Nearly Unidirectional Microwave Networks and Quantum Graphs

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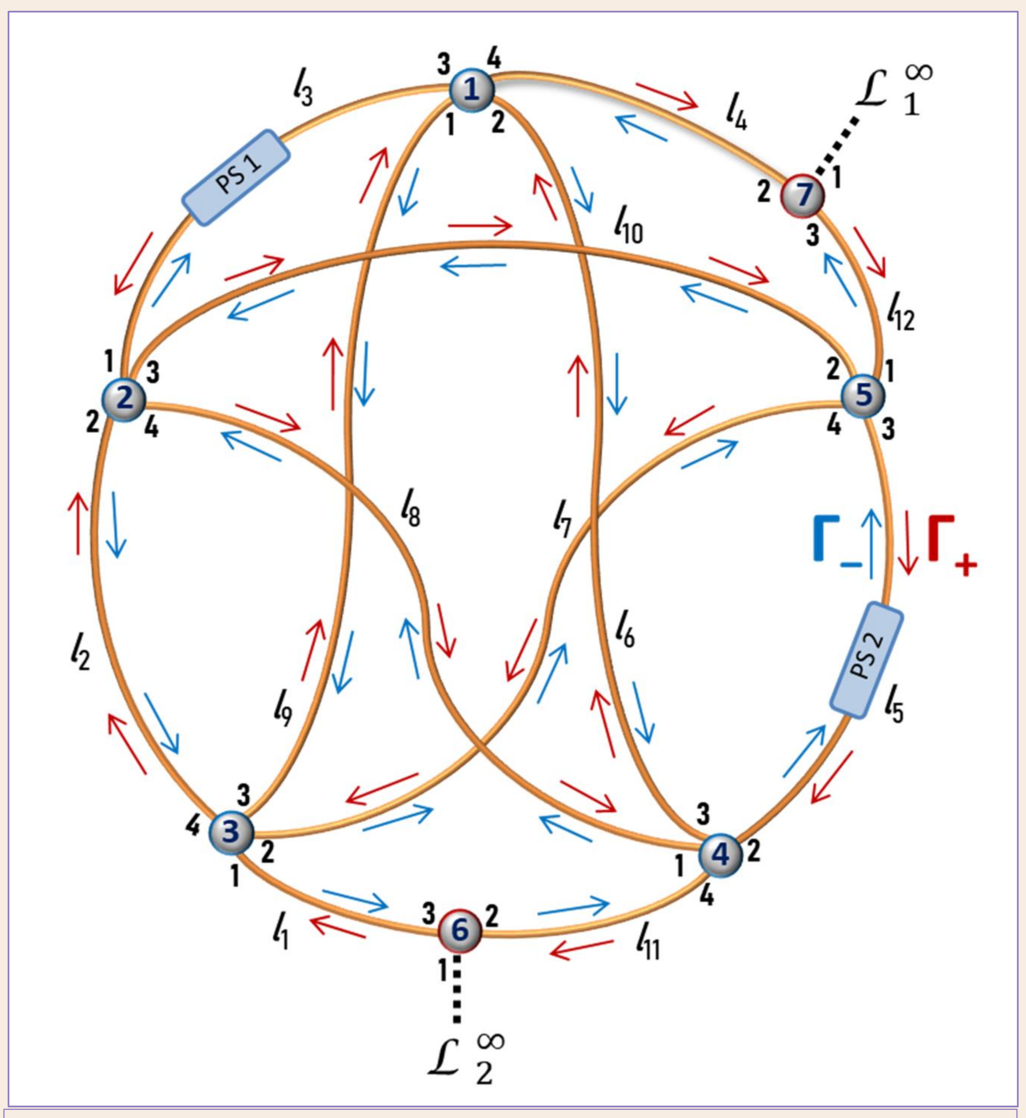
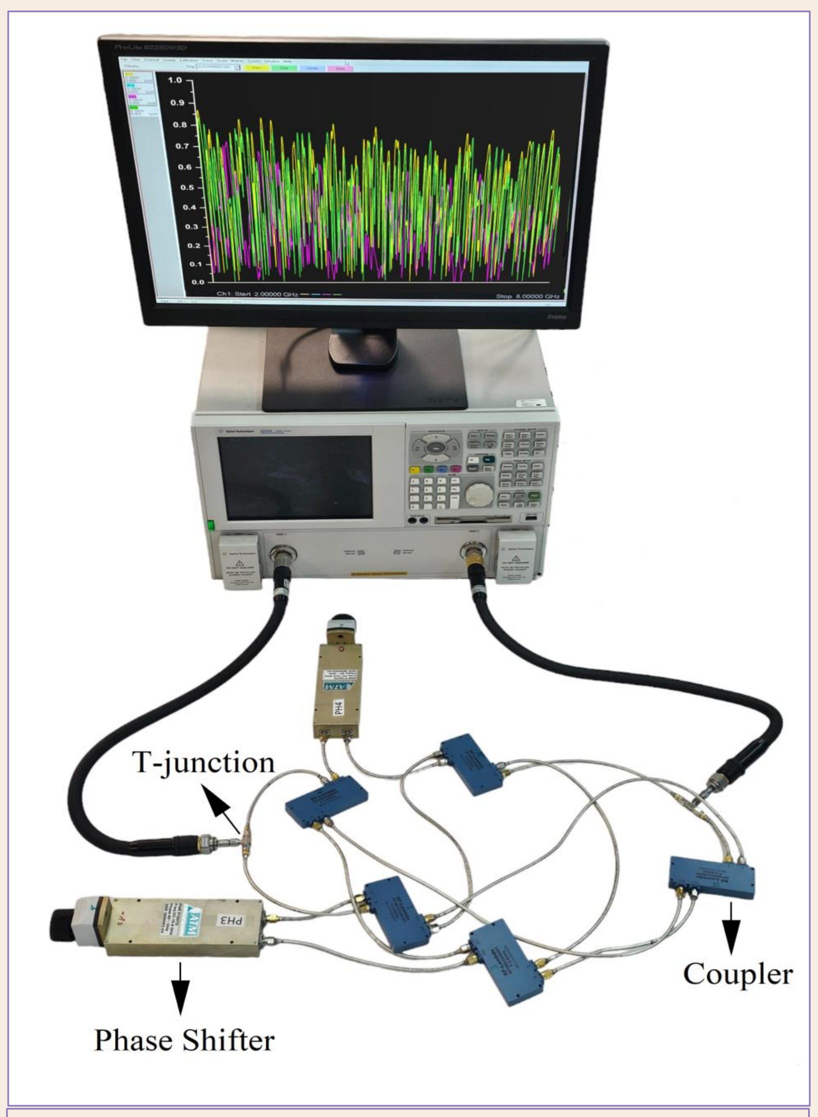
Abstract

