given article is commented on. Articles printed **in bold and underlined** constitute the single-topic cycle of publications *Electromagnetic radiation of variable frequency*.

- **Articles published in the Philadelphia List journal without IF, recognized by MNiSW:**
 - {16} W. T. Chyla, *Medical pharmacology from antiquity to the present (Farmakologia medyczna od starożytności do współczesności)*. Farmacja Polska, vol. 70(5), pp. 258-266 (2014). [IV-6]
 - {17} W. T. Chyla, *Contemporary pharmacopeias and their harmonization (Farmakopee współczesne i ich harmonizacja)*. Farmacja Polska, vol. 70(3), pp. 134-136 (2014). [IV-7]
 - {18} W. T. Chyla, *American pharmacopeia – The US Pharmacopeia and National Formulary (Farmakopea amerykańska – the US Pharmacopeia and National Formulary)*. Farmacja Polska, vol. 70(4), pp. 179-182 (2014). [IV-8]
 - {19} W. T. Chyla, *Environmental factors that affect accurate weighing in pharmaceutical laboratories (Warunki środowiskowe wpływające na dokładne ważenie w laboratoriach farmaceutycznych)*. Farmacja Polska, vol. 70(8), pp. 428-435 (2014). [IV-9]
 - {20} W. T. Chyla, *Metrological parameters of non-automatic balances used in pharmaceutical laboratories (Parametry metrologiczne wag nieautomatycznych stosowanych w laboratoriach farmaceutycznych)*. Farmacja Polska, vol. 70(11), pp. 651-661 (2014). [IV-10]
 - {21} W. T. Chyla, *Significant figures and rounding rules according to FDA (Cyfry znaczące oraz zaokrąglanie liczb według FDA)*. Farmacja Polska, vol. 71(3), pp. 194-199 (2015). [IV-11]
 - {22} W. T. Chyla, *International regulations concerning weighing in pharmacology and in the pharmaceutical industry in the context of quality assurance systems (Międzynarodowe dokumenty regulujące ważenie w farmacji i w przemyśle farmaceutycznym w kontekście systemów zarządzania jakością)*. Farmacja Polska, vol. 71(4), pp. 246-255 (2015). [IV-12]
- **Articles published in the MNiSW-recognized national journals:**
 - {23} W. T. Chyla, *Metrology in modern global economy (Metrologia we współczesnej gospodarce globalnej)*. Nauka, vol. 4, pp. 139-169 (2014). [III-6]
 - {24} W. T. Chyla, *The origins and the present of the NIST with references to the situation of metrology in Poland (Geneza i współczesność NIST z odniesieniami do realiów funkcjonowania metrologii w Polsce)*. Nauka, vol. 2, pp. 93-114 (2012). [III-5]
* Przedruk w Zarządzaniu Jakością, vol. 1/2013 (31), pp. 88-102 (2013).
 - {25} W. T. Chyla, *Avogadro project (IAC) and redefinition of the mole (Projekt Avogadro (IAC) i redefinicja jednostki liczności materii)*. Wiadomości Chemiczne, vol. 66, no. 7-8, pp. 767-787 (2012). [III-7]
 - {26} W. T. Chyla, *Origins, organization and priorities of modern metrology (Geneza, organizacja i priorytety współczesnej metrologii)*. Zarządzanie Jakością, vol. 1/2013 (31), dodatek specjalny Metrologia 2013, pp. 30-39 (2013). [III-8]
 - {27} W. T. Chyla, *National metrology institutes in Europe and in the world (Krajowe instytuty metrologiczne w Europie i na świecie)*. Zarządzanie Jakością, vol. 1/2013 (31), dodatek specjalny Metrologia 2013, pp. 62-73 (2013). [III-9]
 - {28} W. T. Chyla, *Developments in the contemporary scientific metrology (Kierunki zmian we współczesnej metrologii naukowej)*. PAR – Pomiar, Automatyka, Robotyka, vol. 10/2013, pp. 144-149 (2013). [III-10]
- **Articles published in other scientific and technical journals and in conference materials:**
 - {29} W. T. Chyla, *Simultaneous gravitational and refractive lensing*. International Journal of Astronomy and Astrophysics, vol. 2, pp. 76-80 (2012). [IV-5]
 - {30} W. T. Chyla, *Discussion of the 2010 BIPM director's annual report (Omówienie sprawozdania Dyrektora Międzynarodowego Biura Miar z działalności BIPM w roku 2010)*. Metrologia – Biuletyn Głównego Urzędu Miar, vol. 6, no. 1, March 2011, pp. 23-38. [III-11]
 - {31} W. T. Chyla, *Discussion of the 2009 BIPM director's annual report (Omówienie raportu Dyrektora Międzynarodowego Biura Miar z działalności BIPM w roku 2009)*. Metrologia – Biuletyn Głównego Urzędu Miar, no. 1(16), March 2010, pp. 19-32. [III-12]
 - {32} W. T. Chyla, *Discussion of the 2008 BIPM annual report (Omówienie raportu Międzynarodowego Biura Miar za rok 2008)*. Metrologia – Biuletyn Informacyjny GUM, no. 1(12), March 2009, pp. 3-12. [III-13]
 - {33} W. T. Chyla, *Reform of the international system of units SI (Reforma międzynarodowego układu jednostek miar SI)*. Metrologia w systemach zarządzania – 7. Praktyczne aspekty metrologii, 23-25 września 2013 r., Kielce-Ameliówka, pp. 31-42. [III-14]

