## Longitudinal coupling between electrically driven spin-qubits and a resonator

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## Motivation

Electron/hole-spin qubits $\rightarrow$ long coherence times and scalability
Electric field + Spin orbit interaction $\rightarrow$ Access spin states [1]
Floquet qubits more robust against noise than their static counterparts [2]
Experiment shows that shuttling electron spins enhances their coherence [3]


## Floquet spin-qubit states

- If $H_{e l}(t+T)=H_{e l}(t)$, with $T=2 \pi / \Omega$, solutions to Schrödinger equation $i \partial_{t}|\Psi(t)\rangle=H_{e l}(t)|\Psi(t)\rangle$ are Floquet states $\left|\Psi_{j}(t)\right\rangle$ written as,

$$
\left|\Psi_{j}(t)\right\rangle=e^{-i \epsilon_{j} t}\left|\psi_{j}(t)\right\rangle, j=0,1,2 \ldots
$$

where the periodic Floquet mode $\left|\psi_{j}(t)\right\rangle$ obey:

$$
\left(H_{e l}(t)-i \partial_{t}\right)\left|\psi_{j}(t)\right\rangle=\epsilon_{j}\left|\psi_{j}(t)\right\rangle,\left|\psi_{j}(t+T)\right\rangle=\left|\psi_{j}(t)\right\rangle
$$

Qubit states $\left\{\left|\Psi_{\tilde{q}}^{F}(t)\right\rangle\right\}_{\tilde{q}=0,1}$ : Floquet states with most overlap with the lowest two instantaneous ground states of $H_{e l}(t)\left|\Psi_{q}^{\text {inst }}(t)\right\rangle=\epsilon_{q}^{\text {inst }}(t)\left|\Psi_{q}^{\text {inst }}(t)\right\rangle$


Assuming a driven harmonic potential in adiabatic regime with parameters $E_{0}=0.1, \Omega=0.3$ and $\omega_{0}=1$, here we show the plots of $\left|\left\langle\Psi_{i}^{F}(t) \mid \Psi_{1(2)}^{\mathrm{inst}}(t)\right\rangle\right|^{2}$ with instantaneous states $\left|\Psi_{1(2)}^{\text {inst }}(t)\right\rangle$ and the Floquet states that had the most overlap to choose $\left\{\left|\Psi_{i}^{F}(t)\right\rangle\right\}_{i=11,12}$ as the qubit states.

Hamiltonian in the Floquet qubit subspace
$H_{s-p}(t)=\left[g_{z}(t) \tau_{z}^{F}+\left(g_{+}(t) \tau_{+}^{F}+\right.\right.$ h.c. $\left.)\right]\left(a^{\dagger}+a\right)+\omega_{c} a^{\dagger} a$,
$g_{z}(t)=\frac{\boldsymbol{R}_{c}}{2} \cdot \frac{d}{d t}\left[\left\langle\psi_{1}(t)\right| \boldsymbol{m}\left|\psi_{1}(t)\right\rangle-\left\langle\psi_{0}(t)\right| \boldsymbol{m}\left|\psi_{0}(t)\right\rangle\right]$,
$g_{+}(t)=i e^{i \epsilon_{q} t} \boldsymbol{R}_{c} \cdot\left(\epsilon_{q}-i \frac{d}{d t}\right)\left\langle\psi_{1}(t)\right| \boldsymbol{m}\left|\psi_{0}(t)\right\rangle, \boldsymbol{m}=\left[\begin{array}{ll}\sigma_{x} & -\sigma_{y}\end{array}\right]^{T}$
Longitudinal term $g_{z}(t) \rightarrow 0$ in static case

## Longitudinal readout

- The readout of the Floquet spin-qubit can be achieved faster [4] by utilizing longitudinal spin-photon couplings instead of transverse interactions due to better pointer separation at initial times as,

$$
\langle a(t)\rangle=-i\left(g_{z}(k=1) / \kappa\right)\left\langle\tau_{z}^{F}\right\rangle\left(1-e^{-\kappa t / 2}\right)
$$

Dependence of the coupling on the QD trajectory


Coupling strength as path is varied from a line to a circle. For Ge/SiGe hole-spin $g_{z} / 2 \pi \approx 10 \mathrm{MHz}$ and $g_{z} / 2 \pi \approx 4 \mathrm{MHz}$ for GaA s electron spin