We experimentally investigate the undirected open microwave network with internal absorption composed of two coupled directed halves, unidirectional networks Γ+ and Γ−, corresponding to two possible directions of motion on their edges. We measure the two-port scattering matrix and evaluate the spectral statistics and elastic enhancement factor. The comparison of experimental resonances with theoretical predictions by Weyl's law shows double degeneracy. Despite time-reversal symmetry, the missing level spectral statistics and elastic enhancement factor are close to the Gaussian unitary ensemble predictions. We used numerical calculations for the open nondissipative quantum graph possessing the same structure as the microwave network to investigate the doublet structures in the spectrum which otherwise would not be experimentally resolved. We show that the doublet size distribution is close to the Poisson distribution.

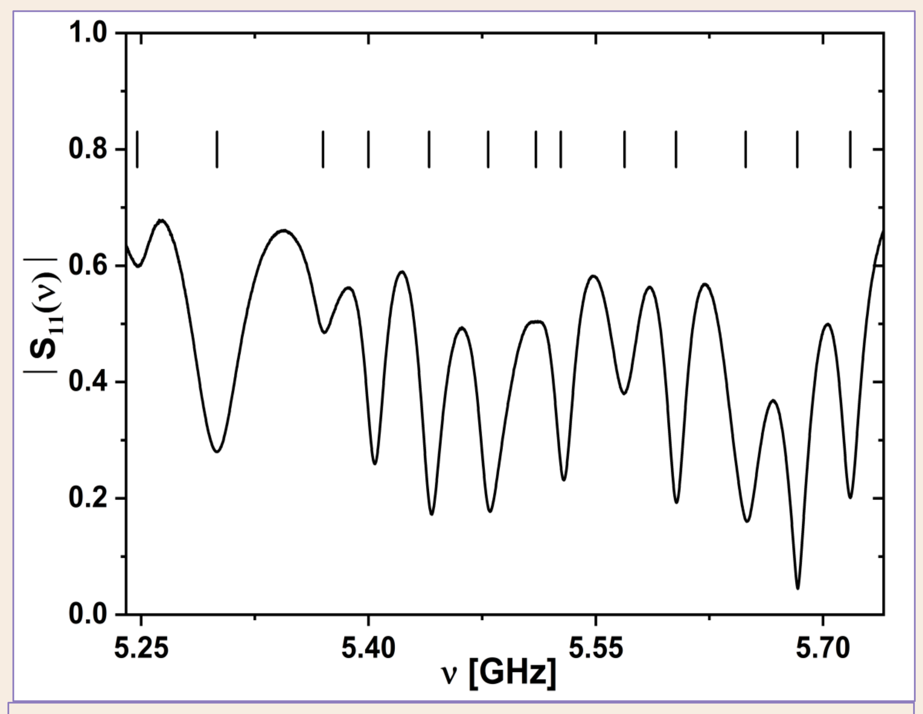
Experimental Setup and Results



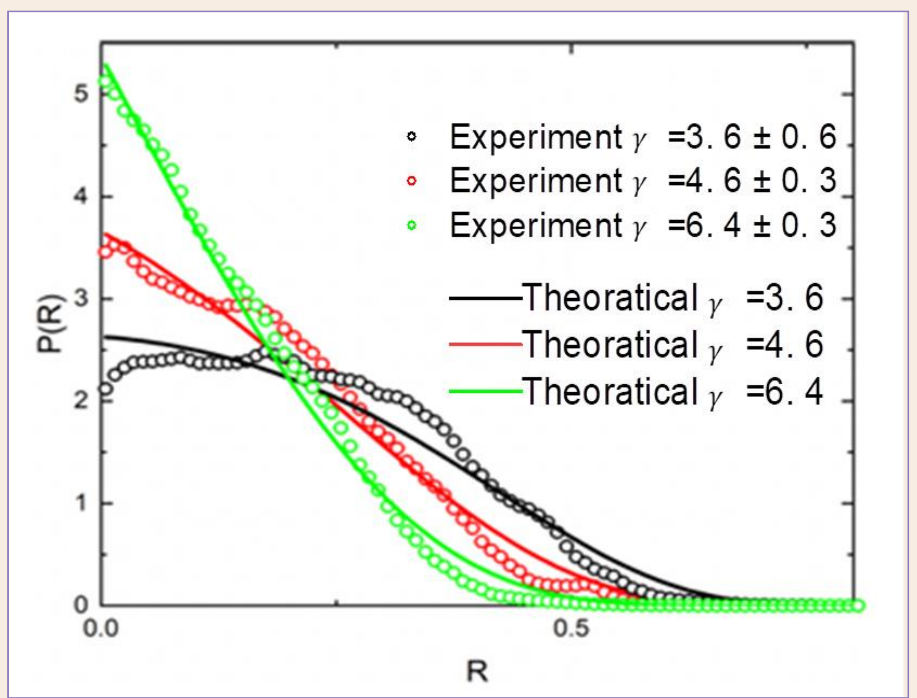
Undirected microwave network simulating an undirected quantum graph possessing two coupled unidirectional graphs Γ+ (red arrows) and Γ− (blue arrows).



