Autoreferat

1. Name and Family Name

Włodzimierz Ungier

2. Diploma, academic degrees.

PhD in 1978

PhD Thesis In the Institute of Physics of Polish Academy of Science, Warsaw:

”Phonon replicas of bound exciton recombination”

Supervised by Prof. Dr Maciej Suffczyński.

Master Degree in 1971

Warsaw University, Faculty of Physics,

Supervised by Prof. Dr Maciej Suffczyński.

3. My current academic career.

Since 1971 I have been working in the Institute of Physics of Polish Academy of Sciences in Warsaw. Additionally in the period 1990-2002 I was employed by Ministry of Education as scientific secretary of Physics Olympiad.

4. Description of achievements underlying the habilitation

The achievements underlying the habilitation consist of a collection of five papers from the period of 2007-2014, concerning the absorption of microwave radiation energy electric component by electron gas with Rashba coupling in spin resonance conditions.

Introduction

The possibility of electron spin manipulation is one of the most important problem in spintronics. Many proposed calculation schemes based on spin use the oscillating magnetic field. This field of frequency corresponding to electron spin resonance (ESR) induces quantum transitions between different electron spin states and the ESR pulse can generate magnetization of the electron system.

During the last few decades it was shown that spin-orbit mechanisms, introduced by Rashba[1] and Dresselhaus[2] , which couple the carrier momentum $\hbar\mathbf{k}$ and its spin allow for efficient control of electron spin also by the electric field affecting the electron momentum. The idea that band spin orbit coupling, described by Hamiltonian $\alpha_R(\mathbf{k}\times\mathbf{n})\boldsymbol{\sigma}$ (α_R is a characteristic constatnt of material, vector $\mathbf{n}$ defines direction of the built-in electric field and $\boldsymbol{\sigma}$ is the Pauli spin operator), causes spin transitions to be allowed under the action of electric component of radiation has been put forward long ago by Rashba[3] for crystals with broken mirror symmetry, for example in crystals of wurtzite structure. This idea, called the electric-dipole (ED) ESR, depends on calculation of matrix elements due to electric dipole transitions between different spin states of electrons and is related to displacement current. The ED ESR was realized experimentally in the case of electrons bound to donors.[4] Another mechanism of electric excitation of ESR , the current-induced (CI) ESR,[5] is connected with the drift current driven by electric component of radiation. Exactly this CI ESR mechanism is the subject-matter considered in five articles presented here, and making habilitation basis.

The CI ESR has been employed to study *g*-factors, *g*-factor anisotropies and spin relaxation, mainly in two-dimensional semoconductor structures. Greate contribution to these investigations was made by two research groups, the group of Z. Wilamowski in PAS and the group of W. Jantsch in Johannes Kepler

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[1] E.I. Rashba, Sov. Phys. Solid State. **1**, 366 (1959); Y.L. Bychkov, E.I. Rashba, J. Phys. C **17**, 6039 (1984)
[2] G. Dresselhaus, Phys. Rev. **100**, 580 (1955)
[3] E.I. Rashba, Sov. Phys. Solid State. **2**, 1109 (1960)
[4] M. Dobrowolska, A. Witowski, J.K. Furdyna, T. Ichiguchi, H.D. Drew, P.A. Wolff, Phys. Rev. B, **29**, 6652 (1984).
[5] E. Michaluk, J. Błoniarz, M. Pabich, Z. Wilamowski and A. Mycielski, Acta Phys. Pol A 110 (2) 263 (2006)

University in Linz. One of the main accomplishments of these groups was tuning of ESR by constant electric current and associated constant Rashba field, [6] what was the direct confirmation of CI ESR mechanism.

In moderately impure systems, in the regime of frequent collisions with imperfections an electron dissipates energy absorbed from an oscillating electric field. The power absorbed in ESR depends on the electron momentum relaxation time τ, as well as on the transverse spin relaxation time T_2. Moreover, in contradiction to paramagnetic resonance, in case of two dimensional structures it strongly depends on the sample orientation in respect to the direction of external magnetic field. In the ESR participate only electrons occupying uncompensated spin states. At low temperatures these states correspond to energy near the Fermi level. ESR investigations thus enable the analysis of electron spin dynamics.

The five enclosed works concern the resonant absorption of radiation by electron gas in samples placed in microwave cavity. The main interest there is an absorption due to the electric component of microwave radiation. As it was shown in paper 1) and confirmed by Edelstein[7], the oscillating electric field interacting with electron momentum drives resonance few orders of magnitude more efficiently than the magnetic one.

At the beginning we have assumed that the only channel of energy transfer during CI ESR is due to Rashba magnetic absorption. In the frame of Bloch[8] description this absorption is proportional to imaginary part of magnetic susceptibility. However, in observed signals [5] one can see the influence of dispersive (real) component of susceptibility. Explanation of this fact required more precise description of CI ESR.

