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Summary of professional achievements

Institute of Physics of the Polish Academy of Sciences

Warsaw, May 12, 2015

Contents

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Curriculum Vitae

1.1. Personal data

First and last name	Tomasz Sowiński
[redacted]	
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1.2. Education and scientific degrees

Sept. 22, 2008	PhD degree in physics Faculty of Physics, University of Warsaw PhD thesis *"Interaction of the two-level systems with the quantized electromagnetic field"* prepared under supervision of prof. dr hab. Iwo Biaynicki-Birula.
June 21, 2005	MSc in theoretical physics Faculty of Physics, University of Warsaw Thesis *"Complete classical and quantum description of motion in the rotating harmonic trap"* prepared under supervision of prof. dr hab. Iwo Biaynicki-Birula.
2000 – 2005	Student at the Faculty of Physics, University of Warsaw Specialization: theoretical physics. Graduated with honours.

1.3. Employment in academic institutions

2013 –	Assistant professor (pol. adiunkt) Center for Theoretical Physics of the Polish Academy of Sciences
2010 –	Assistant professor (pol. adiunkt) Quantum Optics Group Institute of Physics of the Polish Academy of Sciences
2009 – 2012	Assistant professor (pol. adiunkt) Faculty of Biology and Environmental Sciences Cardinal Stefan Wyszynski University in Warsaw
2005 – 2009	Assistant Center for Theoretical Physics of the Polish Academy of Sciences
2003 – 2005	Laboratory technician for informatics Center for Theoretical Physics of the Polish Academy of Sciences

1.4. Longer scientific visits

2012 – 2013	Visiting Scientist (9 months) ICFO – The Institute of Photonic Sciences Castelldefels (Barcelona), Spain within KOLUMB PostDoc Scholarship granted by the Foundation for Polish Science
2011	Visiting Scientist (3 months) ICFO – The Institute of Photonic Sciences Castelldefels (Barcelona), Spain within European Union Project "NAME-QUAM"

1.5. Bibliometric statistics

Bibliometric data as of the date	May 12, 2015 r.
Total number of articles published or accepted for publication	23
Number of articles after PhD degree	19
Number of articles in the Web of Science database	22
Total number of citations according to the Web of Science	127
Number citations without self-citations according to the Web of Science	102
Hirsch Index according to the Web of Science	8
Total Impact Factor*)	65,61
Total number of citations according to the Google Scholar	228

*) Total Impact Factor counted under assumption that the Impact Factor of the articles published in 2014 and 2015 is the same as in 2013.

Tomasz [signature]
12 maja 2015 r.

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Scientific achievement

2.1. Title of the achievement

MODELING OF PHYSICAL PHENOMENA
IN SYSTEMS OF ULTRA-COLD BOSONS
CONFINED IN OPTICAL LATTICES

2.2. Series of articles forming the achievement

[H1] T. Sowiński
"Creation on demand of higher orbital states in a vibrating optical lattice"
Phys. Rev. Lett. **108**, 165301 (2012).
Impact Factor: 7.943.

[H2] T. Sowiński[1)], M. Łącki, O. Dutta, J. Pietraszewicz, P. Sierant, M. Gajda, J. Zakrzewski, M. Lewenstein
"Tunneling-Induced Restoration of the Degeneracy and the Time-Reversal Symmetry Breaking in Optical Lattices"
Phys. Rev. Lett. **111**, 215302 (2013).
Impact Factor: 7.728.

[H3] T. Sowiński
"Exact diagonalization of the one dimensional Bose-Hubbard model with local 3-body interactions"
Phys. Rev. A **85**, 065601 (2012).
Impact Factor: 3.042.

[H4] T. Sowiński
"One-dimensional Bose-Hubbard model with pure three-body interactions"
Cent. Eur. J. Phys. **12**, 473 (2014).
Impact Factor: 1.077.

[1)] As declared by the co-authors, my contribution is on the level of 55%.

[H5] T. Sowiński[2)], R. W. Chhajlany
"Mean-field approaches to the Bose-Hubbard model with three-body local interaction"
Phys. Scripta **T160**, 014038 (2014).
Impact Factor: 1.296.

[H6] T. Sowiński
"Quantum phase transition in a shallow one dimensional optical lattice"
J. Opt. Soc. Am. B **32**, 670 (2015).
Impact Factor: 1.806.

2.3. Detailed description of the achievement

2.3.1. Justification of the choice of the articles

After obtaining the PhD degree I have changed the subject of my research and I have started to explore physics of ultra-cold gases confined in optical traps. Therefore, almost all my articles published after 2008 are closely related to this topic. However, they are devoted to many different problems like spin dynamics (Einstein-de Haas effect), orbital effects, the role of long-range interactions, etc. Many of these articles were prepared in a spread national and international collaboration. My habilitation achievement concerns ultra-cold, spinless bosons confined in optical lattices and interacting via short-range interactions.

2.3.2. Introduction

Quantum engineering, i.e. coherent manipulation of the matter and radiation on the atomic scale, is rapidly developing field of modern physics. Its development, apart from the obvious benefits in our everyday life, has a very deep importance for our understanding of the Nature. These highly precise experiments give many possibilities to measure various properties of the quantum system with a very high accuracy and control its dynamics on a quantum level. Nowadays experimental methods of quantum engineering allow controlling individual atoms confined in the so-called optical lattices – a specially organized laser beams which form a trapping periodic potential of almost arbitrary shape. From this point of view the concept of the optical lattice is not only an excellent tool for quantum optics, but also it can be a milestone in deep understanding of the many-body systems, which solid state physicists are trying to explain for many decades. One of the major advantage of the optical lattice experiments is fact that, unlike in the solid lattice case, almost all parameters of the optical lattice (its shape, depth, and distance between sites) can be experimentally controlled. Moreover one can also control strength of the mutual interactions

[2)]As declared by the co-author, my contribution is on the level of 50%.

between particles. These all possibilities make concept of optical lattice a very useful tool to study changes of different properties of the system by changing its various parameters. It is also worth to notice that in practice it is possible to prepare the experimental system in such a way that it can be nearly ideal experimental realization of the various theoretical Hubbard-like models. In this way the optical lattices are becoming nothing else but the dedicated quantum simulators for problems of the condensed matter physics [1].

The Hubbard-like models

The starting point of our studies on spinless bosons confined in optical lattice is a quite general Hamiltonian which can be written in the second quantization formalism as

$$\hat{\mathcal{H}} = \int \mathrm{d}\boldsymbol{r}\, \hat{\Psi}^\dagger(\boldsymbol{r}) \left[-\frac{\hbar^2\boldsymbol{\nabla}^2}{2m} + V(\boldsymbol{r})\right] \hat{\Psi}(\boldsymbol{r}) + \frac{g}{2}\int \mathrm{d}\boldsymbol{r}\, \hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}^\dagger(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r})\hat{\Psi}(\boldsymbol{r}), \qquad (2.1)$$

where $V(\boldsymbol{r})$ is an external potential forming the optical lattice, and g is a coupling constant which is proportional to the s-wave scattering length a_0 controlled experimentally. Bosonic field operator $\hat{\Psi}(\boldsymbol{r})$ anihilates a particle at point $\boldsymbol{r}$ and it fulfills standard commutation relations $\left[\hat{\Psi}(\boldsymbol{r}), \hat{\Psi}^\dagger(\boldsymbol{r}')\right] = \delta^{(3)}(\boldsymbol{r}-\boldsymbol{r}')$, $\left[\hat{\Psi}(\boldsymbol{r}), \hat{\Psi}(\boldsymbol{r}')\right] = 0$.

For simplicity, in the following, I will assume that the optical lattice is qubic and formed by three perpendicular pairs of laser beams with the laser wavelength λ and some intensties. Therefore, the periodic potential has simple, separable form:

$$V(\boldsymbol{r}) = V_x\cos^2(kx) + V_y\cos^2(ky) + V_z\cos^2(kz), \qquad (2.2)$$

where $k = 2\pi/\lambda$. The parameters V_x, V_y i V_z are controlled independently.

The Hamiltonian (2.1) is very general and without further assumptions and simplifications the detailed analysis of its properties is impossible. The standard procedure which leads to the simplified and effective description originates in the decomposition of the field operator $\hat{\Psi}(\boldsymbol{r})$ in the single-particle basis of maximally localized Wannier functions $\mathcal{W}_{\boldsymbol{i}}^{\boldsymbol{\alpha}}(\boldsymbol{r})$. In the general case of any periodic potential, finding the Wannier functions can be quite hard task. However, in the case of separable potential (2.2) they have a product form $\mathcal{W}_{\boldsymbol{i}}^{\boldsymbol{\alpha}}(\boldsymbol{r}) = W_{i_x}^{\alpha_x}(x)W_{i_y}^{\alpha_y}(y)W_{i_z}^{\alpha_z}(z)$. One-dimensional Wannier functions $W_i^\alpha(\xi)$ can be easily find since single-particle part of the Hamiltonian has a form

$$H = -\frac{\hbar^2}{2m}\frac{\mathrm{d}^2}{\mathrm{d}\xi^2} + V_0\cos^2\left(\frac{2\pi}{\lambda}\xi\right). \qquad (2.3)$$

In this case, the eigenproblem of finding the Bloch functions $(H - E_q^\alpha)\psi_q^\alpha(\xi) = 0$ is an example of the well known Mathieu equation. The Wannier functions $W_i^\alpha(\xi)$ are simple superpositions of the Bloch functions [2, 3]

$$W_k^\alpha(\xi) = \frac{V}{2\pi}\int_{q\in\mathrm{BZ}} \mathrm{d}q\, \mathrm{e}^{-iqR_k}\psi_q^\alpha(\xi), \qquad (2.4)$$

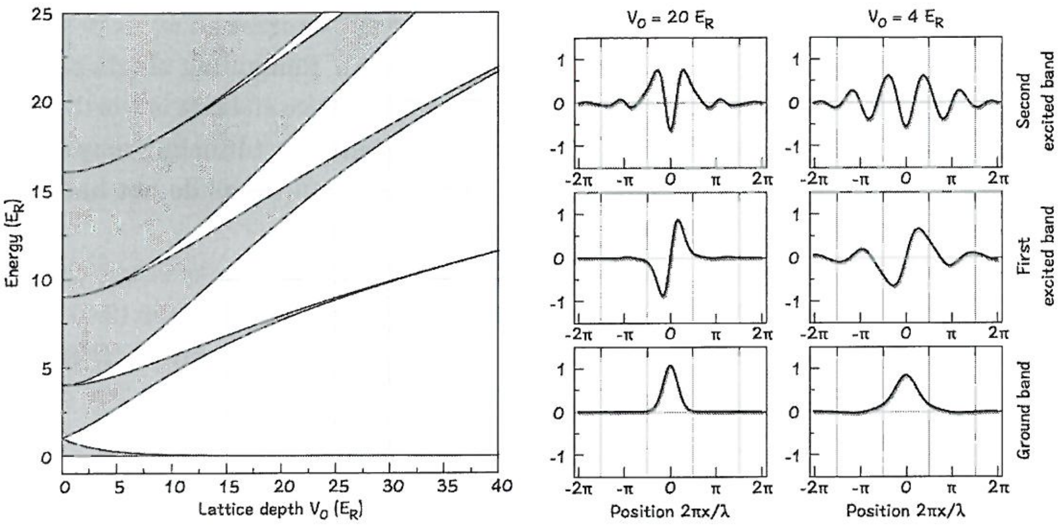

Figure 2.1: (left panel) Band structure of the one-dimensional optical lattice described by the Hamiltonian (2.3) as a function of the depth of the optical lattice V_0. As expected, the narrowing of the bands is observed when the lattice depth become larger. For clarity, the zero point energy is chosen on the level of the ground-energy of the lowest band. (right panel) Wannier functions (2.4) localized in the chosen lattice site as functions of the depth of the optical lattice and the band index α. For deep enough lattices, the shapes of Wannier functions imitate shapes of eigenfunctions of one-dimensional harmonic oscillator.

where R_k is the position of the chosen lattice site, q is a quasi-momenta of the Bloch function $\psi_q^\alpha(\xi)$, and the integration is done over the whole Brillouin zone. The spectrum of the Hamiltonian (2.3) and shapes of the first three Wannier functions for different lattice depths V_0 are presented in Fig. 2.1.

One decomposes the field operator in the basis of the Wannier functions as follows:

$$\hat{\Psi}(r) = \sum_{i,\alpha} \mathcal{W}_i^\alpha(r)\hat{a}_{\alpha i}, \tag{2.5}$$

where $\hat{a}_{\alpha i}$ annihilates a boson in a single-particle state described with the wavefunction $\mathcal{W}_i^\alpha(r)$. The decomposition allows us to rewrite the Hamiltonian (2.1) to the multi-band Hubbard-like form:

$$\hat{\mathcal{H}} = \sum_\alpha \sum_{i,j} t_{i-j}^{(\alpha)} \hat{a}_{\alpha i}^\dagger \hat{a}_{\alpha j} + \sum_{ijkl} \sum_{\alpha\beta\gamma\delta} U_{ijkl}^{(\alpha\beta\gamma\delta)} \hat{a}_{\alpha i}^\dagger \hat{a}_{\beta j}^\dagger \hat{a}_{\gamma k} \hat{a}_{\delta l}. \tag{2.6}$$

Parameters $t_n^{(\alpha)}$ and $U_{ijkl}^{(\alpha\beta\gamma\delta)}$ are appropriate matrix elements of the original Hamiltonian (2.1)

$$t_l^{(\alpha)} = \int \mathrm{d}^3 r\, \overline{\mathcal{W}}_i^\alpha(r)\, H_0\, \mathcal{W}_{i+l}^\alpha(r), \tag{2.7a}$$

$$U_{ijkl}^{(\alpha\beta\gamma\delta)} = \frac{g}{2} \int \mathrm{d}^3 r\, \overline{\mathcal{W}}_i^\alpha(r) \overline{\mathcal{W}}_j^\beta(r) \mathcal{W}_k^\gamma(r) \mathcal{W}_l^\delta(r), \tag{2.7b}$$

where $H_0 = -\frac{\hbar^2}{2m}\nabla^2 + V(r)$. The diagonal parameters $t_0^{(\alpha)}$ express an average single-particle energies in appropriate Wannier states $\mathcal{W}_i^{(\alpha)}(r)$. Remaining single-particle elements $t_{n\neq 0}^{(\alpha)}$ are tunneling amplitudes between given lattice sites. It is worth noticing, that due to the construction of the Wannier functions, the tunneling amplitudes do not change the lattice band. In contrast, the interaction terms do not have this property and in principle they may couple any bands.