2. The cumulative list of presentations at conferences and seminars⁵

- {34} W. T. Chyla, W. D. Deering, *How charge accumulation in the double-barrier quantum well affects its spectral and transport properties*. 39th Annual Meeting of the American Physical Society, Texas Section, University of North Texas, Denton, TX, Nov. 1-2 1991.
- {35} W. T. Chyla, *Investigation of spectral and transport properties of the charged double-barrier quantum well*. Physics Department, University of North Texas, Nov. 14, 1991.
- {36} W. T. Chyla, *Manifestation of the generalized Berry phase in the natural shape of molecular electronic spectral lines*. Department of Physics, University of North Texas, Feb. 12, 1991.
- {37} W. T. Chyla, *Entropy and temperature of nuclear fusion*. Physics Department, University of North Texas, Sept. 28, 1989.
- {38} W. T. Chyla, *Yukawa-type potential of the heavy quark interaction*. Department of Physics, University of North Texas, March 30, 1989.
- {39} W. T. Chyla, *Electrical conductivity in quantum well systems*. Physics Department, University of North Texas, Jan. 26, 1989.
- {40} W. T. Chyla, *Supersymmetry for non-relativistic hamiltonians*. Physics Department, University of North Texas, Oct. 27, 1988.
- {41} W. T. Chyla, *The band mechanism of active transport of sodium ions across cell membranes*. Department of Physics, University of North Texas, Oct. 20, 1987.
- {42} W. T. Chyla, *Semiconductor nanostructures (Nanostruktury półprzewodnikowe)*. Regional Seminar on the Didactics of Physics and Astronomy, Regional Methodology Center, Olsztyn, 25 X 1997.
- {43} W. T. Chyla, *The principle of least time (Zasada najmniejszego czasu)*. Seminar on the Didactics of Physics, Division of Physics, Agricultural-Technical Academy in Olsztyn (presently University of Warmia and Mazury in Olsztyn), 7 IX 1996.
- {44} W. T. Chyla, *Modelling of nanostructure potentials (Modelowanie potencjału nanostruktur)*. Department of Physics, Warsaw University, branch in Białystok (presently University of Białystok), Oct. 1993.
- {45} W. T. Chyla, *Weighing instruments producer's association in Poland*. 63rd GA CECIP, 3 V 2013, Rotterdam, Netherlands.

