pairing in a system of a few attractive fermions in a harmonic trap

tomasz sowiński

Ministry of Science and Higher Education
Republic of Poland

abstract

We study a strongly attractive system of a few fermions confined in a one-dimensional harmonic trap, interacting via two-body contact potential. Performing exact diagonalization of the Hamiltonian we analyze the ground state and the thermal state of the system in terms of one- and two-particle reduced density matrices. We show how for strong attraction the correlated pairs emerge in the system. We find that the fraction of correlated pairs depends on temperature and we show that this dependence has universal properties analogous to the gap function known from the theory of superconductivity.

the model

- two distinguishable flavors of fermions (↑ and ↓)
- both flavors have equal masses
- both in the same one-dimensional harmonic confinement
- opposite spins do interact via sort range δ-like potential

$$\hat{\mathcal{H}} = \sum_\sigma \int \mathrm{d}x\, \hat\Psi^\dagger_\sigma(x)\left[-\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}x^2} + \frac{1}{2}x^2\right]\hat\Psi_\sigma(x)$$
$$+\, g\int \mathrm{d}x\, \hat\Psi^\dagger_\downarrow(x)\hat\Psi^\dagger_\uparrow(x)\hat\Psi_\uparrow(x)\hat\Psi_\downarrow(x)$$

(anti-)commutation relations

- the same spins

$$\left\{\hat\Psi_\sigma(x),\hat\Psi^\dagger_\sigma(x')\right\} = \delta(x-x')$$
$$\left\{\hat\Psi_\sigma(x),\hat\Psi_\sigma(x')\right\} = 0$$

- opposite spins

$$\left[\hat\Psi_\uparrow(x),\hat\Psi^\dagger_\downarrow(x')\right] = 0$$
$$\left[\hat\Psi_\uparrow(x),\hat\Psi_\downarrow(x')\right] = 0$$

the method

- we fix the number of fermions $\mathcal{N}_\uparrow$ and $\mathcal{N}_\downarrow$

$$\left[\hat{\mathcal{N}}_\uparrow,\hat{\mathcal{H}}\right] = \left[\hat{\mathcal{N}}_\downarrow,\hat{\mathcal{H}}\right] = 0 \qquad \hat{\mathcal{N}}_\sigma = \int \mathrm{d}x\,\hat\Psi^\dagger_\sigma(x)\hat\Psi_\sigma(x)$$

- we decompose the field operator in the single-particle basis of the harmonic oscillator eigenfunctions

$$M \sim 12$$

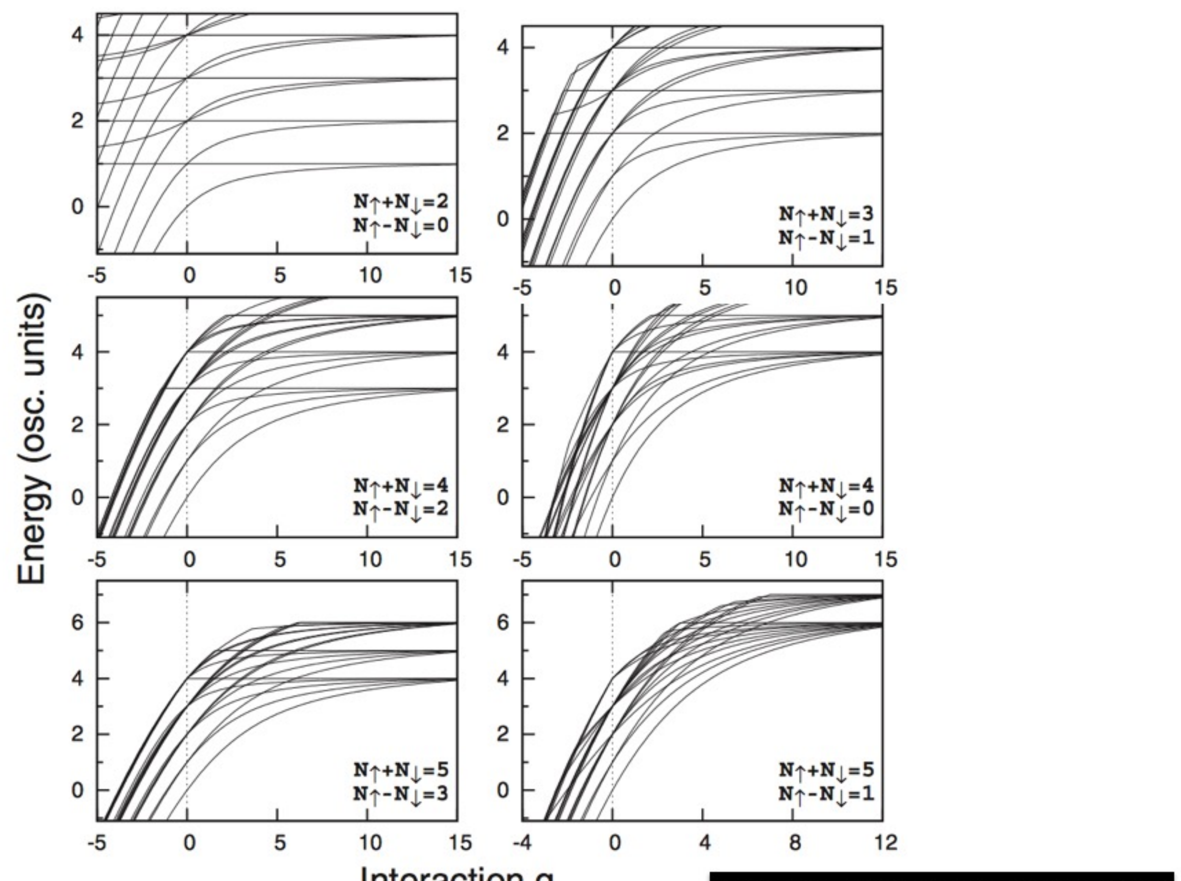
$$\hat\Psi_\sigma(x) = \sum_{n=1}^{M}\hat a_{\sigma n}\,\phi_n(x)$$