Since the Wannier functions form a complete basis in the space of single-particle states, the Hamiltonian (2.6) is fully equivalent to the initial Hamiltonian (2.1). However, the set of Wannier states is countable set of well localized states. Therefore, there is a quite natural route to obtain simplified models.

Standard Bose-Hubbard model

When the ultra-cold gas of weekly interacting bosons is confined in a very deep optical lattice (in practice $V_0 > 30E_R$) the typical interaction energies are much smaller than the energy gap between lattice bands and it is not possible to promote particles to higher bands. In consequence, the particles will occupy only the ground band of the optical lattice. At the same time, the tunnelings to the distant neighbors are negligible small when compared with the tunnelings to the nearest neighbor sites (details in Sec. 2.3.6.). Therefore, one can simplify the Hamiltonian (2.6) to the form

$$\hat{\mathcal{H}}_{\mathrm{BH}} = E_0 \sum_i \hat{a}_i^\dagger \hat{a}_i - t \sum_{\{i,j\}} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_i \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i, \tag{2.8}$$

where for simplicity we introduced $E_0 = t_0^{(0)}$, $t = t_1^{(0)}$, $U = U_{iiii}^{(0000)}$ and $\hat{a}_i = \hat{a}_{(0,0,0)i}$. The symbol $\{.\}$ is understood as summation over nearest neighbor sites. Properties of the ground-state of the system described by the Hamiltonian (2.8) are well understood and were analyzed with many complementary methods [4, 5, 6, 7, 8, 9, 10]. One of the most important consequences of the Hamiltonian (2.8) is the existence of two quantum phases in which the system can be found depending on the parameters of the model: *(i)* the superfluid phase, when the single-particle tunnelings between sites dominate the interactions; *(ii)* the Mott-Insulator phase, when the interactions dominate the tunnelings. It is matter of fact that the system (2.8) can be found in this phase only for commensurate densities ρ (the average number of particles in given lattice site). In 2002, in spectacular experiment with ultra-cold Rubidium atoms, these two quantum phases where observed and the controlled quench through the quantum phase transition point was performed [11]. Few years later the analogous experiment with ultra-cold fermions was done [12, 13]. In this way, an experimental realization of the famous Hubbard model, known in condensed matter physics from over 50 years, was achieved [14].

Experiments described in [11, 12, 13] had the fundamental impact to the progress in quantum engineering. They have proved that systems of ultra-cold atoms confined in optical lattices may play a role of quantum simulators, i.e. programmable experimental setups for different theoretical models known in physics and previously considered only as some interesting *toy-models*. From that time, the incessant race in theoretical extensions of Hubbard-like models and possible experimental realizations is present in the area of ultra-cold atoms [P1]. My habilitation scientific achievement directly fits in to this context.

2.3.3. Creation on demand of higher orbital states – [H1]

One of the possible extensions of the Hubbard model is taking into account the influence of higher bands of the optical lattice. This possibility is important since new methods of promoting ultra-cold atoms to higher orbital states were invented. In the first experiments of this kind with rubidium ^{87}Rb atoms the two-photon Raman processes stimulated by specially arranged counter propagating laser beams were used [15]. If the frequency difference between beams is tuned to the energy gap between bands of the periodic potential then the occupation of the atoms will oscillate between coupled states. The second path for creating orbital states originates in the resonant band-dependent tunneling between lattice sites [16]. As was mentioned above, in the simplest optical lattice arrangement (2.2) the band-dependent tunnelings are not present. Therefore, in the experiment [16] a more complicated geometry of the superlattice (the periodic potential with staggered deep and shallow sites) was used. When lattice depths are tuned appropriately one can induce the process of tunneling from the ground-band of the shallow site to the excited-band of the deep site. These two experiments have opened a completely new path of exploring ultra-cold gasses [17].

In the article [H1] I have presented a completely different receipt for controlled promoting of atoms to the higher orbital states. It originates on the parametric resonance phenomenon and it is described below. All calculations and numerical simulations in [H1] were done for realistic system of ultra-cold chromium atoms ^{52}Cr confined in the optical lattice created by the laser beams with $\lambda = 523\,\mathrm{nm}$. All quantitative results are determined under these assumptions.

To get better understanding of the new way of creating of the orbital states in optical lattices let us concentrate on the structure of the Hamiltonian (2.6) with a few lowest bands of the periodic potential. If we assume that the lattice depth in the Z direction is large then all excited states in this direction are not accessible and the lowest orbital states are the single particle states p_x and p_y (single excitations in X or Y direction respectively). If this is the case, the Hamiltonian (2.6) can be written as a sum of non-local single-particle Hamiltonian $\mathcal{H}_t$ and local, two-body interaction

Hamiltonian $\mathcal{H}_{\mathtt{int}}$ of the form:

$$\hat{\mathcal{H}}_{\mathtt{int}} = \sum_i \Bigg[\sum_\sigma E_\sigma \hat{n}_i^\sigma + \frac{U_{\sigma\sigma}}{2}\hat{n}_i^\sigma(\hat{n}_i^\sigma - 1) + \sum_{\sigma\neq\sigma'} U_{\sigma\sigma'}\hat{n}_i^\sigma \hat{n}_i^{\sigma'} \tag{2.9a}$$

$$+ \frac{U_{sx}}{2}\hat{a}_i^{\dagger 2}\hat{b}_i^2 + \frac{U_{sy}}{2}\hat{a}_i^{\dagger 2}\hat{c}_i^2 + \frac{U_{xy}}{2}\hat{b}_i^{\dagger 2}\hat{c}_i^2 + \text{H.c.}\Bigg], \tag{2.9b}$$

where $\sigma \in (s, x, y)$ runs over indices of the bands considered ($s = (0,0,0)$, $x = (1,0,0)$, $y = (0,1,0)$). For simplicity, I introduce notation $\hat{a}_i = \hat{a}_{si}$, $\hat{b}_i = \hat{a}_{xi}$, $\hat{c}_i = \hat{a}_{yi}$, $\hat{n}_{\sigma i} = \hat{a}^\dagger_{\sigma i}\hat{a}_{\sigma i}$, $E_\sigma = t_0^{(\sigma)}$, $U_{\sigma\sigma'} = U_{iiii}^{(\sigma\sigma'\sigma\sigma')}$. Detailed form of the single-particle part of the Hamiltonian $\mathcal{H}_{\mathrm{t}}$ is given and discussed in [H1], but it has no importance in further analysis.

At this point it is worth noticing that in the typical static experimental scenario the interaction terms in line (2.9b) have no importance and they can be neglected. It comes from the fact that typically the parameters U_{sx} and U_{sy} are essentially smaller than the corresponding energy gaps $\Delta E_\sigma = E_\sigma - E_s$. Therefore, due to the energy conservation, they cannot lead to a significant occupation of the related orbital states. Moreover, in the case of the optical lattice with different lattice depths in X and Y directions, the interaction energy U_{xy} is much smaller than the energy difference $\Delta E_{xy} = |E_x - E_y|$. In consequence, the last term of the local Hamiltonian can also be neglected. The values of these parameters for the example of a symmetric lattice (calculated directly from the shape of the Wannier functions) are presented in Fig. 2.2. Let us note, that neglecting of the mentioned terms in the Hamiltonian leads directly to the conservation of the number of particles in each orbital state. Then the only consequence of the multi-orbital interactions is the energy cost of the "density-density" interactions caused by the terms $\hat{n}_\sigma\hat{n}_{\sigma'}$ in the first line of the Hamiltonian. The key observation described in [H1] is the conclusion that in some dynamical scenarios these commonly neglected terms can be important since they may crucially change the properties of the system. Particularly, they can be exploited for creating orbital states on demand.

Let us consider the most simple situation at which the system of ultra-cold bosons is confined in a very deep optical lattice, in the Mott insulating phase far from the transition point, with an average filling $\rho = 2$. Additionally, let us assume that the lattice depth is not symmetric in X and Y directions and it is equal $V_x = 32E_R$ and $V_y = 20E_R$ respectively. In such a case, the single-particle tunnelings are completely dominated by interactions and they can be neglected. In consequence, the state of the system is a product state with respect to the lattice sites and the dynamics in each lattice site can be treated independently. Now, let us assume that the depth of the lattice in X direction is a periodic function of time with small amplitude and well defined frequency, $V_x(t)/E_R = 32 + A\sin(\omega t)$. Due to the structure of the Hamiltonian (2.9) the entire dynamics takes place in the subspace spanned by three two-body states $|200\rangle = \frac{1}{\sqrt{2}}\hat{a}^{\dagger 2}|\text{vac}\rangle$, $|020\rangle = \frac{1}{\sqrt{2}}\hat{b}^{\dagger 2}|\text{vac}\rangle$, $|002\rangle = \frac{1}{\sqrt{2}}\hat{c}^{\dagger 2}|\text{vac}\rangle$. In this subspace the

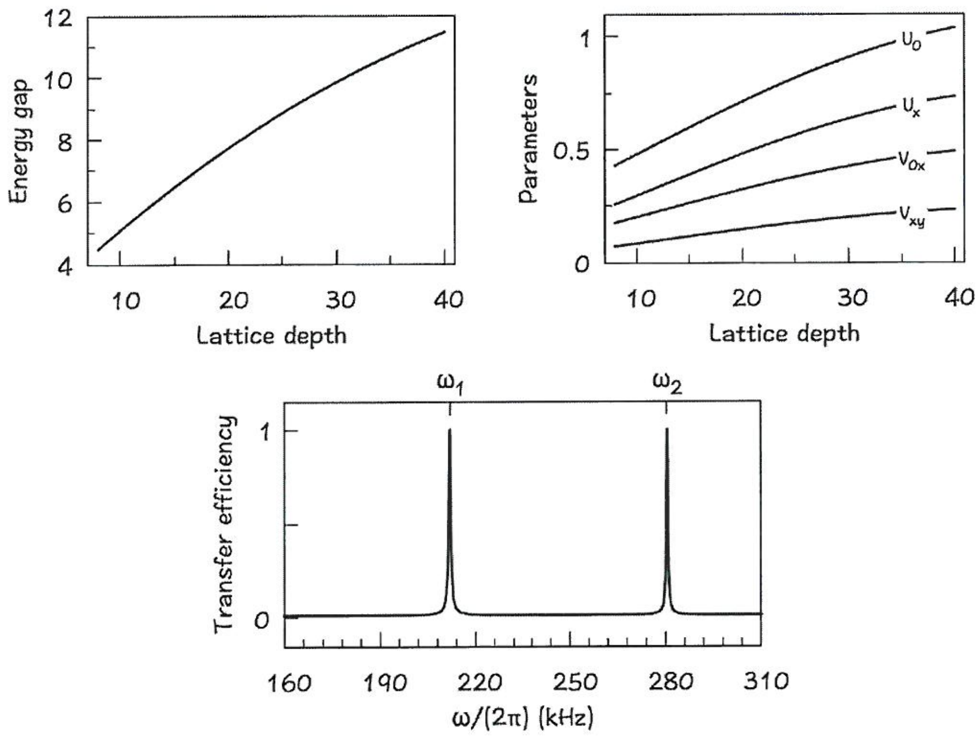

Figure 2.2: (upper panel) Parameters of the Hamiltonian (2.9) as functions of the lattice depth calculated directly from the shapes of the Wannier functions in a symmetric lattice ($V_x = V_y$) and $g = 1$. As it can be seen, in the typical experimental situation, the typical interaction energies are at least ten times smaller than the energy gap between lattice bands. (bottom panel) Transfer efficiency as a function of the frequency of the vibrating lattice ω. Two distinguishable peaks are related to the resonant condition at which the complete transfer of atoms to the excited band is possible. *The figure adopted from [H1].*

Hamiltonian has a simple matrix form

$$\hat{\mathcal{H}}_{\text{sub}}(t) = \begin{bmatrix} 2E_s + U_{ss} & U_{sx} & U_{sy} \\ U_{sx} & 2E_x + U_{xx} & U_{xy} \\ U_{sy} & U_{xy} & 2E_y + U_{yy} \end{bmatrix}. \tag{2.10}$$

All parameters of this Hamiltonian depend on time through nontrivial dependence on the lattice depth. If the frequency of the vibrations ω is tuned to the energy difference between the energy of the ground-state $2E_s+U_{ss}$ and the energy of the one of the excited states $2E_x+U_{xx}$ ($2E_y+U_{yy}$) calculated in the static situation ($A \equiv 0$), the parametric resonance phenomena induced by off-diagonal elements of the matrix (2.10) will take place. In consequence the system will oscillate between coupled many-body states. The bottom panel in the Fig. 2.2 shows the transfer efficiency, defined as the highest depletion of the initial state for a given frequency ω and amplitude $A = 4$. The positions of the resonant frequencies agree with the energy difference between considered many-body states. The full width at half maximum for the both resonances is about $\delta\omega/(2\pi) \approx 700\,\mathrm{Hz}$. At this point it is worth noticing, that the full transfer from the ground-band to the excited-band takes place in few milliseconds (details in [H1]). Therefore, it is much faster than the experimentally obtained decay time of hundreds of milliseconds [15]. It means that the mechanism of orbital states creation is quite fast and indeed can be exploited.

