Majorana-magnon interactions in topological Shiba chains

Pei-Xin Shen [1*], Vivien Perrin [2*], Mircea Trif [1], Pascal Simon [2]

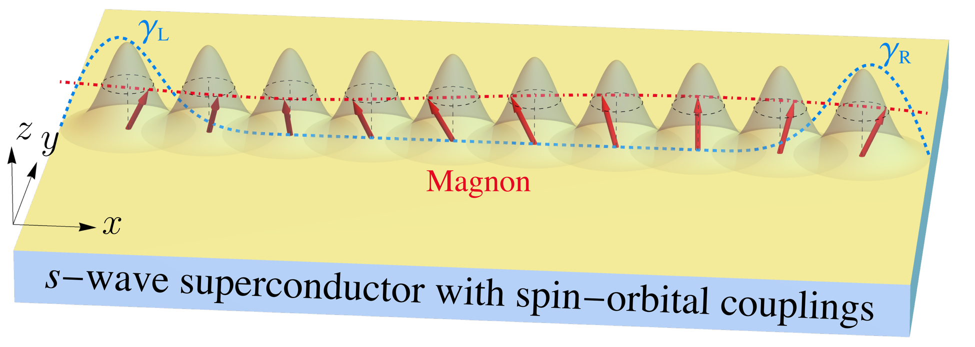
[1]International Research Centre MagTop, Institute of Physics, Polish Academy of Sciences, Aleja Lotnikow 32/46, PL-02668 Warsaw, Poland
[2]Université Paris-Saclay, CNRS, Laboratoire de Physique des Solides, 91405, Orsay, France
Phys. Rev. Res. **5**, 033207 (2023)

Abstract

A chain of magnetic impurities deposited on the surface of a superconductor can form a topological Shiba band that supports Majorana zero modes and hold a promise for topological quantum computing. Yet, most experiments scrutinizing these zero modes rely on transport measurements, which only capture local properties. Here we propose to leverage the intrinsic dynamics of the magnetic impurities to access their non-local character. We use linear response theory to determine the dynamics of the uniform magnonic mode in the presence of external ac magnetic fields and the coupling to the Shiba electrons. We demonstrate that this mode, which spreads over the entire chain of atoms, becomes imprinted with the parity of the ground state and, moreover, can discriminate between Majorana zero mode and trivial zero mode located at the ends of the chain. Our approach offers a non-invasive alternative to the scanning tunnelling microscopy techniques used to probe Majorana zero modes. Conversely, the magnons could facilitate the manipulation of Majorana zero modes in topological Shiba chains.

Model of ferromagnetic Yu-Shiba-Rusinov chain

A chain of ferromagnetically coupled adatoms on a 2D s-wave superconductor harboring Majorana zero modes. The uniform magnonic mode interacts with the Majorana zero modes, altering its dynamics.



The Hamiltonian describing a chain of N classical spins coupled to an s-wave superconductor can be written as,

$$H_{\mathrm{tot}} = \frac{1}{2}\int \mathrm{d}\boldsymbol{r}\hat{\Psi}^{\dagger}(\boldsymbol{r})(\mathcal{H}_{\mathrm{el}} + \mathcal{H}_{\mathrm{el-m}})\hat{\Psi}(\boldsymbol{r}) + H_{\mathrm{m}}\,,$$

$$\mathcal{H}_{\mathrm{el}} = \left(\frac{p^2}{2m} - \mu + \lambda_R(p_x\sigma_y - p_y\sigma_x)\right)\tau_z + \Delta\tau_x\,,$$

$$\mathcal{H}_{\mathrm{el-m}} = -J\sum_{j=1}^{N}(\boldsymbol{S}_j\cdot\boldsymbol{\sigma})\delta(\boldsymbol{r}-\boldsymbol{r}_j)\,,$$

$$H_{\mathrm{m}} = \sum_{\langle i,j\rangle} J_{\mathrm{ex}}\boldsymbol{S}_i\cdot\boldsymbol{S}_j - \sum_{j=1}^{N}\left(\frac{K_z}{2}(S_j^z)^2 - \gamma H S_j^z\right)\,.$$

