Visibility of microwave absorption as a probe of Majorana bound states non-locality

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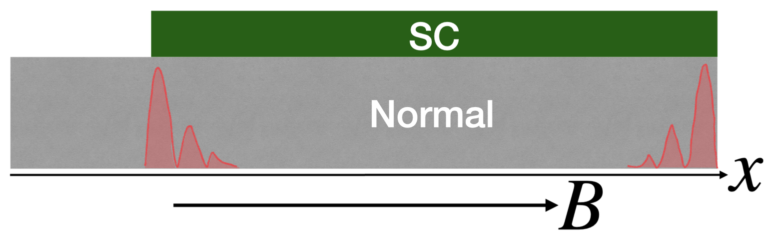
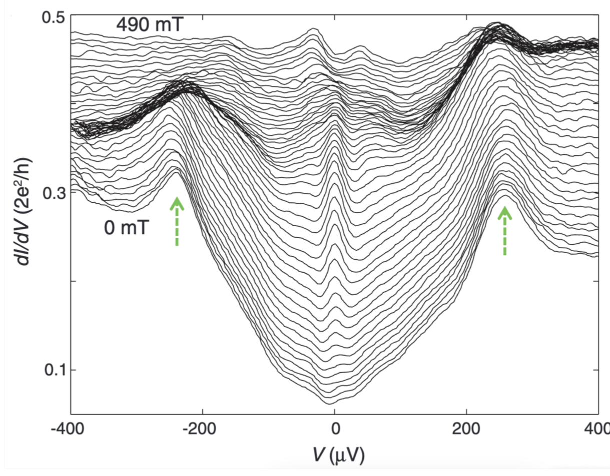
Majorana bound state (MBS)



Topologically protected zero energy MBS (in red) in spin-orbit nanowire [1]



Zero-bias conductance peak: *A predicted signature of Majorana bound states [2]*

Ambiguity in the detection of topologically protected MBS:

- Trivial Andreev bound state (ABS) shows similar signature like zero-bias conductance peak and zero energy for a range of magnetic fields. [3]
- Quasi-Majorana bound state (QMBS) arise due to defects or imperfect gate voltages leading to "local topological states" which are difficult to control.