The theoretical analysis described above was also generalized to other experimental scenarios in [H1]. Among others, the dynamics of the system with initially symmetric lattice depths was considered. As it was shown, in this case the states $|020\rangle$ and $|002\rangle$, due to the off-diagonal terms and vanishing energy gap, are not good approximations for the excited eigenstates of the Hamiltonian (2.10). In this case, atoms can be efficiently transferred to the symmetric or antisymmetric superpositions of these states $|\pm\rangle = (|020\rangle \pm |002\rangle)/\sqrt{2}$ via symmetric or antisymmetric modulations in both directions. One of the most interesting results presented in [H1] is the recipe for creation orbital states of the form $|\pm\mathrm{i}\rangle = (|020\rangle \pm i|002\rangle)/\sqrt{2}$. It is possible by a specially arranged sequence of vibrations in X and Y direction. All these cases were studied with details in [H1].

Predictions described in [H1] were also tested against approximations of the model. It was done by performing simulations in the generalized models. It was shown that the tunnelings to neighboring sites, when taken into account, do not change transfer efficiency. It was also shown that higher bands of the optical lattices, neglected in the model studied, do not change the conclusions. As it is described in details in [H1], all these simulations confirm validity of the approximations.

Finally let me mention that the model studied completely neglects additional term in the time-dependent Schrödinger equation related to the time evolution of the single-particle basis. As was shown in [18] this additional correction leads to the spreading of the resonance but it does not change the maximal value of the transfer efficiency.

2.3.4. Properties of bosons loaded to the orbital states of the optical lattice – [H2]

It is natural, that controlled transfer of atoms to the higher orbital states of the optical lattice opens a new experimental possibilities. Indeed, properties of atoms in higher states can be completely different from those in the ground-band. For example, in the square lattice in each lattice site the two single-particle states p_x and p_y are degenerate. The question, how this degeneracy influences the properties of many-body system has inspired many discussions and has brought many interesting results [19, 20, 21, 22, 23, 24].

The article [H2] was inspired by the theoretical paper [25], where the authors analized the system of ultra-cold bosons loaded to the first excited band of the asymmetric two-dimensional optical lattice described by the potential

$$V(x,y) = V_x \sin^2(k_x x) + V_y \sin^2(k_y y). \tag{2.11}$$

The parameters of the lattice were chosen in such a way that the tunnelings are possible only in the one direction, i.e. we assume that $V_y \gg V_x$. At the same time the single-particle degeneracy of the orbitals p_x and p_y is restored by tuning the wave vectors of the lasers forming the lattice. Such a possibility is well visible in the harmonic approximation of the lattice site. In the vicinity of the minimum the potential can be approximately written as $V(x,y) \sim (V_x k_x^2)x^2 + (V_y k_y^2)y^2$. Therefore, by choosing the laser wavelengths to fulfill the condition $V_x/V_y = (k_y/k_x)^2$ the lattice site become rotationally symmetric and orbitals p_x and p_y are degenerated. In such a case the Hamiltonian (2.6) of bosons loaded to the first excited band can be reduced to the form:

$$\hat{\mathcal{H}}_{\texttt{orbit}} = \sum_i \hat{H}_i - \sum_{\{i,j\}} \left(t_x \hat{a}^\dagger_{xi} \hat{a}_{xj} + t_y \hat{a}^\dagger_{yi} \hat{a}_{yj} \right). \tag{2.12a}$$

The local part of this Hamiltonian has a form [3)]:

$$\hat{H}_i = \sum_\sigma \left[E_\sigma \hat{n}_i^\sigma + \frac{U_{\sigma\sigma}}{2} \hat{n}_i^\sigma (\hat{n}_i^\sigma - 1) \right] + \frac{U_{xy}}{2} \left[4\hat{n}_i^x \hat{n}_i^y + \hat{a}^{\dagger 2}_{xi} \hat{a}^2_{yi} + \hat{a}^{\dagger 2}_{yi} \hat{a}^2_{xi} \right]. \tag{2.12b}$$

The summation runs over excited orbitals $\sigma \in \{x, y\}$. The tunneling of bosons is allowed only in the X direction, but due to the different shapes of excited states in these directions, orbitals p_x and p_y tunnel with different amplitudes, $t_x < 0$, $t_y > 0$ and $|t_x| > |t_y|$. The ratio t_x/t_y is determined by the lattice depths in X and Y direction (see details in [H2]). It is quite obvious that the Hamiltonian (2.12) commutes with the operator of the total number of particles $\hat{N} = \hat{N}_x + \hat{N}_y$, where $\hat{N}_\sigma = \sum_i \hat{n}_i^\sigma$. However, due to the last two terms in the contact interactions, it does not commute with $\hat{N}_x$ and $\hat{N}_y$ separately. As it is seen, the interaction terms responsible for transfer of bosons between orbitals conserve the parity of the operators $\hat{N}_\sigma$. Therefore the Hamiltonian (2.12) has an additional Z_2 symmetry controlled by the operator $\hat{\mathcal{S}} = \exp(i\pi \hat{N}_y)$. With this observation, the ground-state of the system can be found by looking for ground-states in two eigensubspaces of the operator $\hat{\mathcal{S}}$. Let me name these state by $|\mathtt{G}_{\mathtt{even}}\rangle$ and $|\mathtt{G}_{\mathtt{odd}}\rangle$, respectively.

It can be shown that independently on the lattice depth, in the harmonic approximation, the parameters of the Hamiltonian (2.12) related to the contact interactions fulfill the condition $U_{xx} = U_{yy} = 3U_{xy}$. It means that the local part of the Hamiltonian can be rewritten in the form [25]:

$$\hat{H}_i = \frac{U_{yy}}{2} \left[\hat{n}_i \left(\hat{n}_i - \frac{2}{3} \right) - \frac{1}{3} \hat{L}^2_{zi} \right], \tag{2.13}$$

[3)]Note that the shape of the lattice is tuned in such a way the degeneracy between orbitals is restored, $E_x = E_y$. Therefore, all single-particle terms in the Hamiltonian can be neglected.

where $\hat{n}_i = \hat{n}_i^x + \hat{n}_i^y$ oraz $\hat{L}_{zi} = i\left[\hat{a}_{xi}^\dagger \hat{a}_{yi} - \hat{a}_{yi}^\dagger \hat{a}_{xi}\right]$ is a counterpart of the local angular momentum operator. These observation leads directly to the conclusion, that in the harmonic approximation, local Hamiltonian commutes with the local angular momentum operator and has rotational symmetry. The numerical analysis based on this assumption presented in [25] shows that in the limit of vanishing but non-zero tunneling ($|t_x| \rightsquigarrow 0$), when the average filling $\rho = 3/2$, the many-body ground-state of the system has anti-ferro-orbital ordering, i.e. the expectation value of the staggered angular momentum operator $\hat{\mathcal{L}} = \sum_i (-1)^i \hat{L}_{zi}$ is non-zero. It means, that in the limit of small tunnelings, the ground-state is degenerated and due to the spontaneous breaking of the symmetry $\hat{\mathcal{S}}$ induced by tunneling the time-reversal symmetry is also broken.

The essence of our work [H2] is showing that going beyond the harmonic approximation changes the scenario described above completely. In general, different interaction parameters calculated directly from the Wannier functions do not obey the particular condition $U_{xx} = U_{yy} = 3U_{xy}$. Whenever lattice depths in X and Y directions are different and the degeneracy between orbitals p_x and p_y is restored by tuned laser wavelengths, the exact inequalities $3U_{xy} < U_{xx} < U_{yy}$ hold. In consequence, the local part of the Hamiltonian can not be written in the form (2.13) and the local angular momentum operator $\hat{L}_{zi}$ does not commute with the Hamiltonian. One would think, that this apparently technical and quantitatively small change modifies previous results insignificantly. However, the results obtained for the exact Hamiltonian (2.12) and for the Hamiltonian in the harmonic approximation differ not only quantitively but also qualitatively.

The first difference between the exact and the approximated model is visible in the limit of small tunneling, $|t_x| \rightsquigarrow 0$. The orbitals p_x and p_y are degenerated on the level of the single-particle part of the Hamiltonian. However, due to the contact interactions, this degeneracy is lifted. For the filling $\rho = 3/2$, the lowest energy many-body state is some specific combination of an insulating phase in p_y orbital and half-filled superfluid phase in p_x orbital [4)]. In the limit of huge tunnelings, when all interactions can be neglected, all bosons occupy the p_y orbital and form the superfluid phase with filling $\rho = 3/2$. In these two limits the ground-state of the system is not degenerate and it is the eigenstate of the symmetry operator $\hat{\mathcal{S}}$. When the tunneling is changed, the ground-state of the system smoothly goes from one phase to the other. In the left panel of the Fig. 2.3 the average densities in each orbital ($\rho_\sigma = \sum_i \langle \hat{n}_{\sigma i}\rangle / L$) and hopping correlation ($h_\sigma = \sum_i \langle \hat{a}_{\sigma i}\rangle / L$) as a functions of normalized tunneling are presented. The results were obtained by exact diagonalization of the Hamiltonian in the lattice with $L = 8$ sites.[5)]

[4)]This observations is a natural consequence of two facts: $3U_{xy} < U_{xx} < U_{yy}$ and $|t_x| > |t_y|$.

[5)]The insulating phase in given orbital is characterized by a vanishing hopping h_σ and integer filling ρ_σ. In the superfluid phase, large nonlocal single-particle correlations are present and they are characterized by a non-vanishing hopping h_σ.

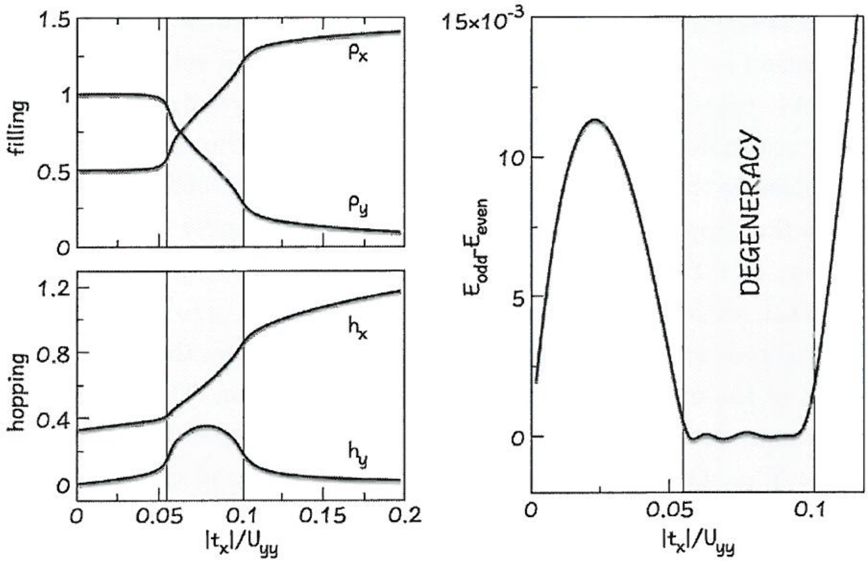

Figure 2.3: (left panel) The average filling of the orbitals ρ_σ and the hopping correlation h_σ as functions of normalized tunneling. The average filling of the lattice $\rho = 3/2$. (right panel) The energy difference between ground-states calculated in eigen-subspaces of the symmetry operator $\hat{\mathcal{S}}$ as a function of the normalized tunneling. For well defined and finite range of tunnelings (gray area) the ground-state of the system is degenerated. All plots obtained from the exact diagonalization of the Hamilotnian with $L = 8$ sites. *The figure adopted from [H2].*

The most interesting scenario, which is absolutely not present in the harmonic approximation, is realized in the vicinity of the point where both orbitals are significantly occupied ($\rho_x \approx \rho_y$). From the numerical analysis based on the exact diagonalization of the Hamiltonian for finite lattice size L it follows, that in the well defined range of tunnelings the ground-state of the system is degenerate. In practice, the method gives us the energy difference between the ground-states in the eigenspaces of $\hat{\mathcal{S}}$. For given lattice size L the ground-state become degenerate exactly in L points and the range of tunnelings where it happens is of finite size and it is well defined (right panel in Fig. 2.3). Moreover, the energy difference $|E_{\text{odd}} - E_{\text{even}}|$ calculated in points where the degeneracy is not present decreases with L. On this basis we conclude that in the thermodynamic limit $L \to \infty$ the many-body ground state of the system is degenerate for any tunneling in this range.