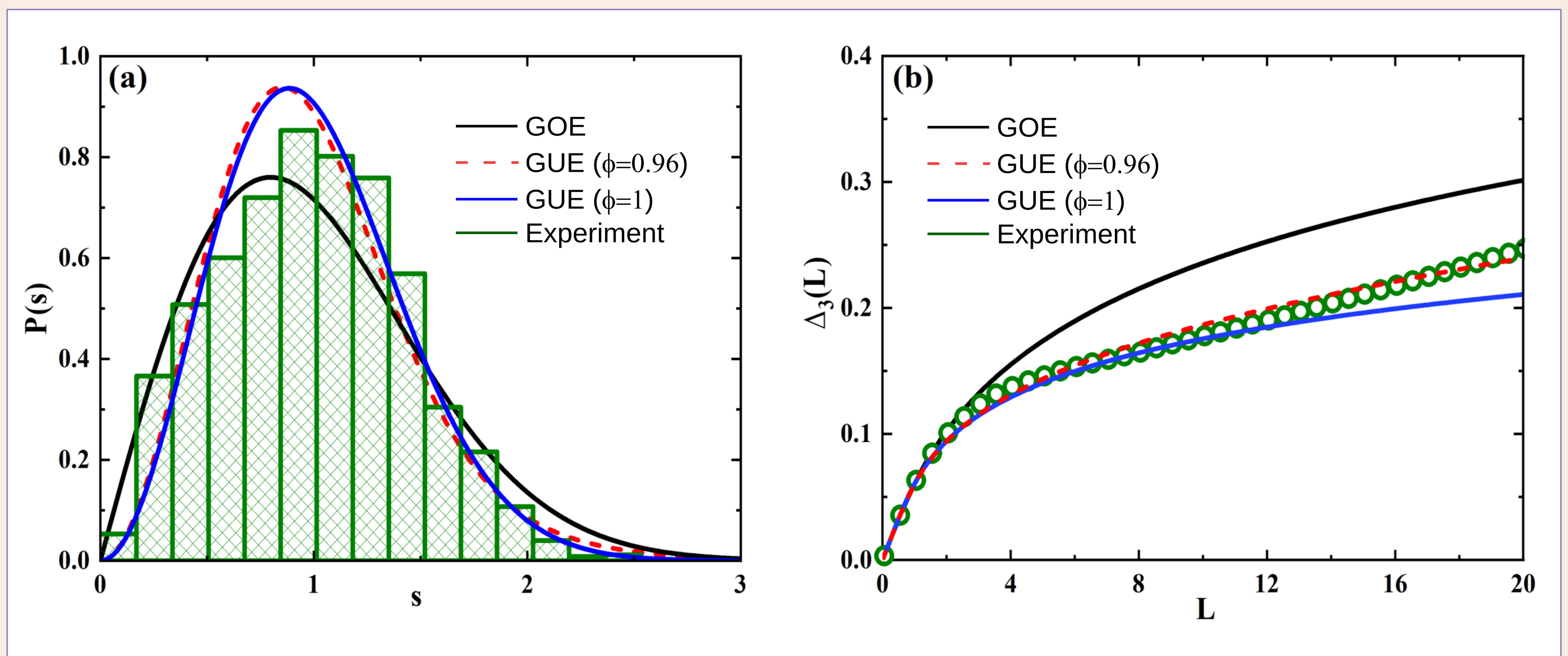
Undirected microwave network connected via HP 85133-616 and HP85133-617 flexible microwave cables to a vector network analyzer (VNA), Agilent E8364B.



