

# Stochastic quantum hydrodynamics for Bose-Einstein condensates

Matter wave field:  $\Psi(\mathbf{x}) = \sqrt{n(\mathbf{x})} e^{i\theta(\mathbf{x})}$        $\mathbf{v}(\mathbf{x}) = \frac{\hbar}{m} \nabla \theta(\mathbf{x})$       interaction  $g$

Continuity:  $\frac{\partial n}{\partial t} = -\nabla \cdot [n\mathbf{v}] + \sqrt{-i\frac{g}{\hbar}} \xi(t)$

Incompressible hydrodynamics

Euler equation:  $\frac{\partial \mathbf{v}}{\partial t} + (\mathbf{v} \cdot \nabla) \mathbf{v} = -\nabla \left[ V_{\text{trap}}(\mathbf{x}) + gn - \frac{\hbar^2}{2m\sqrt{n}} \nabla^2 \sqrt{n} - \frac{\sqrt{i\hbar g}}{2} \xi(t) \right]$

Quantum pressure      Quantum noise

Teething problems: shocks

(no viscosity, lack of a minimal length scale)

Speculation: Can one include quantum turbulence in a meaningful averaged way?

