Time-continuous measurement of position and momentum of a particle

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Model of continuous measurement

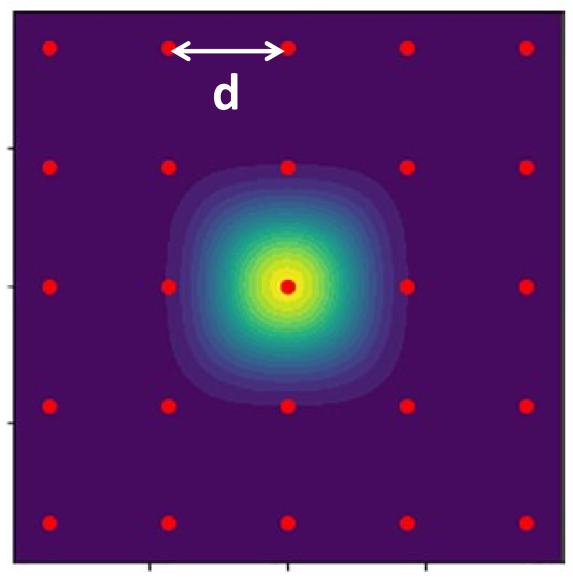
We formulate a model of a quantum particle continuously monitored by detectors measuring simultaneously its position and momentum. We implement the postulate of wave-function collapse by assuming that upon detection the particle is found in one of the meters' states chosen as a discrete subset of coherent states. The dynamics, as observed by the meters, is thus a random sequence of jumps between coherent states.

Detectors:

$$\langle x|\alpha_{mn}\rangle = \frac{1}{\pi^{1/4}} e^{-\frac{(x-x_m)^2}{2}} e^{ik_n x} \stackrel{\text{def}}{=} |\alpha\rangle$$

Measurement – Jump operators :

$$C_\alpha = \sqrt{\gamma}|\alpha\rangle\langle\alpha|$$



Gorini-Kossakowski-Sudarshan-Lindblad equation

Open system formalism - In the quantum statistical description we treat the particle as a (small) open system coupled to the "reservoir" of detectors.

$$\rho_s = i[\rho_s, H_s] - \frac{1}{2}\sum (C^+_\alpha C_\alpha\, \rho_s + \rho_s C^+_\alpha C_\alpha) + \sum C_\alpha\, \rho_s C_\alpha{}^+$$

To solve the above GKSL equation we use the Quantum Monte Carlo Wavefunction method. It may provide a computational advantage as well as possible additional physical insight from studying the preaveraged single trajectories. The density operator is obtained by averaging over many realizations of a single wavefunction's dynamics:

$$\rho_S(t) = \overline{\phi^*(t)\phi(t)}$$

Quantum Monte Carlo Wavefunction Method

Implementation:

$$\phi^{(1)}(t+\delta t) = \left(1 - i\delta t\left(H_S - \frac{i}{2}C^+_\alpha C_\alpha\right)\right)\phi(t)$$

$$\delta p_\alpha = \delta t\,\langle\phi(t)|C^+_\alpha C_\alpha|\phi(t)\rangle = \delta t\,\gamma|\langle\alpha|\phi(t)\rangle|^2$$

(i) With probability $1 - \sum_\alpha \delta p_\alpha$ the wave function is the one obtained from nonunitary evolution (with necessary normalization),

$$|\phi(t+\delta t)\rangle = \frac{|\phi^{(1)}(t+\delta t)\rangle}{||\,|\phi^{(1)}(t+\delta t)\rangle\,||}.$$

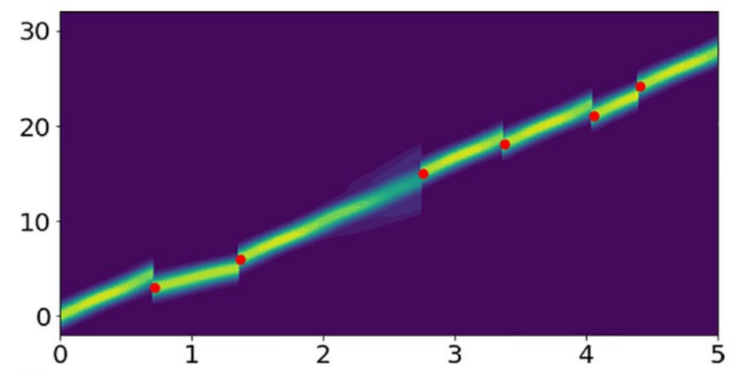
(ii) One of the meters clicks with probability $\delta p_\alpha/\delta p$ and the particle jumps to the measured state

$$|\phi(t+\delta t)\rangle = \frac{C_\alpha|\phi(t)\rangle}{||C_\alpha|\phi(t)\rangle||} = |\alpha\rangle.$$