$$\left[-\frac{1}{2}\frac{\mathrm{d}^2}{\mathrm{d}x^2}+\frac{1}{2}x^2\right]\phi_n(x) = \left(n+\frac{1}{2}\right)\phi_n(x)$$

- we calculate all matrix elements of the Hamiltonian
- we perform an exact diagonalization and we find egienstates and corresponding eigenenergies

$$\hat{\mathcal{H}}|\mathtt{G}_i\rangle = \mathcal{E}_i|\mathtt{G}_i\rangle$$

spectrum of the Hamiltonian



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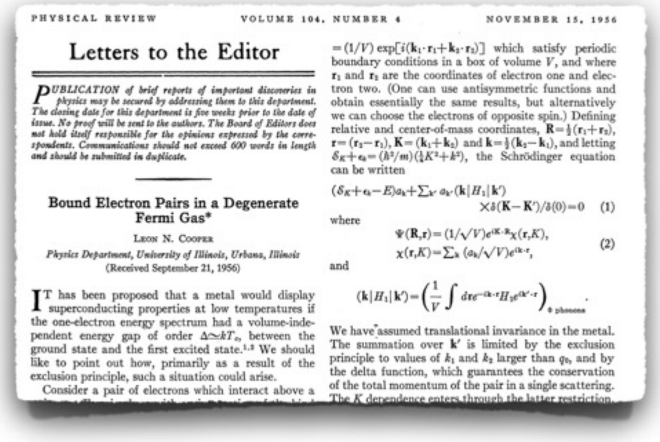
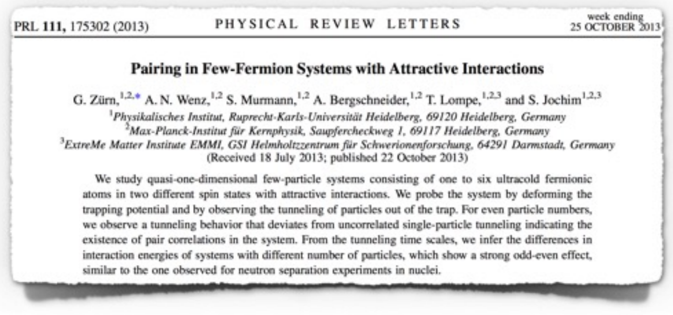
on the track of pairs...