Hence, first we defined power absorption as the time derivative of the one electron Hamiltonian averaged over the period of microwave field oscillations. Next, we used the Drude[9] model which is very powerfull in description of electron conductivity. Similarly to the theory of spin-Hall effect given by Chudnovsky, [10] we extended Drude model by spin-dependent part of the electron velocity. It caused the arising of electric Rashba field (complementary to magnetic Rashba field) being the correction to electric

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[6] Z. Wilamowski, H. Malissa, F. Schaffler, and W. Jantsch, Phys. Rev. Lett. **98**, 187203 (2007).
[7]V.M. Edelstein, Phys. Rev. B **81**, 165438 (2010)
[8] F. Bloch, Phys. Rev. 70, 460 (1946)
[9] P. Drude, Ann. Phys. (Leipzig) **1**, 566 (1900)
[10] E.M. Chudnovsky, Phys. Rev. Lett. **99**, 206601 (2007)

microwave field in Lorentz force in Drude equation of electron motion. The role of this spin-dependent field, unknown until now in literature, is mainly effective for the Larmor frequency.

Investigation of the mechanism of absorption have pointed out that the power of absorbed energy, due to electric component of microwave radiation, should be described by the Joule-heat with electron current induced by external rf (radio-frequency) electric field and spin dependent electric Rashba field for the first time recognized and introduced in the 3-rd paper. Moreover, the results of calculations confirmed the presence of dispersive component of magnetic susceptibility in the observed absorption power. The successful transformation of Joule-heat expression into the sum of the oscillatory part of kinetic energy of electrons and the part of magnetic absorption induced by magnetic Rashba field, treated earlier as the only channel of CI ESR absorption.

An interesting result, being in contradiction with the Faraday law , was obtained for two dimensional sample in which electric current is induced in spite of the zero flux of rf magnetic field through the sample plane. Unfortunately, until now, this result remains unconfirmed by experiment.

1) ***W. Ungier, W. Jantsch and Z. Wilamowski***

„Spin resonance absorption in a 2D electron gas"

Acta Physica Polonica **112**, 345 (2007)

(this work was presented on the conference Jaszowiec 2007)

The paper presents the outline of the mechanism of electro-magnetic spin resonance in a two-dimensional electron gas with Rashba coupling. The main asumption used in theory is that the microwave absorption is effected by electron magnetization under action of microwave magnetic field $\mathbf{H}_1$ ~exp(-*iωt)* combined with Bychkov-Rashba field $\mathbf{H}_{BR}$. The last field is induced by electron current driven by electric component of microwave radiation $\mathbf{E}_1$ ~exp(-*iωt)*.

A simple geometric configuration of an experiment is considered, in which the constant external magnetic field $\mathbf{H}_0$ and the microwave electric field $\mathbf{E}_1$ are parallel and they both are in-plane vectors perpendicular to **n** which defines the direction of electron confinement. Thus the cyclotron resonance is absent. The Fourier amplitude of Bychkov- Rashba field can be expressed as

$\mathbf{H}_{BR} \sim \alpha_{BR}\mathbf{j}\times\mathbf{n} \sim \alpha_{BR}\tau[(1+i\omega\tau)/(1+\omega^2\tau^2)]\mathbf{E}_1\times\mathbf{n}$. $\mathbf{H}_{BR}$ and $\mathbf{E}_1$ are the in-plane vectors, perpendicular to each other. Rotating the sample around axis parallel to $\mathbf{E}_1$ (and $\mathbf{H}_0$), observing the relation $\mathbf{H}_1 \perp \mathbf{E}_1$, it is possible to get the following configuration of magnetic fields: $\mathbf{H}(t) = [H_1(t), H_{BR}(t), H_0]$. The electron magnetization $\mathbf{M}$ can be obtained using linearized Bloch equations. The momentary absorption power $P(t) = \mathbf{M}(t)d\mathbf{H}(t)/dt$, averaged over the period $2\pi/\omega$, defines the mean power absorbed per unit area of the sample (H_1, H_{BR} and E_1 are the Fourier amplitudes of the appropriate fields):

$$dP/dA \cong \frac{\omega}{2}\chi''(\omega)\mu_0[|H_1|^2 + |H_{BR}|^2 - 2\,\mathrm{Im}(H_1 H_{BR}^*)]$$

$$= \frac{\omega}{2}\chi''(\omega)\mu_0[|H_1|^2 + \frac{\beta^2}{(1+\omega^2\tau^2)}|E_1|^2 + \frac{2\beta}{(1+\omega^2\tau^2)}|E_1||H_1|] \,, \qquad (*)$$

where $\beta = e\tau\alpha_{BR}/g\mu_0\mu_B\hbar$

(*in the equation 2 of discussed work the definition of* $\mathbf{H}_{BR}$ *lacks* $\hbar$ *in denominator*).

In the calculations of the power it was taken into account the phase difference between the electric and magnetic rf fields inside the microwave cavity, as well as the fact, that the resonant absorption line, $\omega \approx \omega_0$ is sharp (for the quantum well SiSi/Ge $\omega_0 T_2 \approx 10^4$, with T_2 - the transverse spin relaxation time).

From the equation (*) one can see that calculation of the absorption power requires the knowledge of relative phase difference between H_1 and H_{BR} fields. However, the knowledge of phase difference between electric field $\mathbf{E}_1$ and $\mathbf{j}$ is not necessary. So it is not essential, whether spin resonance is caused by electric field, or by electric dipole transitions.

