Fate of the false vacuum in 1D Bose gases: Simulating the early universe

King Lun Ng [1,2]

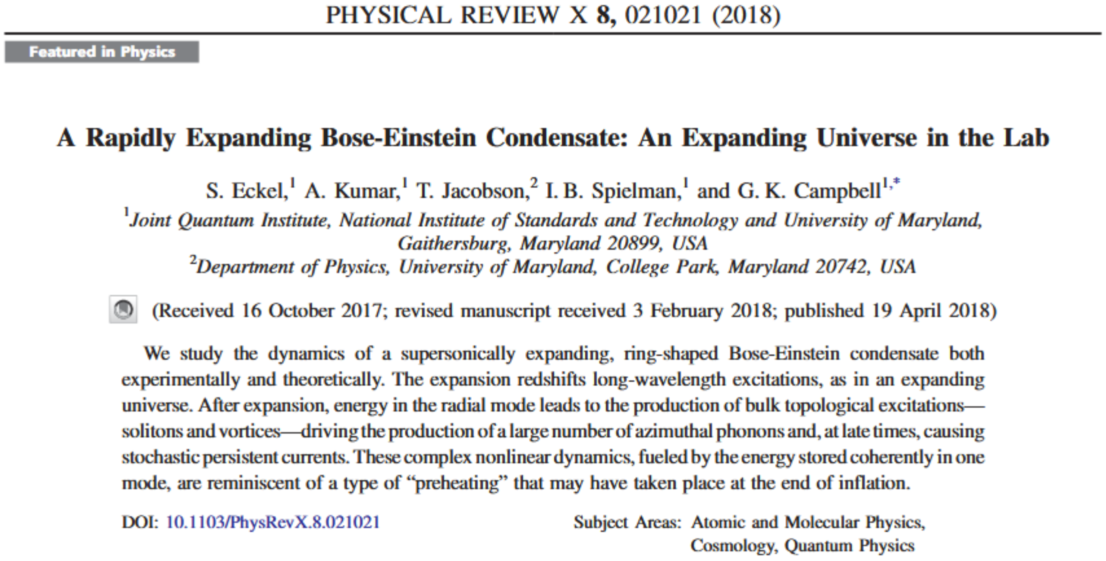
Peter D. Drummond [2], Margaret Reid [2], Bogdan Opanchuk [2], Manushan Thenabadu [2]

1. *IFPAN Quantum Noise Group, Institute of Physics, Polish Academy of Sciences, Warszawa, Poland*
2. *Centre for Quantum Technology Theory, Swinburne University of Technology, Melbourne, Australia*

Workshops: Nonequilibrium phenomena in superfluid systems, March 2023, Warsaw

1.The proposed experiment:
How to simulate the early universe in BEC?

Simulating the early universe in BEC



PHYSICAL REVIEW X **8**, 021021 (2018)

Featured in Physics

A Rapidly Expanding Bose-Einstein Condensate: An Expanding Universe in the Lab

S. Eckel,[1] A. Kumar,[1] T. Jacobson,[2] I. B. Spielman,[1] and G. K. Campbell[1,*]
[1]*Joint Quantum Institute, National Institute of Standards and Technology and University of Maryland, Gaithersburg, Maryland 20899, USA*
[2]*Department of Physics, University of Maryland, College Park, Maryland 20742, USA*

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We study the dynamics of a supersonically expanding, ring-shaped Bose-Einstein condensate both experimentally and theoretically. The expansion redshifts long-wavelength excitations, as in an expanding universe. After expansion, energy in the radial mode leads to the production of bulk topological excitations—solitons and vortices—driving the production of a large number of azimuthal phonons and, at late times, causing stochastic persistent currents. These complex nonlinear dynamics, fueled by the energy stored coherently in one mode, are reminiscent of a type of “preheating” that may have taken place at the end of inflation.

DOI: 10.1103/PhysRevX.8.021021

Subject Areas: Atomic and Molecular Physics,
Cosmology, Quantum Physics

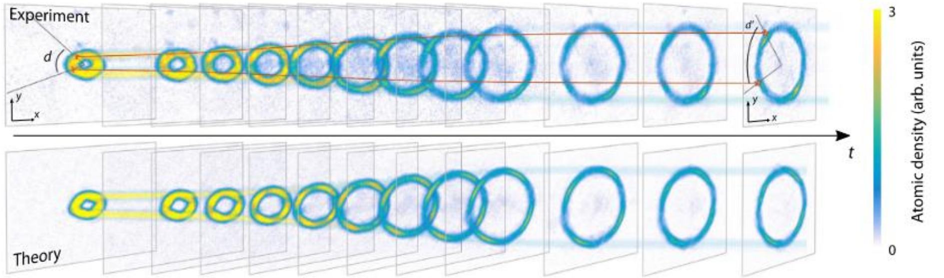


FIG. 1. Measured (top) and simulated (bottom) supersonic expansion of the ring with scale factor $a = R_f/R_i = 4.1(3)$, where $R_f = 46.4(1.4)$ μm [$R_i = 11.3(4)$ μm] is the final (initial) radius [44]. An initial distance d transforms into a larger distance d'. The time elapsed in the figure is approximately 15 ms.

Figure captured from Eckel et al., *PRX* 8, 021021(2018)

Examples:

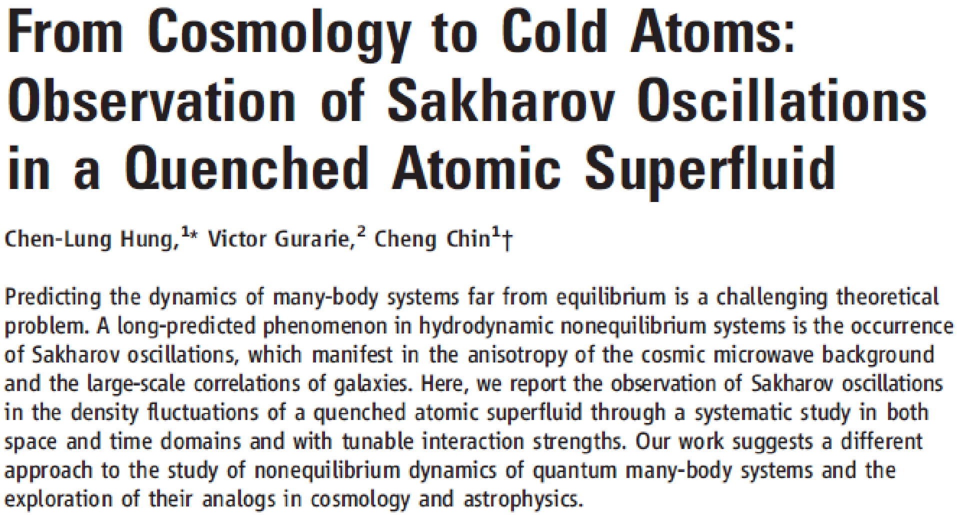
1.Early universe expansion (Eckel *et al.*, *PRX* 8, 021021(2018))

2.Cosmic microwave background (CMB) radiation (Hung *et al.*, *Science* 341 (2013))

3.Dynamics in curved spacetime (Viermann *et al.*, *Nature* 611 (2022))

Etc…

Simulating the early universe in BEC



From Cosmology to Cold Atoms: Observation of Sakharov Oscillations in a Quenched Atomic Superfluid

Chen-Lung Hung,[1]* Victor Gurarie,[2] Cheng Chin[1]†

Predicting the dynamics of many-body systems far from equilibrium is a challenging theoretical problem. A long-predicted phenomenon in hydrodynamic nonequilibrium systems is the occurrence of Sakharov oscillations, which manifest in the anisotropy of the cosmic microwave background and the large-scale correlations of galaxies. Here, we report the observation of Sakharov oscillations in the density fluctuations of a quenched atomic superfluid through a systematic study in both space and time domains and with tunable interaction strengths. Our work suggests a different approach to the study of nonequilibrium dynamics of quantum many-body systems and the exploration of their analogs in cosmology and astrophysics.