Here, $\boldsymbol{\tau} = (\tau_x, \tau_y, \tau_z)$ [$\boldsymbol{\sigma} = (\sigma_x, \sigma_y, \sigma_z)$] are Pauli matrices acting in the Nambu (spin) space, and $\hat{\Psi}(\boldsymbol{r}) = [\hat{\psi}_{\uparrow}(\boldsymbol{r}), \hat{\psi}_{\downarrow}(\boldsymbol{r}), \hat{\psi}^{\dagger}_{\downarrow}(\boldsymbol{r}), -\hat{\psi}^{\dagger}_{\uparrow}(\boldsymbol{r})]^{\mathrm{T}}$ is the electronic field operator at position $\boldsymbol{r}$. In addition, J, J_{ex}, K_z, γ, and H are the coupling between the spins and the condensate, the (direct) Heisenberg exchange between the spins, the local easy-axis anisotropy, the gyromagnetic ratio, and the applied magnetic field.

Motivations

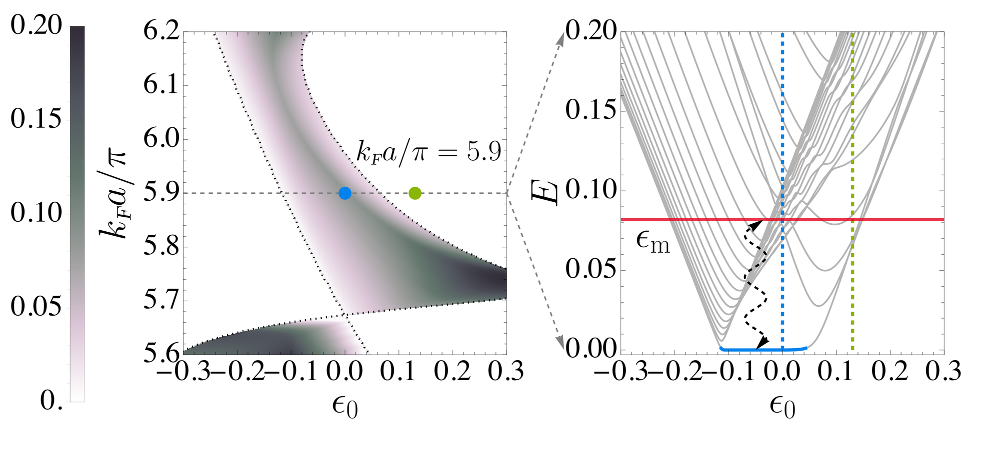
- The origin of zero-bias peaks is still under heavy debate.
- Harness the collective spin dynamics of magnetic impurities.
- Use spin susceptibility to reveal the signature of Majorana zero mode, and differentiate it from the trivial zero mode.
- Investigate the effect of disorders in the exchange couplings.

Phase diagram and Majorana zero modes

For a general state $\psi(\boldsymbol{r})$, the BdG equation $(\mathcal{H}_{\mathrm{el}} + \mathcal{H}_{\mathrm{el-m}})\psi(\boldsymbol{r}) = E\psi(\boldsymbol{r})$ can be reduced into a closed set of equations for the spinor at the impurity positions,

$$[\boldsymbol{S}_i\cdot\boldsymbol{\sigma} + J_{\mathrm{E}}(0)]\psi(\boldsymbol{r}_i) = -\sum_{j\neq i} J_{\mathrm{E}}(\boldsymbol{r}_{ij})\psi(\boldsymbol{r}_j)\,.$$

When the chain is in the deep-dilute limit, we can project it onto the Yu-Shiba-Rusinov states and obtain a $2N\times 2N$ effective tight-binding Hamiltonian $\mathcal{H}_{\mathrm{eff}}$. The Hamiltonian $\mathcal{H}_{\mathrm{eff}}$ belongs to the Altland-Zirnbauer symmetry class D and is characterized by a $\mathbb{Z}_2$ topological invariant. By tuning $k_F a$ and ϵ_0, the system can enter a superconducting topological phase supporting Majorana zero modes,



The dotted lines in the left panel indicate the boundary between topological (gray shaded) and non-topological (white) phases. The magnitude of the gap can be inferred from the shaded degree.