$\mathcal{N}_\uparrow = \mathcal{N}_\downarrow = \mathcal{N}/2$

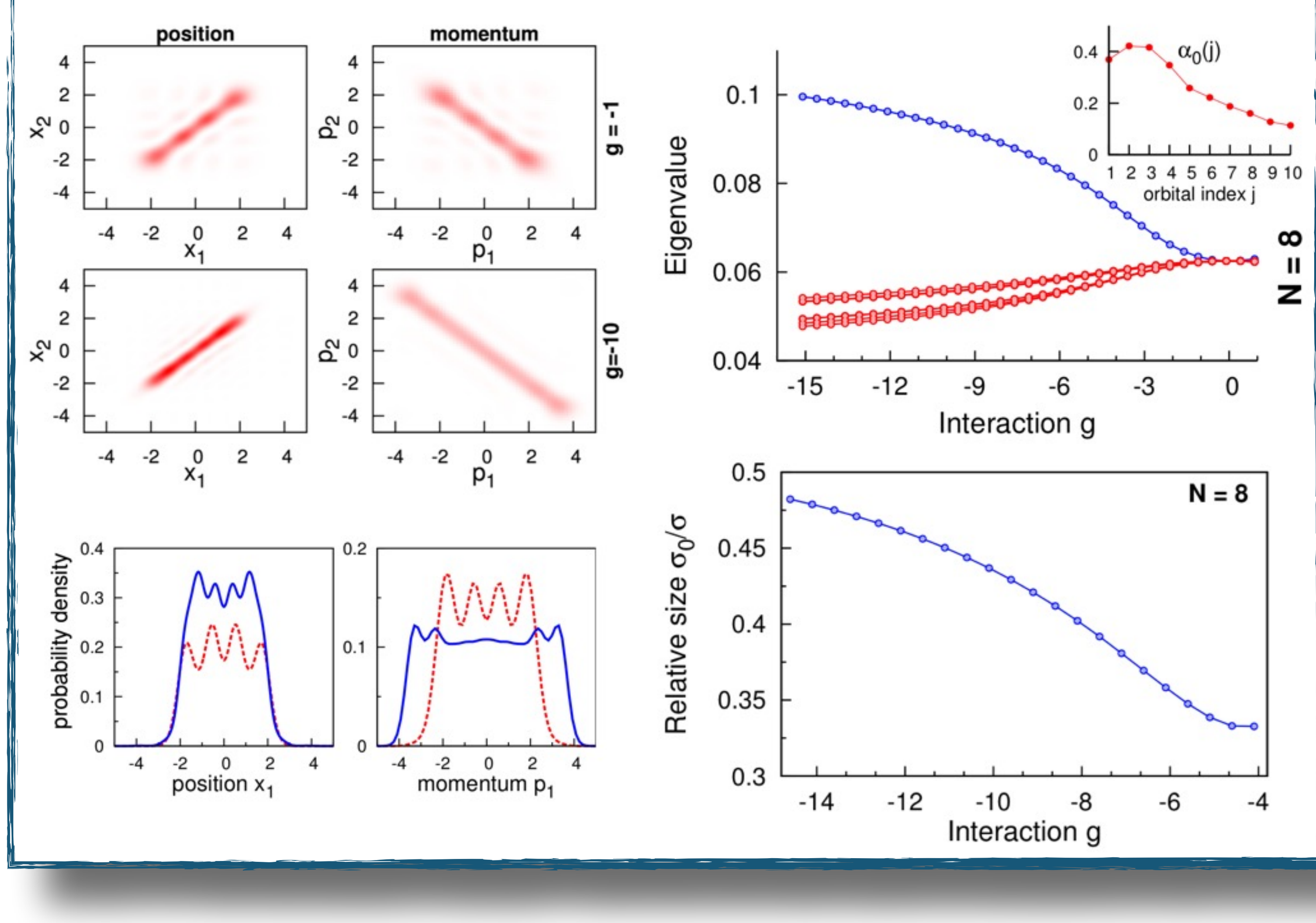
two-fermion reduced density matrix

$$\rho^{(2)}(x_1',x_2';x_1,x_2) = \frac{4}{\mathcal{N}}\langle\mathtt{G}_0|\hat\Psi^\dagger_\uparrow(x_1')\hat\Psi^\dagger_\downarrow(x_2')\hat\Psi_\downarrow(x_2)\hat\Psi_\uparrow(x_1)|\mathtt{G}_0\rangle$$

spectral decomposition of $\rho^{(2)}$

$$\rho^{(2)}(x_1',x_2';x_1,x_2) = \sum_i \lambda_i\,\Phi_i(x_1,x_2)\Phi_i^*(x_1',x_2')$$

the dominant orbital...