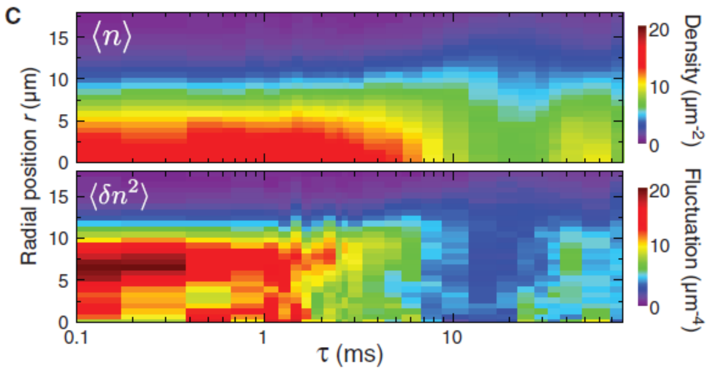


Figure captured from Hung et al., Science 341 (2013)

Examples:

1. Early universe expansion (Eckel *et al.*, *PRX* 8, 021021(2018))

2. Cosmic microwave background (CMB) radiation (Hung *et al.*, *Science* 341 (2013))

3. Dynamics in curved spacetime (Viermann *et al.*, *Nature* 611 (2022))

Etc…

Fate of the false vacuum

- Coleman [1]: decay of relativistic scalar field; from metastable false vacuum to stable true vacuum

$$\partial_t^2\psi - c\nabla^2\psi = -\partial_\psi V(\psi)$$

- Bubble nucleation at speed c

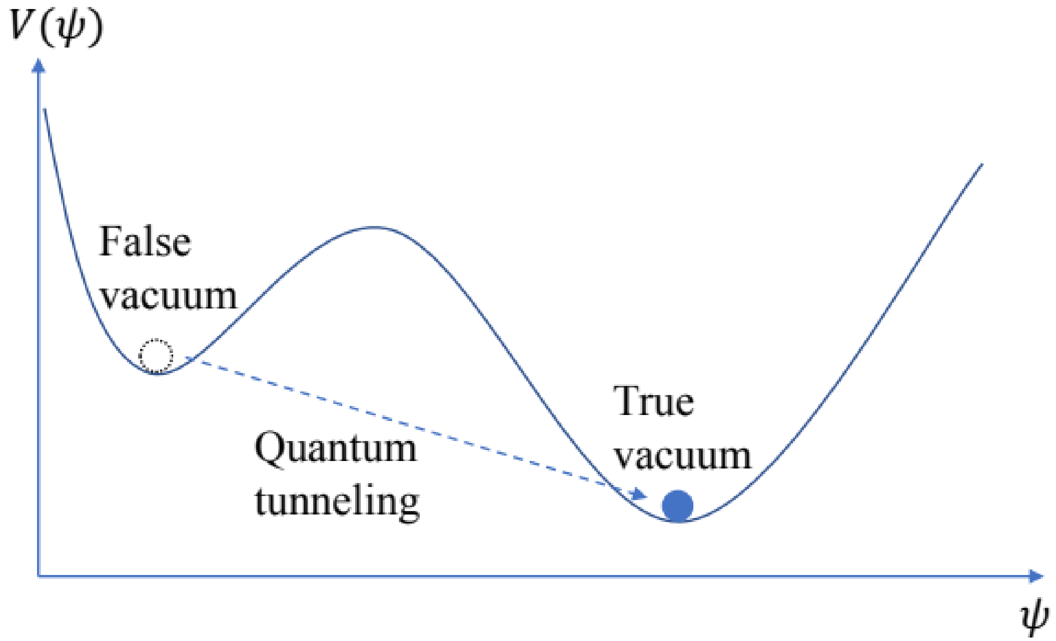


Fig: Illustration of false vacuum decay

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1. S. Coleman, *Phys. Rev. D* 15, 2929 (1977).

Proposed experiment of the false vacuum

- The Hamiltonian

$$\hat{H} = \sum_{j=1}^{2}\int dx \left\{\left(-\hat{\Psi}_j^{\dagger}\frac{\hbar^2\nabla^2}{2m}\hat{\Psi}_j + \frac{g}{2}\hat{\Psi}_j^{\dagger 2}\hat{\Psi}_j^{2}\right) - \nu\left(t\right)\left(\hat{\Psi}_2^{\dagger}\hat{\Psi}_1 + \hat{\Psi}_1^{\dagger}\hat{\Psi}_2\right)\right\}$$

- Modulated time-dependent sinusoidal coupling

$$\nu\left(t\right) = \nu + \delta\hbar\omega\cos\omega t$$

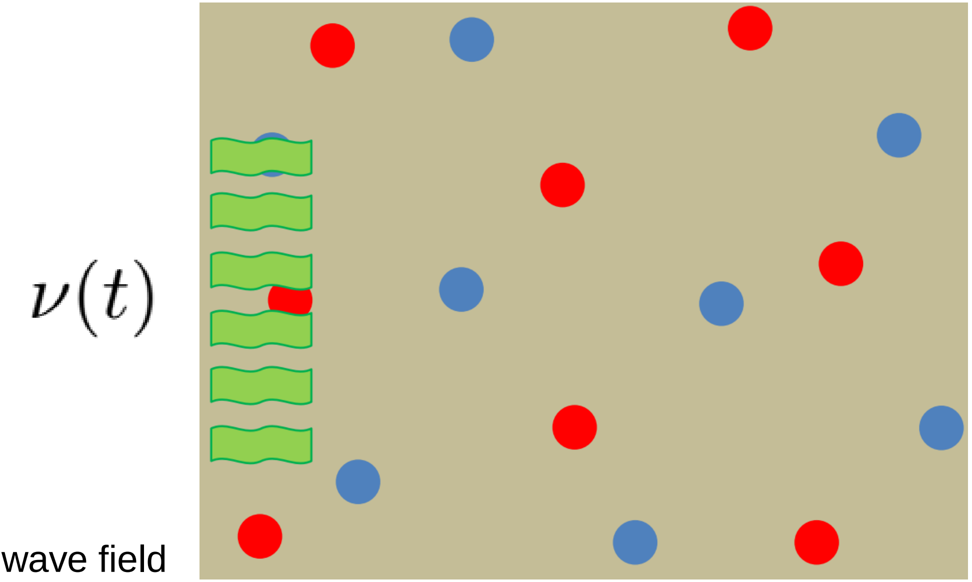

Fig: Modulated coupling by external cw microwave field

Kapitza pendulum: Phase potential creation

- Create metastable+stable potential:
  $\nu \quad \rightarrow \quad \nu(t)$
- Applying high driving frequency at the pivot point of a rigid pendulum
- metastable false vacuum -> small perturbation angle at lower position
- stable true vacuum -> upper vertical position

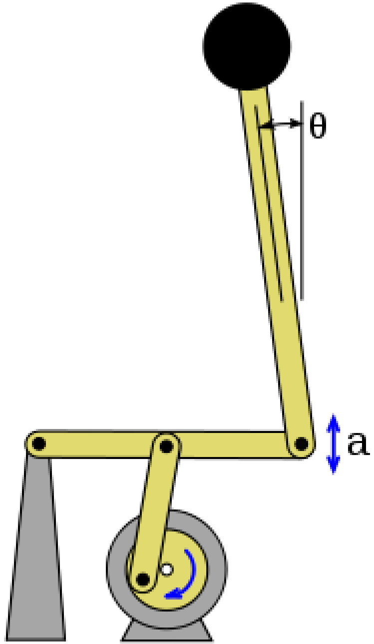

Fig: Illustration of Kapitza pendulum (Figure source:https://en.wikipedia.org/wiki/Kapitza%27s_pendulum)

Relative phase of the BEC

- Two component BEC with phase difference: $\boxed{\phi_a = \phi_1 - \phi_2 - \pi}$
- Phase potential in the condition of "Kapitza pendulum":

$$\boxed{U(\phi_a) = \omega_0^2 \left[\cos(\phi_a) + \frac{\lambda^2}{2} \sin^2(\phi_a) \right]}$$

- Characteristic frequency due to the coupling amplitude:

$$\omega_0 = 2\sqrt{\nu g \rho_c}/\hbar$$

- Fast oscillation amplitude:

$$\lambda = \delta\sqrt{2g\rho_c/\nu}$$

- BEC in false vacuum: $\boxed{\phi_a = 0}$
- BEC in true vacuum: $\boxed{\phi_a = \pm\pi}$