The right panel is a line cut at $k_F a/\pi = 5.9$. The blue (green) dashed line corresponds to the blue (green) dot in the left panel and lies in a topological (normal) phase. The curved arrow depicts the interaction between the Majorana zero mode and the uniform magnonic excitation ϵ_{m}. The parameters are $N = 30$, $\xi_0 = 10a$ and $\lambda_R = 0.05v_F$.

Ferromagnetic lattice dynamics

We can establish the dispersion of the magnetic fluctuations by employing a Holstein-Primakoff transformation. In the limit of large S, the transformation reads,

$$S_j^+ = \sqrt{2S}a_j\,,\quad S_j^- = \sqrt{2S}a_j^{\dagger}\,,\quad S_j^z = S - a_j^{\dagger}a_j\,,$$

with a_j ($a_j^{\dagger}$) being the magnonic annihilation (creation) operator satisfying $[a_j, a_{j'}^{\dagger}] = \delta_{jj'}$. In this work we are interested in triggering the dynamics of the uniform magnonic mode,

$$a_0 = \frac{1}{\sqrt{N}}\sum_{j=1}^{N} a_j \quad \text{with energy} \quad \epsilon_{\mathrm{m}} = K_z S - \gamma H\,,$$

because the following motivations,

1. it represents the lowest energy magnon;
2. it exhibits a constant amplitude along the wire.

Projecting magnetic Hamiltonian onto the uniform mode a_0, we find:

$$H^0_{\mathrm{el-m}} \approx \frac{J}{N}\left[n_0\Sigma_z - \sqrt{2NS}(a_0^{\dagger}\Sigma_+ + a_0\Sigma_-)\right]\,,$$

where Σ_ν is the total spin operator along the $\nu = x, y, z$ axis, $\Sigma_\pm = (\Sigma_x \pm \mathrm{i}\Sigma_y)/2$, and $n_0 = a_0^{\dagger}a_0$. Using the expression for normalization factor $\mathcal{N}$ of the Yu-Shiba-Rusinov wavefunction,

$$\frac{1}{\mathcal{N}} = \frac{\Delta}{JS}\frac{2\alpha^2}{(1+\alpha^2)^2}\,,$$

effectively entails to substituting, $J \to \Delta/S$ and $\Sigma_\nu \to 2\mathcal{N}\alpha^2(1+\alpha^2)^{-2}\Sigma_\nu \equiv \widetilde{\Sigma}_\nu$. The amplitude of a_0 in the frequency space becomes:

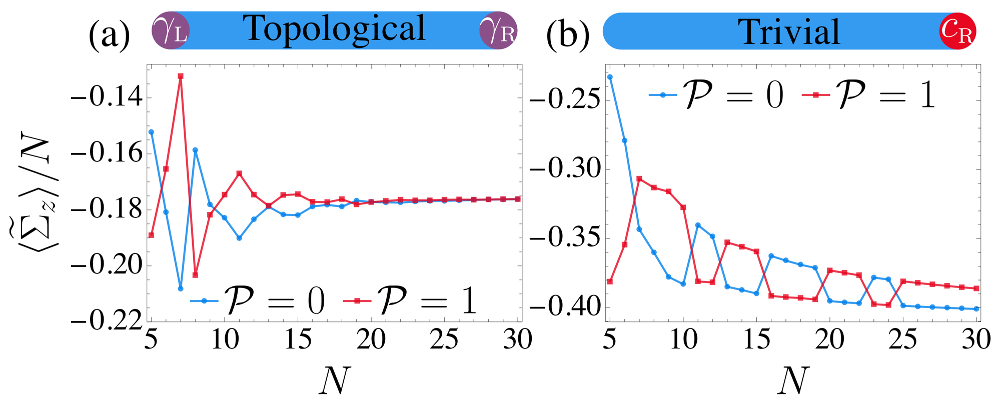
$$a_0(\omega) = \frac{\mathrm{i}h_0}{\omega - \left[\epsilon_{\mathrm{m}} + \frac{\Delta}{NS}\langle\widetilde{\Sigma}_z\rangle + \frac{2\Delta^2}{NS}\Pi_{+-}(\epsilon_{\mathrm{m}})\right] + \mathrm{i}\kappa_{\mathrm{m}}}\,,$$