structure of the orbitals

one-fermion reduced density matrix

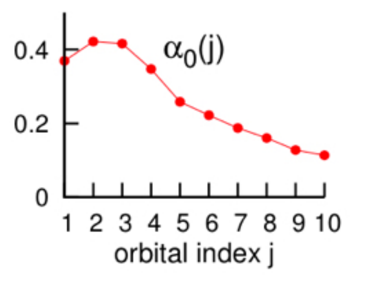
$$\rho^{(1)}(x_1';x_1) = \frac{2}{\mathcal{N}}\langle\mathtt{G}_0|\hat\Psi^\dagger_\sigma(x_1')\hat\Psi_\sigma(x_1)|\mathtt{G}_0\rangle = \sum_i \eta_i\,\varphi_i(x_1)\varphi_i^*(x_1')$$

single-fermion orbitals

decomposition of the two-body orbitals in the basis spanned by one-body orbitals

dominant orbital (strongly correlated)

$$\Phi_0(x_1,x_2) \sim \sum_j \alpha_0(j)\varphi_j(x_1)\varphi_j(x_2)$$



higher orbitals (trivial correlations)

$$\Phi_k(x_1,x_2) \sim \varphi_i(x_1)\varphi_j(x_2) \pm \varphi_j(x_1)\varphi_i(x_2)$$

thermal properties... $\hat\rho_{\mathrm{T}} = \mathcal{Z}^{-1}\mathrm{e}^{-\hat{\mathcal{H}}/k_BT}$

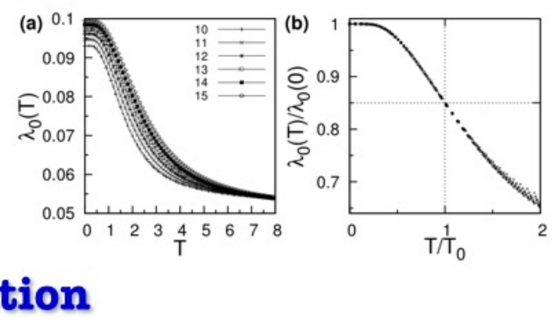
$$\rho^{(1)}_{\mathrm{T}}(x_1';x_1) = \frac{2}{\mathcal{N}}\,\mathrm{Tr}\left[\hat\rho_{\mathrm{T}}\hat\Psi^\dagger_\sigma(x_1')\hat\Psi_\sigma(x_1)\right]$$

$$\rho^{(2)}(x_1',x_2';x_1,x_2) = \frac{4}{\mathcal{N}}\,\mathrm{Tr}\left[\hat\rho_{\mathrm{T}}\hat\Psi^\dagger_\uparrow(x_1')\hat\Psi^\dagger_\downarrow(x_2')\hat\Psi_\downarrow(x_2)\hat\Psi_\uparrow(x_1)\right]$$

universal dependence on temperature

$$\frac{\lambda_0(T)}{\lambda_0(0)} = \chi\left(\frac{T}{T_0}\right)$$

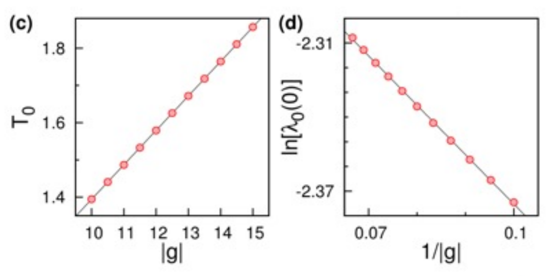
some universal function



universal dependence on interaction

$$\lambda_0(0) = \mathbf{\Lambda}\,\mathrm{e}^{-0.5/|g|}$$

some universal number



references

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Tomasz Sowiński[1,2], Mariusz Gajda[1,2] and Kazimierz Rzążewski[2]

[1] Institute of Physics of the Polish Academy of Sciences - Al. Lotników 32/46, 02-668 Warszawa, Poland
[2] Center for Theoretical Physics of the Polish Academy of Sciences - Al. Lotników 32/46, 02-668 Warszawa, Poland

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