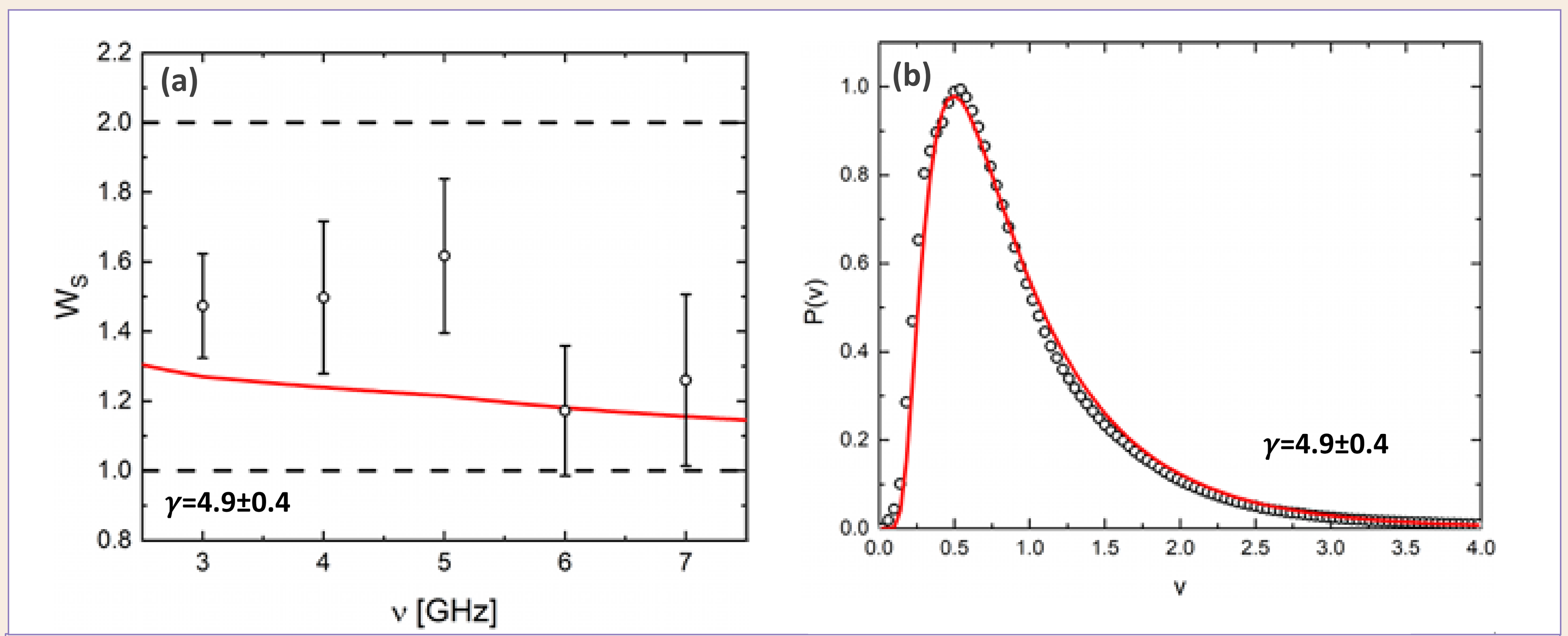
The measured spectrum of the network in the frequency range ν = [5.25, 5.75] GHz.