3. The cumulative list of translations⁶

- {46} 1992 – 1993 Freelance translator of technical texts, SCITRAN, Santa Barbara, CA.
- {47} P. Howarth, F. Redgrave, *Metrology in short*, 3rd ed. (Metrologia w skrócie, wyd. 3). EURAMET 2008.
- {48} M. Abraham, *Smart meters. Electricity smart metering in Canada (Inteligentne mierniki. Inteligentne pomiary energii elektrycznej w Kanadzie)*. OIML Bulletin, vol. LI, no.1, January 2010.
- {49} National Measurement Office, *A strategy for the National Measurement System 2010 – 2014 (Strategia dla Brytyjskiego Krajowego Systemu Miar na lata 2010 – 2014)*. DIUS, May 2009.
- {50} J. L. Guasch, J. L. Racine, I. Sanchez, M. Diop, *Evaluating compliance with standards: The conformity assessment framework*, chapter 4 in *Quality systems and standards for a competitive edge (Badanie zgodności z normami: Struktura systemu oceny zgodności, rozdział 4 w Systemy jakości i normy jako źródło przewagi konkurencyjnej)*. World Bank, Washington, DC, 2007 .
- {51} European Association of National Metrology Institutes, *EMRP Call 2010 – Industry & Environment. Guidance for costing a JRP (Konkurs EMRP 2010 – Przemysł i środowisko. Wytyczne do kalkulacji kosztów JRP)*. EURAMET 2010.

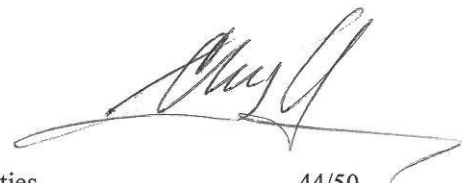
⁵ International conferences and seminars {34}–{41}; national {42}–{44}; other {45}.

⁶ Translations from Polish into English {46}; from English into Polish {47} – {61}.

- {52} European Metrology Research Program (EMRP), *A series of financial documents, JRP-Contract, JRP-Consortium Agreement and associated documents (Seria dokumentów finansowych EMRP, Kontrakt JRP, Umowa Konsorcjum JRP wraz z aneksami i dokumentami towarzyszącymi)*. EURAMET 2009.
- {53} M. Kühne, *Message of the BIPM director on the 2011 World Metrology Day and the International Year of Chemistry (Przesłanie Dyrektora BIPM z okazji Światowego Dnia Metrologii i Międzynarodowego Roku Chemii w 2011 r.)*. Metrologia – Biuletyn GUM, vol. 6, no. 2, May 2011, pp. 4-6.
- {54} S. Patoray, *Message of the BIML director on the 2011 World Metrology Day and the International Year of Chemistry (Przesłanie Dyrektora BIML z okazji Światowego Dnia Metrologii i Międzynarodowego Roku Chemii w 2011 r.)*. Metrologia – Biuletyn GUM, vol. 6, no. 2, May 2011, pp. 7-8.
- {55} BIPM & OIML, *The joint BIPM and OIML press release on the 2011 World Metrology Day. Chemical measurements for our life, our future (Wspólny komunikat prasowy BIPM i OIML z okazji Światowego Dnia Metrologii w 2011 r. Pomiary chemiczne dla naszego życia, naszej przyszłości)*. Metrologia – Biuletyn GUM, vol. 6, no. 2, May 2011, pp. 9-10.
- {56} J. Wallard, *Message of the BIPM director on the 2010 World Metrology Day. Measurements in science and technology – a bridge to innovation (Przesłanie Dyrektora BIPM z okazji Światowego Dnia Metrologii w 2010 r. Pomiary w nauce i technologii mostem do innowacji)*. Metrologia – Biuletyn GUM, no. 2(17), June 2010, pp. 5-8.
- {57} J. Wallard, *Message of the BIPM director on the 2009 World Metrology Day. Measurements in commerce (Przesłanie Dyrektora BIPM z okazji Światowego Dnia Metrologii w 2009 r. Pomiary w gospodarce – metrologia wspiera rozwój gospodarczy)*. Metrologia – Biuletyn GUM, no. 2(13), June 2009, pp. 13-16.
- {58} P. Klenovsky, *There is a niche market for high quality metrological services (Istnieje niszowy rynek usług metrologicznych wysokiej jakości). Interview with the director of the Czech Metrology Institute (CMI). Zarządzanie Jakością, vol. 1/2013 (31), special edition Metrologia 2013, pp. 45-54 (2013).*
- {59} A. Ö. Altan, *The accredited laboratories in Turkey have an increasing share of the market of routine calibrations (W Turcji laboratoria akredytowane mają coraz większy udział w zaspokajaniu popytu na rutynowe wzorcowania). Interview with the representative of the National Metrology Institute of Turkey (TÜBİTAK UME). Zarządzanie Jakością, vol. 1/2013 (31), special edition Metrologia 2013, pp. 55-59 (2013).*
- {60} R. Wynands, *We are engaged in all international metrological activities (Jesteśmy zaangażowani we wszystkie międzynarodowe przedsięwzięcia metrologiczne). Interview with the representative of the National Metrology Institute of Germany (PTB). Zarządzanie Jakością, vol. 1/2013 (31), special edition Metrologia 2013, pp. 127-131 (2013).*
- {61} T. J. Quinn, *Metrology is the heart of the industry (Metrologia jest sercem przemysłu). Interview with the director emeritus of the BIPM. Zarządzanie Jakością, vol. 1/2013 (31), pp. 6-15 (2013).*