Comparing the first two terms of the equation (*) for identic values of E_1 i H_1 (in V/cm) one can conclude, that the optimal signal due to Bychkov-Rashba field H_{BR} appears four orders stronger than that of optimal pure magnetic field H_1, what is experimentally confirmed by measurements obtained in the microwave cavity (for Si/SiGe $\tau \approx 10^{-11}$s, $\alpha_{BR}/\hbar \approx 4$m/s).

Summary

1. *The simple geometry of an experiment, with* $\mathbf{H}_0$ *parellel to the sample plane (thus with cyclotron resonance absent) was considered.*
2. *Under the assumption that power absorbed by electron gas can be calculated in similar way as in the case of paramagnetic resonance, treating Bytschov-Rashba field on the same footing as the external magnetic microwave field.*
3. *In the case of Bloch solution, the absorbed power is proportional to imaginary part of magnetic susceptibility function.*

2) ***Z. Wilamowski, W. Ungier and W. Jantsch***

„Electron spin resonance in a two-dimensional electron gas induced by current or by electric field"

Physical Review B **78**, 174423 (2008),

and

Virtual J. Nanoscale Science & Technology **18** Issue 23 D Awschalom (2008)

The paper describes absorption of electric component of the microwave radiation in the electron spin resonance stimulated by Bychkov-Rashba field. Similarly to the previous work, the only channel of microwave absorption is caused by the magnetzation of electron system.

In general, the arbitrary configuration of the external fields and of the sample are considered. The dependence of the resonance signal on the constant magnetic field $\mathbf{H}_0$ (bias) to the sample plane is explored for the whole range of θ. For the clarity of description we consider the coordinate systems, in which $\mathbf{H}_0$ has only *z* component, and the second is related to the sample by vector **n** perpendicular to the sample plane, Fig. 1.

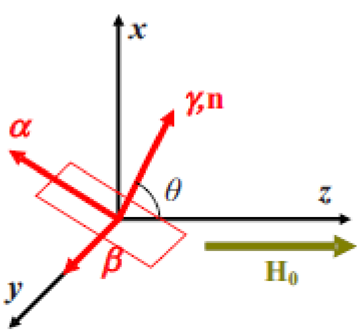

Fig. 1. Coordinate systems; the sample normal **n** is tilted with respect to the static magnetic field $\mathbf{H}_0$, by an angle θ. A new coordinate system α, β, γ is anchored to the sample (indicated by a rectangle).

Considering the presence of cyclotron resonance the active (+) and inactive[11] (-) components of conductivity tensor were defined, $\sigma_\pm = (ne^2/m^*)\tau_\pm$ with $\tau_\pm = \tau_p/[1-i(\omega \pm \omega_c)\tau_p]$ (τ_p - momentum relaxation time, $\omega \approx \omega_c = e\mu_0 H_0 |\cos\theta| / m^*$-cyclotron frequency). Subsequently, the rotating (around $\mathbf{H}_0$) system of coordinates was introduced in which the active and inactive components of Rashba field were expressed by the scalar product of rf electric field vector and especially defined vector $\mathbf{T}_\pm$ with components expressed as combinations of τ_+ i τ_- and $\cos\theta$.

Some configurations of the system were analyzed.

a) For rf electric field **E**~exp(-*iωt)* parallel to constant magnetic field $\mathbf{H}_0$ and for $\theta = \pi/2$ we obtain $\mu_0 H_{BR\pm} = \pm(\alpha_{BR} e E_z / g\mu_B \hbar)\tau_p/(1-i\omega\tau_p)$. Modulus of this expression versus frequency of electric field is plotted in Fig. 2.

[11] E.D. Palik and J.K. Furdyna, Rep. Prog. Phys. **33**, 1193 (1970)

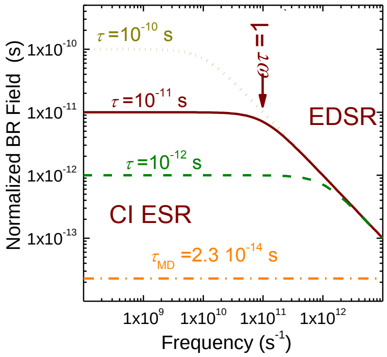

Fig. 2. Frequency dependence of the driving field $|H_{BR\pm}|$ for various momentum relaxation times τ_p .

For the low frequencies ($\omega\tau_p << 1$) $|H_{BR\pm}| \sim \tau_p$, while for the high frequencies ($\omega\tau_p >> 1$) $|H_{BR\pm}| \sim \omega^{-1}$. The above limits correspond to the resonances induced respectively by the drift current (CI ESR) and by ED ESR (considered by Rashba). In the second limit the dependence of $|H_{BR\pm}|$ on electron mobility $e\tau_p / m^*$ disappears and displacement current dominates.

b) The next analysed case, for high frequencies, was that of $\mathbf{E} \| \mathbf{H}_0$ with arbitrary orientation of the sample. The dependence of $\mu_0|H_{BR\pm}|$ vs. the angle θ for different frequencies is plotted on the Fig. 3.

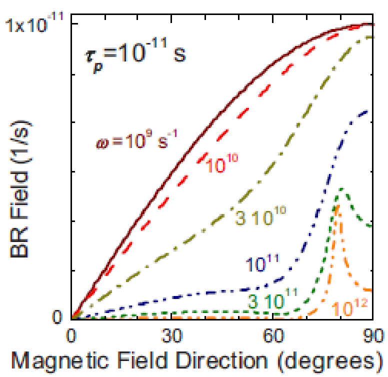

Fig. 3. Dependence of the BR field on the sample orientation.