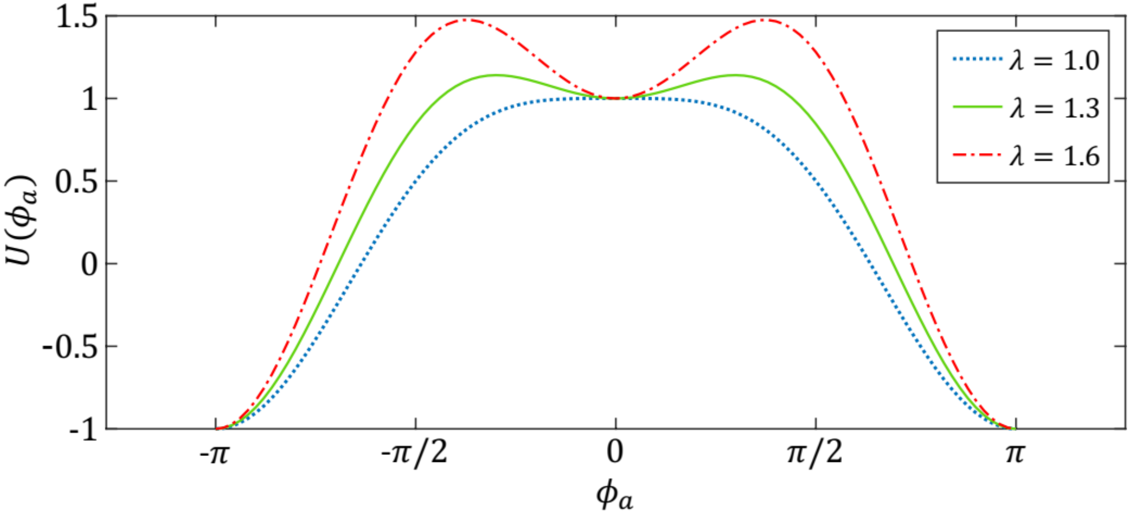

Fig: Phase potential vs relative phase of BEC

Proposed experiment of the false vacuum

- BEC with two occupied hyperfine levels, well mixed with minimized interspecies interaction $g_{12} \approx 0$, and a relative phase π
- For simplicity, we assume intraspecies interaction $g_{11} \approx g_{22} = g$
- This is possible for $^{41}\mathrm{K}$ ($|1\rangle = |F = 1, m_F = 1\rangle$ and $|2\rangle = |F = 1, m_F = 0\rangle$)
- Fialko *et al. 2015 Europhys.Lett. 110 56001*

$$g_{11} \approx g$$

$$g_{22} \approx g$$

$$g_{12} \approx 0$$

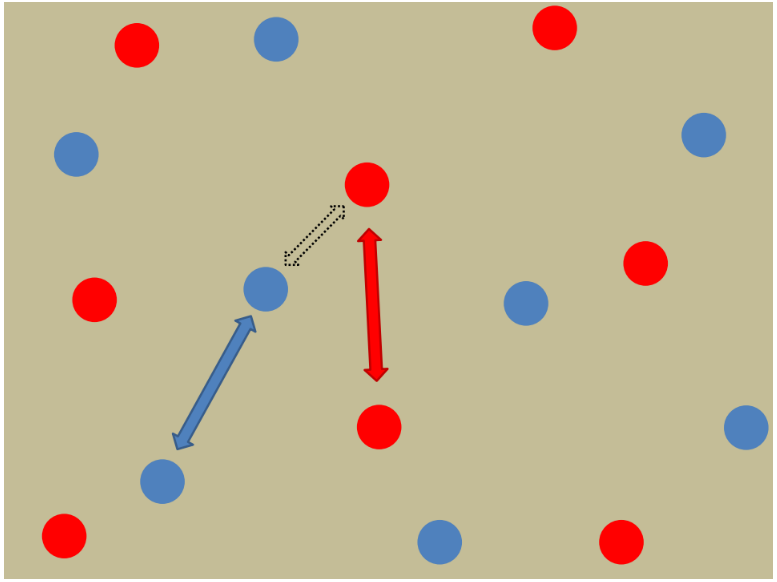

Fig: Illustration of well mixed BEC components

2.Initial state

Initial state (part1): Bogoliubov method

- Assuming component $j = 2$ is in a vacuum state; component $j = 1$ is in thermal equilibrium at temperature T : $\boxed{\hat{\Psi}_1(x,0) = \psi_c + \delta\hat{\Psi}_1}$
- Fluctuations: $\boxed{\delta\hat{\Psi}_1 = \frac{1}{\sqrt{L}}\sum_k \left[u_k \hat{b}_k e^{ikx} - v_k \hat{b}_k^\dagger e^{-ikx}\right]}$
- Bogoliubov coefficients for $k \neq 0$: $u_k = \frac{\epsilon_k + E_k}{2\sqrt{\epsilon_k E_k}}$, $v_k = \frac{\epsilon_k - E_k}{2\sqrt{\epsilon_k E_k}}$
- Free particle energy $E_k = \hbar^2 k^2 / (2m)$ and excitation energy $\epsilon_k = \sqrt{E_k(E_k + 2g\rho_c)}$
- Phonon distribution: $\boxed{\langle \hat{n}_k \rangle = \langle \hat{b}_k^\dagger \hat{b}_k \rangle \equiv n_k = \frac{1}{\exp(\beta\epsilon_k) - 1}}$, $\beta = 1/k_B T$

Initial state (part 2): truncated Wigner Approximation (TWA)

- Long simulation time
- Include thermal and vacuum fluctuations
- Correction of order $1/N^2$ for N particles
- Taking $\hat{b}_k \sim \beta_k \to \beta_{\tilde{k}}$, the corresponding Wigner representation for the BEC fields are (in dimensionless):

$$\boxed{\hat{\Psi}_1 \to \widetilde{\psi}_1 = \widetilde{\psi}_c + \frac{1}{\sqrt{\widetilde{L}}} \sum_{\tilde{k}} (u_{\tilde{k}} \beta_{\tilde{k}} e^{i\widetilde{k}\widetilde{x}} - v_{\tilde{k}} \beta^*_{\tilde{k}} e^{-i\widetilde{k}\widetilde{x}})}$$

$$\boxed{\hat{\Psi}_2 \to \widetilde{\psi}_2 = \frac{1}{\sqrt{\widetilde{L}}} \sum_{\tilde{k}} \alpha_{\tilde{k}} e^{i\widetilde{k}\widetilde{x}}}$$

Initial state (part 2):
truncated Wigner Approximation (TWA)

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$$\boxed{\hat{\Psi}_2 \rightarrow \widetilde{\psi}_2 = \frac{1}{\sqrt{\widetilde{L}}} \sum_{\widetilde{k}} \alpha_{\widetilde{k}} e^{i\widetilde{k}\widetilde{x}}}$$

- Complex Gaussian random variables $\beta_{\widetilde{k}} = \dfrac{\eta_{1,\widetilde{k}}}{\sqrt{2\tanh(\widetilde{\epsilon}_{\widetilde{k}}/8\sqrt{\widetilde{\nu}}\widetilde{\rho}_0^2\tau)}}$ and $\alpha_{\widetilde{k}} = \dfrac{\eta_{2,\widetilde{k}}}{\sqrt{2}}$
- Expectation values of noises: $\langle|\beta_{\widetilde{k}}|^2\rangle = n_{\widetilde{k}} + 1/2$ and $\langle|\alpha_{\widetilde{k}}|^2\rangle = 1/2$, $n_{\widetilde{k}} = (\exp(\beta\epsilon_{\widetilde{k}}) - 1)^{-1}$

Initial state (part 3)

- The BEC is rabi rotated by a microwave pulse to give equal occupation for both spin species with initial relative phase $\phi_1 - \phi_2 = \pi$
- In simulation, this is equivalent to a rotation matrix acting on the BEC fields:

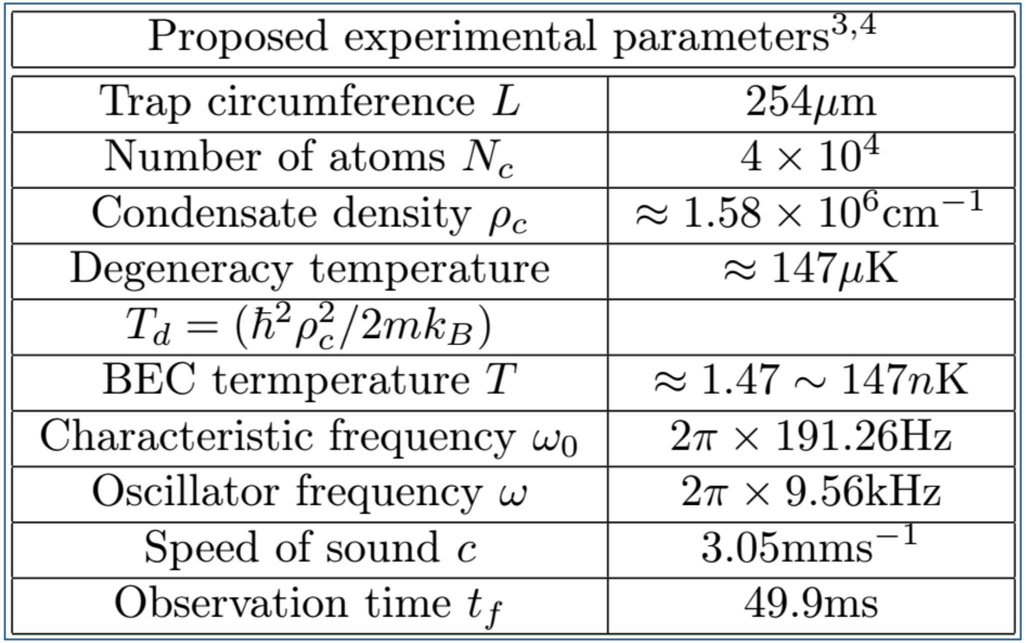
$$\boxed{\begin{pmatrix} \widetilde{\psi}_1' \\ \widetilde{\psi}_2' \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2} & -ie^{-i\phi}\sin\frac{\theta}{2} \\ -ie^{i\phi}\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} \begin{pmatrix} \widetilde{\psi}_1 \\ \widetilde{\psi}_2 \end{pmatrix}}$$