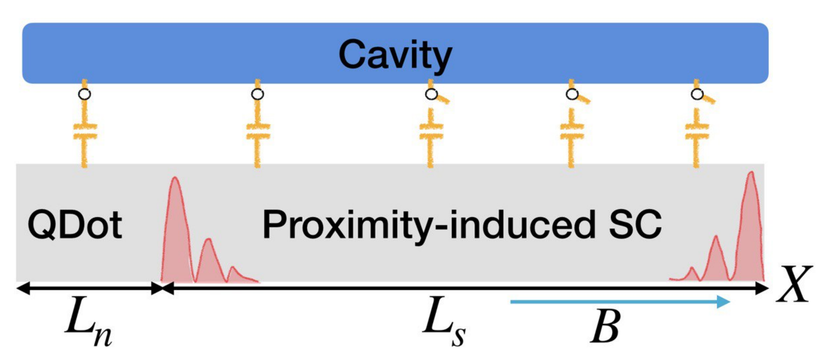
Challenge

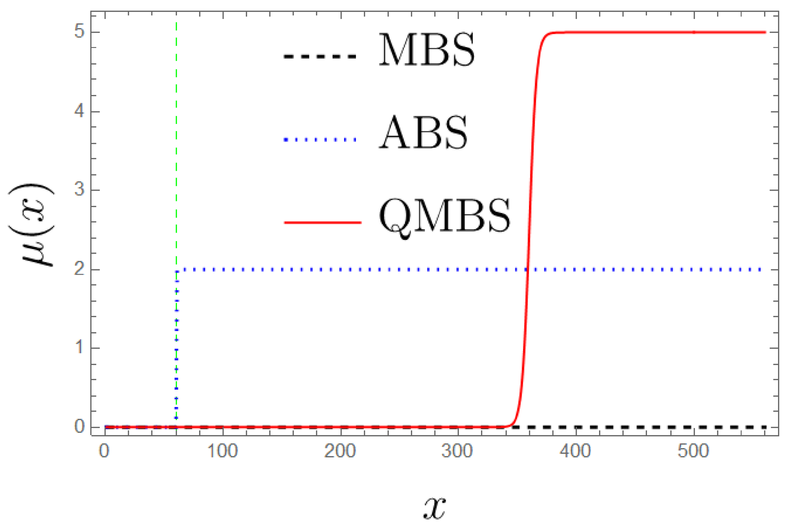
Distinguish Majorana bound states from ABS [3, 4] and QMBS [4] as the latter two also give zero-bias conductance peak

Our solution

Probe the non-locality of Majorana bound state with microwaves!

Model Hamiltonian





(Left) A Rashba nanowire (gray part) with quantum dot (QDot) and partial proximity induced superconductivity is capacitively coupled to a cavity, and has a magnetic field $\mathbf{B}$ *aligned along the wire axis. (Right) Chemical potential profile with the green vertical line at QDot-SC interface.*

- Model Hamiltonian in the tight-binding model is $H_T = H_{el} + H_c + H_{e-c}$ where,