For higher frequencies, and for $\theta \approx 80°$ (in Si/SiGe) one can see the fing erprint of cyclotron resonance.

c) In the case $\omega\tau_p << 1$ the cyclotron resonance is damped for all directions of rf field **E** and for all angles θ.

The electron magnetization as a resultant solution of the Bloch equations, assumed in description of the Rashba-magnetic resonance, is equal to $M_\pm = \chi_\pm(\omega) H_\pm$. $M_\pm$ and $H_\pm$ are the components of complex Fourier amplitudes in the rotating system of coordinantes. The imaginary part of magnetic susceptibility is expressed by the Lorentz shape function f_L: $\chi''_\pm(\omega) = \pm\pi\mu_0\gamma_B M_0 f_L(\pm\omega)$. For the long transverse spin relaxation time T_2 ($\omega_L T_2 >> 1$, ω_L - Larmor frequency) only one component of $H_\pm$ drives the precession of magnetization. For positive g-factor it is H_+. For a weak precession-type excitatons, when the Rabi oscillations decrease and for the low-temperature range, the electron magnetization tends to its equilibrium value $M_0 = (1/2) g\mu_B n_p$, where $n_p = \hbar\omega_L D_s / 2$ is the surface concentration of electrons with uncompensated spins (D_s denotes density of electron states for both spin subbands).

As the final result of the paper, the microwave absorption power per unit area of the sample and for arbitrarily oriented rf electric field **E** was obtained: $dP_M / dA = (1/2)\mu_0\omega(\beta_{BR}e/m^*)^2[\chi''_+(\omega)|\mathbf{T}_+\mathbf{E}|^2 + \chi''_-(\omega)|\mathbf{T}_-\mathbf{E}|^2]$. The vectors $\mathbf{T}_+$ and $\mathbf{T}_-$ define orientation of the sample, as well as they describe the cyclotron-resonance-type dependence of the power. The dependence of absorption signal on angle θ, for the case $\mathbf{E} \| \mathbf{H}_0$, is presented in Fig. 4:

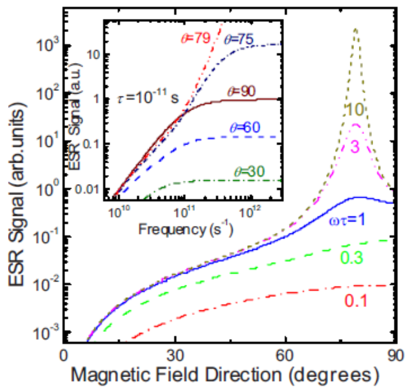

Fig. 4. Dependence of the ESR signal induced microwave electric field on the tilt angle θ of the static magnetic field $\mathbf{H}_0$ relative to the vector **n**. The inset shows the frequency dependence of signal for different angles θ.

The obtained resonance absorption power, treated as a contribution to the Joule heat, together with the equation $dP_M / dA = (1/2)\,\mathrm{Re}(\mathbf{E}^*\hat{\sigma}_s\mathbf{E})$, define tensor $\hat{\sigma}_s$ as a correction to the classical conductivity tensor. The ratio of this correction to the Drude conductivity $\sigma = n_s e^2 \tau_p / m^*$, for $\mathbf{E} \| \mathbf{H}_0$ directed along the sample plane, is approximately eqal to 2×10^{-5} for Si/SiGe. In spite of relatively small value this correction is easily observed in experiment.

Summary

1. *The current state of knowledge about CI ESR is presented.*
2. *All possible configurations of electric and magnetic fields with arbitrary orientation of the sample are considered.*
3. *The expression for power absorption due to the electric component of the microwave energy radiation with explicit dependence on the vector of rf electric field is derived.*
4. *The influence of cyclotron resonance on the absorption signal is shown.*
5. *The Lorentz-shape function is used in description of the signal.*
6. *The resonant correction to the conductivity tensor is defined and evaluated..*

3) ***W. Ungier, Z. Wilamowski and W. Jantsch***

„ Spin-orbit force due to Rashba coupling at the spin resonance condition"

Physical Review B **86**, 245318 (2012)

In the paper the more profound analyse of microvave absorption by two dimensional gas is used. Since the Rashba field is a function of the velocity of each individual electron, the precise description of one electron motion in the Drude model[9] is performed. The closely connected to Drude model conductivity tensor,[12] depending on fixed external magnetic field, describes the cyclotron motion and defines the relative phases of the oscillating current and of microwave electric field. One of the essential and very important element of the analysis (carried over from the preious works) is the decomposition of electron velocity into the part depending on its momentum and the part depending on spin: $\mathbf{v} = \mathbf{v}^{(p)} + \mathbf{v}^{(R)}$, where $\mathbf{v}^{(R)}$ is not equal to zero only for electrons in unpaired spin states.