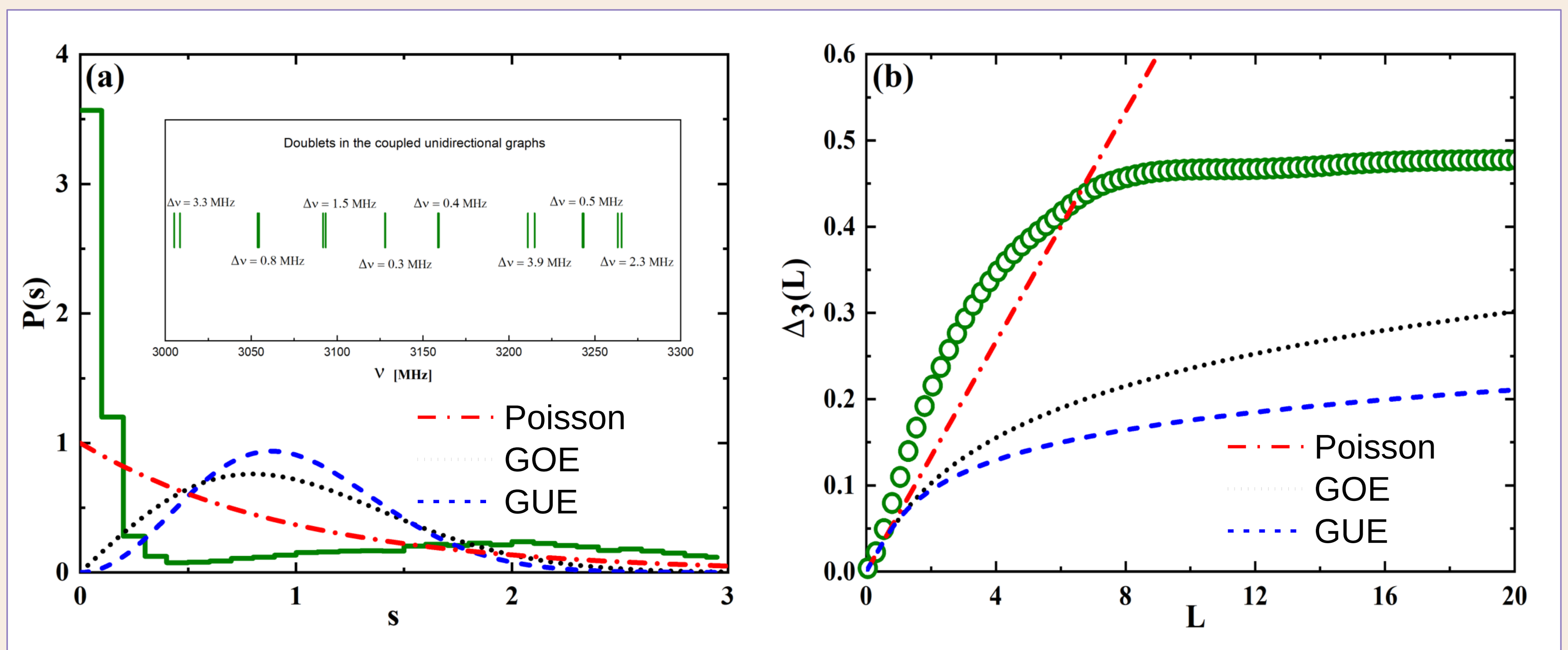
Distributions $P(R)$ of the reflection coefficient R for the microwave network.