$$\Pi_{+-}(\omega) = -\mathrm{i}\int_{-\infty}^{\infty}\mathrm{d}t e^{\mathrm{i}\omega t}\theta(t)\langle[\widetilde{\Sigma}_+(t), \widetilde{\Sigma}_-(0)]\rangle\,,$$

where Π_{+-} is the transverse susceptibility associated with the operator $\widetilde{\Sigma}_\pm$. Therefore, the magnon resonance frequency and its decay, are respectively shifted by,

$$\delta\epsilon_{\mathrm{m}} = \frac{\Delta}{NS}\left(\langle\widetilde{\Sigma}_z\rangle + 2\Delta\,\mathrm{Re}\,\Pi_{+-}(\epsilon_{\mathrm{m}})\right)\,,\quad \delta\kappa_{\mathrm{m}} = -\frac{2\Delta^2}{NS}\mathrm{Im}\,\Pi_{+-}(\epsilon_{\mathrm{m}})\,,$$

which represents one of our **main results**. The magnitude of these changes is determined by the superconductor gap Δ reduced by the total number of spins in the chain NS.



We show $\langle\widetilde{\Sigma}_z\rangle/N$ for the two parities of Majorana zero modes in the left panel. While each parity exhibit different values of $\langle\widetilde{\Sigma}_z\rangle/N$ when N is small, they become exponentially indiscernible for $Na \gg \xi_0$. However, for a fine-tine trivial zero mode, we see that $\langle\widetilde{\Sigma}_z\rangle$ is different for the two parities even for a large number of impurities.

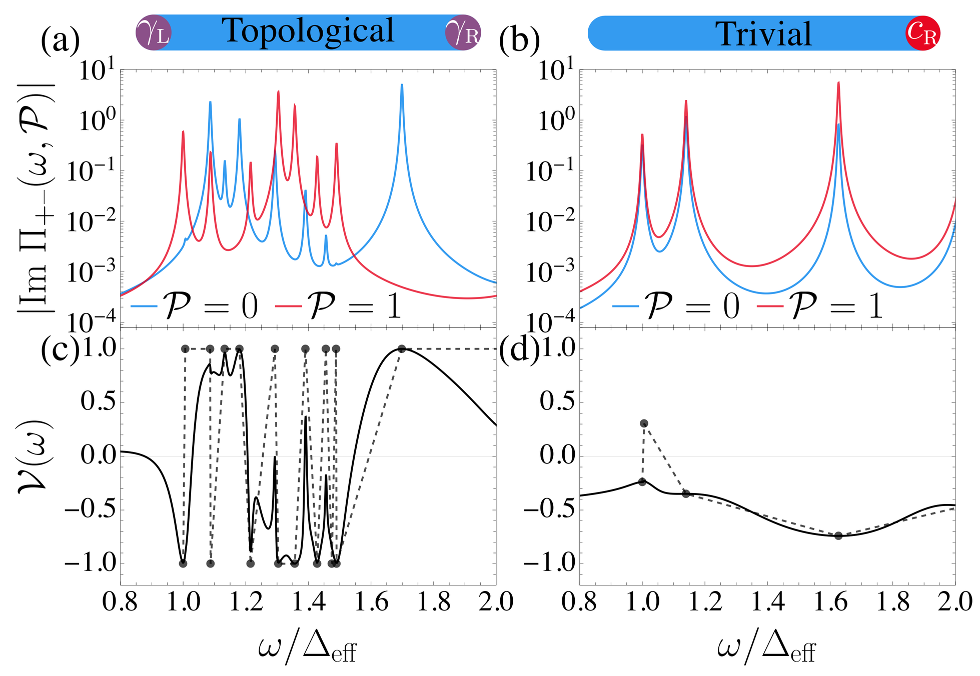
Spin-susceptibility

When $\Delta_{\mathrm{eff}} < \omega < 2\Delta_{\mathrm{eff}}$, the susceptibility is dominated by,