The obtained time derivative of the momentum part of velocity with added damping term is $d\mathbf{v}^{(p)}/dt = (e/m^*)(\mathbf{E}_1 + c^{-1}\mathbf{v}^{(p)} \times \mathbf{B}_{0\perp} + c^{-1}\mathbf{v}^{(R)} \times \mathbf{B}_{0\perp}) - (\mathbf{v}^{(p)} - \mathbf{v}^{(p)}_{rel})/\tau$ (*).

It is assumed in the Eq. (*) that relaxation of momentum part of the velocity happens much more often than the spin relaxation, so that momentum part tends to $\mathbf{v}^{(p)}_{rel} = -\mathbf{v}^{(R)}(\boldsymbol{\sigma})$, which corresponds to the extrmum condition $\partial\mathcal{H}/\partial\mathbf{v}^{(p)} = 0$.

After adding spin dependent part of the velocity $d\mathbf{v}^{(R)}/dt = v_R\mathbf{n} \times d\boldsymbol{\sigma}/dt$ to both sides of the equation (*) one obtains the generalized Drude equation $d\mathbf{v}/dt = (e/m^*)(\mathbf{E}_1 + e^{-1}\mathbf{F}^{(SO)} + c^{-1}\mathbf{v} \times \mathbf{B}_{0\perp}) - \mathbf{v}/\tau$ with entirely new spin dependent force $\mathbf{F}^{(SO)} = v_R m^* \mathbf{n} \times d\boldsymbol{\sigma}/dt$. In contradiction to the forces described earlier by S.Q. Shen[12] and E.M. Chudnovsky[10] this force depends on the speed of spin variation. The second important fact is that the phase difference between the rf $\mathbf{E}_1$ and that of oscillatory component of velocity is constant , i.e. $\mathbf{v}_\omega = \mathrm{const}$ in $\mathbf{v} = \mathbf{v}_0 + \mathrm{Re}[\mathbf{v}_\omega \exp(-i\omega t)]$ (every change of initial $\mathbf{v}_0$ after collision can be interpreted as a change of the coordinate system of the electron oscillations; on the other hand , as long as $\tau << T_2$, the succesive collisions during period T_2 do not perturb the electron spin precession ; small deviations from the precession axis connected with often varying Rashba field $\mathbf{B}_{R0}$ can be neglected - M. Duckheim and D. Loss[14]) .

The momentary absorption power of electric component of microwave radiation by one electron is defined as a partial derivative of the Hamiltonian $\partial\mathcal{H}/\partial t = e\mathbf{E}_1(t)\mathbf{v}(t)$ (which depends on time through

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[12] B. Lax, H.J. Zeiger, and R.N. Dexter, Physica 20, 818 (1954)
[13] S.Q. Shen, Phys. Rev. Lett. **95**, 187203 (2005)
[14] M. Duckheim and D. Loss, Nat. Phys, **2, 195 (2006)**

the vector potential **A)**. Averaging over the period $2\pi/\omega$ and taking into account currents of all electrons, the obtained power of the absorption (per unit of sample area) can be expressed via the Fourier amplitudes: $P_E(\omega) = (1/2)\,\mathrm{Re}\{\mathbf{E}^*_{1\omega}\mathbf{j}_\omega\}$. Due to the spin dependent force $\mathbf{F}^{(SO)}$, which can influence the electron current, the obtained power has two components, $P_E(\omega) = P^{(J)}_\omega + P^{(SO)}_\omega$, where $P^{(J)}_\omega = (1/2)\,\mathrm{Re}\{\mathbf{E}^*_{1\omega}\hat{\sigma}(\omega)\mathbf{E}_{1\omega}\}$ denotes the classical Joule heat and $P^{(SO)}_\omega = (1/2)\,\mathrm{Re}\{\mathbf{E}^*_{1\omega}\hat{\sigma}(\omega)e^{-1}\mathbf{F}^{(SO)}_\omega\}$ is the resonant correction to the absorbed power. This is the main result of this paper.

The second important result is the proof of the equation $P_E(\omega) = n_e m^* <|\mathbf{v}_\omega|^2> / 2\tau + P^{(M)}_\omega$ (<X> denotes the average over all occupied electron states) which exhibits that besides the kinetic part of the electron motion in the transfer of absorbed energy also participates magnetic absorption driven by Rashba field, $P^{(M)}_\omega = (\omega/2)\,\mathrm{Im}(\mathbf{B}^*_{R\omega}\mathbf{M}_\omega)$. Thus the magnetic absorption, treated in the previous works as the only channel of energy transfer at the resonance, participates in the energy absorption at the cost of kinetic energy of electron system.

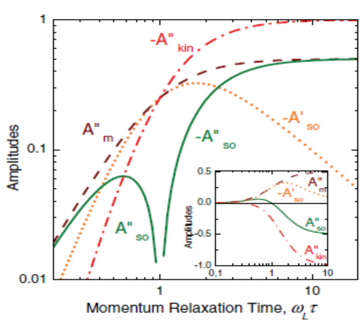

Fig. 1. Dependence of the CIESR amplitudes on momentum relaxation time for $\theta=\pi/2$. The solid line stands for the total electric absorption amplitude and the dotted for the dispersion amplitude. The dashed line describes the energy transfer to the magnetic energy and the dashed-dotted line stands for the reduction of the Joule heat at spin resonance. Inset: dependencies in linear scale, demonstrating the signs of all amplitudes.