4. Reviews of monographs

- {62} T. Quinn, *From artefacts to atoms. The BIPM and the search for ultimate measurement standards (Od artefaktów do atomów. Międzynarodowe Biuro Miar i poszukiwanie ostatecznych wzorców pomiarowych)*. Oxford University Press, 2012. Zarządzanie Jakością, vol. 1/2013 (31), pp. 116-118 (2013).
- {63} R. Nater, A. Reichmuth, R. Schwartz, M. Borys, P. Zervos, *Dictionary of weighing terms. A guide to the terminology of weighing (Słownik terminów wagarskich. Przewodnik po terminologii stosowanej w ważeniu)*, Springer, 2013. PAR – Pomiary, Automatyka, Robotyka, vol. 4/2014, p. 62 (2014).



PART VI

COPIES OF DOCUMENTS

1. Copies of diplomas (ad PART I)

- 1992 – Doctoral diploma (PhD), University of North Texas, USA.
1994 – Nostrification of the doctoral degree, Department of Physics, Warsaw University.
- 1990 – 1993 US Particle Accelerator School, certificates of completion.
 - * 1993 Harvard University, Cambridge, USA.
 - * 1992 Stanford University, Stanford, USA.
 - * 1991 University of Illinois, Urbana-Champaign, USA.
 - * 1990 Harvard University, Cambridge, USA.
- 1987 – Diploma Master of Arts in Physics, University of Southern California, Los Angeles, USA.
- 1974 – Diploma Master of Science in Chemistry, Department of Chemistry, Warsaw University.



2. Copies of publications (ad PARTS II, III, IV)

2.1. Copies of articles that constitute the single-topic cycle of publications (ad PART II).

- Articles on *Electromagnetic radiation of variable frequency* published in Philadelphia List journals with IF:

- [1] W. T. Chyla, *I. Geometrical optics of variable frequency light rays: Theoretical basis*. Canadian Journal of Physics, vol. 78, pp. 721-745 (2000). {9}
- [2] W. T. Chyla, *II. Geometrical optics of variable frequency light rays in special relativistic and nonrelativistic situations: The general law of reflection*. Canadian Journal of Physics, vol. 78, pp. 747-754 (2000). {10}
- [3] W. T. Chyla, *III. Geometrical optics of variable frequency light rays in the general relativistic regime: Combined gravitational and refractive lensing*. Canadian Journal of Physics, vol. 78, pp. 755-767 (2000). {11}
- [4] W. T. Chyla, *Refraction in a relativistic medium*. Optik – International Journal of Light and Electron Optics, vol. 124, pp. 1477-1479 (2013). {4}
- [5] W. T. Chyla, *Group velocity of the electromagnetic wave packet in a relativistic dispersive medium*. Journal of Electromagnetic Waves and Applications, vol. 27, pp. 938-943 (2013). {3}
- [6] W. T. Chyla, *Refraction-dependent kinematic shift of spectral lines*. European Physical Journal Plus, vol. 130, paper 78, pp. 1-6 (2015). {1}
- [7] W. T. Chyla, *On generation of collimated high-power gamma beams*. Laser and Particle Beams, vol. 24, pp. 143-156 (2006). {8}