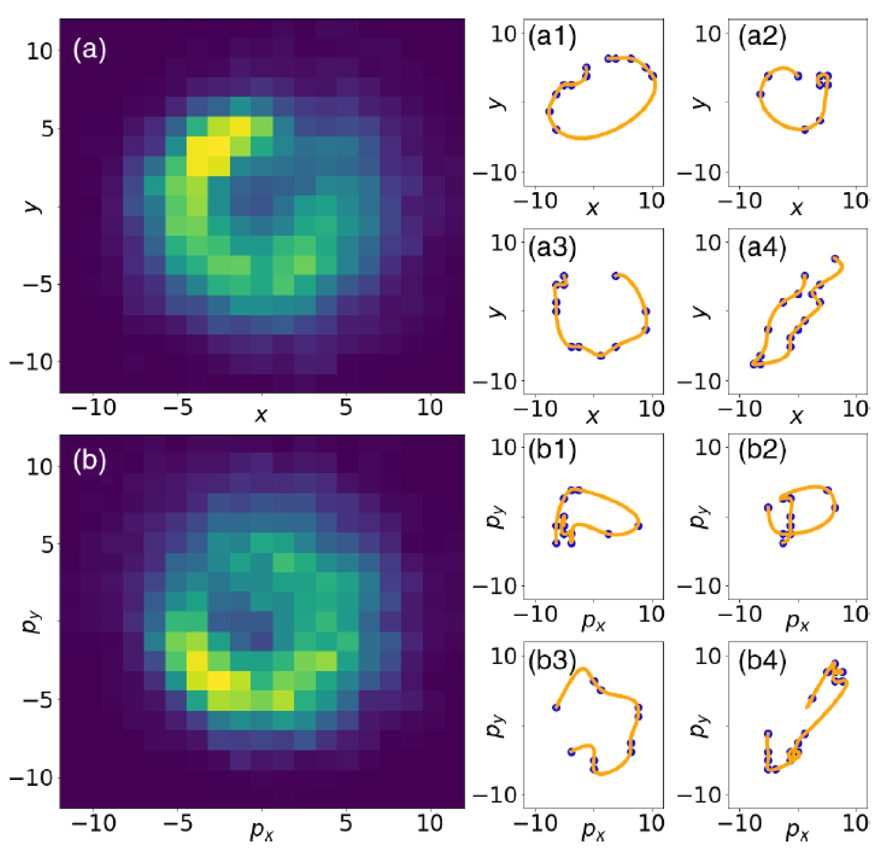
J. Dalibard, Y. Castin, K. Mølmer, Wave-Function Approach to Dissipative Processes in Quantum Optics, Phys. Rev. Lett. **68**, 580 (1992).
K. Mølmer, Y. Castin, J. Dalibard, Monte Carlo wavefunction method in quantum optics, J. Opt. Soc. Am. B **10**, 524 (1993).

Examples of observed dynamics of a particle

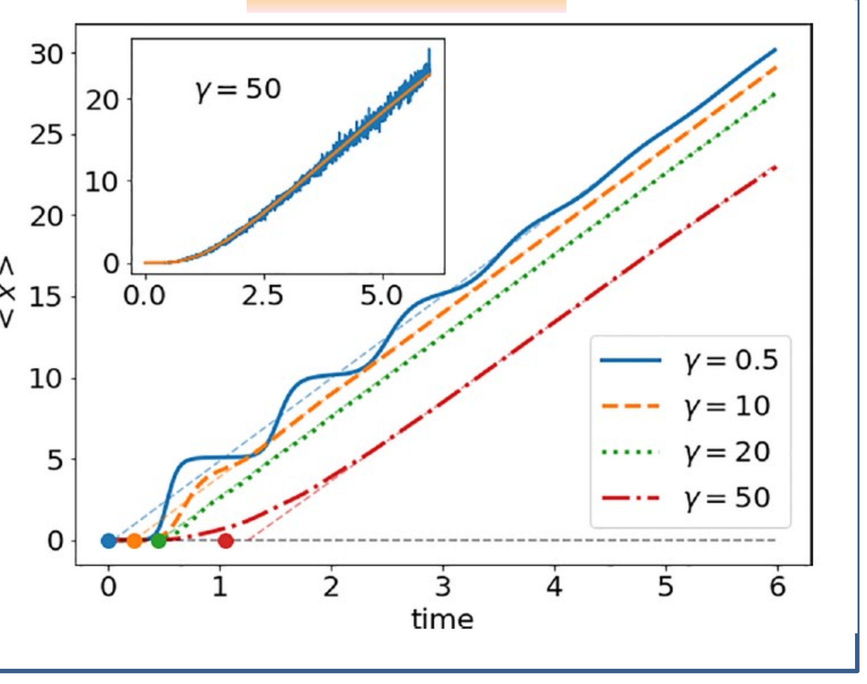
Free particle – a single trajectory



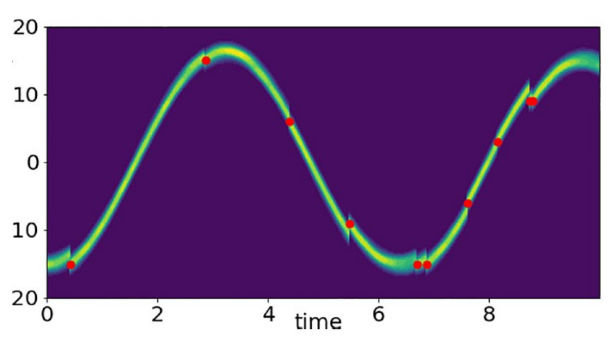
A particle with angular momentum in a stationary state of h.o. at t=0

