Nuclear spin squeezing induced by light for fermions in optical lattices

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Spin squeezed states are states with spin fluctuations in some direction perpendicular to spins eigendirecion minimised below Heisenberg limit. Such states are particularly convenient from metrological point of view.
Recently a method of squeezing of the collective spin composed of N pseudo-½-spins induced by light was proposed [1].
Here the attempts towards extending this model to longer on-site spins $(s>1/2)$ are presented.

Squeezing:

Components of angular momentum satisfy Heisenberg uncertainty principle. Coherent state minimises it with equal variations:

$$\Delta J_x\,\Delta J_y = \frac{1}{2}|\langle J_z\rangle| \qquad \Delta J_x = \Delta J_y = \Delta J_C$$

Squeezed state has uncertainty of one of the components smaller than ΔJ_C:

$$\Delta J_{\min}\,\Delta J_{\max} \geq \frac{1}{2}|\langle J_z\rangle| \qquad \Delta J_{\min} < \Delta J_C < \Delta J_{\max}$$

Squeezing is measured by squeezing parameter ξ^2:

$$\xi^2 = 2J\frac{\Delta J^2_{\min}}{\langle J\rangle^2}$$

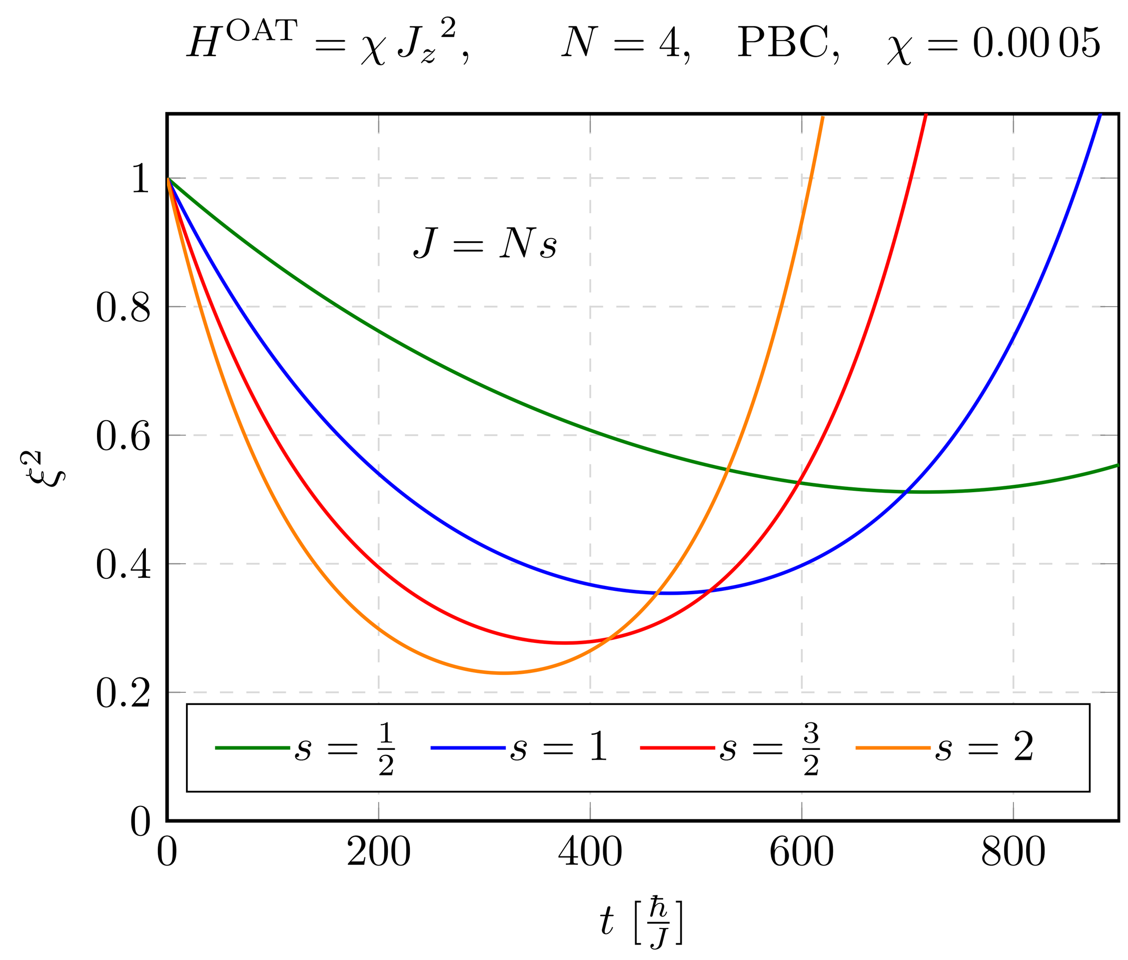
For coherent $\xi^2=1$, for squeezed $\xi^2<1$.