In the range of restored degeneracy both states $|\mathtt{G}_{\text{even}}\rangle$ and $|\mathtt{G}_{\text{odd}}\rangle$ have exactly the same energy. From the model point of view, any of their superposition $|\mathtt{G}\rangle = \cos(\theta)|\mathtt{G}_{\text{even}}\rangle + \sin(\theta)e^{i\phi}|\mathtt{G}_{\text{odd}}\rangle$ is an equally good ground-state of the system. Nevertheless, since we are dealing with the many-body system, it is quite natural that in the limit of the macroscopic system ($L \to \infty$) different superpositions will respond differently to the external interactions with environment which can not be neglected. Induced by this interaction, the system will spontaneously select well defined superposition which is the most robust for the external disturbance [6]. This phenomenological observation can be formulated more formally via einselection principle [26]. According to this principle, the macroscopic state that is realized physically should exhibit as low entanglement as possible. In other words, the state which is selected should be as close to the product state (with respect to the lattice sites) as possible. In practice, to find appropriate superposition, we numerically minimize the entanglement entropy as a function of angles θ and ϕ. Finally, without going into details described in [H2], we find two states $|\mathtt{G}_\pm\rangle = (|\mathtt{G}_{\mathtt{even}}\rangle \pm i|\mathtt{G}_{\mathtt{odd}}\rangle)/\sqrt{2}$ which are the closest states to the product state. Note, that in the case studied the states are *complex* superpositions of the eigenstates of the symmetry operator. This observation leads directly to the non-vanishing expectation value of the staggered angular momentum operator $\hat{\mathcal{L}}_z = \sum_j (-1)^j \hat{L}_{zj}$. In consequence, in the range of tunnelings where the degeneracy of the ground-state is restored, due to the spontaneous symmetry breaking mechanism, the system is found in the anti-ferro-orbital order and breaks the time-reversal symmetry. All results obtained with the exact diagonalization of the Hamiltonian were confirmed by co-authors in the large scale calculations with DMRG method.

At this point it is worth noticing, the article [H2] was nominated by the director of the Institute of Physics as the best publication in the Institute in 2013.

2.3.5. Bose-Hubbard models with three-body interactions – [H3-H5]

Higher orbitals of the optical lattice can be taken into account also in an effective manner. It is possible, whenever any interaction term leading to the change of the orbital is much smaller then the energy gap between considered states. In such a case, higher orbital states play a role of virtual, intermediate states and their influence can be taken into account as corrections to the ground-band states in the second order of perturbation theory. Since contact interactions considered in the model have a short-range and local form, they can be effectively included to the Bose-Hubbard model (2.8) by generalizing the energy interaction parameter U to the parameter which depends on occupation $\tilde{U}(n)$. Obviously, the new parameter has to obey natural requirements $\tilde{U}(0) = \tilde{U}(1) = 0$ and $\tilde{U}(2) = U$. Therefore, the lowest nonvanising correction is proportional to $(n-2)$ and in consequence the expansion of the interaction energy has a form:

$$\tilde{U}(n) = U + \frac{W}{3}(n-2) + \ldots. \tag{2.14}$$

[6] The similar effect is present in the Ising model without external magnetic field. The ground-state manifold is two-dimensional and it is spanned by the eigenstates of the spin-reversal operator, which commutes with the Hamiltonian. In the thermodynamic limit the system spontaneously select the state with the largest magnetization which breaks the immanent symmetry of the Hamiltonian.

In this way one obtains the effectice Hubbard-like model of the form:

$$\hat{\mathcal{H}}_{\text{eff}} = E_0 \sum_i \hat{a}_i^\dagger \hat{a}_i - t \sum_{\{i,j\}} \hat{a}_i^\dagger \hat{a}_j + \frac{U}{2} \sum_i \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i + \frac{W}{6} \sum_i \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i^\dagger \hat{a}_i \hat{a}_i \hat{a}_i. \tag{2.15}$$

Formally, effective correction W can be viewed as an additional, local, three-body interaction. It is worth noticing that parameter W, as well as higher corrections, were determined in recent experiment with ultra-cold rubidium atoms ^{87}Rb confined in optical lattice [27]. It means that the extended model (2.15) is not only a theoretical divagation but it has also some experimental meaning.

From the effective approach point of view, the three-body interactions term W is directly related and determined by the two-body one and therefore it can not be controlled independently [28, 29]. However, this rigorous conjecture is valid only in the case of short-range interactions. As was shown in [30], whenever long-range interactions are considered, an effective three-body interaction can be tuned independently from the two-body one in a huge range of parameters. Moreover, as was shown recently in [31], it is possible to use ultra-cold spinor atoms in Mott-Insulator phase to simulate Bose-Hubbard Hamiltonian with local three-body interactions. All these together mean that it is quite important to perform general studies on the properties of the Hamiltonian (2.15).

The first theoretical analysis of the Bose-Hubbard model with three-body interactions was done in the perturbative mean-field theory framework [32, 33]. In these articles, it was shown that three-body interactions lead directly to the shift of the boundary between insulating and superfluid phases. However, in this approach the lowest boundary for filling $\rho = 1$ is insensitive to the three-body interaction. The perturbative mean-field approach is directly related to the [H5] described below. In other work [34] the two-dimensional version of the model was studied within Monte Carlo method. This work focuses on the first-order phase transition induced by changes of the chemical potential (changes of the total number of particles in the system). Also, the model in the one-dimensional case was partially studied in [35] via DMRG method (Density Matrix Renormalization Group). It was shown that, in contrast to the mean-field approach, the boundary of the first insulating lobe (for $\rho = 1$) is changed when three-body interactions are considered. This subtle analysis has shown that properties of the Hamiltonian (2.15) should be analyzed with more accurate methods than simple mean-field approach of independent lattice sites.

Exact diagonalization of the one-dimensional model – [H3]

I used an exact diagonalization of the one-dimensional Hamiltonian (2.15) to study its properties mainly in the vicinity of the phase transition point [H3]. Here, in contrast to previous analysis [35], calculations were performed also for higher filling $\rho = 2$ where three-body interactions play a crucial role in stabilizing an insulating phase. Due to the experimental circumstance presented in [27], both repulsive and attractive three-body interactions were studied in the paper.

The method of an exact diagonalization of the Hamiltonian belongs to the "brute-force" methods. On the one hand, it is exact since it does not contain any approximations. On the second, it is highly limited by operational resources of nowadays computers since it is based on diagonalization of huge matrices which dimension scales exponentially with the size of the problem. The method is based on the observation that whenever the system is in the insulating phase then the finite energy gap Δ for adding/subtracting a particle to/from the system is present. The gap decreases with increasing tunneling and it becomes equal to 0 in the quantum phase transition point t_c/U. When performing an exact diagonalization of the Hamiltonian with fixed number of particles N and fixed size of the lattice L one finds the ground-state of the system $|\mathtt{G}_{N,L}\rangle$ and its energy $E(L,N)$. Then one defines upper and bottom border of the insulating phase [7]:

$$\mu_+(\rho, L) = E(L, \rho L + 1) - E(L, \rho L), \tag{2.16a}$$

$$\mu_-(\rho, L) = E(L, \rho L) - E(L, \rho L - 1). \tag{2.16b}$$

It is quite obvious that both quantities strongly depend on the size of the system L. Therefore, calculations for given t, U, and W are repeated for different sizes (in this case $L = 5, \ldots, 8$). It is worth mentioning that the diagonalization is performed in a full many-body Fock basis, i.e. it is assumed that each site of the lattice can be occupied by $0, \ldots, N$ bosons. In the next step, borders $\mu_\pm$ are extrapolated to the infinite-size limit $L \to \infty$. The extrapolation is possible since the quantities considered depend linearly on the system size [H3]. The energy cost in the thermodynamic limit is defined as a difference of the quantities obtained in this way, $\Delta = \mu_+ - \mu_-$. Without going into detalis described in [H3], the quantum phase transition point t_c from the insulating to the superfluid phase is determined from the condition $\Delta = 0$. At the same time, both quantities $\mu_\pm$ determine the border on the phase diagram between insulating and superfluid phase (Fig. 2.4).

The results obtained in [H3] show that for higher filling $\rho = 2$ the size of the insulating lobe is very sensitive to the presence of three-body interactions. It is visible even in the limit of vanishing tunneling $t \to 0$, where an analytical result can be obtained. In this limit, for $\rho = 1$, the energy cost Δ does not depend on W. However for $\rho = 2$ it is equal $\Delta = U + W$. An increasing sensitivity for three-body interactions is quite natural when microscopic picture is considered. Any tunneling process which leads to destruction of the insulating phase with $\rho = 2$ has to compete not only with two-body interactions but also with three-body ones.

Exact diagonalization of the Hamiltonian gives also a possibility to check numerically what is the universality class of the model at the quantum phase transition

[7] The quantities $\mu_\pm$ are direct counterparts of the chemical potential defined for finite-size systems.

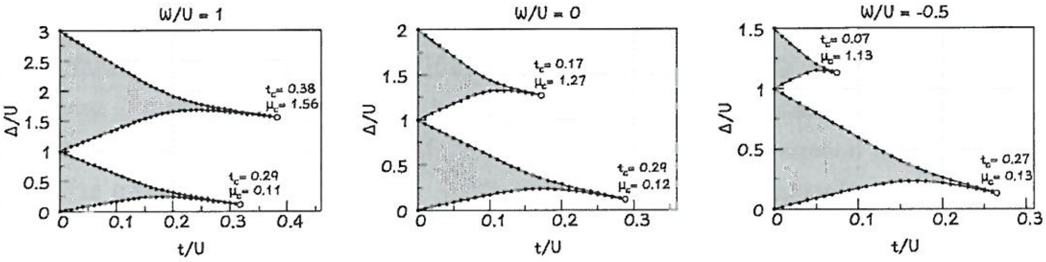

Figure 2.4: Phase diagrams for the one-dimensional model (2.15) for three different three-body interactions W/U. Note, the first insulating phase ($\rho = 1$) is almost insensitive to the presence of three-body interactions. However, the shape of the second insulating lobe ($\rho = 2$) crutialy depends on these interactions. The lobe is enlarged (shrunk) for repulsive (attractive) three-body forces when compared to the lobe predicted by the standard Bose-Hubbard model. *The figure adopted from [H3].*

point. It is known that at the tip of the Mott lobe the one-dimensional Bose-Hubbard model belongs to the universality class of the XY spin model. It means that the phase transition from the insulating to the superfluid phase is of the Berezinskii-Kosterlitz-Thouless (BKT) type [36, 37, 38]. It is characterized by the exponential decay of the correlation length which can be related to the characteristic decay of the energy cost Δ on the insulating side:

$$\ln\left(\frac{\Delta}{U}\right) \sim \frac{1}{\sqrt{1-t/t_c}}, \qquad \text{for } t < t_c. \tag{2.17}$$

The numerical results (details in [H3]) show that the universality of the phase transition does not depend on additional local three-body interactions. Independently on the sign and amplitude of three-body interactions W, the quantum phase transition point from the insulating to the superfluid phase always belongs to the BKT class.

DMRG method for the model without two-body interactions – [H4]

The observation, that in the one-dimensional model the universality class of the phase transition from the insulating to the superfluid phase does not depend on the three-body interactions, have inspired me to study an extreme version of the model – the model with huge three-body interactions, $U/W \to 0$. In the article [H4] I study properties of the one-dimensional model defined via the following Hamiltonian [42, 43]:

$$\hat{\mathcal{H}}_{\mathsf{3body}} = -t\sum_i \hat{a}_i^\dagger(\hat{a}_{i-1} + \hat{a}_{i+1}) + \frac{W}{6}\sum_i \hat{a}_i^\dagger\hat{a}_i^\dagger\hat{a}_i^\dagger\hat{a}_i\hat{a}_i\hat{a}_i. \tag{2.18}$$

Here, in contrast to [H3], the ground-state of the model is studied on a large scale ($L \approx 100$). Obviously, for such a big system, the exact diagonalization method fails. Therefore, this analysis is performed using DMRG method. The DMRG algorithm was invented in the 90s of the previous century and till now it is viewed as the most effective large-scale method for one-dimensional problems [44]. Without going into details (for example, see the beautiful review [45]), as a result of the algorithm one gets the ground-state energy $E(L, \rho, \kappa)$ calculated for a given chain size L and fixed total number of bosons $\rho \cdot L + \kappa$. In principle, it is also possible to estimate the value of any k-order correlation function of the form $\langle \mathtt{G}|\hat{O}_{i_1} \cdots \hat{O}_{i_k}|\mathtt{G}\rangle$, where $\hat{O}_i$ is an arbitrary operator acting locally on site i (in practice k is not large). Without any additional cost, due to the construction of the algorithm, it is also possible to calculate entanglement entropy of the subsystem of the size l, $\mathcal{S}(l, L) = -\mathrm{Tr}\,(\hat{\rho}_l \ln \hat{\rho}_l)$. The operator $\hat{\rho}_l = \mathrm{Tr}_{L-l}|\mathtt{G}\rangle\langle\mathtt{G}|$ is a reduced density matrix of the subsystem of the size l obtained by tracing out remaining degrees of freedom from the projection operator $|\mathtt{G}\rangle\langle\mathtt{G}|$. As discussed in [H4], the entanglement entropy of the subsystem can be used for independent estimation of the position of the phase transition point from insulating to superfluid phase.

In the first part of the article [H4] the analysis of the ground-state of the system was performed in close analogy to the concept described in [H3]. The ground-state energy $E(L, \rho, \kappa)$ calculated for $\rho \in \{2, 3\}$, $\kappa \in \{-1, 0, 1\}$, and different system sizes $L = 32, 48, \ldots, 128$ was used to determine the boundaries of the insulating phase $\mu_\pm$. In consequence, the energy cost Δ for adding/subtracting a particle to/from the system is also calculated. In this way, one obtains the insulating lobes on the phase diagram of the model and determines the positions of the phase transition points for the first two insulating phases (Fig 2. in [H4]). Due to the huge accuracy of calculations performed, the energy cost Δ scales exactly as it is predicted for BKT universality class (2.17) (left panel in Fig. 2.5).