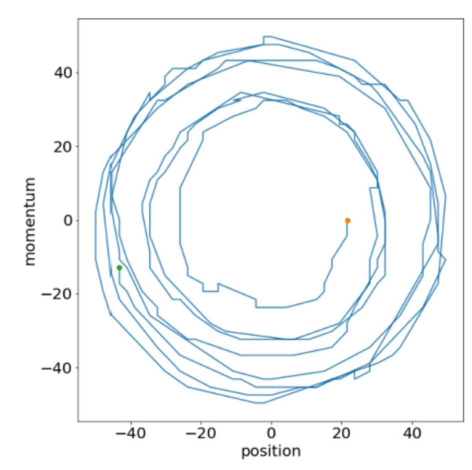


Zeno effect

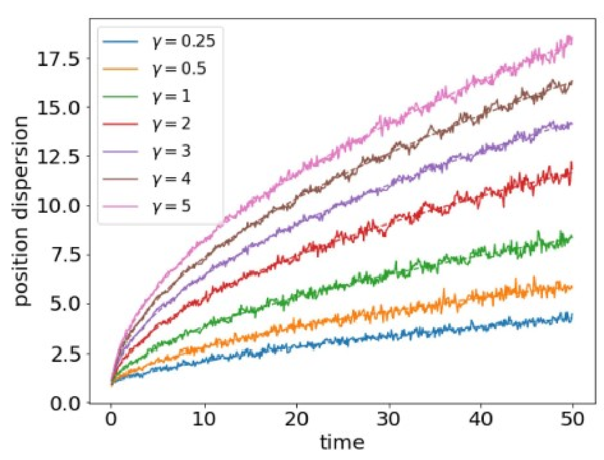


Particle in a harmonic potential





Dispersion of position:

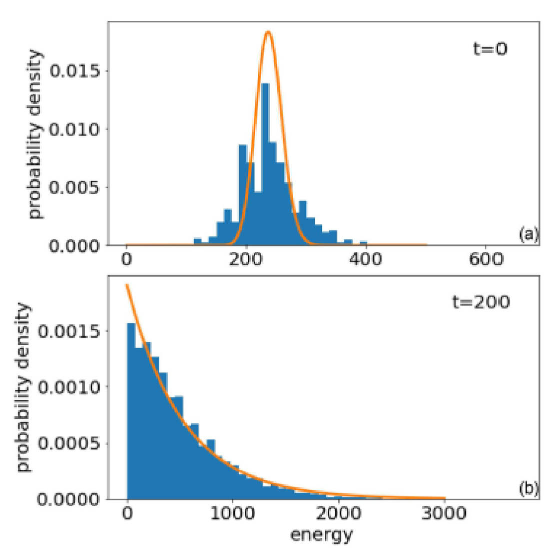


$$\delta^2(t) \approx Dt + \delta_0^2$$

$$D = \gamma\sum_{j,i}(dj)^2 e^{\frac{-d^2(j^2+i^2)}{2}} \approx \gamma\frac{2\pi}{d^2}$$

Heating:

$$\langle E(t)\rangle = \delta^2(t) + E_0 = Dt + (\delta_0^2 + E_0)$$



Continuously monitored dynamics as a cassical stochastic process

$$\frac{d}{dt}x = p + \xi_x(t) \qquad \frac{d}{dt}p = -x + \xi_p(t)$$

$$\frac{d}{dt}x = p + \xi_x(t) \qquad \frac{d}{dt}p = \xi_p(t)$$

$$\langle\xi_x(t)\xi_x(t')\rangle = D_x\,\delta(t-t')$$

$$\langle\xi_p(t)\xi_p(t')\rangle = D_p\,\delta(t-t')$$

$$\delta_x^2(t) = \delta_x^2(0) + \frac{1}{2}(D_x + D_p)\,t \qquad \delta_p^2(t) = \delta_p^2(0) + \frac{1}{2}(D_x + D_p)\,t$$

$$\delta_x^2(t) = \delta_x^2(0) + \delta_p^2(0)t^2 + D_x t + \frac{1}{3}D_p t^3 \qquad \delta_p^2(t) = \delta_p^2(0) + D_p t$$

F. Gampel, M. Gajda, Continuous simultaneous measurement of position and momentum of a particle, Phys Rev. A **107**, 012420 (2023)
F. Gampel, M. Gajda, On Repeated Measurements of a Quantum Particle in a Harmonic Potential, Acta Phys. Pol. A **143**, S131 (2023)