$$H_{el} = \sum_{\sigma,\sigma'}\sum_{j=1}^{L} c^\dagger_{j,\sigma}\left[(2t_h(x_j) - \mu(x_j))\,\delta_{\sigma\sigma'} + V_Z(x_j)\,\sigma^x_{\sigma\sigma'}\right] c_{j,\sigma'} + \sum_{j=1}^{L}\left(\Delta(x_j)\,c^\dagger_{j,\uparrow}c^\dagger_{j,\downarrow} + h.c.\right)$$
$$-\sum_{\sigma,\sigma'}\sum_{j=1}^{L-1}\left[c^\dagger_{j+1,\sigma}\left(t_h(x_j)\delta_{\sigma\sigma'} - i\alpha(x_j)\sigma^y_{\sigma\sigma'}\right)c_{j,\sigma'} + h.c.\right],$$
$$H_{e-c} = \sum_\sigma\sum_j g(x_j)c^\dagger_{j,\sigma}c_{j,\sigma}(a^\dagger + a), \quad H_c = \omega_c\, a^\dagger a,$$

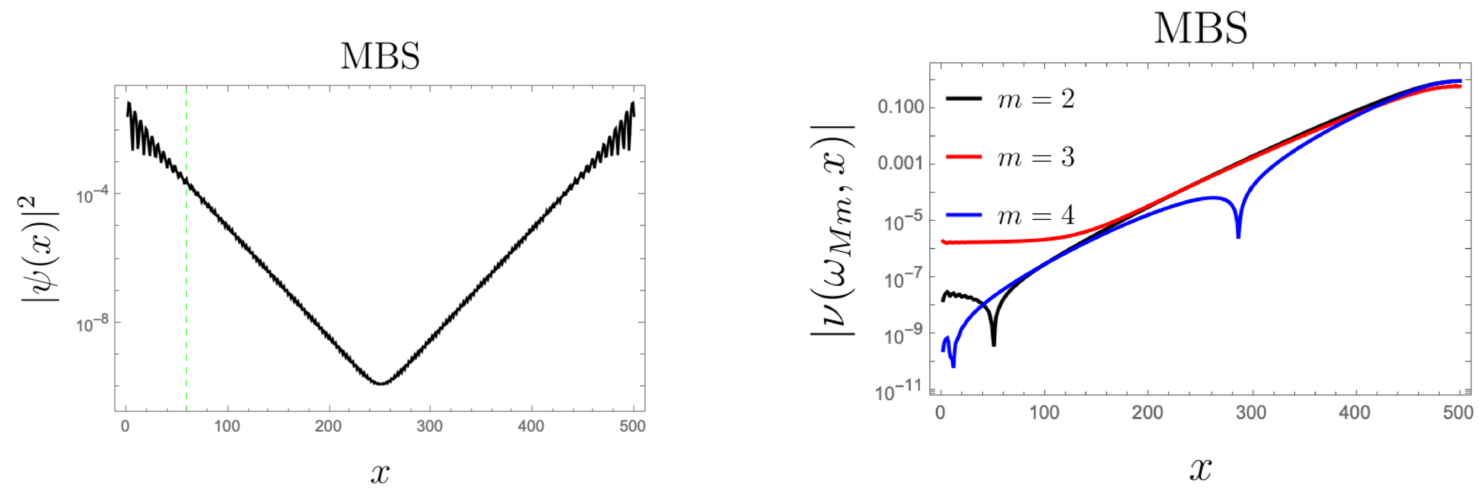
  $c_{j,\sigma}, c^\dagger_{j,\sigma}$: electron operators with spin σ at site j, $\alpha(x_j)$: spin-orbit strength, $V_Z(x_j)$: Zeeman energy, $\mu(x_j)$: chemical potential, $\Delta(x_j)$: proximity superconducting gap, ω_c: cavity frequency, $a(a^\dagger)$: photon operators, and $g(x_j)$: cavity-wire coupling strength.