The dependence on momentum relaxation time of amplitudes (coefficients) of the real and imaginary parts of magnetic susceptibility function, A'_{SO} and A''_{SO} (at $\theta = \pi/2$), respectively, are presented in Fig. 1. The amplitudes A''_M oraz A''_{kin} correspond to the magnetic absorption $P^{(M)}_\omega$ and to the difference $n_e m^* < |\mathbf{v}_\omega|^2 > / 2\tau - P^{(J)}_\omega$, respectively. The classical Joule heat is due to the current induced only by external electric force $e\mathbf{E}_1$, while velocity amplitudes $\mathbf{v}_\omega$ result from effective total force $e\mathbf{E}_1 + \mathbf{F}^{(SO)}$. The presence of A'_{SO} explains asymmetry of the CI ESR signal observed in experiments.[5]

Summary

1. *The equation of electron motion is derived in the spin-extended Drude model. The spin dependent force as a correction to rf electric external force is introduced.*
2. *The momentary absorption power due to the electric component of microwave radiation by an electron is defined as a time derivative of the Hamitonian. The obtained mean absorption power is the sum of the classical Joule heat and the resonant part due to the introduced electric spin dependent correction.*
3. *It is shown that the magnetic resonant absorption driven by Rashba field is explicitly present.*
4. *The analysis of induced currents in the limit of low frequencies is presented.*
5. *The obtained expression for power contains a dispersive part of magnetic susceptibility which explains the observed shape of absorption signal.*

4) ***W. Ungier and W. Jantsch***

"Rashba fields in a two-dimensional electron gas at electromagnetic spin resonance"

Physical Review B **88**, 115406 (2013)

This paper describes two dimensional electron gas under the influence of microwave radiation, simultaneously the electric $\mathbf{E}_1$ and magnetic field $\mathbf{B}_1$. Thus in the magnetic resonance the spin precession is driven by the external field $\mathbf{B}_1$ and the „internal" Rashba field $\mathbf{B}_R$. In this case the power absorbed by one electron from the radiation, averaged over the period $2\pi/\omega$, is equal to $P_\omega = P_\omega^{(E)} + P_\omega^{(M)} = (e/2)\,\mathrm{Re}\,\mathbf{E}_{1\omega}^*\mathbf{v}_\omega + (\omega/2)(g/2)\,\mathrm{Im}\,\mathbf{B}_{1\omega}^*\boldsymbol{\mu}_\omega$.

The considered two-dimensional system is oriented arbitrarily relatively to direction of external constant magnetic field $\mathbf{B}_0$. $\mathbf{B}_1$ is assumed to be the in-plane vector (this allows linearization of equations of electron motion with regard to the oscillating quantities) and perpendicular to $\mathbf{E}_1$, Fig. 1.

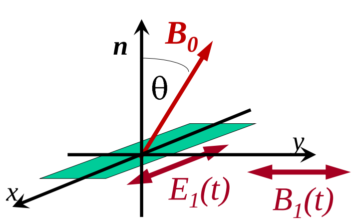

Fig. 1. Directions od external fields in relation to the sample plane.

Then system of equations for the Fourier amplitudes describing electron motion is the following: $\boldsymbol{\mu}_{\omega} = (\delta n)^{-1}\hat{\chi}(\omega)[(g/2)\mathbf{B}_{1\omega} + \mathbf{B}_{R\omega}]$, $\mathbf{v}_{\omega}^{(p)} + \mathbf{v}_{\omega}^{(R)} = (ne)^{-1}\hat{\sigma}(\omega)(\mathbf{E}_{1\omega} + \mathbf{E}_{R\omega})$ and $\mathbf{v}_{\omega}^{(R)} = (\alpha_R/\hbar\mu_B)\mathbf{n}\times\boldsymbol{\mu}_{\omega}$, where

$\mathbf{B}_{R\omega} = (\alpha_R m^*/\hbar\mu_B)\mathbf{n}\times\mathbf{v}_{\omega}^{(p)}$ and $\mathbf{E}_{R\omega} = -i\omega(\alpha_R m^*/e\hbar\mu_B)\mathbf{n}\times\boldsymbol{\mu}_{\omega}$. The Rashba electric field is assumed to be equal $\mathbf{E}_{R\omega} = e^{-1}\mathbf{F}_{\omega}^{(SO)}$, where the force $\mathbf{F}^{(SO)}$ is defined in the previous paper 3). In order to solve the above equations it suffices to find the momentum part of electron velocity. This velocity is obtained up to α_R^2 order and is defined by the following function of $\mathbf{E}_1$ and $\mathbf{B}_1$: $\mathbf{v}_{\omega}^{(p)} = (ne)^{-1}\hat{\sigma}(\omega)\{[\hat{I} - \hat{\Delta}_E(\omega)]\mathbf{E}_{1\omega} - \hat{\Lambda}(\omega)(g/2)\mathbf{B}_{1\omega}\}$, where for ω equal to the Larmor frequency $\Delta_E \approx 3\times(10^{-3} - 10^{-2})$ and $\Lambda \approx 1.5\times(10^{-5} - 10^{-4})$ in SiSi/Ge. Evaluation of Δ and Λ allows to define scale of dependencies of the resonance signals on the electric and magnetic microwave field components.