In the second part of the article [H4], the position of the critical point was determined in completely independent way which does not originate on the energetic arguments. The method exploits universal properties of the entanglement entropy $\mathcal{S}$ in finite-size systems. It is known, that in the thermodynamic limit $L \to \infty$, an entanglement entropy of the subsystem is logarithmically divergent with the size of the subsystem l whenever long-range correlations are manifested in the ground-state of the system, i.e. the system is in the superfluid phase. In contrast, if the system remains in the insulating phase all correlations have finite range. Therefore, for large enough sizes of the subsystem, an entanglement entropy saturates on some finite value. This observation has some consequences also for the finite-size systems ($L < \infty$). It can be shown that in such a case the entanglement entropy of the subsystem treated as a function of the subsystem size l has the following form [46, 47]:

$$\mathcal{S}(l, L) = \frac{\mathtt{c}}{3\kappa} \ln\left[\frac{\kappa L}{\pi} \sin\left(\frac{\pi l}{L}\right)\right] + s(L) + \mathcal{O}\left(\frac{l}{L}\right), \tag{2.19}$$

where $\kappa = 1$ or $\kappa = 2$ for periodic and open boundary conditions respectively. The parameter $\mathtt{c}$ is called the central charge of the model. If the system remains in the insulating phase the central charge is equal to 0. However, it is greater than 0 whenever long-range correlations are present in the system (like in the superfluid phase).

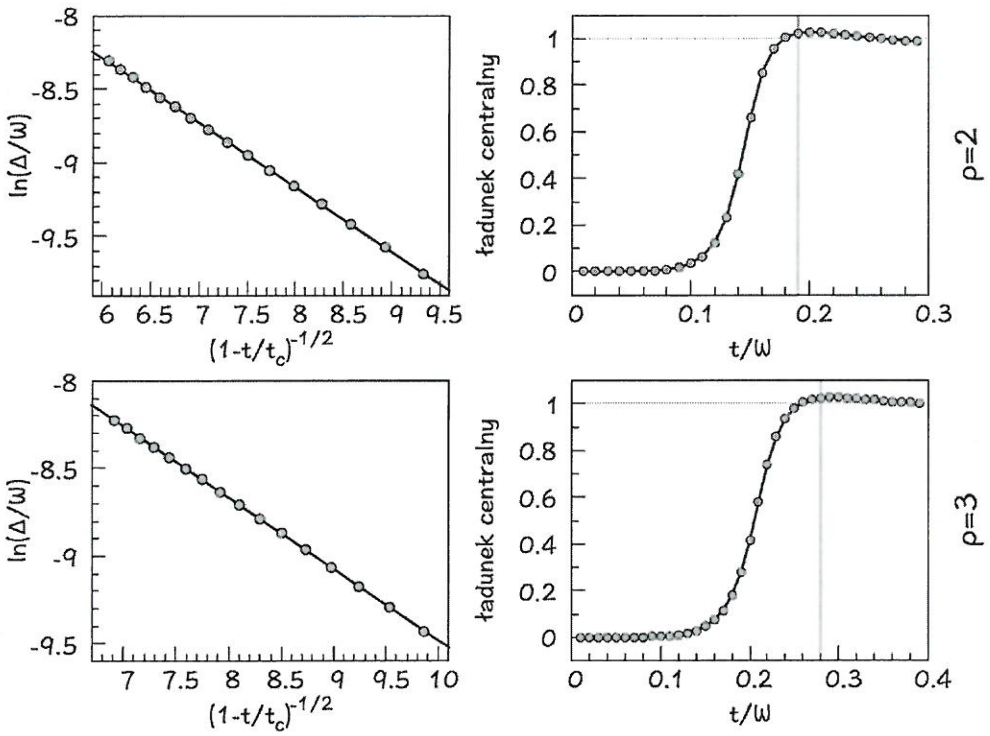

Figure 2.5: (left panel) The energy cost Δ determined in the insulating phase as a function of the tunneling t. In chosen scaling the numerical points fit to the linear regression predicted for BKT universality class (2.17). The numerical points were obtained with DMRG method for the system size $L = 128$. (right panel) The central charge c as a function of the tunneling t obtained from the scaling of the entanglement entropy (2.19). In the vicinity of the phase transition point the central charge changes its value from 0 (in insulating phase) to 1 (in superfluid phase) and it reaches the maximal value at the critical point. *The figure adopted from [H4].*

It can be shown, that in the ideal superfluid phase in the limit of infinite size $L \to \infty$ the central charge is equal to 1. As it is shown in [H4], all this theoretical predictions are clearly visible in numerical results obtained for the system studied with the DMRG method. First, the entanglement entropy scales appropriately with the size of the subsystem. The numerical results for $\mathcal{S}(l, L)$, if plotted as a function of $\ln[\sin(\pi l/L)]$, fit to the linear regression (Fig 4. in [H4]). With this observation, the central charge c as a function of the tunneling can be estimated. The central charge determined with the help of this procedure vanishes deeply in the insulating phase and saturates at 1 far in the superfluid phase (right panel in Fig. 2.5). In the vicinity of the phase transition point one observes a rapid change in the behavior of the central charge and the maximal value of c is reached near the position of the critical point determined with the previous method. Such behavior of the central charge is very similar to the situation observed in the standard Bose-Hubbard model [10]. It is believed that non monotonicity in the central charge behavior is a direct consequence of the finite size of the system, and in the thermodynamic limit it smoothly flows to "step-like" behavior.

The Gutzwiller anzatz approach - [H5]

As it was mentioned above, the model (2.15) was analyzed in the perturbative mean-field approach in [32, 33]. The method based on the assumption that the superfluid phase appears in the system when the $U(1)$ symmetry of the model is spontaneously broken and the non-vanishing value of the local order parameter $\Phi_i = \langle \hat{a}_i \rangle$ is present. The existence of the non-vanishing order parameter Φ_i is not consistent with the conservation of total number of particles $\hat{N} = \sum_i \hat{a}_i^\dagger \hat{a}_i$. Therefore, the analysis has to be carried out in the grand canonical ensemble. The average number of particles in the system is controlled by the chemical potential μ. The assumption on the translational invariance of the system together with the following approximation of tunneling terms[8)]:

$$\hat{a}_i^\dagger \hat{a}_i \approx |\Phi|^2 + \overline{\Phi}\hat{\delta} + \Phi\hat{\delta}^\dagger = \Phi\hat{a}_i^\dagger + \overline{\Phi}\hat{a}_i - |\Phi|^2, \tag{2.20}$$

bring the Hamiltonian (2.15) to the sum of independent Hamiltonians for each site separately

$$\hat{\mathcal{H}}_{\mathrm{MF}} = \sum_i \left[\frac{U}{2}\hat{a}_i^\dagger\hat{a}_i^\dagger\hat{a}_i\hat{a}_i + \frac{W}{6}\hat{a}_i^\dagger\hat{a}_i^\dagger\hat{a}_i^\dagger\hat{a}_i\hat{a}_i\hat{a}_i - tz\Phi\left(\hat{a}_i^\dagger + \hat{a}_i - \Phi\right) - \mu\hat{a}_i^\dagger\hat{a}_i \right]. \tag{2.21}$$

The coordination number z defines the number of neighboring sites in the lattice.

Directly from energetic arguments one finds that the ground-state of the Hamiltonian (2.21) can be obtained for vanishing or not vanishing order parameter Φ depending on the value of parameters t, μ, U i W. It is a matter of fact, that vanishing value of the order parameter Φ consequently leads to the integer value of the average number of particles $\langle \sum_i \hat{a}_i^\dagger \hat{a}_i \rangle / N$, i.e. the ground-state of the model (2.15) can be found in the insulating phase only for integer filling ρ. The border in the phase diagram between the insulating and superfluid phase $t_c(\mu)$ can be found by solving well defined energetic condition in which both phases have the same energy (detalis in [32, 33] and [H5]). The solid line in Fig. 2.6 represents the border between phases for different values of three-body interactions.

The perturbative mean-field approach is sufficient to determine the boundary between quantum phases of the ground-state, but it has some fundamental limitations. It is based on approximate decomposition (2.20) which is not valid in deep superfluid phase dominated by single-particle tunnelings. Therefore, the value of the order parameter is not well determined in this phase. Moreover, the method assumes ideal

[8)]Directly from the definition of the order parameter one finds that the annihilation operator can be written as $\hat{a}_i = \Phi + \hat{\delta}_i$, where $\hat{\delta}_i$ is some well defined operator with vanishing expectation value.

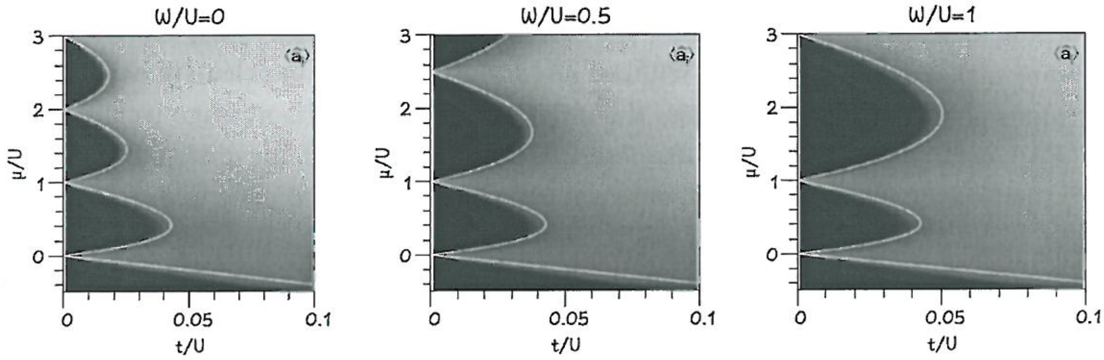

Figure 2.6: Phase diagrams of the model (2.15) obtained with mean-field methods for different values of the three-body interaction W. White line represents the border between phases obtained with perturbative mean-field theory. The order parameter determined numerically with the Gutzwiller ansatz is presented in the background. Both method determine the same position of the border between insulating (black area) and superfluid phase (light area). *The figure adopted from [H5].*

translational invariance of the system. Therefore, it is hard to check the stability of the solution to the local differences in the density of particles. In the article [H5] we compare all pretictions of the perturbative mean-field approach with the results obtained in the framework of, so called, Gutzwiller ansatz [39, 40, 41] which has no mentioned disadvantages but still it belongs to the class of the mean-field methods.

The Gutzwiller ansatz approach belongs to the variational methods. Therefore, the starting point of the analysis is the definition of the whole family of probe functions for the ground-state of the system. In this case we assume that the ground-state of the system $|\Psi\rangle$ is a product state with respect to the lattice sites, $|\Psi\rangle = \prod_i |\psi_i\rangle$. The state of the subsystem $|\psi_i\rangle$ is decomposed in the local Fock base cut at a large enough number of particles $n_{\max}$. Strictly, the ground-state of the system is assumed to be of the form:

$$|\Psi\rangle = \prod_i \sum_{n=0}^{n_{\max}} \alpha_i(n)|n\rangle_i, \tag{2.22}$$

where $\alpha_i(n)$ is the probability amplitude of finding n bosons in the i-th lattice site. The normalization condition for these amplitudes has a form $\sum_n |\alpha_i(n)|^2 = 1$. It is quite obvious that the product state (2.22) cannot describe any non-local correlations in the superfluid phase. Nevertheless, the superfluid long-range coherence is captured at this level by non-trivial superpositions of local Fock states and the local value of the order parameter $\Phi_i = \langle \hat{a}_i \rangle$ can be determined.

The amplitudes of the state which mimics the ground-state of the system in the family of probe functions (2.22) are determined by minimizing the expectation value of the Hamiltonian (2.15). In the article [H5] the minimization was performed numerically with imaginary-time method on the square 8×8 lattice with periodic boundary conditions and with $n_{\max} = 4$. Without going into details described in [H5], as a result of this minimization one obtaines the colection of amplitudes $\{\alpha_i(n)\}$ related to the state with the lowest energy. Without any difficulties one can calculate expectation values of different operators like the order parameter Φ_i, the average local density $\langle\hat{a}_i^\dagger\hat{a}_i\rangle$, etc. The numerical results obtained fully confirm that the order parameter is transitionally invariant, i.e. Φ_i does not depend on i. Shaded plots in Fig. 2.6 show the value of Φ as a function of the parameters of the Hamiltonian for different values of three-body interactions. The border between insulating phase (the black area with $\Phi = 0$) and superfluid phase (gray-scaled area with $\Phi \neq 0$) is clearly visible and its position coincidences with the border determined with the perturbative method (white line).

2.3.6. Quantum phase transition in a shallow optical lattice – [H6]

In my recent paper [H6], I discuss a different regime of experimental parameters where interactions between particles are still very weak but the periodic potential of the optical lattice is very shallow. This problem seems to be difficult for two reasons. The first is related to enlarged spreading of the Wannier functions which directly increases importance of the tunnelings to the further sites $t_i^{(0)}$ ($|i| > 1$). The second comes from the fact that the band gap between the ground-band and a excited-band becomes smaller and it may be of the same size as typical interactions between particles. Then, the influence of higher bands of the periodic potential has to be taken into account. From the point of view of the Hubbard-like model the description of the system becomes quite complicated and one can have conviction that some other methods should be adopted. One possibility is to exploit some methods, working directly in the configuration space of confined particles. This idea was adopted recently with the so-called hybrid quantum Monte Carlo method and used to study the phase transition from the insulating to the superfluid phase in a very shallow optical lattice [48].