- We define **visibility of microwave absorption associated with the two parities** $\nu(\omega, x_j)$ [5] from left end of wire to position x_j of wire as

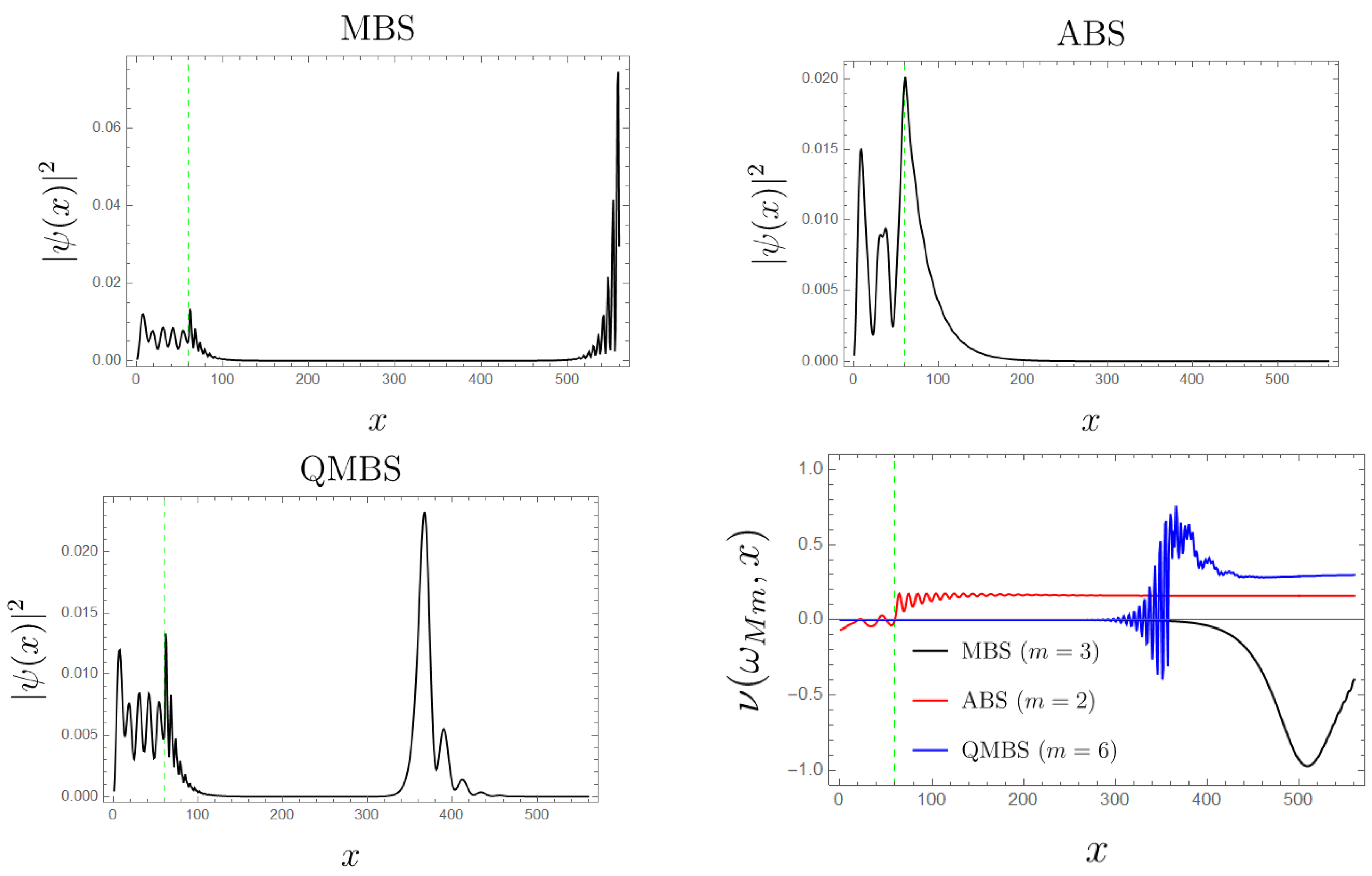
$$\nu(\omega, x_j) = \frac{\mathrm{Im}[\chi_o(\omega, x_j)] - \mathrm{Im}[\chi_e(\omega, x_j)]}{\mathrm{Im}[\chi_o(\omega, x_j)] + \mathrm{Im}[\chi_e(\omega, x_j)]},$$