where $\theta = \pi/2$ and $\phi = -\pi/2$

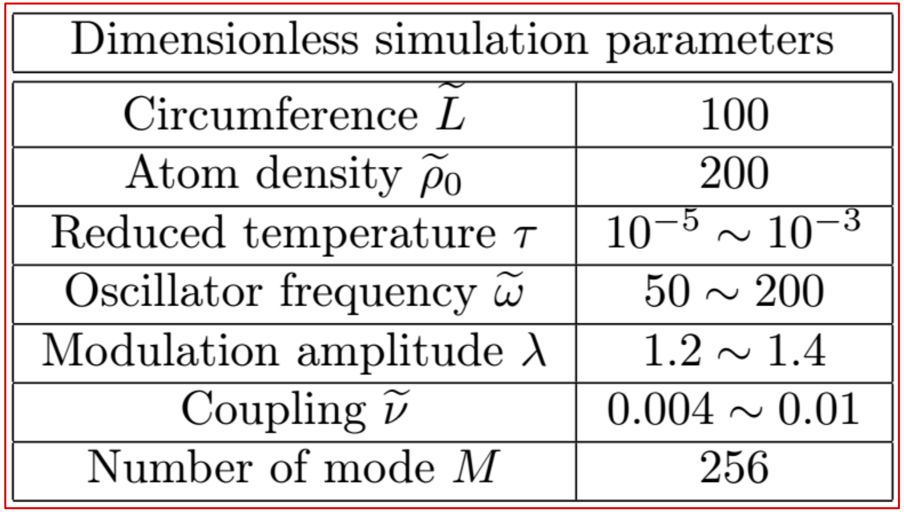
- Initial conditions $\boxed{\langle|\widetilde{\psi}_1'|^2\rangle = \langle|\widetilde{\psi}_2'|^2\rangle}$ and $\boxed{\phi_1 - \phi_2 = \pi}$

3.Decay of false vacuum

Some parameters



Proposed experimental parameters[3,4]	
Trap circumference L	254μm
Number of atoms N_c	4×10^4
Condensate density ρ_c	$\approx 1.58 \times 10^6$cm^{-1}
Degeneracy temperature	$\approx 147\mu$K
$T_d = (\hbar^2 \rho_c^2 / 2mk_B)$	
BEC termperature T	$\approx 1.47 \sim 147n$K
Characteristic frequency ω_0	$2\pi \times 191.26$Hz
Oscillator frequency ω	$2\pi \times 9.56$kHz
Speed of sound c	3.05mms^{-1}
Observation time t_f	49.9ms



Dimensionless simulation parameters	
Circumference $\widetilde{L}$	100
Atom density $\widetilde{\rho}_0$	200
Reduced temperature τ	$10^{-5} \sim 10^{-3}$
Oscillator frequency $\widetilde{\omega}$	$50 \sim 200$
Modulation amplitude λ	$1.2 \sim 1.4$
Coupling $\widetilde{\nu}$	$0.004 \sim 0.01$
Number of mode M	256

3. A. Kumar *et al.*, *Phys. Rev. A* 95, 021602(R) (2017).

4. M. Kunimi and I. Danshita, *Phys. Rev. A* 99, 043613 (2019).

The decay of false vacuum and the bubble nucleation of true vacua

- The Wigner field trajectory in real time

$$\frac{d\widetilde{\psi}_j}{d\widetilde{t}} = -i\left[-\sqrt{\widetilde{\nu}}\widetilde{\nabla}^2\widetilde{\psi}_j + \tilde{g}\widetilde{\psi}_j|\widetilde{\psi}_j|^2\right] + i\frac{\sqrt{\widetilde{\nu}}}{2}\left[1+\sqrt{2}\lambda\widetilde{\omega}\cos(\widetilde{\omega}\widetilde{t})\right]\widetilde{\psi}_{3-j}$$

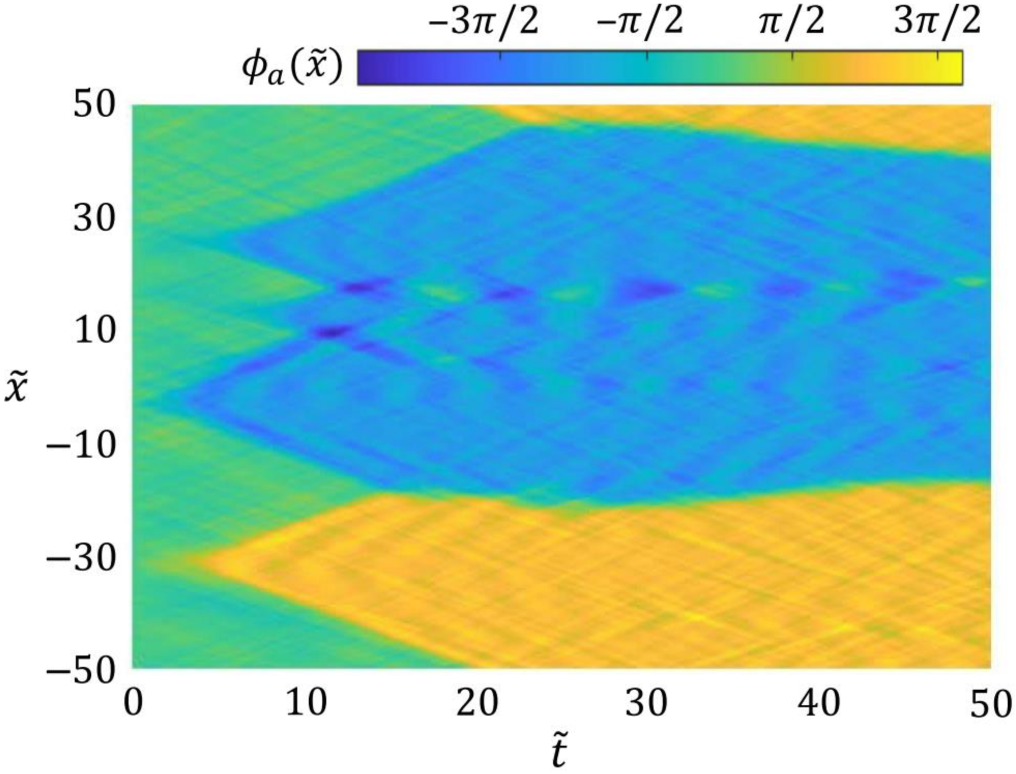


Fig: Decay of 1D false vacuum from a single trajectory simulation with reduced temperature $\tau = 10^{-5}$, corresponds to $T \sim 1.5\mathrm{nK}$.