$$\Pi_{+-}(\omega, \mathcal{P}) = -\sum_{E_n>0}\frac{(-1)^{\mathcal{P}}\mathcal{O}^{\mathcal{P}+}_{0n}\mathcal{O}^{\mathcal{P}-}_{n0}}{\omega - E_n - (-1)^{\mathcal{P}}E_0 + \mathrm{i}\eta}\,,$$

$$\mathcal{O}^{\mathcal{P}\pm}_{nm} = \sum_{j=1}^{N}\left[\Phi_n^{\dagger}(\boldsymbol{r}_j)\delta_{\mathcal{P}1} + \overline{\Phi}_n^{\dagger}(\boldsymbol{r}_j)\delta_{\mathcal{P}0}\right]\sigma_\pm\Phi_m(\boldsymbol{r}_j)\,,$$

where $\mathcal{O}^{\mathcal{P}\pm}_{nm}$ is the parity-dependent matrix element, $\Phi_n(\boldsymbol{r})$ is the wavefunction pertaining to the Bogoliubon with energy E_n. We show $\Pi_{+-}(\omega, \mathcal{P})$ for Majorana zero modes and trivial zero modes. While both exhibit a peak structure because of the resonances at $\omega = E_n - E_0$, their distinction is encoded in their amplitudes.



To quantify it, we define the visibility of the spin susceptibility associated with the two parities as follows:

$$\mathcal{V}(\omega) \equiv \frac{\mathrm{Im}\,\Pi_{+-}(\omega,0) - \mathrm{Im}\,\Pi_{+-}(\omega,1)}{\mathrm{Im}\,\Pi_{+-}(\omega,0) + \mathrm{Im}\,\Pi_{+-}(\omega,1)}\,,$$

which is shown in the bottom panel for two distinct zero modes.

Inversion symmetry and quantized visibility

We see that $\mathcal{V}(\omega)$ oscillates between -1 and 1 in the topological regime, while it does not in the trivial regime. This significant difference can be traced back to the symmetry of the pristine system: the effective Hamiltonian H_{eff} is invariant under the symmetry operation

$$\mathcal{S} = \tau_z \otimes \mathcal{I}\,,$$

where $\mathcal{I}$ is the inversion operator that maps site j into $N+1-j$. Hence, the n^{th} single-particle eigenstate is either symmetric or anti-symmetric under $\mathcal{S}$, corresponding to the eigenvalues $S_n = 1$ and $S_n = -1$, respectively. This reflects onto the transitions matrix elements which satisfy,

$$\mathcal{O}^{\mathcal{P}\pm}_{0n} = (-1)^{\mathcal{P}}S_0S_n\mathcal{O}^{\mathcal{P}\pm}_{0n}\,.$$