  where $\chi_{o(e)}(\omega, x_j) = \sum_{\epsilon_m>0}\left(\frac{|\mathcal{M}^o_{mM}(x_j)|^2}{\omega \pm \epsilon_M - \epsilon_m + i\eta}\right)$ is the susceptibility [6], $\mathcal{M}^e_{mM}(x_j) = \sum_{\sigma,l'=1}^{l'=j}\left[u^*_{M\sigma}(x_{l'})u_{m\sigma}(x_{l'}) - v^*_{M\sigma}(x_{l'})v_{m\sigma}(x_{l'})\right]$ and $\mathcal{M}^o_{mM}(x_j) = \sum_{\sigma,l'=1}^{l'=j}\left[v_{M\sigma}(x_{l'})u_{m\sigma}(x_{l'}) - u_{M\sigma}(x_{l'})v_{m\sigma}(x_{l'})\right]$. Here u, v come from $c^\dagger_{j,\sigma} = \sum_{n\geq 0}\left[u^*_{n\sigma}(x_j)b^\dagger_n + v_{n\sigma}(x_j)b_n\right]$ with $b_n, b^\dagger_n$ as Bogoliubov operators.

- **MBS in isolated SC has same exponential scaling behavior as visibility** $\nu(\omega, x_j)$:

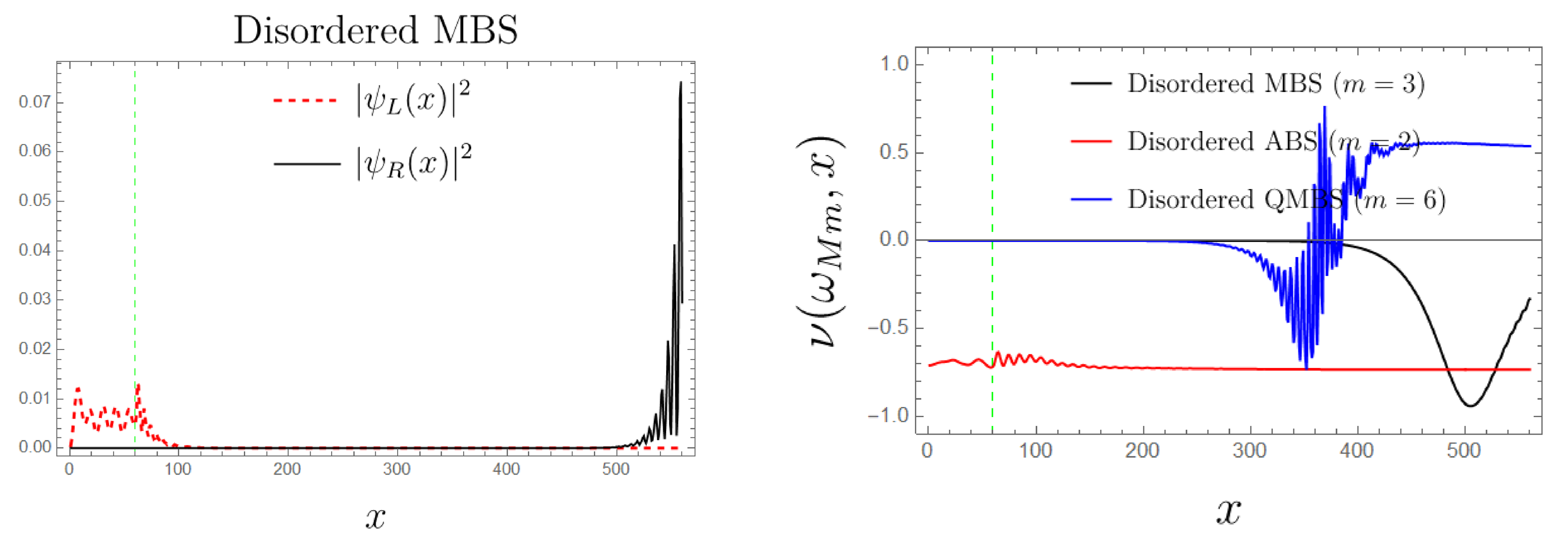


Majorana, Andreev, quasi-Majorana bound states



- **MBS: Visibility captures the non-local nature of MBS as it peaks only when the cavity couples to both spatially separated wavefunction peaks.**
- ABS: In contrast to MBS, cavity couples to the peak that is localized near interface leading to visibility peak near interface.
- QMBS: Visibility peaks as the cavity couples to the peaks at edges of topological phase and not at SC edge.

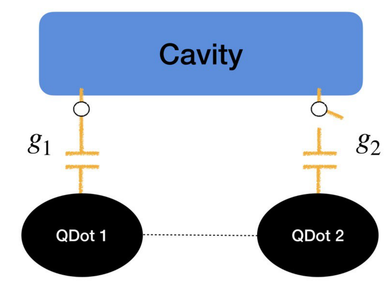
Gaussian disorder in the SC part



- Disorder strength less than minimal superconducting gap.
- **MBS: Visibility retains its character due to the topological robustness of MBS against disorder.**
- ABS: Visibility is affected as ABS is not topologically protected against disorder.
- QBS: Visibility maintains its qualitative features.

Poor man's Majorana

- Poor man's Majorana [7]: Lack topological protection, but mimics Majorana properties
- Visibility $\nu(\omega) = (2g_1g_2)/(g_1^2 + g_2^2)$ where, $g_{1,2}$ is dot-cavity coupling strength, shows the non-local coupling nature as it requires $g_1 \neq 0$ and $g_2 \neq 0$.



Conclusions and outlook

- MBS has unique visibility profile due to its non-local nature that helps to distinguish it from ABS & QMBS.
- Visibility has same scaling behavior as MBS wavefunction (results not shown here).
- We also considered the effect of barrier (results not shown here).
- Possibility of applying this non-local visibility to other Majorana platforms?

References

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