2.2. Copies of articles in the field of metrology (ad PART III).

- **Articles published in Philadelphia List journals with IF:**

- [1] W. T. Chyla, *Evolution of the international metric system of units SI*. Acta Physica Polonica A, vol. 120, pp. 998-1011 (2011). {7}
- [2] W. T. Chyla, *On the proposed redefinition of the mole*. Metrologia, vol. 49, pp. L11-L13 (2012). {6}
- [3] W. T. Chyla, *On the structure of the New SI definitions of base units*. Metrologia, vol. 49, pp. L17-L19 (2012). {5}
- [4] W. T. Chyla, *The problem of scale in electric metrology of nanostructures in the context of the New SI redefinition of the base electric unit*. Acta Physica Polonica A, vol. 125, no. 1, pp. 3-12 (2013). {2}

- **Articles published in the MNiSW-recognized national journals:**

- [5] W. T. Chyla, *The origins and the present of the NIST with references to the situation of metrology in Poland (Geneza i współczesność NIST z odniesieniami do realiów funkcjonowania metrologii w Polsce)*. Nauka, vol. 2, pp. 93-114 (2012). {24}
* Reprint in Zarządzanie Jakością, vol. 1/2013 (31), pp. 88-102 (2013).
- [6] W. T. Chyla, *Metrology in modern global economy (Metrologia we współczesnej gospodarce globalnej)*. Nauka, vol. 4, pp. 139-169 (2014). {23}
- [7] W. T. Chyla, *Avogadro project (IAC) and redefinition of the mole (Projekt Avogadro (IAC) i redefinicja jednostki liczności materii)*. Wiadomości Chemiczne, vol. 66, no. 7-8, pp. 767-787 (2012). {25}
- [8] W. T. Chyla, *Origins, organization and priorities of modern metrology (Geneza, organizacja i priorytety współczesnej metrologii)*. Zarządzanie Jakością, vol. 1/2013 (31), special edition Metrologia 2013, pp. 30-39 (2013). {26}
- [9] W. T. Chyla, *National metrology institutes in Europe and in the world (Krajowe instytuty metrologiczne w Europie i na świecie)*. Zarządzanie Jakością, vol. 1/2013 (31), special edition Metrologia 2013, pp. 62-73 (2013). {27}
- [10] W. T. Chyla, *Developments in the contemporary scientific metrology (Kierunki zmian we współczesnej metrologii naukowej)*. PAR – Pomiar, Automatyka, Robotyka, vol. 10/2013, pp. 144-149 (2013). {28}

- **Articles published in the national technical journal:**

- [11] W. T. Chyla, *Discussion of the 2010 BIPM director's annual report (Omówienie sprawozdania Dyrektora Międzynarodowego Biura Miar z działalności BIPM w roku 2010)*. Metrologia – Biuletyn Głównego Urzędu Miar, vol. 6, no. 1, March 2011, pp. 23-38. {30}
- [12] W. T. Chyla, *Discussion of the 2009 BIPM director's annual report (Omówienie raportu Dyrektora Międzynarodowego Biura Miar z działalności BIPM w roku 2009)*. Metrologia – Biuletyn Głównego Urzędu Miar, vol. 1(16), March 2010, pp. 19-32. {31}
- [13] W. T. Chyla, *Discussion of the 2008 BIPM annual report (Omówienie raportu Międzynarodowego Biura Miar za rok 2008)*. Metrologia – Biuletyn Informacyjny GUM, no. 1(12), March 2009, pp. 3-12. {32}

- **Article published in conference materials:**

- [14] W. T. Chyla, *Reform of the international system of units SI (Reforma międzynarodowego układu jednostek miar SI)*. Metrologia w systemach zarządzania – 7. Praktyczne aspekty metrologii, 23-25 września 2013 r., Kielce-Ameliówka, pp. 31-42. {33}