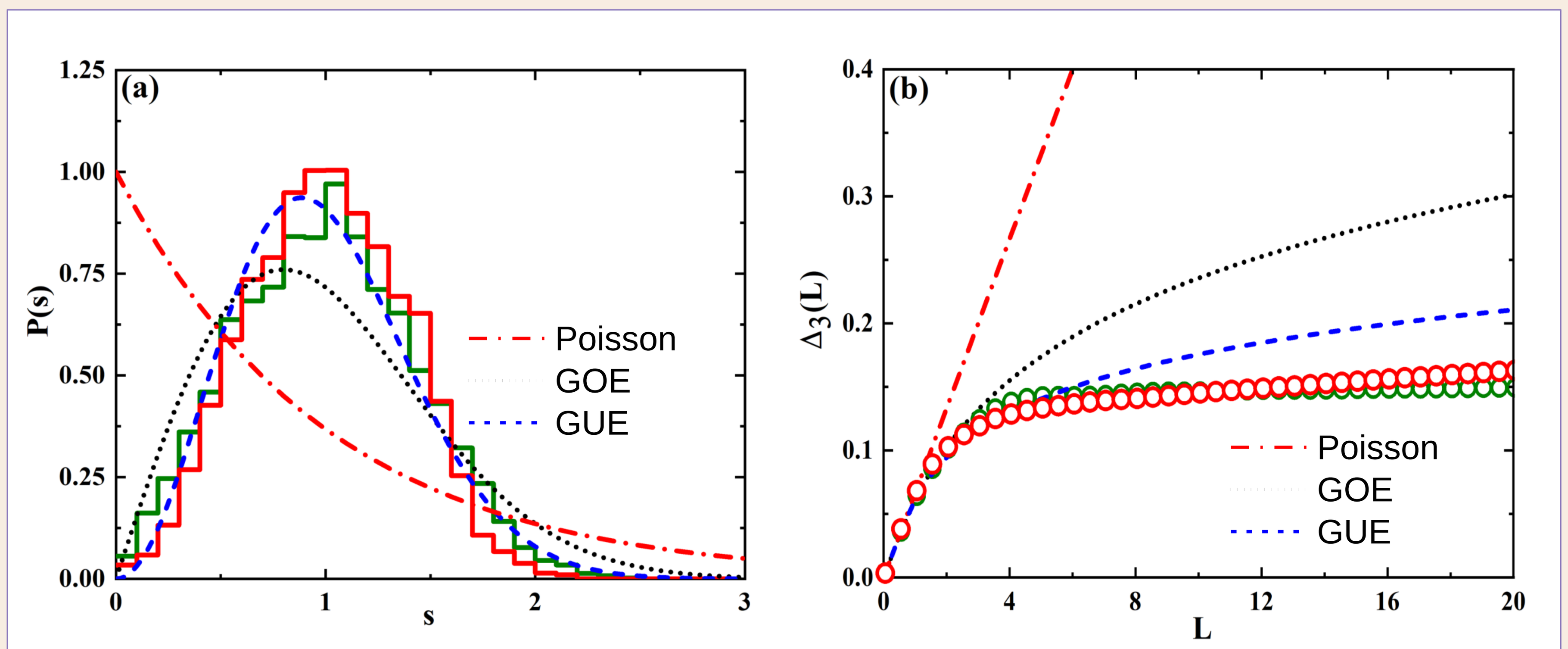
(a) Experimental and theoretical results for the nearest-neighbor spacing distribution obtained for the undirected microwave networks. (b) Experimental and theoretical results for the spectral rigidity obtained for the undirected microwave networks.



(a) The elastic enhancement factor W_S (black open circles). The expected theoretical values for GUE systems are marked by red solid line.
(b) Experimental distribution $P(v)$ of the imaginary part of the diagonal elements of the Wigner's $\tilde{K}$ (black open circles). The theoretical distribution $P(v)$ is marked by red solid line.



The numerical nearest-neighbor spacing distribution $P(s)$ (green histogram) and (b) the spectral rigidity Δ_3 (L) (green open circles) calculated for the undirected graph Γ.



(a) The numerical nearest-neighbor spacing distribution $P(s)$ evaluated from the spectra of the graph Γ. under the assumption that the doublets are not resolved and are treated as singlet states (green histogram). The distribution $P(s)$ evaluated for the simplified closed graph Γ' which was composed of ten edges and five couplers (red histogram). The graph Γ' is characterized by the spectrum of exactly doubly degenerate states. (b) The spectral rigidity $\Delta_3(L)$ evaluated from the spectra of the graph Γ (green circles) is compared with the one evaluated for the graph Γ' (red circles).

Summary and Conclusions

- We experimentally investigated an undirected open microwave network with internal absorption composed of two coupled unidirectional networks Γ+ and Γ− corresponding to two possible directions of motion on their edges.
- Though the networks are characterized by time reversal symmetry their missing level nearest-neighboring distribution and the spectral rigidity do not obey GOE predictions.
- We used numerical calculations for open quantum graphs simulating microwave networks with no internal absorption to investigate their spectral statistics and doublets, which were not experimentally resolved.
- The numerically obtained spectral characteristics shows significant deviation from the GUE predictions. We show that the doublet size distributions are close to Poisson distribution.

References

- O. Farooq, A. Akhshani, M. Ławniczak, M. Białous, and L. Sirko, Phys. Rev. E 110, 014206 (2024)
- M. Akila and B. Gutkin, J. Phys. A: Math. Theor. 48, 345101 (2015)

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