In my recent paper [H6], I tried to formulate the problem of shallow lattices in the old-fashioned Hubbard-like description approach. The main goal is to find the most relevant corrections to the standard Bose-Hubbard model. For simplicity, I assume that the lattice is shallow only in the one spatial direction. In the remaining directions the lattice is very deep, therefore one can assume that the dynamics of bosons is frozen. With this assumption, the model (2.6) reduces to the one-dimensional case. The subtle analysis shows that in the vicinity of the quantum phase transition point (for filling $\rho = 1$) the only correction that is relevant, even for very shallow lattice ($V \approx 4E_R$), comes from the tunnelings to the next-nearest neighbor $t_2^{(0)}$. In particular, all other corrections that originate in the higher bands' physics, as well as in the inter-site interactions, are less important, and at the first approximation they can be omitted.

To justify these non-obvious statements let me first show that the influence of higher orbitals of the optical lattice can be neglected. An influence of higher bands depends on the interactions which can promote bosons to higher orbitals. The most relevant term of this kind is controlled by the parameter $U_{\mathrm{IB}} = U^{(ppss)}_{iiii}$, where s and p denote ground and excited band respectively. This term controls the process of a direct promotion of two bosons from the ground-band $|\mathrm{g}\rangle = (\hat{a}^\dagger_{si})^2|\mathrm{vac}\rangle$ to the first excited band in the same site of the lattice $|\mathrm{e}\rangle = (\hat{a}^\dagger_{pi})^2|\mathrm{vac}\rangle$. In the subspace of two-body states in the chosen lattice site and reduced to the two lowest bands the Hamiltonian has a form:

$$\hat{\mathcal{H}}_{\mathrm{loc}} = 2\widetilde{\Delta}|\mathrm{e}\rangle\langle\mathrm{e}| + 2U_{\mathrm{IB}}\left(|\mathrm{g}\rangle\langle\mathrm{e}| + |\mathrm{e}\rangle\langle\mathrm{g}|\right), \tag{2.23}$$

where $\widetilde{\Delta} = E_p - E_s$ denotes the single-particle band-gap. Due to the inter-band interaction term U_{IB}, the contribution of the excited orbital $|\mathrm{e}\rangle$ to the ground-state $|\mathrm{G}\rangle$ of the Hamiltonian (2.23) has a form

$$|\langle\mathrm{e}|\mathrm{G}\rangle|^2 = \frac{x^2}{2x^2 + 2 + 2\sqrt{1+x^2}}, \qquad x = \frac{2U_{\mathrm{IB}}}{\widetilde{\Delta}}. \tag{2.24}$$

This quantity is a proper measure of the influence of higher orbital to the properties of the ground-state of the system. One can estimate its value in the vicinity of the quantum phase transition point. It is known that in the one-dimensional case (for filling $\rho = 1$) the critical tunneling is in the region $t/U \gtrsim 0.25$. The band-gap $\widetilde{\Delta}$ depends on the lattice depth and even for very shallow lattices (as shown in [H5]) fulfills the condition $t/\widetilde{\Delta} < 0.1$. At the same time, the ratio of he inter-band interaction to the ordinary two-body interaction in the ground-band U_{IB}/U is always smaller than 0.5. These three facts together lead directly to the conclusion that in the vicinity of the phase transition the condition $U_{\mathrm{IB}}/\widetilde{\Delta} < 0.2$ is satisfied. It means that even for very shallow lattices the excited state contribution $|\langle\mathrm{e}|\mathrm{G}\rangle|^2 < 4\%$. Therefore, the influence of higher orbitals can be neglected. At this moment it is worth noticing, that relatively small contribution of the higher bands is caused by the conservation of the energy. In the vicinity of the phase transition, the energy cost of transfer bosons from the ground to the excited band is much larger than the typical interaction energy which lead to this promotion. However, this argument is not general and in the case studied it is valid only for small densities. For larger densities all interactions are enhanced by a factor $\sqrt{n}$ and in consequence they can become larger than the band-gap $\widetilde{\Delta}$.

After we excluded influence of the higher bands we consider processes which act within the ground-band of the lattice. These processes have completely different character since they are not suppressed by the conservation of energy. There is no single-particle cost to move particle between lattice sites. Therefore, these processes, even if their characteristic energy is small, can lead to a large non-local correlations and in consequence they can play a substantial role in the vicinity of the phase transition where the insulating phase is destroyed. The most relevant process of this kind (and the only parameter in the limit of a very deep lattice) is obviously the single-particle tunneling to the neighboring site t. In the case of shallow lattice, one can find the most important correction to this process by comparing the importance of the tunneling to the next-nearest neighbors $t' = t_2^{(s)}$ and interaction-induced tunneling $U_{\mathrm{IS}} = U_{iiij}^{(ssss)}$, for $j = i \pm 1$. The detailed analysis (described in [H6]) shows that in the vicinity of the phase transition point the ratio $|U_{\mathrm{IB}}|/t$ is substantially smaller than the ratio $|t'|/t$. It means that in the vicinity of the phase transition of the one-dimensional model the most relevant correction to the standard Bose-Hubbard model is related to the next-nearest neighbor tunneling t'. Therefore, the extended Hubbard-like model has a form:

$$\hat{\mathcal{H}}_{\mathtt{Shallow}} = -t\sum_i \hat{a}_i^\dagger(\hat{a}_{i-1} + \hat{a}_{i+1}) - t'\sum_i \hat{a}_i^\dagger(\hat{a}_{i-2} + \hat{a}_{i+2}) + \frac{U}{2}\sum_i \hat{a}_i^\dagger\hat{a}_i^\dagger\hat{a}_i\hat{a}_i. \tag{2.25}$$

The ratio t'/t is determined by the depth of the optical lattice and it can be calculated directly from (2.7a) (details in [H6]). One can show that the ratio is always negative and it decays to zero when the lattice become infinitely deep.

In the limit of deep lattice ($V_0 \to \infty$) the next-nearest neighbor tunnelings are neglected and the properties of the system are fully determined by the total number of bosons N and the only one dimensionless parameter t/U. When the parameter is tuned to the well defined critical value, the system undergoes the phase transition from the insulating to the superfluid phase. In general, the parameter t/U can be controlled experimentally in different ways. Typically the changes of t/U are controlled by changing the lattice depth. Then, the single-particle tunneling t changes its value together with two-body interaction U. In other scenario one can tune the interaction coupling g without changing the shape of the lattice. Then, only the interaction U varies. From the standard Bose-Hubbard model point of view both scenarios are equally good since the critical value t_c/U can be reached in both ways.

When extended models with larger number of parameters are considered, the situation is more complicated and different scenarios can lead to different results. It is quite well visible in the model with next-neighbor tunnelings (2.25). In this case, the properties of the ground-state are controlled by two independent dimensionless parameters t/U and t'/U. When the depth of the optical lattice changes, both tunnelings, their ratio, as well as the interaction U vary. Therefore, it is hard to determine the moment when the system crosses the quantum phase transition point. However, when the experiment is performed in the lattice with given depth V_0, the ratio t/t' is fixed and the transition through the critical point can be tuned by varying interaction U. In this scenario the ratio t'/U is explicitly determined by the lattice depth and the ratio t/U. After all, the model has only one parameter of control t/U. The additional parameter t'/U plays only additional role in fixing the value of critical tunneling. This simple observation can have some importance when very subtle experimental quench through the phase transition point are considered. In the article [H6] it is assumed that the second experimental scenario is realized.

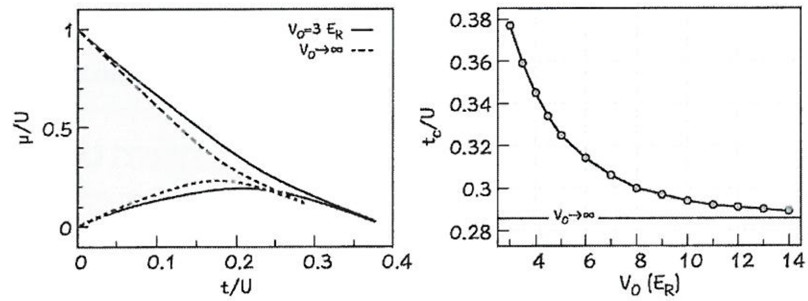

Figure 2.7: (left panel) The phase diagram of the model (2.25) with the borders of the first insulating lobe $\rho = 1$ in to limiting cases: very shallow lattice (solid line) and very deep lattice when the next-nearest neighbor tunneling can be neglected (dashed line). (right panel) The critical value of the tunneling t_c as a function of the depth of the optical lattice. Note, for shallow lattices the critical tunneling t_c is larger. *The figure adopted from [H6].*

In [H6] the Hamiltonian (2.25) was studied with the exact diagonalization method in close analogy to the analysis of the model with local three-body interactions in [H3]. Therefore, I will limit myself to discuss the final results obtained. It should be mentioned that although the model is valid only in the vicinity of the phase transition its properties were studied in the whole range of the tunnelings. The theoretical model (2.25), without any derivation, was studied recently in the mean-field approach [49] and in the two-dimensional case with the quantum rotor approach [50]. The general analysis in a whole range of interactions enable us to compare the results obtained with different methods.

The results presented in [H6] shows that the critical value of the tunneling t_c/U is substantially changed in the shallow optical lattice. In Fig. 2.7 the phase diagrams for two different lattice depths are compared. On the right panel, the critical value of the tunneling as a function of the lattice depth is shown. Contrary to the naive intuition, when the next-nearest neighbor tunneling is included the area of the insulating phase is enlarged (for more shallow lattice the critical tunneling is larger). It comes from the fact that both tunnelings t and t' have opposite signs and some kind of destructive interference of both processes is present in the system. This fact was also noticed in higher dimensions [49, 50]. This observation validates also the model assuming that the critical tunneling is in the region where $t/U \gtrsim 0.25$. The shift of the position of the critical point in the phase diagram takes place also in the direction of the chemical potential. However, the value μ_c/U decreases with the value t'. This behavior is opposite in higher dimensions [49, 50].

Tomasz Sowiński

12 maja 2015 r.

3

Other scientific achievements

Detailed references [P1–P23] can be find in the paragraph 4.1.

3.1. Before obtaining PhD degree

My scientific career started in 2003 during the third year of studies at Faculty of Physics of the University of Warsaw. At this time I was employed in the Center for Theoretical Physics of the Polish Academy of Sciences and I started collaboration with professor Iwo Białynicki-Birula. At that time the main objective of my scientific work was a deep analysis of classical and quantum dynamics of particles confined in the anisotropic harmonic trap rotating along arbitrary axis, not necessary parallel to one of main directions of the trap. This research was motivated by recent experiments on trapping Bose-Einstein condensates and single ions in harmonic traps.

As a result we found previously unknown areas of trap parameters when the dynamics is unstable. Additionally, we have predicted phenomenon of gravity-induced resonances in harmonic trap [P22,P23]. In the framework of loghatirmic Schrödinger equation, we have shown that the size and positions of the regions of unstable dynamics strongly depend on interactions [P23]. Moreover, we gave a simple prescription for constructing complete set of stationary states for any linear system from classical trajectories [P21]. All these results were finally summarized in my diploma thesis under supervision of prof. Białynicki-Birula. I was graduated with the suma cum laude note in 2005.

In my PhD thesis a systematic description of two-level systems (qubits) interacting with external quantized electromagnetic field was developed. This problem, in the general framework, is still awaiting for exact solution. It has a fundamental importance in understanding of physical properties of qubits, e.g., characteristic decay time to the ground-state, the sensitivity to the external perturbations, etc.

Our approach to the problem [P20] exploits the formalism of the quantum field theory, where the central role is played by the Feynman propagators and diagrams. This approach, in contrast to other methods exploited previously, enables one to carry on the perturbative calculations effectively. In particular, we have calculated the polarizability and susceptibility up to the fourth order of perturbation theory (previous result was known only in the second order; the result in fourth order was incorrect). This result lead directly, via the analytical continuation and the linear response theory, to the general formulas for the atomic polarizability and the dynamic of single spin susceptibility, i.e. quantities which characterize the intensity of the system response to the external perturbation. At this point it was possible to resolve also some ambiguities concerning the sign prescription that arise in the literature in the phenomenological treatment. We also generalized the method to the systems with higher number of internal degrees of freedom (qudits) [P20].

3.2. After obtaining PhD degree

After obtaining my PhD in theoretical physics I continued my employment in the Center for Theoretical Physics of the Polish Academy of Sciences till 2009. In that time I generalized the results from my PhD to the situation when the system is in contact with a thermostat [P19].

In 2009, when I was employed in the Faculty of Biology and Environmental Sciences of the Cardinal Stefan Wyszynski University, I have started collaboration with professor Mariusz Gajda from the Institute of Physics, Polish Academy of Sciences. In our first paper we discussed validity of the Gross-Pitaevskii (GP) equation by comparing its predictions with exact dynamics of two ultra-cold bosons confined in harmonic trap [18].

In 2010-2011 we worked on the problem of Einstein–de Haas (EdH) effect induced by the long-range dipolar forces in the spinor condensates. With the simplified but still realistic model, we have shown that the selective promotion of atoms confined in the harmonic trap to states with non-vanishing orbital angular momentum is possible [P16]. In our next paper [P17] we have shown that the orbital states created via EdH effect have some topological properties which are protected and conserved when the geometry of the external trap is changed dynamically.