One axis twisting:

One Axis Twisting (OAT) is the simplest protocol of angular momentum (AM) squeezing. It is induced by Hamiltonian proportional to the square of a spin component perpendicular to the AM direction:

$$H^{\mathrm{OAT}} = \chi\, {J_z}^2$$

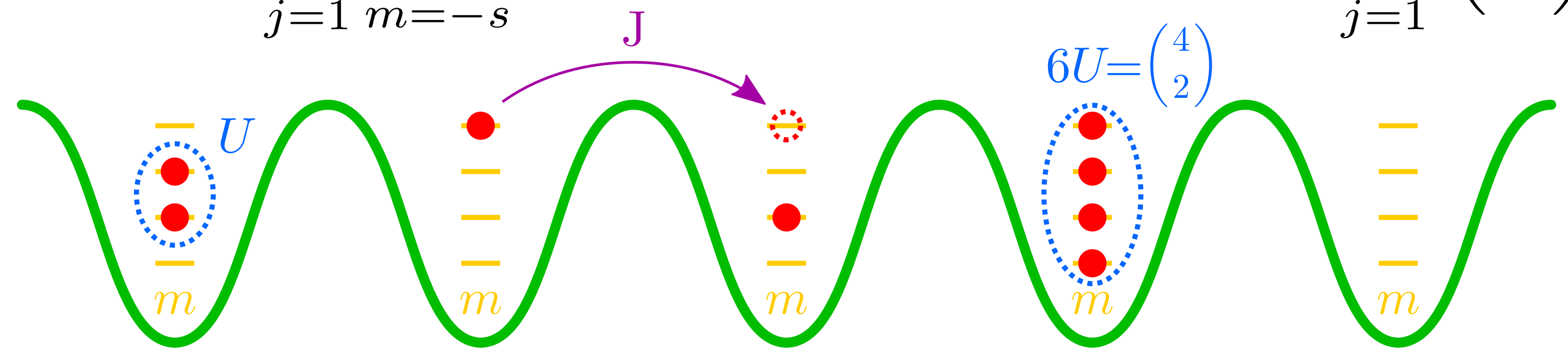
We can see that, the longer the spin J, the faster and deeper is the squeezing – longer on-site spin s promise a potential improvement for the same number of atoms N.



Fermi-Hubbard model:

We consider N fermionic spins s (atoms heaving nuclear spins s) in N-sited optical lattice. Collective AM is a sum of on-site spins. The possible processes are: tunnelling of fermions between neighbouring sites with strength J and repulsion of atoms occupying the same site with strength U. We assume that only ground state of each well can be occupied, so atoms in the same site have to have different spins (Pauli exclusion):

$$H^{\mathrm{FH}} = -\mathrm{J}\sum_{j=1}^{N}\sum_{m=-s}^{+s} a^\dagger_{j+1,m}a_{j,m} + a^\dagger_{j,m}a_{j+1,m} + U\sum_{j=1}^{N}\binom{n_j}{2}$$

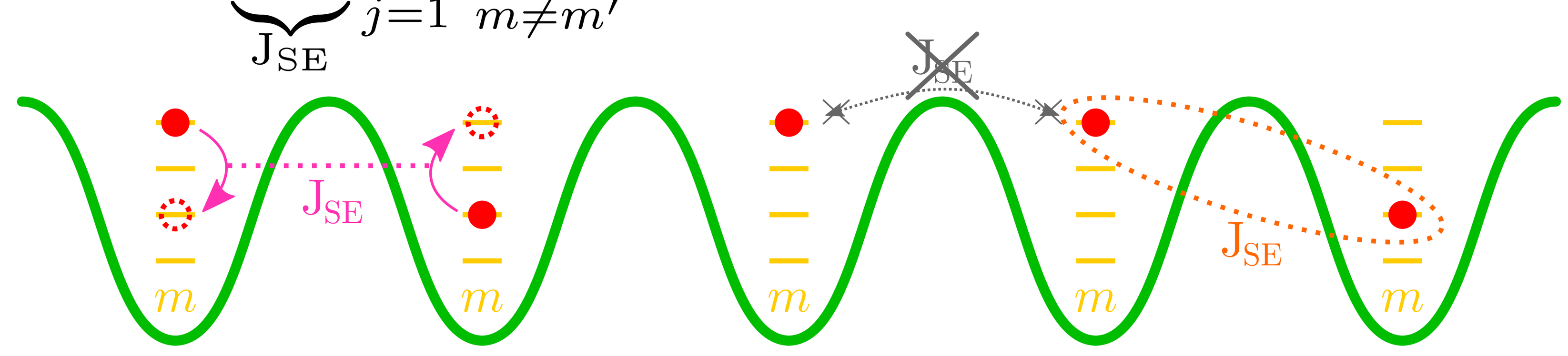


Effective Spin exchange model:

In case of very strong on site repulsion $U>>J$, one can restrict its self only to states with one atom in each site, with transitions via double-occupied states treaten as perturbation – effective Spin Exchange model:

$$\left(S_j^{m\to n} = a^\dagger_{j,n}a_{j,m}\right)$$

$$H^{\mathrm{SE}} = \underbrace{\frac{2\mathrm{J}^2}{U}}_{\mathrm{J_{SE}}}\sum_{j=1}^{N}\sum_{m\neq m'}\left(-n_{j,m}n_{j+1,m'} + S_j^{m'\to m}S_{j+1}^{m\to m'}\right)$$