The decay of false vacuum and the bubble nucleation of true vacua

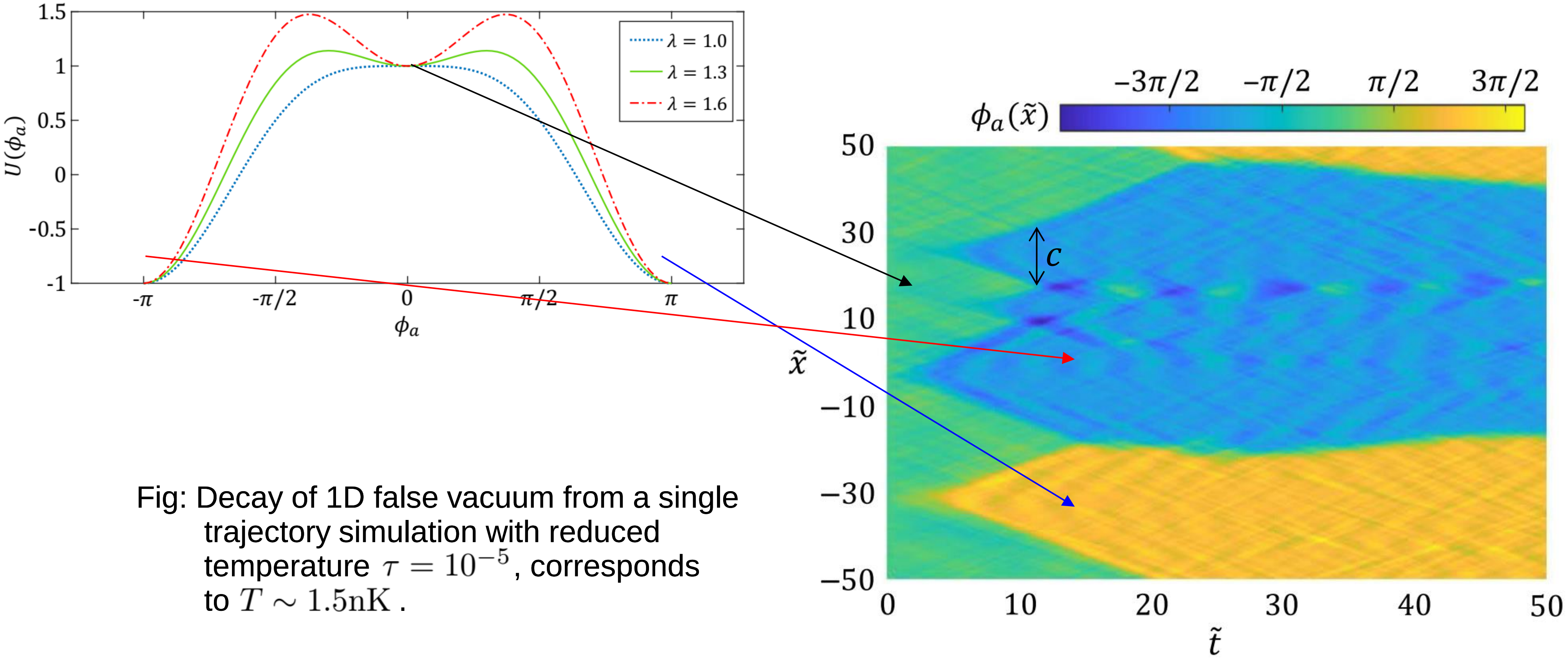


Fig: Decay of 1D false vacuum from a single trajectory simulation with reduced temperature $\tau = 10^{-5}$, corresponds to $T \sim 1.5\mathrm{nK}$.

1D false vacuum at finite temperatures

- Relative number density distribution: $\boxed{p_z(\widetilde{x}) = \frac{I_2(\widetilde{x}) - I_1(\widetilde{x})}{I_2(\widetilde{x}) + I_1(\widetilde{x})}}$,

where $I_1(\widetilde{x}) = |\frac{\widetilde{\psi}_1 + \widetilde{\psi}_2}{\sqrt{2}}|^2 - \frac{1}{2\Delta\widetilde{x}}$ and $I_2(\widetilde{x}) = |\frac{\widetilde{\psi}_1 - \widetilde{\psi}_2}{\sqrt{2}}|^2 - \frac{1}{2\Delta\widetilde{x}}$

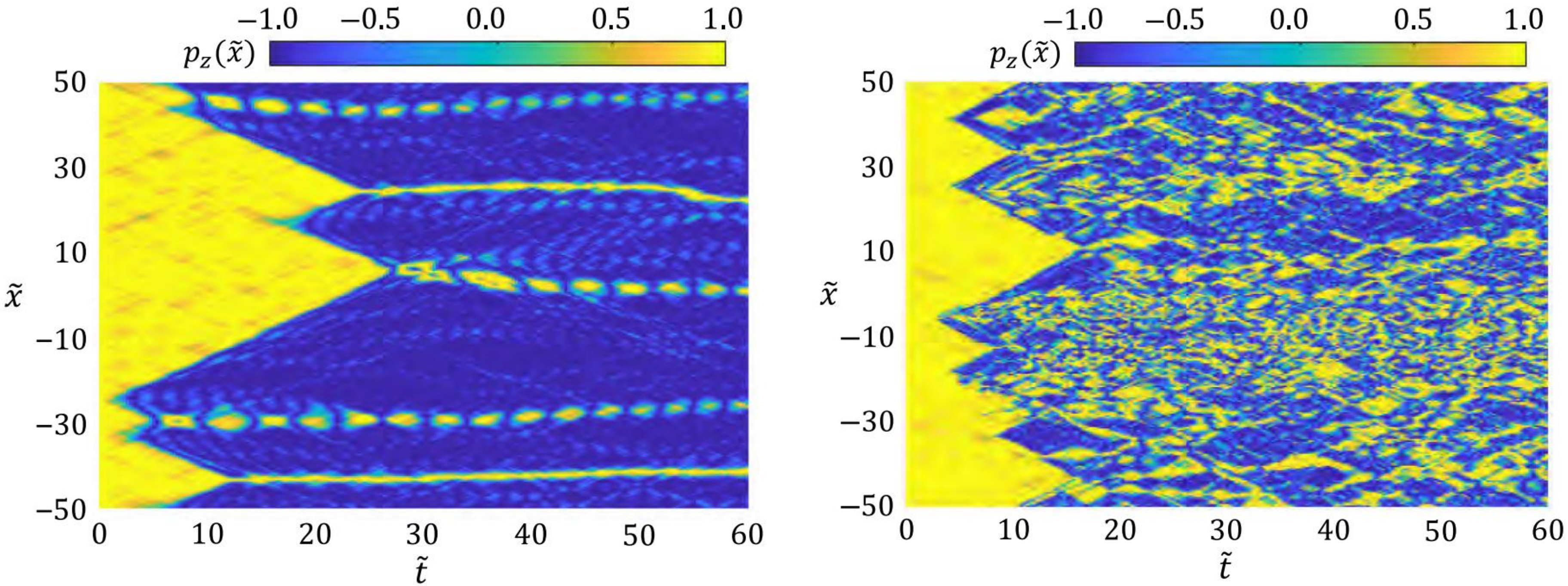
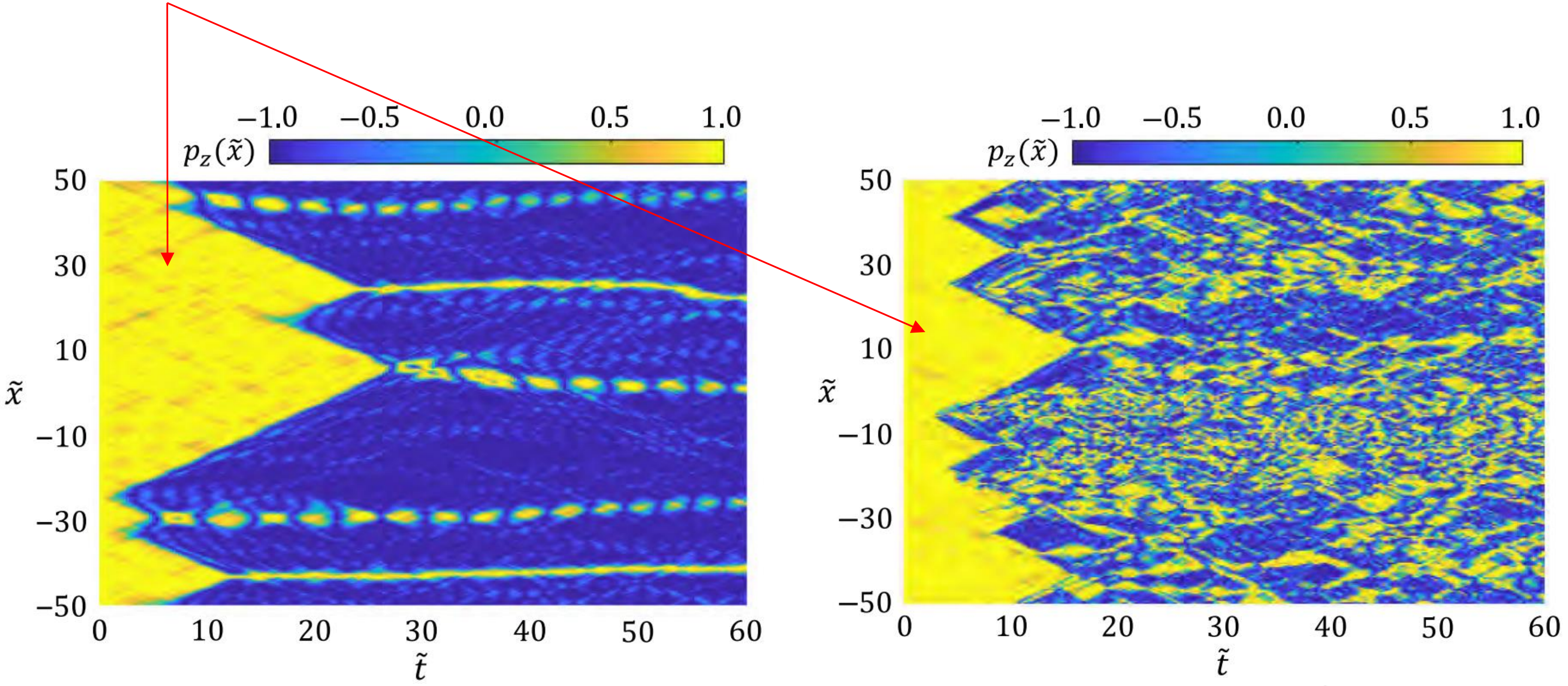