2.3. Copies of articles in other fields (ad PART IV).

- **Articles published in Philadelphia List journals with IF¹:**

- [1] W. T. Chyla, W. D. Deering, *Expulsion of carriers from the double-barrier quantum well and investigation of its spectral consequences*. Applied Physics A, vol. 55, pp. 218-225 (1992). {12}
- [2] W. T. Chyla, W. D. Deering, *Electrostatic correction to the square-potential model of nanostructures significantly affects current resonances*. Applied Physics A, vol. 58, pp. 91-98 (1994). {13}
- [3] W. T. Chyla, *The hyperbolic potential of the heavy-quark interaction*. Il Nuovo Cimento A, vol. 105, pp. 619-623 (1992). {14}
- [4] W. T. Chyla, *The semi-classical hopping model of the electrical conductivity of semiconductors*. Journal of Physics and Chemistry of Solids, vol. 41, pp. 797-799 (1980). {15}

- **Article published in a new international journal:**

- [5] W. T. Chyla, *Simultaneous gravitational and refractive lensing*. International Journal of Astronomy and Astrophysics, vol. 2, pp. 76-80 (2012). {29}

- **Articles published in a Philadelphia List journal without IF, recognized by MNiSW:**

- [6] W. T. Chyla, *Medical pharmacology from antiquity to the present (Farmakologia medyczna od starożytności do współczesności)*. Farmacja Polska, vol. 70(5), pp. 258-266 (2014). {16}
- [7] W. T. Chyla, *Contemporary pharmacopeias and their harmonization (Farmakopee współczesne i ich harmonizacja)*. Farmacja Polska, vol. 70(3), pp. 134-136 (2014). {17}
- [8] W. T. Chyla, *American pharmacopeia – The US Pharmacopeia and National Formulary (Farmakopea amerykańska – the US Pharmacopeia and National Formulary)*. Farmacja Polska, vol. 70(4), pp. 179-182 (2014). {18}
- [9] W. T. Chyla, *Environmental factors that affect accurate weighing in pharmaceutical laboratories (Warunki środowiskowe wpływające na dokładne ważenie w laboratoriach farmaceutycznych)*. Farmacja Polska, vol. 70(8), pp. 428-435 (2014). {19}
- [10] W. T. Chyla, *Metrological parameters of non-automatic balances used in pharmaceutical laboratories (Parametry metrologiczne wag nieautomatycznych stosowanych w laboratoriach farmaceutycznych)*. Farmacja Polska, vol. 70(11), pp. 651-661 (2014). {20}
- [11] W. T. Chyla, *Significant figures and rounding rules according to FDA (Cyfry znaczące oraz zaokrąglanie liczb według FDA)*. Farmacja Polska, vol. 71(3), pp. 194-199 (2015). {21}
- [12] W. T. Chyla, *International regulations concerning weighing in pharmacology and in the pharmaceutical industry in the context of quality assurance systems (Międzynarodowe dokumenty regulujące ważenie w farmacji i w przemyśle farmaceutycznym w kontekście systemów zarządzania jakością)*. Farmacja Polska, vol. 71(4), pp. 246-255 (2015). {22}



¹ Articles [1] – [2] were published after PhD, but they are based on the doctoral dissertation. Articles [3] – [4] were published before PhD, but their subjects are entirely different from the doctoral dissertation.

2.4. Copies of reviews of monographs.

- {62} T. Quinn, *From artefacts to atoms. The BIPM and the search for ultimate measurement standards (Od artefaktów do atomów. Międzynarodowe Biuro Miar i poszukiwanie ostatecznych wzorców pomiarowych)*. Oxford University Press, 2012. Zarządzanie Jakością, vol. 1/2013 (31), pp. 116-118 (2013).
- {63} R. Nater, A. Reichmuth, R. Schwartz, M. Borys, P. Zervos, *Dictionary of weighing terms. A guide to the terminology of weighing (Słownik terminów wagarskich. Przewodnik po terminologii stosowanej w ważeniu)*, Springer, 2013. PAR – Pomiary, Automatyka, Robotyka, vol. 4/2014, p. 62 (2014).



THE END
OF THE SELF-REVIEW OF RESEARCH ACTIVITIES

A handwritten signature in black ink, appearing to read 'W. Chyla', with a long horizontal flourish extending to the right.