In 2011, after joining the international scientific project NAME-QUAM, I had a short-time visit (3 months) in the Quantum Optics Group in ICFO - The Institute of Photonic Sciences (Barcelona, Spain) led by prof. Maciej Lewenstein. During the visit I have started my research on possible extensions of Hubbard-like model describing properties of ultra-cold gases confined in optical lattices. Till now, this is the main subject of my scientific activity. My first task in the group was to accurately estimate parameters of a model describing gas of ultra-cold molecules (for example RbCs, KLi, etc.) interacting via electrically induced long-range dipolar forces. To make this calculations precisely I developed a new method for calculating matrix elements of the interaction Hamiltonian from shapes of Wannier functions for given optical lattice. Having these parameters the properties of the Hamiltonian were analyzed with an exact diagonalization approach. This analysis shows that in a very narrow window of optical lattice parameters it is possible to find the ground-state in the so called pair-superfluid phase. This prediction was confirmed by coworkers in a large-scale calculations performed with MERA algorithm (Multiscale Entanglement Renormalization Ansatz). The results were published in Phys. Rev. Lett. [P15].

In 2012 I started my nine-month postdoctoral research (granted by the Foundation for Polish Sciences) in the group of prof. Lewenstein. At that time, a new idea of combining the both recently developed paths was launched. Namely, we started to analyze if the EdH mechanism can be observed in the optical lattice scheme. It turned out that this concept is very productive and many interesting results were obtained. In [P13] we derive the first extended Hubbard-like model taking into account the dynamics of spin of confined particles. We explore the quantum phases of the ground-state of the system with mean-field approach and we support these results with, already well established, exact diagonalization approach. In this way we show that depending on the parameters the state of the system can be found in the exotic superposition of many-body states: Mott-Insulator state in one of the spin component and superfluid phase in remaining component. In [P10] we discuss an influence of the anisotropy and anharmonicity of the lattice site to the existence of the EdH mechanism. Due to the high selectivity of the long-range magnetic interactions (caused by a very small interaction energy) these two properties of the lattice site (typically neglected by other authors) have a curtail importance for producing quantum phases with an orbital angular momentum. This observation is related to the fact that magnetic interactions couple states from the ground-band of the lattice with the states from the excited band, where anharmonicity and anisotropy cannot be neglected. It is worth noticing, that the results obtained in [P10] and [P13] were basis of the PhD thesis of J. Pietraszewicz prepared under my co-supervision.

In the context of orbital physics in the optical lattices, we have analyzed extended Fermi-Hubbard model which describes the system of ultra-cold polar fermions in two-dimensional optical lattice. In our paper [P11] we show that, in the case of fermionic systems, the single-band description is invalid even for quite typical experimental situations. Therefore, the extension of the model by taking into account higher orbitals become necessary. The analysis of the derived extended model shows that some exotic, strongly correlated quantum phases may appear in the system when interactions between fermions become strong.

Our few-year experience in developing and studying the extended Hubbard-like models resulted in the idea of composing a review article on the subject. The review was written in a huge international collaboration of coauthors with different experience and different interests. Therefore, it covers almost all aspects of the issue – from the simplest extensions of the standard Bose-Hubbard model, through the models taking into account higher orbitals of the periodic potentials and internal degrees of freedom, to the models with long-range interactions. At the moment the review article is accepted for publication in the Reports on Progress in Physics and it is awaiting publication [P1].

In 2013 I have started collaboration with the group of prof. Wiesław Leoński from University of Zielona Góra. One of the subjects developed in the group is question on possible applications of strongly-correlated quantum systems to the problems of quantum information theory. In our joint paper [P5] we study quantum correlations in the system of ultra-cold spin-1 bosons confined in optical superlattice (in insulating phase). We use the effective-spin approach and we classify all possible correlations that can be observed. We show that adiabatic change of parameters controlled experimentally can lead to "switching" the system between states with different correlations.

Recently, it turned out that the numerical method of an exact diagonalization of the Hamiltonian can be a very useful tool to analyze systems of a few strongly-correlated fermions. Already during my postdoc in ICFO, we studied spectrum of the many-body Hamiltonian describing two-flavor mixture of fermions interacting via short-range interactions and confined in a one-dimensional harmonic trap [P9]. The problem is closely related to the running experiments in the J. Selim's group in Heidelberg [51, 52, 53, 54]. In these experiments it is possible to precisely control a number of confined fermions, to change quasi-adiabatically mutual interactions, etc. Moreover, in the final phase of the experiment, the probability of finding a fermion on a given single-particle level of harmonic confinement can be measured. In our work [P9] we show that for strong enough interactions, due to the quasi-degeneracy of the many-body ground-state of the system, experimental tuning of interactions cannot be treated as adiabatic and can lead to observable consequences. In addition, we checked how the properties of the system will change when possible anharmonicity of the trap as well as spin-dependent interactions are present in the system.

The same model of few fermions in the regime of attractive forces was studied in our recent paper [P3]. The main motivation of this work was the question if one can find some tracks showing that pairing correlations (similar to the Cooper pairing [55]) can be observed in the system of a few particles. After exact diagonalization of the Hamiltonian for a two-component mixture of few fermions we analyze the ground-state of the system in the language of the one- and the two-particle density matrices. We show how for a strong attraction the fraction of correlated pairs of opposite spins emerge in the system. We find that the fraction of correlated pairs depends on a temperature and we show that this dependence has universal properties analogous to that known from the BCS theory [56]. Due to the connection of our results with currently developed experiments, we proposed experimental scheme to validate our predictions.

In this way I have started the next, for me completely unknown, path of my research on strongly correlated few-body systems of ultra-cold particles. This quite fresh idea can be very fruitful since it forms some kind of bridge between quantum optics of two, three particles and the condensed matter and the nuclear physics where collective behavior of many-particle systems are responsible for spectacular phenomena having no counterparts in classical world. It seems that good understanding of this missing puzzle, i.e. properties of mesoscopic number of quantum particles, may be a milestone in understanding of fundamental quantum phenomena. Recently, I decided to intensify my research in this direction and from 2015, as the leader of the

Iuventus Plus grant, this is my main subject of studies.

In 2015 I was coauthored the article [P4] where we studied and compared properties of the soliton solutions of the one-dimensional GP equation with exactly solvable Lieb-Liniger model of interacting bosons. My contribution to this work is relatively small and it is limited to validation of some temporary hypothesis.

Tomasz [signature]

12 maja 2015 r.

4

Scientific activity

4.1. Full list of scientific papers

Articles [P2], [P6], [P7], [P8], [P12] and [P14] form a habilitation achievement and were discussed in Paragraph 2.
Other articles were discussed in Paragraph 3.

After PhD degree

[P1] O. Dutta, M. Gajda, P. Hauke, M. Lewenstein, D.-S. Lühmann, B. A. Malomed, T. Sowiński, J. Zakrzewski
"Non-standard Hubbard models in optical lattices"
Rep. Prog. Phys. (accepted, in press) (2015).

[P2] T. Sowiński
"Quantum phase transition in a shallow one dimensional optical lattice"
J. Opt. Soc. Am. B **32**, 670 (2015).

[P3] T. Sowiński, M. Gajda, K. Rzążewski
"Pairing in a system of a few attractive fermions in a harmonic trap"
Europhys. Lett. **109**, 26005 (2015).

[P4] T. Karpiuk, T. Sowiński, M. Gajda, K. Rzążewski, M. Brewczyk
"Correspondence between dark solitons and the type II excitations of Lieb-Liniger model"
Phys. Rev. A **91**, 013621 (2015).

[P5] A. Barasiński, W. Leoński, T. Sowiński
"Ground-state entanglement of spin-1 bosons undergoing superexchange interactions in optical superlattices"
J. Opt. Soc. Am. B **31**, 1845 (2014).

[P6] T. Sowiński
"One-dimensional Bose-Hubbard model with pure three-body interactions"
Cent. Eur. J. Phys. **12**, 473 (2014).

[P7] T. Sowiński, R. W. Chhajlany
"Mean-field approaches to the Bose-Hubbard model with three-body local interaction"
Phys. Scripta **T160**, 014038 (2014).

[P8] T. Sowiński, M. Łącki, O. Dutta, J. Pietraszewicz, P. Sierant, M. Gajda, J. Zakrzewski, M. Lewenstein
"Tunneling-Induced Restoration of the Degeneracy and the Time-Reversal Symmetry Breaking in Optical Lattices"
Phys. Rev. Lett. **111**, 215302 (2013).

[P9] T. Sowiński, T. Grass, O. Dutta, M. Lewenstein
"Few interacting fermions in a one-dimensional harmonic trap"
Phys. Rev. A **88**, 033607 (2013).

[P10] J. Pietraszewicz, T. Sowiński, M. Brewczyk, M. Lewenstein, M. Gajda
"Spin dynamics of two bosons in an optical lattice site: a role of anharmonicity and anisotropy of the trapping potential"
Phys. Rev. A **88**, 013608 (2013).

[P11] O. Dutta, T. Sowiński, M. Lewenstein
"Orbital physics of polar Fermi molecules"
Phys. Rev. A **87**, 023619 (2013).

[P12] T. Sowiński
"Exact diagonalization of the one dimensional Bose-Hubbard model with local 3-body interactions"
Phys. Rev. A **85**, 065601 (2012).

[P13] J. Pietraszewicz, T. Sowiński, M. Brewczyk, J. Zakrzewski, M. Lewenstein, M. Gajda
"Two component Bose-Hubbard model with higher angular momentum states"
Phys. Rev. A **85**, 053638 (2012).

[P14] T. Sowiński
"Creation on demand of higher orbital states in a vibrating optical lattice"
Phys. Rev. Lett. **108**, 165301 (2012).

[P15] T. Sowiński, O. Dutta, P. Hauke, L. Tagliacozzo, M. Lewenstein
"Dipolar molecules in optical lattices"
Phys. Rev. Lett. **108**, 115301 (2012).

[P16] T. Świsłocki, T. Sowiński, M. Brewczyk, M. Gajda
"Creation of topological states of a Bose-Einstein condensate in a square plaquette of four optical traps"
Phys. Rev. A **84**, 023625 (2011).

[P17] T. Świsłocki, T. Sowiński, J. Pietraszewicz, M. Brewczyk, M. Lewenstein, J. Zakrzewski, M. Gajda
"Tunable dipolar resonances and Einstein-de Haas effect in a 87Rb-atom condensate"
Phys. Rev. A 83, 063617 (2011).

[P18] T. Sowiński, M. Brewczyk, M. Gajda, K. Rzążewski
"Dynamics and decoherence of two cold bosons in a one-dimensional harmonic trap"
Phys. Rev. A **82**, 053631 (2010).

[P19] T. Sowiński
"Two-level atom at finite temperature"
Acta Phys. Polon. A **116**, 994 (2009).

Before PhD degree

[P20] I. Białynicki-Birula, T. Sowiński
"Quantum electrodynamics of qubits"
Phys. Rev. A **76**, 062106 (2007).

[P21] T. Sowiński
"Wave functions of linear systems"
Acta Phys. Polon. B **38**, 2173 (2007).

[P22] I. Białynicki-Birula, T. Sowiński
"Gravity-induced resonances in a rotating trap"
Phys. Rev. A **71**, 043610 (2005).

[P23] I. Białynicki-Birula, T. Sowiński
"Solutions of the logarithmic Schrdinger equation in rotating harmonic trap"
Nonlinear Waves: Classical and Quantum Aspects (F. Kh. Abdullaev and V. V. Konotop (eds.)) p. 99-106, Kluwer Acad. Amsterdam (2004).

Only in ArXiv repository

[X1] T. Sowiński, R. W. Chhajlany, O. Dutta, L. Tagliacozzo, M. Lewenstein
"Violation of the universality hypothesis in ultra-cold atomic systems"
`ArXiv:1304.4835` (2013).

[X2] T. Sowiński, I. Białynicki-Birula
"Harmonic oscillator in rotating trap: Complete solution in 3D"
`ArXiv:quant-ph/0409070` (2004).

4.2. Scientific grants

As the Principal Investigator

1. *"Strongly correlated systems of few ultra-cold atoms"*
   The scientific grant founded by the Ministry of Science and Higher Education in the period 2015 – 2017, Action: Iuventus Plus IV
   Budget: 185 000 PLN

2. *"Ultra-cold gases confined in optical lattices of different shapes"*
   The scientific grant founded by the National Science Center in the period 2011 – 2014, Action: SONATA I
   Budget: 188 500 PLN

As an investigator

1. *"Thermal phenomena in cold atomic gases"*
   The scientific grant founded by the National Science Center in the period 2012 – 2015, Action: MAESTRO
   PI: prof. dr hab. Kazimierz Rzążewski

2. *"NAME-QUAM: Nanodesigning of atomic and molecular quantum matter"*
   European Union Project funded under 7th Framework Programme in the period 2010 – 2012.
   PI of the polish part: prof. dr hab. Mariusz Gajda.

3. *"Quantum electrodynamics of qubits and qudits"*
   The scientific grant founded by the Ministry of Science and Education in the period 2008 – 2010.
   PI: prof. dr hab. Iwo Białynicki-Birula.

4. *"Quantum Informatics and Engineering"*
   The scientific grant ordered by the Ministry of Science and Higher Education realized by the Laboratory of Physical Foundations of Information Processing in the period 2003 – 2007.
   PI: dr hab. Lech Mankiewicz.

5. *"Electromagnetic phenomena in rotating and accelerated systems"*
   The scientific grant founded by the Ministry of Science and Higher Education in the period 2004 – 2006.
   PI: prof. dr hab. Iwo Białynicki-Birula.