## Decoherence

- Following a Floquet-Born-Markov approach, the rate equation for the Floquet spin-qubit in the interaction picture is

$$
\begin{gathered}
\dot{\rho}_{S}(t)=\sum_{s= \pm, z} \Gamma_{s} \mathcal{D}_{s}\left[\rho_{S}(t)\right] \\
\mathcal{D}_{s}\left[\rho_{S}(t)\right]=\tau_{s}^{F} \rho_{S}(t)\left(\tau_{s}^{F}\right)^{\dagger}-\frac{1}{2}\left\{\left(\tau_{s}^{F}\right)^{\dagger} \tau_{s}^{F}, \rho_{S}(t)\right\} \\
\Gamma_{ \pm}=\left(\frac{\Omega \lambda_{0}}{\omega_{0} \lambda_{S O}}\right)^{2} \sum_{\alpha, k}\left|m_{ \pm, \alpha}(k)\right|^{2}\left(k \pm \gamma_{B}\right)^{2} J_{\alpha}\left[\Omega\left(k \pm \gamma_{B}\right)\right]
\end{gathered}
$$

$\Gamma_{z}=\left(\frac{\Omega \lambda_{0}}{\omega_{0} \lambda_{S O}}\right)^{2} \sum_{\alpha, k}\left|m_{z, \alpha}(k)\right|^{2} k^{2} J_{\alpha}(k \Omega), J_{\alpha}(\omega)$ : bath spectral function


(a) Relaxation $1 / T_{1}=\Gamma_{+}+\Gamma_{-}$and (b) dephasing $1 / T_{2}=\Gamma_{z}$ rates for hole-based (black, blue, red) and electron-based Floquet spin-qubits (green)

Qubit power dissipation: $P \approx \Gamma_{+} \Omega$

## Two qubit CPHASE gate

- For two Floquet spin-qubits, with $\Delta=\omega_{c}-\Omega$, we obtained:

$$
H_{2 q}=\Delta a^{\dagger} a+\left(g_{z, 1} \tau_{z, 1}^{F}+g_{z, 1} \tau_{z, 2}^{F}\right)\left(a^{\dagger}+a\right)
$$

known to realise a CPHASE gate [5] for a gate time $t_{g}=\pi \Delta /\left(8 g_{z, 1} g_{z, 2}\right)$. For $\mathrm{Ge} / \mathrm{SiGe}$ hole-spin $t_{g} \approx 50 \mathrm{~ns}$ and for GaAs electron spin $t_{g} \approx 100 \mathrm{~ns}$, show that $t_{g} \ll T_{1} \sim 0.1 \mathrm{~ms}$

## Geometric origin of the interaction

For general $U(\boldsymbol{r})$, the geometric nature of the interaction is revealed in instantaneous frame defined by the unitary $\mathcal{U} \equiv \mathcal{U}[\boldsymbol{E}(t)]$,

$$
H_{s-p}=E_{c, \beta} \dot{E}_{\alpha}\left(m_{\alpha \beta}^{z} \tau_{z}^{F}+m_{\alpha \beta}^{+} e^{i \epsilon_{q} t} \tau_{+}^{F}+\text { h.c. }\right)\left(a^{\dagger}+a\right)
$$

where $m_{\alpha \beta}^{z}(t), m_{\alpha \beta}^{+}(t) \propto \mathcal{F}_{\alpha \beta}^{E}$ where, $\mathcal{F}_{\alpha \beta}^{E}=\partial_{\alpha} \mathcal{A}_{\beta}^{E}-\partial_{\beta} \mathcal{A}_{\alpha}^{E}+i\left[\mathcal{A}_{\alpha}^{E}, \mathcal{A}_{\beta}^{E}\right]$ is the Berry curvature and $\mathcal{A}_{\alpha}^{E}=i \mathcal{U}^{\dagger} \partial_{E_{\alpha}} \mathcal{U}$ is the Berry connection

## Conclusion \& Outlook

- Defined Floquet spin-qubit, unraveled tunable transverse and longitudinal spin-photon coupling to a resonator, constructed a CPHASE two-qubit gate, and estimated the coherence in the presence of ohmic noise
- Revealed the geometric nature of interaction using adiabatic perturbation theory
- Future: Generalise to multiple qubits and evaluate the phonon-induced decoherence


## References

[1] Golovach et al., PRA 81, 022315 (2010)
[2] Huang et al., PRA 15, 034065 (2021)
[3] Mortemousque et al., PRXQ 2, 030331 (2021)
[4] Didier et al., PRL 115, 20360 (2015)
[5] Harvey et al., PRB 97, 235409 (2018)

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