# Quantifying the soliton phase in ultracold 1d Bose gases

Herbert Potrykus[1,2], Igor Nowicki[1,2], Piotr Deuar[1]

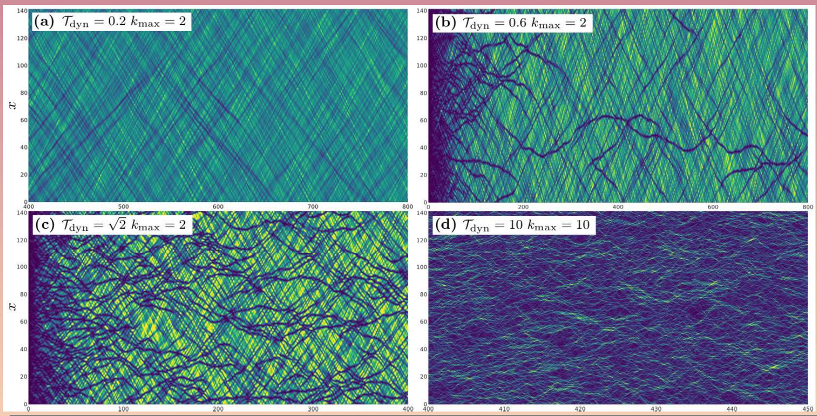
1 Institute of Physics, Polish Academy of Sciences, Warsaw, Poland
2 Warsaw University, Warsaw, Poland

*Known: 1d ultracold gases with g>0 host dark solitons in thermal equilibrium.*

*Question: under what conditions do they dominate the state?*

$$\hbar\frac{d\Psi(x)}{dt} = (-i-\gamma)\mathcal{P}_{k_{\max}}\left[\left(-\frac{\hbar^2}{2m}\nabla^2-\mu+g|\Psi(x)|^2\right)\Psi(x)+\sqrt{2\gamma k_BT\hbar}\,\eta(x,t)\right]$$

PSGPE – projected stochastic Gross-Pitaevskii equation generates the thermal states




Two essential paremeters of the uniform Bose gas:

Lieb-Liniger interaction strength $\gamma_{LL}=\frac{mg}{\rho\hbar^2}$ relative temperature ($T_d$=degeneracy) $\tau=\frac{T}{T_d}=\frac{2k_BTm}{\hbar^2\rho^2}$

Scale to natural (healing length) units:

$\xi_\rho=\hbar/\sqrt{mg\rho}=1/(\rho\sqrt{\gamma_{LL}})$ $\quad t_{\xi\rho}=\hbar/(g\rho)=1/(\gamma_{LL}\rho^2)$ $\quad m$

And normalize to unit density at *T=0*: $\psi(x)=\frac{\phi(x)}{\gamma_{LL}^{1/4}}$

**Leads to: scaled PSGPE**

$$\frac{d\phi(x)}{dt}=(-i-\gamma)\mathcal{P}_{k_{\max}}\left[\left(-\frac{1}{2}\nabla^2-\mu+|\phi(x)|^2\right)\phi(x)+\sqrt{\mathcal{T}_{\rm dyn}}\sqrt{\gamma}\,\eta(x,t)\right]$$

coupling to high energy tails generally small; chempot sets density; thermal complex noise

One essential dynamics parameter: **"dynamical" temperature**

$$\mathcal{T}_{\rm dyn}=\frac{\tau}{\sqrt{\gamma_{LL}}}=\frac{2k_BT\sqrt{m}}{\hbar\rho^{3/2}\sqrt{g}}$$

## Counting solitons, try 1: match dips to perfect soliton profile

$$\phi(x)=\sqrt{\bar{n}}e^{i\Theta}\left[iu+\sqrt{1-u^2}\tanh\left((x-x_0)\sqrt{g\bar{n}(1-u^2)}\right)\right]$$

perfect gray soliton

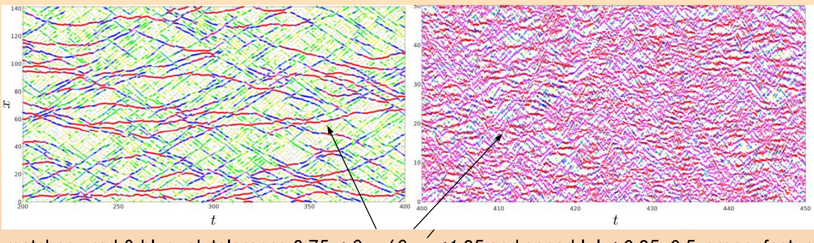
Matching procedure:

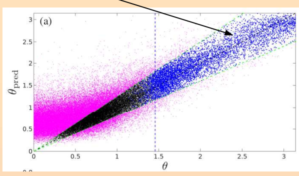
0. consider only lowest minima within a ±ξ range
1. guess mean background density $n_{est}$, choose range $r$
2. match actual minimum density $n_{0,meas}$ and depth $d_{meas}$ to perfect soliton $n_{0,perf}$, $d_{perf}$ to get velocity $u$ and background density *nbar*
3. compare phase jump $\theta_{pred}(u,nbar)$ predicted from *u&nbar* to actual jump $\theta_{meas}$
4. count as soliton if e.g. u<0.5 and the match is sufficient $0.75<\theta_{pred}/\theta_{meas}<1.25$
5. compare mean accepted *nbar* to the guess $n_{est}$, and redo with better $n_{est}$ if needed.