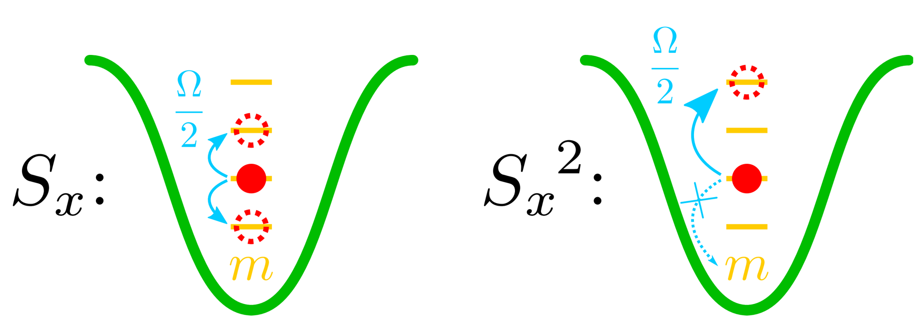


Processes allowed: interaction of nearest neighbours and exchange of spin between nearest neighbours, both with strength $\mathrm{J_{SE}}$. Processes possible only for neighbours with different magnetisation m (Pauli exclusion).

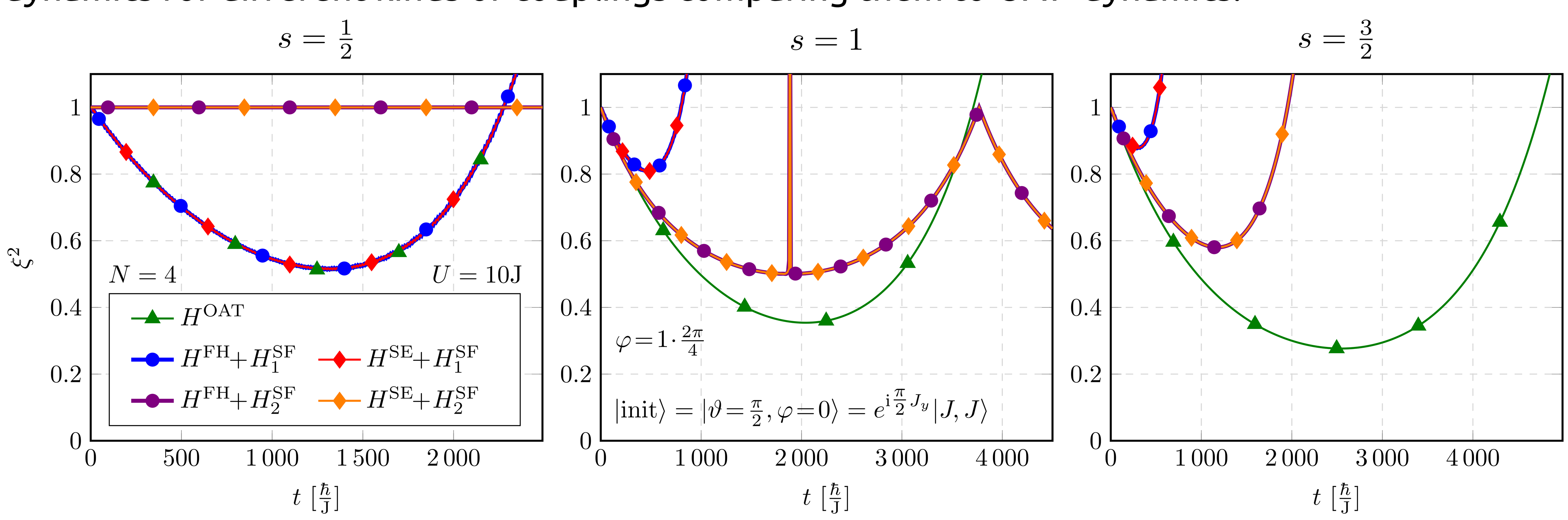
Coupling to the light – Spin-flip:

$$H_1^{\mathrm{SF}} = \Omega_1\sum_{j=1}^{N}\sin(j\varphi)\,S_{z,j}$$

$$H_2^{\mathrm{SF}} = \Omega_2\sum_{j=1}^{N}\sin(j\varphi)\left(S_{z,j}\right)^2$$



The on-site spin states can be changed by interaction with light, the amplitude of these interactions differ among atoms due to different spatial positions. We check the spin squeezing dynamics for different kinds of couplings comparing them to OAT dynamics.



Observations: The H_1^{SF} coupling gives exactly OAT for spin $s=1/2$. Also for longer spins s all the couplings reveal squeezing, however less efficient (shallower) than OAT. The simulation performed using full Fermi-Hubbard Hamiltonian stays in perfect agreement with simulations using Spin Exchange one – for the chosen parameters effective SE model is valid.

Conclusion – The model of light induced OAT does not simply generalise to longer atomic spins – the same couplings give qualitatively different results for different spins s. Search for effective OAT for longer spins require further detailed investigation.