In the case $\mathbf{E}_1 = 0$ and $\mathbf{B}_1 \neq 0$ the Fourier amplitudes $\mathbf{v}_{\omega}^{(p)}$ and $\mathbf{v}_{\omega}$ for the electrons in uncompensated spin states do not vanish. For the in-plane vector $\mathbf{B}_1$ a contradiction with the Faraday induction law arises; no matter that the flux of rf field $\mathbf{B}_1$ through the sample plane is equal to zero, the electric current is induced.

The total power absorbed by the electron system (equation 14 of the paper) is the function of $\mathbf{v}_{\omega}^{(p)}$ which defines Rashba magnetic field driving spin precession and in consequence, the electric Rashba field. The part of power due to electric rf field consists of classical Joule heat and the resonant correction connected with additional current induced by Rashba electric field. The part due to magnetic rf field consists of the well known magnetic absorption expression with the correction due to Rashba magnetic field. Both Rashba corrections give contributions to the magneto-electric expression $\delta P_{\omega}^{(1)}$, linear in α_R, present when magnetic and electric rf field affect the sample simultaneously. This expression vanishes when one od the rf fields can be neglected. In the second order of approximation, proportional to α_R^2, the resonant Rashba correction has two components, electric and magnetic one. They both contribute to the power absorbed either independently or with simultaneous presence of the electric and magnetic components of microwave radiation. For the identical values (in V/e) of amplitudes of $\mathbf{E}_1$ and $\mathbf{B}_1$ rf fields the contribution to absorption signal in SiSi/Ge is seven orders, and magneto-electric is two orders weaker than the electric signal CI ESR. The line-shapes of resonant absorption for various relaxation rates and diverse orientation of the sample are shown below:

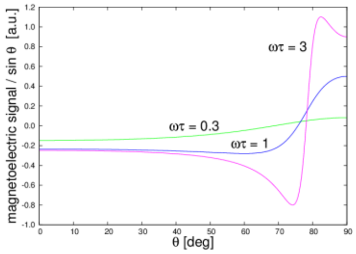

Fig. 1. The magneto-electric signal $\delta P_{\omega}^{(1)}$

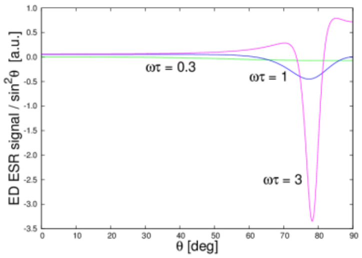

Fig. 2. The electric part of signal $\delta P_{\omega}^{(2)}$

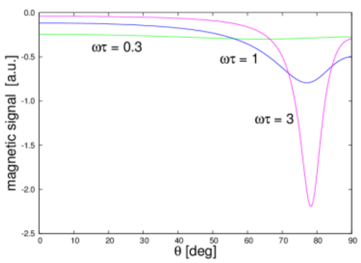

Fig. 3. The magnetic part of signal $\delta P_{\omega}^{(2)}$

The extrema of electric and magnetic signals arise at the vicinity of cyclotron resonance. The electric signal is more sensitive than the magnetic one to momentum relaxation time what can be explained by the proportionality of the Rashba field B_R to the electron mobility $e\tau/m^*$.

Taking into account that the electric part of $\delta P_{\omega}^{(2)}$ dominates remaining signals, the approximate equality holds $\delta P_{\omega}^{(2)} = (1/2)\,\mathrm{Re}(\mathbf{E}_{1\omega}^{*}\mathbf{j}_{R\omega})$, where $\mathbf{j}_{R\omega} = (\delta n/n)\hat{\sigma}(\omega)\mathbf{E}_{R\omega}$ is the current of electrons in states with uncompensated spin. The sign of $\delta P_{\omega}^{(2)}$ depends on the relative direction and phase of $\mathbf{j}_R(t) = \mathrm{Re}\,\mathbf{j}_{R\omega}\exp(-i\omega t)$ in respect to $\mathbf{E}_{1\omega}(t)$. The negative value of $\delta P_{\omega}^{(2)}$ for the frequency near to cyclotron resonance corresponds to the reduction of total power absorption $P_{\omega}^{(tot)}$ (equation 14 in the paper).

Summary

1. *The paper consider absorption of microwave energy by two dimensional sample under symultaneous influence of the electric and magnetic radiation components.*
2. *The momentum dependent electron velocity was derived as a function of the external rf fields up to the square of small Rashba parameter α_R .*
3. *The absorption power is calculated up to α_R^2 for the magneto-electric, electric and pure magnetic signals. Graphs of these signals are shown vs. sample orientation relative to the constant magnetic field.*
4. *The negative value of electric resonant signal is explained.*

5) ***W. Ungier***

"Rashba coupling in three-dimensional (wurtzite structure) electron gas at electric-dipole spin resonance"

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The paper (5) considers electric CI ESR in three dimensions. As in previously discussed works (3) and (4) the resonance is described using the Drude model of electron motion and conductivity tensor formalism. In contradiction to description in two dimensions the tensor character of the electron effective mass and Lande factor *g* are taken into account. Thus the expressions for the Rashba fields $\mathbf{B}_R$ and $\mathbf{E}_R$ (introduced in paper(4)) depend on the longitudinal and transverse effective masses and *g*-factor. The Lande factor defines *z* axis ($\mathbf{e}_z$ in Fig. 1) of the rotating coordinate system which is very convenient for description of electron spin precession. The sample system coordinates (x_s, y_s, z_s) with z_s being parallel to wurtzite c- axis (**n**) is presented on Fig. 1.