Fig: (left) reduced temperature $\tau = 1 \times 10^{-5}$; (right) $\tau = 3 \times 10^{-4}$

1D false vacuum at finite temperatures

- False vacuum and true vacua (bubble universes)



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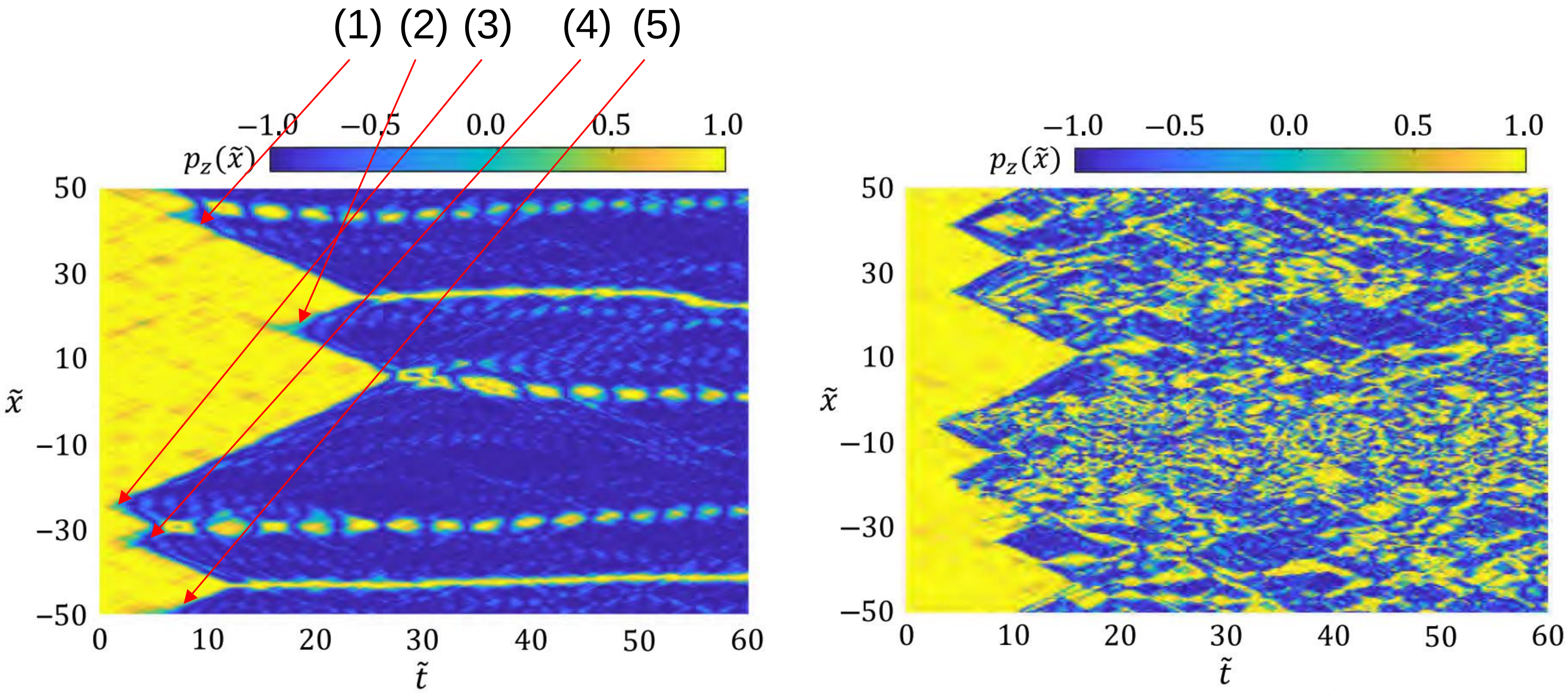
(1) (2) (3) (4) (5)

Fig: (left) reduced temperature $\tau = 1 \times 10^{-5}$; (right) $\tau = 3 \times 10^{-4}$

1D false vacuum at finite temperatures

- Domain walls and oscillons

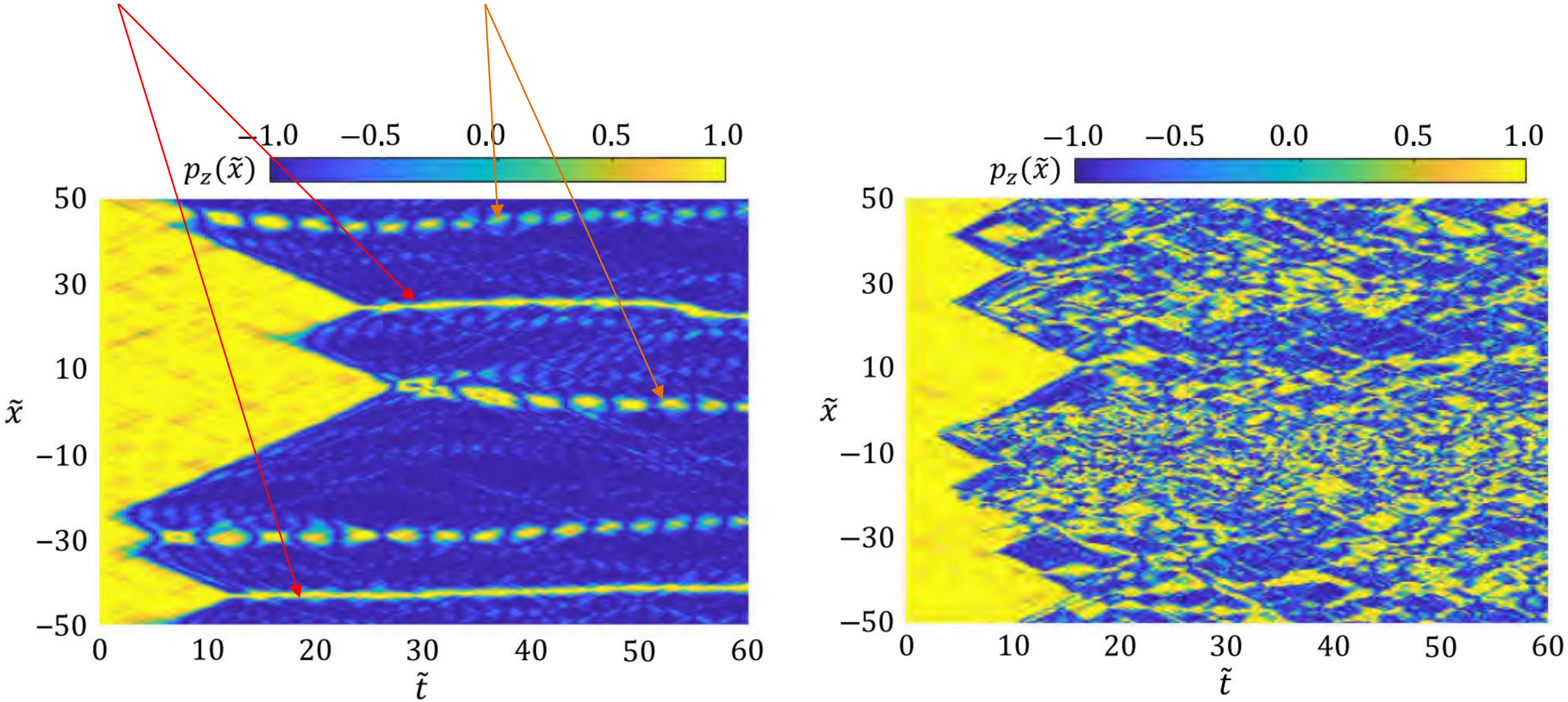

Fig: (left) reduced temperature $\tau = 1 \times 10^{-5}$; (right) $\tau = 3 \times 10^{-4}$

4.Decay rate

Tunneling rate: quantify bubble nucleation

- Average cosine of the relative phase:

$$\langle \cos\phi_a \rangle = \frac{1}{\widetilde{L}} \int^{\widetilde{L}} \cos\phi_a(\widetilde{x})\, d\widetilde{x}$$