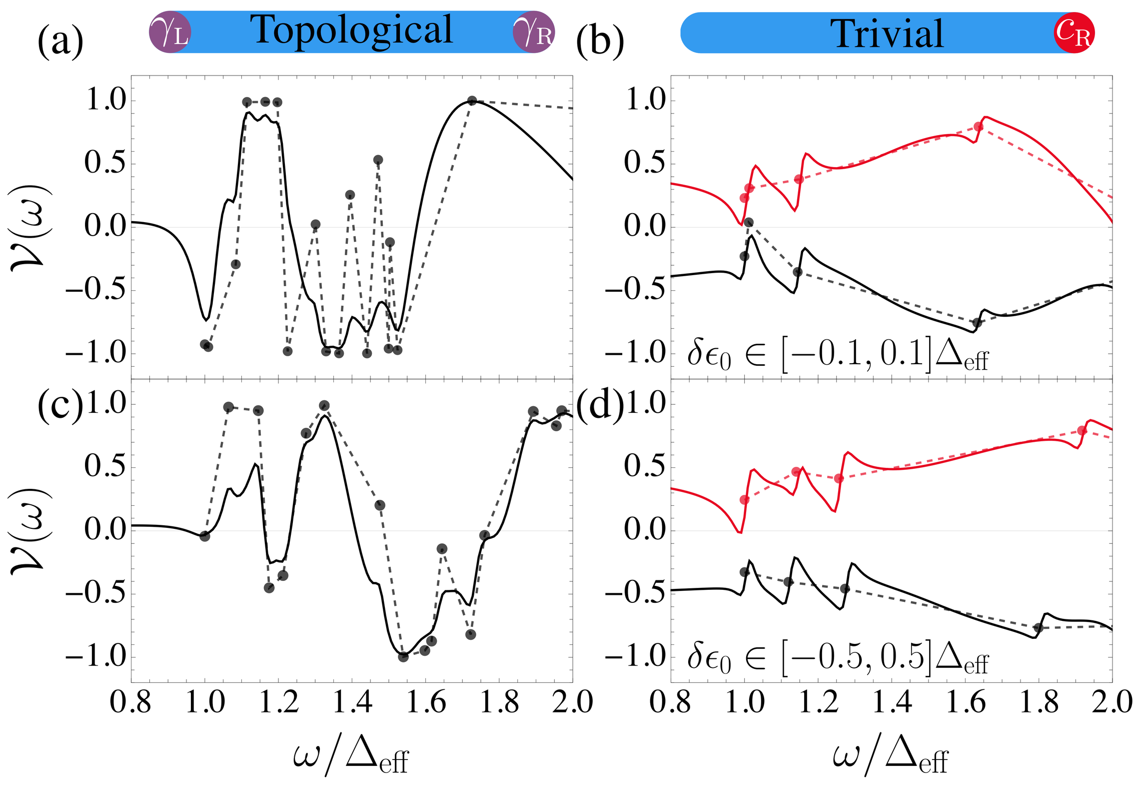
Therefore, $\mathcal{O}^{\mathcal{P}\pm}_{0n} \neq 0$ only when $(-1)^{\mathcal{P}}S_0S_n = 1$, which means one of the parities always gives a vanishing contribution for any transition. The amplitude of the visibility at the resonances $\omega_n \equiv E_n - E_0$ in the limit $\eta \to 0$ becomes,

$$\mathcal{V}(\omega_n) = S_0S_n \equiv \pm 1\,.$$

On the other hand, an accidental zero-energy mode located at one edge in the trivial regime severely breaks the inversion symmetry, rendering the visibility arbitrary. This behaviour is also intimately related to the non-locality of the Majorana zero modes, as opposed to the locality of the trivial zero modes.

Robustness against disorders

To test how deviations from the pristine inversion symmetry alters the visibility $\mathcal{V}(\omega)$, we have added random disorder in the individual Shiba energy ϵ_0 along the chain.



We see that the oscillation of the visibility remains intact for the Majorana zero modes, albeit with a reduced amplitude. The trivial zero modes visibility, on the other hand, is practically unaffected by disorder because they are local and therefore insensitive to the interference pattern of the bulk modes.

Conclusions and outlook

We have studied the interaction between the Majorana zero modes and magnons in ferromagnetically aligned magnetic impurities coupled to a spin-orbit coupling s-wave superconductor.

- Unravelled the non-local Majorana zero modes imprints onto the uniform magnonic mode.
- Demonstrated their intimate connection with the spatial symmetry of the chain.
- Discriminated the effect of Majorana zero modes and trivial zero modes from the magnonic response.
- Showed the robustness of the response against moderate onsite disorder.

There are several possible future directions:

- Interface magnons with chiral Majorana modes in 2D Yu-Shiba-Rusinov impurity lattice.
- Use the magnonic mode actively for processing quantum information with Majorana zero modes.
- Extend current machine learning techniques to detect topological structures based on the data of spin susceptibility.

References

[1] P.-X. Shen, V. Perrin, M. Trif, and P. Simon, Majorana-Magnon Interactions in Topological Shiba Chains, Phys. Rev. Res. **5**, 033207 (2023).

[2] F. Küster, S. Brinker, R. Hess, D. Loss, S. S. P. Parkin, J. Klinovaja, S. Lounis, and P. Sessi, Non-Majorana Modes in Diluted Spin Chains Proximitized to a Superconductor, Proceedings of the National Academy of Sciences 119, e2210589119 (2022).

Acknowledgement

This work is supported by the Foundation for Polish Science through the international research agendas program co-financed by the European Union within the smart growth operational program, and by the National Science Centre (Poland) OPUS 2021/41/B/ST3/04475.