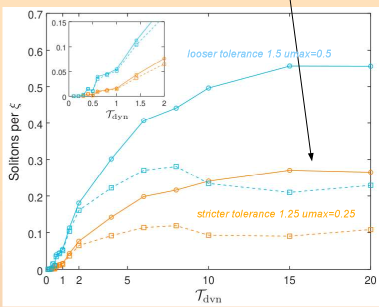
$$d_{\rm pred}=\bar{n}(1-u^2)\tanh^2\left(r\sqrt{\frac{\bar{n}}{\bar{n}_{\rm est}}}\sqrt{1-u^2}\right)$$
$$n_{0,\rm pred}=\bar{n}u^2$$

$$\theta_{\rm pred}=2\tan^{-1}\left[\frac{\sqrt{1-u^2}}{u}\tanh\left(r\sqrt{\frac{\bar{n}}{\bar{n}_{\rm est}}}\sqrt{1-u^2}\right)\right]$$

Conclusions about counting:
- appearance of solitons at low T caught OK
- saturation by rubbish at high T
- need to consider soliton history





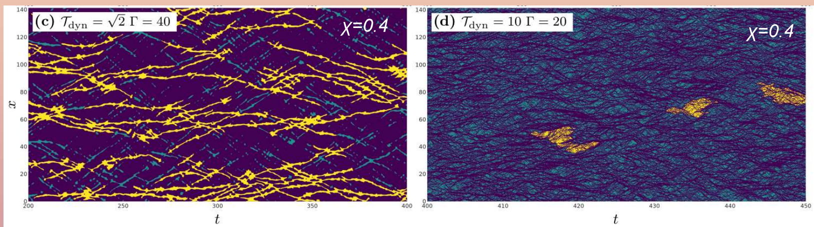




matches: red & blue: ok tolerance $0.75<\theta_{pred}/\theta_{meas}<1.25$ and speed $|u|<0.25, 0.5$, green: fast, pink/cyan/yellow/gray: poor fits

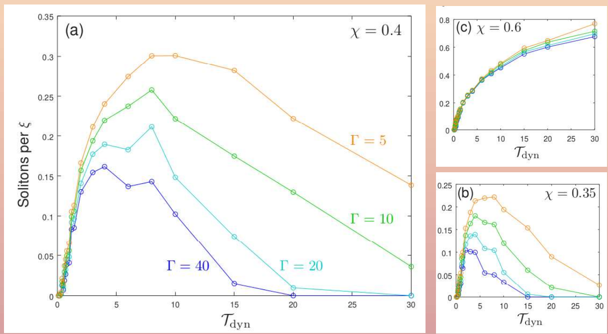
## Counting solitons, try 2: fill troughs, keep long-lived ones

*χ = max. density threshold "inside" soliton*

*Γ = minimum connected volume to treat as soliton*

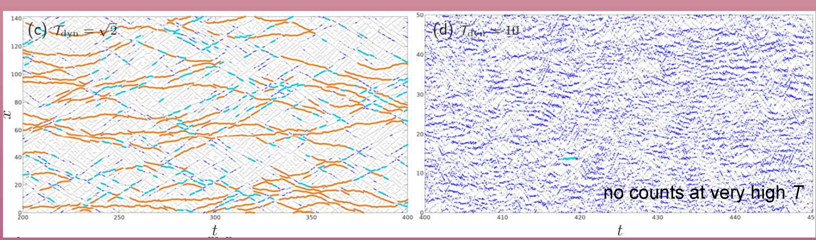



works well at low *T* — catches meaningless lumps at high *T*



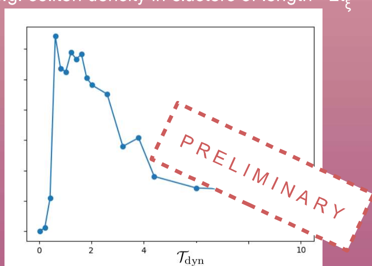

problems again with characterising the high *T* end

## Counting solitons, try 3: keep only long clusters of dips




Clustering procedure:
- only consider dips deemed good enough (e.g. $0.67<\theta_{pred}/\theta_{meas}<1.5$ and speed $|u|<0.5$, above)
- collect dips into chains, if dips at successive sample times are closer than $\epsilon\xi$ (above: ε=2/3)
- Above: Orange – chains of length $>5t_\xi$, cyan: length$>2t_\xi$

e.g. soliton density in clusters of length $>2t_\xi$




This approach appears to work as required:
- catches all solitons at low T
- ceases to count them at high T

$$0.3\lesssim\mathcal{T}_{\rm dyn}=\frac{\tau}{\sqrt{\gamma_{LL}}}\lesssim\mathcal{O}(5)$$

We acknowledge the support of:
the National Science Centre, Poland, projects nr. 2018/31/B/ST2/01871 and 2012/07/E/ST2/01389