- Threshold value for bubble nucleation

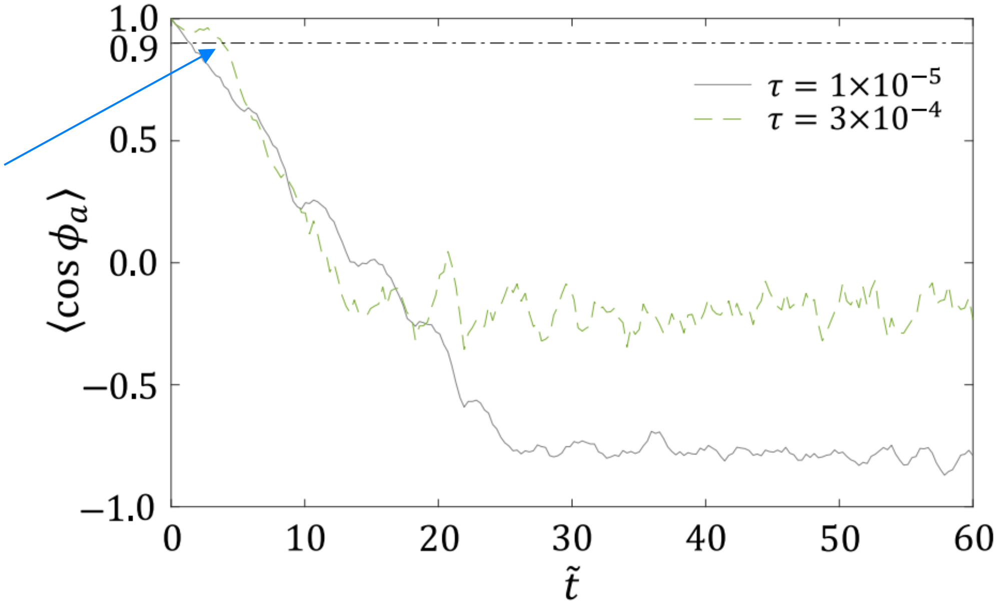

Fig: Example of average cosine of the BEC relative phase at two temperatures

Tunneling rate: survival probability

- Average cosine of the relative phase:

$$\langle \cos\phi_a \rangle = \frac{1}{\widetilde{L}} \int^{\widetilde{L}} \cos\phi_a(\widetilde{x})\, d\widetilde{x}$$

- Threshold value for bubble nucleation
- Survival probability and tunneling rate[1]

$$\mathcal{F} = \exp(-\Gamma\widetilde{t})$$

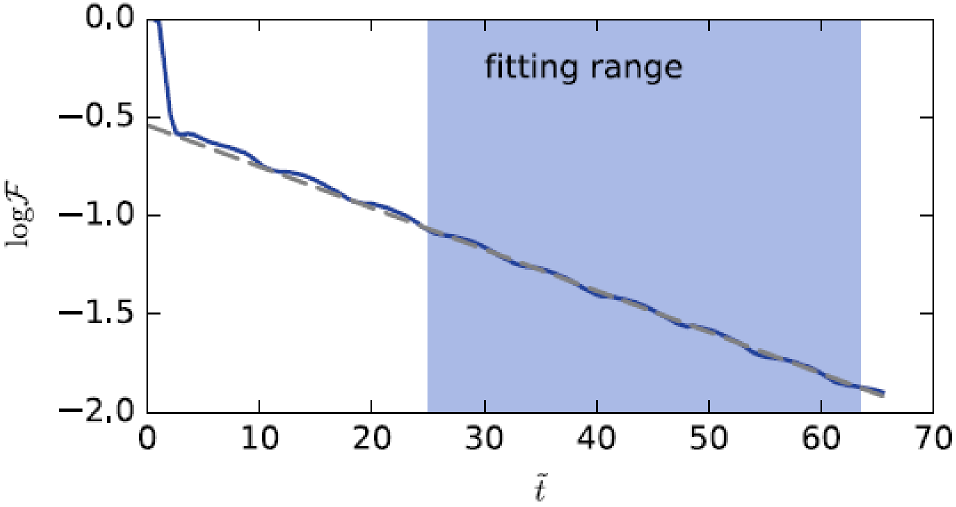

Fig: Fitting the slope of survival probability of false vacuum (captured from Fialko et al., J. Phys. B: At. Mol. Opt. Phys. 50, 024003 (2017))

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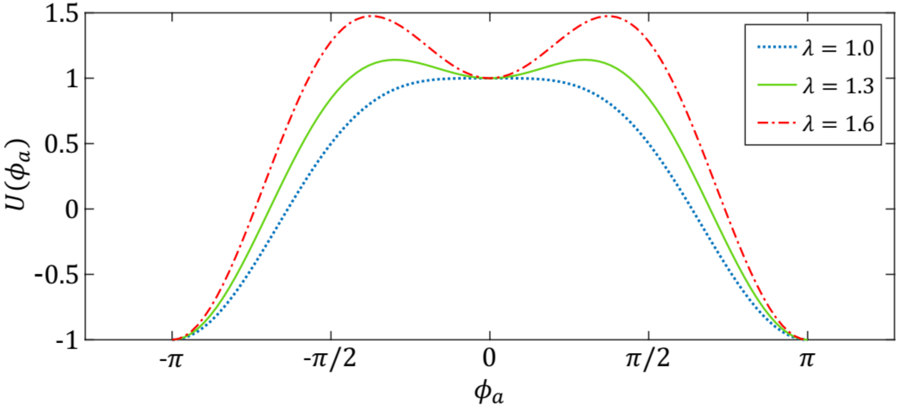
[1] S. Takagi, *Macroscopic Quantum Tunneling* (Cambridge University Press 2002).

Tunneling rate

- Statistical results from 80000 Wigner trajectories
- Coherent state with no thermal effect included
- Various external coupling $\tilde{\nu}$
- Various oscillation amplitude λ