4.3. Scientific conferences and symposia

Oral presentations

1. *"Tunneling-Induced Restoration of the Degeneracy and the Time-Reversal Symmetry Breaking in Optical Lattices"*
   21th Central European Workshop on Quantum Optics, Brussels
   June 25, 2014. (contributed talk)

2. *"Universality of extended Bose-Hubbard models with local three-body interactions"*
   The Third Poznań Symposium on Quantum Engineering, Information, and Nonlinear Optics (QEINO 2013)
   October 16, 2013. (invited talk)

3. *"One-dimensional extended Bose-Hubbard models with local three-body interactions"*
   20th Central European Workshop on Quantum Optics, Stockholm
   June 16, 2013. (contributed talk)

4. *"Dipolar molecules in optical lattices"*
   The Second Poznań Workshop on Quantum Engineering, Quantum Information, and Semi-Quantum Biology
   October 16, 2012. (invited talk)

5. *"One-dimensional Bose-Hubbard model with local three-body interactions"*
   Quantum Technologies Conference III, Warszawa
   September 15, 2012. (invited talk)

6. *"Density-dependent processes of dipolar molecules in an optical lattice"*
   Quantum Technologies Conference II, Kraków
   September 30, 2011. (invited talk)

7. *"Exact dynamics and decoherence of two cold bosons in a harmonic trap"*
   Quantum Technologies Conference, Toruń
   September 2, 2010. (invited talk)

8. *"Physical properties of qubits at finite temperature"*
   LFPPI Symposium, Sopot
   April 25, 2009.

9. *"Quantum field theory methods applied to qubits"*
   4th LFPPI Symposium in Łódź
   April 4, 2008.

Conference posters

1. *"Spontaneous breaking of the time reversal symmetry in optical lattices"*
   Workshop on Coherent Control of Complex Quantum Systems (C3QS 2014)
   Okinawa, April 14–17, 2014.

2. *"Creation on demand of higher orbital states in a vibrating optical lattice"*
   International Conference on Frontiers of Cold Atoms and Related Topics
   Hong Kong, May 14–17, 2012.

3. *"Density-dependent processes of dipolar molecules in an optical lattice"*
   Bose-Einstein Condensation (BEC 2011)
   Sant Feliu, September 10–16, 2011.

4. *"Dynamics and decoherence of two cold bosons in a 1D harmonic trap"*
   Many-Body Quantum Dynamics in Closed Systems
   Barcelona, September 7–9, 2011.

5. *"Two-level atom at finite temperature"*
   Quantum Optics VII - Quantum Engineering of Atoms and Photons
   Zakopane, June 8–12, 2009.

6. *"Quantum electrodynamics of qubits"*
   (a) Foundations of Quantum Physics Conference
       Bad Honnef, September 21–26, 2008.
   (b) Control, Constraints and Quanta
       Będlewo, October 10–16, 2007.
   (c) Photons, Atoms, and Qubits (PAQ07)
       London, September 2–5, 2007.

7. *"Gravity-induced resonances in the rotating harmonic trap"*
   Quantum Optics VI - Quantum Engineering of Atoms and Photons
   Krynica, June 13–18, 2005.

4.4. Seminar lectures

Over **30** seminar lectures on running scientific projects presented in polish scientific centers (Institutes of the Polish Academy of Sciences, University of Warsaw, Jagiellonian University, Adam Mickiewicz University, University of Zielona Góra).

4.5. Organizing activity

- (2005 – 2011) member of the Main Committee of the Physics Olympiad.
  Organizing manager of the Committee in the season 2009/2010.

- Secretary of the annual Quantum Technologies Conference (`www.QuantumTech.ifpan.edu.pl`).
- member of the Organizing Committee of the symposium "From Geometry and Chaos to Quantum Information and Neurobiology" (`www.cft.edu.pl/SymposiumMarek`), April, 24-25 2015 r.

Tomasz Jasiński
12 maja 2015 r.

5

Teaching activity

5.1. Supervisor functions

PhD co-supervisor

- Joanna Pietraszewicz
  "Ultra-cold bosonic atoms with weak magnetic interactions confined in optical latices"
  Supervisor: prof. dr hab. Mariusz Gajda.
  Institute of Physics of the Polish Academy of Sciences (2013).

Bachelor supervisor

- Marcin Obidziński
  "Motion of the charged particle in crossed electric and magnetic fields"
  Faculty of Mathematics and Natural Sciences UKSW (2010).

- Piotr Majblat
  "Estimation of areas of polygons with Monte Carlo approach"
  Faculty of Mathematics and Natural Sciences UKSW (2009).

5.2. Classes for students

2005/2006

- Theoretical Physics I - exercises (30 hours, in polish)
  Faculty of Mathematics and Natural Sciences UKSW

- Physics I - exercises (30 hours, in polish)
  Faculty of Christian Philosophy UKSW

2006/2007

- Physics I - exercises (30 hours, in polish)
  Faculty of Christian Philosophy UKSW

2007/2008

- Introduction to Nucleus and Particle Physics - exercises (30 hours, in polish)
  Faculty of Mathematics and Natural Sciences UKSW

2008/2009

- Introduction to Nucleus and Particle Physics - exercises (30 hours, in polish)
  Faculty of Mathematics and Natural Sciences UKSW

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2009/2010

- Mathematics I - exercises (90 hours, in polish)
  Faculty of Biology and Environmental Sciences UKSW
- Mathematics II - exercises (60 hours, in polish)
  Faculty of Biology and Environmental Sciences UKSW
- Physics II - exercises (60 hours, in polish)
  Faculty of Biology and Environmental Sciences UKSW

2010/2011

- Mathematics I - exercises (90 hours, in polish)
  Faculty of Biology and Environmental Sciences UKSW
- Information Technology - exercises (90 hours, in polish)
  Faculty of Biology and Environmental Sciences UKSW

2011/2012

- Mathematics I - exercises (90 hours, in polish)
  Faculty of Biology and Environmental Sciences UKSW
- Information Technology - exercises (90 hours, in polish)
  Faculty of Biology and Environmental Sciences UKSW
- Information Technology - exercises (30 hours, in polish)
  Faculty of Biology and Environmental Sciences UKSW

5.3. Activity in community organizations

2009 – 2013	Member of the "Almukantarat"Astronomy Club In the period 2001–2003 member of the Club Council.

5.4. Popular science articles

- T. Sowiński
  "Modelowanie Rzeczywistości – electronic manual"
  Polish version of the manuals to programs attached to the book:
  I. Białynicki-Birula and I. Białynicka-Birula *"Modelowanie Rzeczywistości"*,
  Wydawnictwa Naukowo-Techniczne (2006).
- A. Trętowska, Ł. Nowotko, W. Śliwa, G. Wrochna, T. Sowiński, P. Fita
  "CCD Observatory in school. Guide for students, teachers and parents"
  Electronic publication founded by European Commission in program *"Hands-On Universe"* (2005).

- **57** articles about physics published in the oldest polish educational magazine "Young Technician" (pol. "Młody Technik"). Full list on my home webpage.
- T. Sowiński, *"The Earth is flat"* (pol. "Ziemia jest płaska"), Głos Nauczycielski 9/2011, 10-11.
- T. Sowiński, *"Ecological reactors"* (pol. "Ekologiczne reaktory"), Charaktery 3/2010, 78-80.
- T. Sowiński, *"Right of the observation"* (pol. "Racja obserwacji"), Charaktery 5/2009, 76-79.
- T. Sowiński, *"Formula for a miracle"* (pol. "Wzór na cud"), Charaktery 12/2008, 62-65.
- **5** short subject popular science movies.

5.5. Popular science lectures

- **2** talks in the Polish Physical Society in Białystok.
- **8** talks during few editions of the Festival of Science in Warsaw.
- **7** talks in public schools.
- **7** talks for students in the Children University.
- **1** talk in the Copernicus Science Center in Warsaw.
- **1** talk in the N. Copernicus Astronomical Center, Polish Academy of Sciences.

The detailed list of my popular science talks is available on my home webpage.

Tomasz Sowiński
12 maja 2015 r.

6

Awards and honors

2014	Director of the Institute of Physics Prize for the best scientific publication in 2013
2013	3-year scholarship for Outstanding Young Scientists granted by Ministry of Science and Higher Education
2012	KOLUMB PostDoc Scholarship granted by the Foundation for Polish Science
2009	Best lecture of the XIII Science Festival in Warsaw
2008	"Master of science popularization Golden Mind 2008" Title awarded by the President of the Polish Academy of Sciences
2005, 09, 10	Nominated for the title "Popularizer of Science" awarded by the Polish Press Agency and the Ministry of Science and Higher Education
2004/05	Ministry of National Education Scholarship

Tomasz Sowiński
12 maja 2015 r.

Bibliography

[1] R. P. Feynman, Int. J. Theore. Phys. **21**, 467 (1982).

[2] G. H. Wannier, Phys. Rev. **52**, 191 (1937).

[3] N. Marzari, Rev. Mod. Phys. **84**, 1419 (2012).

[4] M. P. A. Fisher *et al.*, Phys. Rev. B **40**, 546 (1989).

[5] V. F. Elesin *et al.*, JETP Lett. **60**, 177 (1994).

[6] D. Jaksch *et al.*, Phys. Rev. Lett. **81**, 3108 (1998).

[7] T. D. Kühner *et al.*, Phys. Rev. B **61**, 12474 (2000).

[8] J. Zakrzewski, Phys. Rev. A **71**, 043601 (2005).

[9] T. P. Polak and T. K. Kope, Phys. Rev. B **76**, 094503 (2007).

[10] S. Ejima *et al.*, Phys. Rev. A **85**, 053644 (2012).

[11] M. Greiner *et al.*, Nature (London) **415**, 39 (2002).

[12] U. Schneider *et al.*, Science **322**, 1520 (2008).

[13] R. Jördens *et al.*, Nature (London) **455**, 204 (2008).

[14] J. Hubbard, Proc. R. Soc. London, Ser. A **276**, 283 (1963).

[15] T. Müller *et al.*, Phys. Rev. Lett. **99**, 200405 (2007).

[16] G. Wirth *et al.*, Nature Phys. **7**, 147 (2011).

[17] M. Lewenstein and W. V. Liu, Nature Phys. **7**, 101 (2011).

[18] M. Łącki, J. Zakrzewski, Phys. Rev. Lett. **110**, 065301 (2013).

[19] V. W. Scarola and S. Das Sarma, Phys. Rev. Lett. **95**, 033003 (2005).

[20] W. V. Liu and C. Wu, Phys. Rev. A **74**, 013607 (2006).

[21] J. Larson *et al.*, Phys. Rev. A **79**, 033603 (2009).

[22] A. Collin *et al.*, Phys. Rev. A **81**, 023605 (2010).

[23] F. Pinheiro *et al.*, Phys. Rev. A **85**, 033638 (2012).

[24] F. Pinheiro *et al.*, Phys. Rev. Lett. **111**, 205302 (2013).

[25] X. Li *et al.*, Phys. Rev. Lett. **108**, 175302 (2012).

[26] W. H. Zurek, Rev. Mod. Phys. **75**, 715 (2003).

[27] S. Will *et al.*, Nature (London) **465**, 197 (2010).

[28] T. T. Wu, Phys. Rev. **115**, 1390 (1959).

[29] T. Köhler, Phys. Rev. Lett. **89**, 210404 (2002).

[30] P. R. Johnson *et al.*, New J. Phys. **11**, 093022 (2009).

[31] L. Mazza *et al.*, Phys. Rev. A **82**, 043629 (2010).

[32] B. L. Chen *et al.*, Phys. Rev. A **78**, 043603 (2008).

[33] K. Zhou *et al.*, Phys. Rev. A **82**, 013634 (2010).

[34] A. Safavi-Naini *et al.*, Phys. Rev. Lett. **109**, 135302 (2012).

[35] J. Silva-Valencia and A. M. C. Souza, Phys. Rev. A **84**, 065601 (2011).

[36] V. L. Berezinskii, Sov. Phys. JETP **32**, 493 (1971).

[37] V. L. Berezinskii, Sov. Phys. JETP **34**, 610 (1972).

[38] J. M. Kosterlitz and D. J. Thouless, J. Phys. C **6**, 1181 (1973).

[39] M. C. Gutzwiller, Phys. Rev. Lett. **10**, 169 (1963).

[40] M. C. Gutzwiller, Phys. Rev. **134**, A923 (1964).

[41] M. C. Gutzwiller, Phys. Rev. **137**, A1726 (1965).

[42] J. Silva-Valencia and A. Souza, Eur. Phys. J. B **85**, 161 (2012).

[43] C. A. Avila *et al.*, Phys. Lett. A **378**, 3233 (2014).

[44] S.R., White, Phys. Rev. Lett. **69**, 2863 (1992).

[45] U. Schollwöck, Rev. Mod. Phys. 77, 259 (2005).

[46] P. Calabrese *et al.*, J. Stat. Mech. Theor. Exp. P06002 (2004).

[47] N. Laflorencie *et al.*, Phys. Rev. Lett. **96**, 100603 (2006).

[48] S. Pilati and M. Troyer, Phys. Rev. Lett. **108**, 155301 (2012).

[49] Y. Gao and F. Han, Mod. Phys. Lett. B **22**, 33 (2008).

[50] T. A. Zaleski and T. K. Kopeć, J. Phys. B **43**, 085303 (2010).

[51] F. Serwane *et al.*, Science **332**, 336 (2011).

[52] G. Zürn *et al.*, Phys. Rev. Lett. **108**, 075303 (2012).

[53] G. Zürn *et al.*, Phys. Rev. Lett. **111**, 175302 (2013).

[54] A. N. Wenz *et al.*, Science **342**, 457 (2013).

[55] L. N. Cooper, Phys. Rev. **104**, 1189 (1956).

[56] J. Barden *et al.*, Phys. Rev. **108**, 1175 (1957).

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