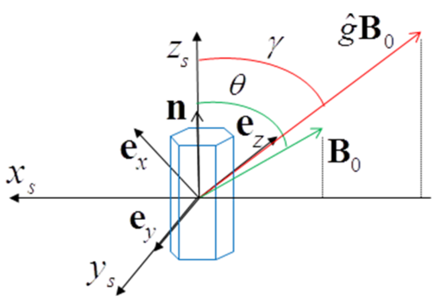

Fig. 1. Considered coordinate systems. Magnetic field $\mathbf{B}_0$ is tilted by θ with respect to the wurtzite crystal *c* axis (vector **n**).

The precision of tensor description is preserved in every expression up to the corrections of order of α_R^2 without referring to any specific system of coordinates. However, the oscilating part of Bloch equations, written in rotating basis $\mathbf{e}_\pm = (\mathbf{e}_x \pm i\mathbf{e}_y)/\sqrt{2}$, separates into two equations. One of them (+) corresponds to the ordinary electron spin resonance. The algebraic solution of these equations for the Fourier amplitudes, $\mu_{\omega\pm} = (\delta n)^{-1}\chi_\pm(\omega)B_{R\omega\pm}$ (μ corresponds to electron magnetic moment, *B* corresponds to the Rashba magnetic field) leads to the conclusion that magnetic susceptibility tensor is diagonal in rotating frame: $\hat{\chi} = \mathrm{diag}(\chi_+, \chi_-, 0)$, where the elements are the simple complex functions $\chi_\pm(\omega) = \mp\gamma M_0 / (\omega \mp \omega_L + i/T_2)$.

In calculations of the absorption power $\delta P_{res}^{(E)}$ the isotropic approximation of the electron mass and the Lande *g*-factor is used. Then the conductivity in rotating frame, as well as the magnetic susceptibility tensors, have only diagonal nonvanishing elements.

For rf **E**(t) parallel to $\mathbf{B}_0$ and for $\theta = 90°$ the space distribution of electron current and of the Rashba fields $\mathbf{B}_R$ and $\mathbf{E}_R$, relatively to rf **E**(t), coincide with that in two-dimensional case. For $\omega\tau > 1$ the resonant correction of the absorbed power is always positive, except for $\theta = 0°$, while in two dimensional case it is negative at the vicinity of the cyclotron resonance, $\theta = \arccos(\omega_L / \omega_c)$.

The obtained power correction $\delta P_{res}^{(E)}$ (equation 19 of discussed paper) contains dispersive contribution from the magnetic susceptibility ($\chi'_\pm$), what is an expected result for the bulk compound ZnO.[5] Fig. 2 presents the composition of ESR line (the line-shapes are described by first derivative of Lorentzian functions). Dashed (pure absorption) line corresponding to ED ESR is centered on the resonance field B=3454.5G for donor electrons, and dotted line (dispersive-like component), corresponding to CI ESR is centered on B=3456.8G – resonance field for conduction electrons.

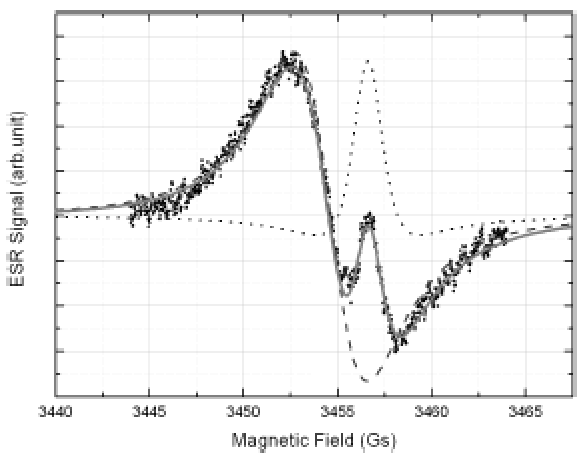

Fig. 2 (from the paper of E. Michaluk *et al.*[5])

At the end of discussed paper the tensor components in choosen coordinate systems, as well as the transformation matrices between those systems are given. This allows to define tensor components in those choosen systems and hence precise calculation of $\delta P_{res}^{(E)}$ taking into account the anisotropy of the electron effective mass and of the Lande factor.

Summary

1. *The paper describes the contribution to absorption power originating from the electric component of microwave radiation in three-dimensional electron gas. The anisotropy of electron effective mass and Lande g-factor are accounted for.*
2. *In calculations of resonant correction of the power* $\delta P_{res}^{(E)}$ *an isotropic approximation of electron mass and g-factor is used. The obtained result for rf* $\boldsymbol{E}(t)$ *parallel to* $\boldsymbol{B}_0$ *is compared with analogical correction for two-dimensional case.*
3. *The shape of ESR absorption line measured in ZnO (in section IV of discussed paper there is incorrect ref. 8 instead of 14) can be explained by the presence of dispersive component of magnetic susceptibility in the obtained expression for the power* $\delta P_{res}^{(E)}$.