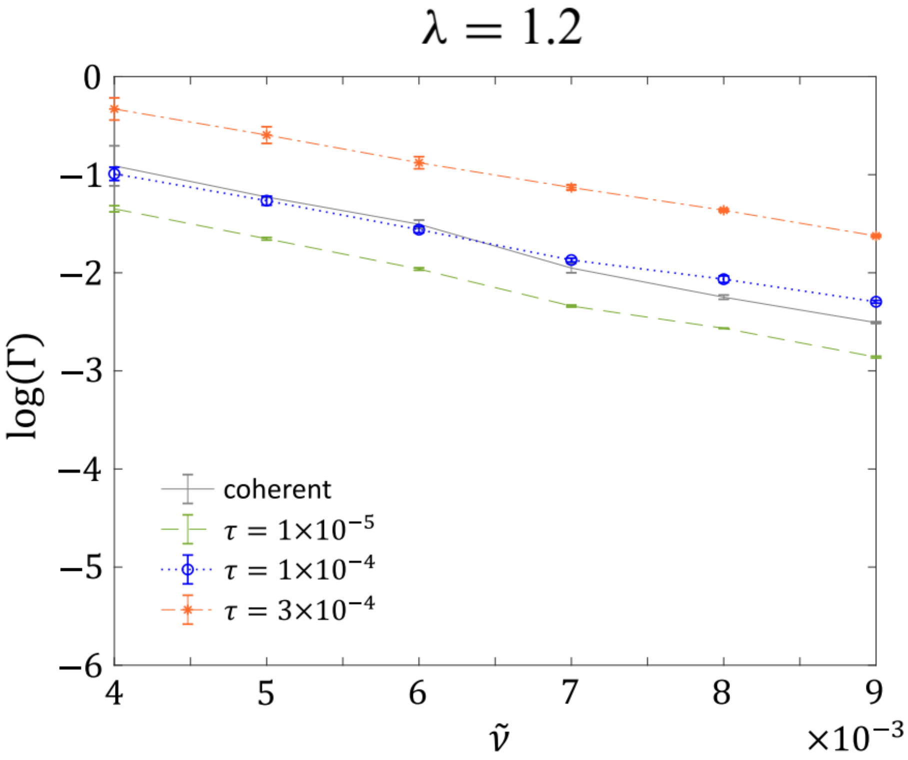
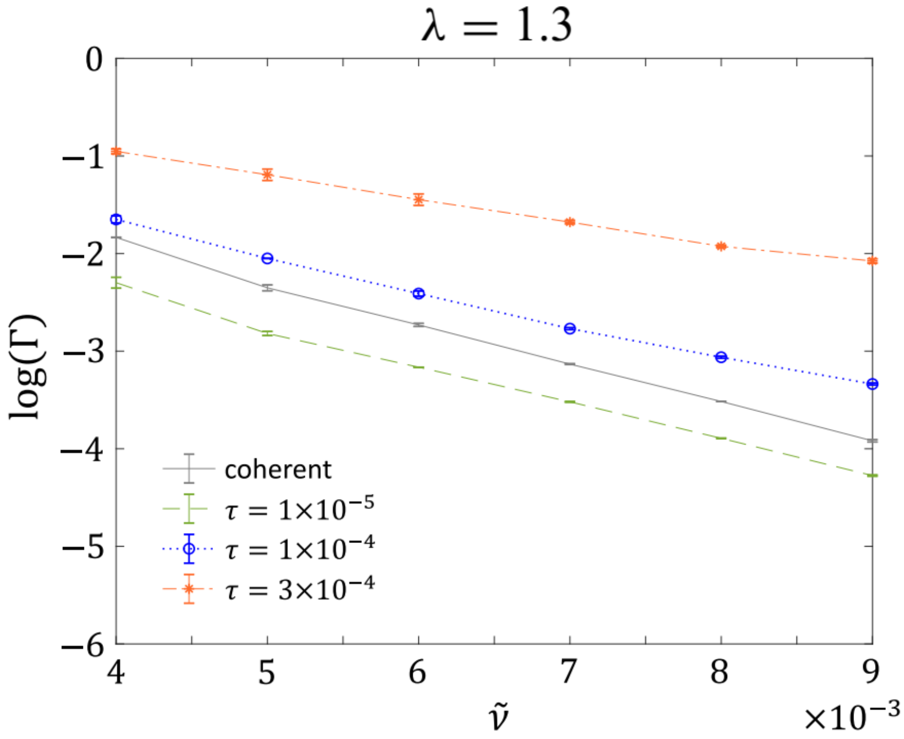
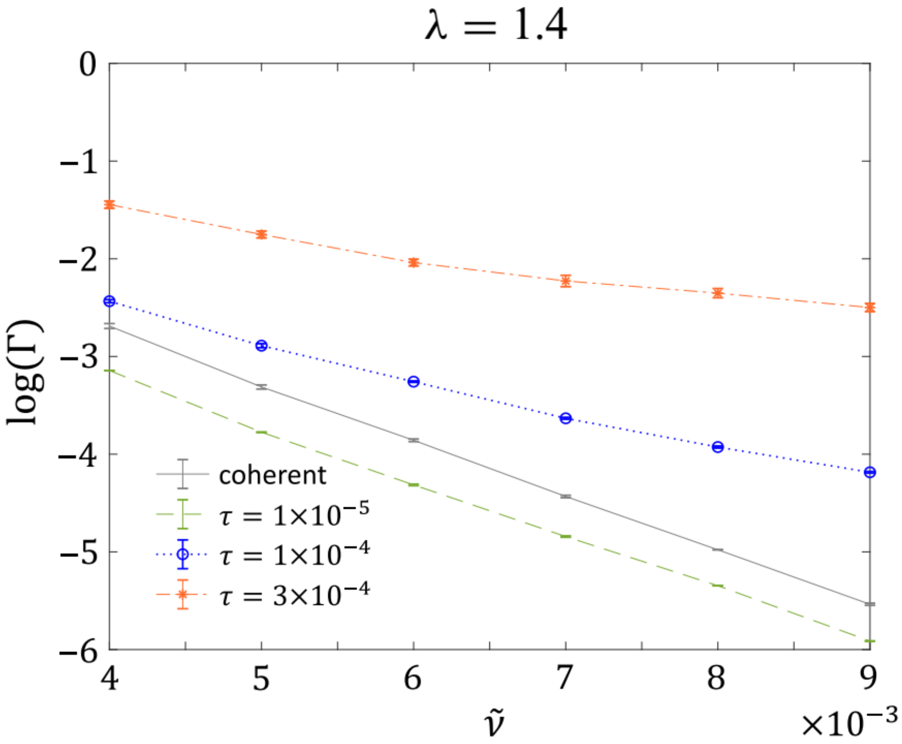


Fig: Tunneling rate at various temperatures

Tunneling rate

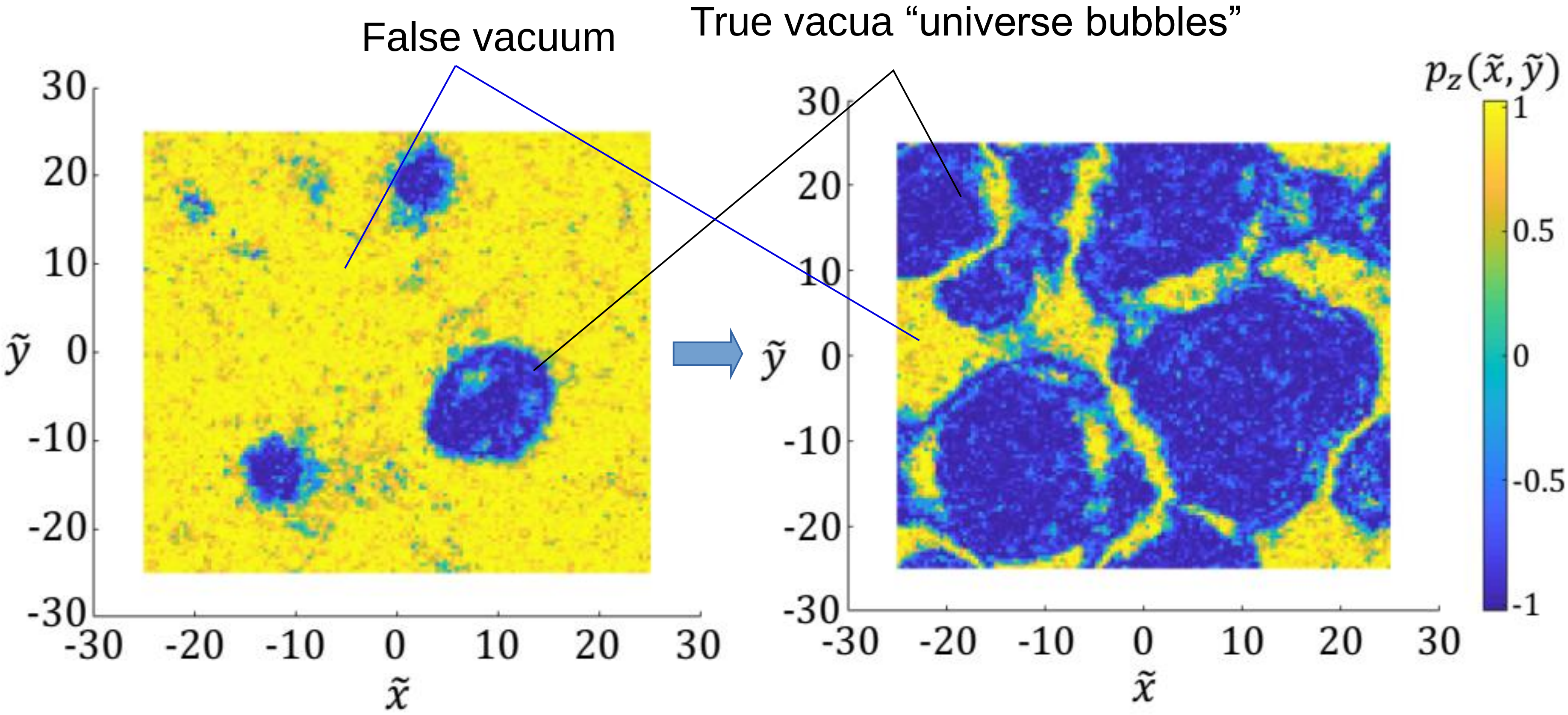
- High oscillation amplitude (deeper phase potential “depth”) reduces tunneling rate
- Strong external coupling reduces tunneling rate
- Tunneling rate is dominated by the thermal fluctuations at high temperature





4.Higher dimensions?

2D false vacuum at finite temperatures


False vacuum

True vacua “universe bubbles”

Fig: Simulation of bubble nucleation in 2D BEC

5.Summary

Summary

- BEC with two spin components as the analogous relativistic quantum field
- Relative phase corresponds to the false/true vacuum
- Components are coupled via modulation microwave
- Thermal fluctuations coexist with true vacua
- Bubble nucleation is accelerated at finite temperature
- Questions: Observables? Oscillons vs domain walls? Vacuum stablization?

Reference

- O. Fialko *et al.*, *EPL* 110, 56001 (2015); O. Fialko *et al.*, *J. Phys. B: At. Mol. Opt. Phys.* 50, 024003 (2017); K.L. Ng *et al., PRX Quantum 2*, 010350 (2021)
- T. P. Billam *et al.*, *Phys. Rev. D* 100, 065016(2019) ; T.P. Billam *et al.*, *Phys. Rev. A* 104 053309 (2021)
- J. Braden *et al.*, *Phys. Rev. Lett.* 123 031601 (2019); J. Braden *et al.*, *JHEP* 2019, 174 (2019)

Thank You