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Correlation aspects of interacting quantum systems in one dimension

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Abstract

We study interacting quantum systems in one spatial dimension in the framework of Bethe ansatz solutions. One example of such is the system composed of interacting bosonic particles restricted to exist in one dimension, which is known as interacting Bose gas or Lieb-Liniger model. Its one-body correlation function can be evaluated in terms of Fredholm type determinants, however we aim at extracting the importance of certain elementary excitations for the correlator. For this, we formulate the one-body function structural components (the so-called form factors) in terms of those excitations and observe their convergence to the exact determinantal result. Furthermore, the addition of a confining potential breaks the integrability of the system. However, in the strongly interacting limit, or Tonks-Girardeau gas, the bosonic system can be mapped into a Fermi gas, restoring the integrability in cases where the wave function of the fermionic counterpart is known. In the asymptotic regime, the particles' momentum distribution can be described by the so-called Tan's contact. Such a quantity is an important conceptual idea to understand correlation and interaction effects over interacting systems. Finally, if we consider a free gas interacting with only one impurity, such a system is also part of the Bethe ansatz integrable model family and it is known as McGuire model. In this context, we evaluate the impurity's correlation function and show its asymptotic convergence with an effective form-factors theory.

One-dimensional Bose gas

Lieb-Liniger model

A one-dimensional system composed of N particles interacting via a δ-like potential is described by the Lieb-Liniger model,

$$H = -\sum_{j=1}^{N} \partial_j^2 + 2c \sum_{j>\ell} \delta(x_j - x_\ell), \tag{1}$$

which can be solved by Bethe ansatz for the set of allowed momenta

$$\lambda_j + \frac{2}{L}\sum_{l=1}^{N} \tan^{-1}\left(\frac{\lambda_j - \lambda_l}{c}\right) = \frac{2\pi}{L} I_j, \quad j = 1, \dots, N. \tag{2}$$

- $c \to \infty$ and $T = 0$:

$$\lambda_j = \frac{2\pi}{L} I_j, \qquad I_j^{GS} = j - \frac{N+1}{2}. \tag{3}$$

One-body function

One-body correlation function at zero temperature,

$$\rho(x,t) = \sum_{\mu \in \mathcal{H}_{N-1}} e^{-i(E_\lambda - E_\mu)t} e^{i(P_\mu - P_\lambda)x} |\langle \mu | \Psi(0) | \lambda \rangle|^2. \tag{4}$$

Question: Is there a specific subspace of $\mathcal{H}_{N-1}$ that provides a sufficiently large contribution to the one-body function (4)?

- Generic excited state: Two holes, (h_1, h_2), and m particle-hole excitations, (p_j, h_j) with $j = 3, \dots, m+2$. The rapidity set $\boldsymbol{\mu}$ for the excited state is then $\boldsymbol{\mu} = \bar{\boldsymbol{\mu}} - \mathbf{h}_{m+2} + \mathbf{p}_m$, yielding

$$\begin{aligned} |\langle \boldsymbol{\mu} | \Psi(0) | \boldsymbol{\lambda} \rangle|^2 = &\frac{1}{2}\left(\frac{2}{L}\right)^{2N-1} \frac{\prod_{j>k=1}^{N} (\lambda_j - \lambda_k)^2 \prod_{j>k=1}^{N+1} (\bar{\mu}_j - \bar{\mu}_k)^2}{\prod_{j=1}^{N} \prod_{k=1}^{N+1} (\lambda_j - \bar{\mu}_k)^2} \\ &\times \prod_{a=1}^{m+2} \frac{\prod_{j=1}^{N} (\lambda_j - h_a)^2}{\prod_{j=1}^{N+1} (\bar{\mu}_j - h_a)^2} \prod_{a=3}^{m+2} \frac{\prod_{j=1}^{N+1} (\bar{\mu}_j - p_a)^2}{\prod_{j=1}^{N} (\lambda_j - p_a)^2} \\ &\times \frac{\prod_{a>b=3}^{m+2} (h_a - h_b)^2 \prod_{a>b=1}^{m+2} (p_a - p_b)^2}{\prod_{a=1}^{m+2} \prod_{b=3}^{m+2} (h_a - p_b)^2}. \end{aligned} \tag{5}$$

- A further particle-hole excitation over the excited state $|\boldsymbol{\mu}\rangle$ (2sp+mph), *i.e.* $|\boldsymbol{\mu} + (p_{m+3}, h_{m+3})\rangle$, originates a form factor smaller than the primary one [1],

$$|\langle \boldsymbol{\mu} + (p_{m+3}, h_{m+3}) | \Psi(0) | \boldsymbol{\lambda} \rangle|^2 < |\langle \boldsymbol{\mu} | \Psi(0) | \boldsymbol{\lambda} \rangle|^2. \tag{6}$$

Trapped Lieb-Liniger

Harmonic trap

Consider a harmonically trapped Lieb-Liniger gas,

$$H = \sum_{j=1}^{N} \left(-\partial_j^2 + x_j^2\right) + g \sum_{j>\ell} \delta(x_j - x_\ell). \tag{7}$$

The addition of an external potential breaks its integrability due to the presence of inhomogeneity.

Strongly interacting limit

When the repulsive interaction between particles becomes infinitely large, the *fermionization* of the bosons occurs. Such a regime is also known as Tonks-Girardeau (TG) gas. It restores the integrability of the system as

$$\Psi_\alpha^{(b)}(x_1, \dots, x_N) = \prod_{i>j} \mathrm{sgn}(x_i - x_j) \Psi_\alpha^{(f)}(x_1, \dots, x_N). \tag{8}$$

Tan's contact

The asymptotic behavior of the momentum distribution is characterized by the contact,

$$\mathcal{C} \equiv \lim_{k \to \infty} k^4 n(k). \tag{9}$$

- Boson-fermion correspondence $\to$ trapped Fermi gas single-particle solutions,

$$\begin{aligned} \mathcal{C} = &\frac{2}{\pi} \int dx\, \mathcal{Z}^{-1} \sum_{\{n_j\}} e^{-\beta \sum_{j=1}^{N} E_{n_j}} \\ &\times \sum_{j \neq k} \left\{ \left[\phi_{n_j}(x) \partial_x \phi_{n_k}(x)\right]^2 - \phi_{n_j}(x) \phi_{n_k}(x) \partial_x \phi_{n_j}(x) \partial_x \phi_{n_k}(x) \right\}. \end{aligned} \tag{10}$$

- Universal scaling [2]:

$$\mathcal{C}_N \propto N^{5/2} - N^{3/4[1+\exp(-2/\tau)]}. \tag{11}$$

Mobile impurity

A system of spin up particles with momenta P_j and coordinates x_j interacting with a spin down particle (the impurity) with momentum P_{imp} and coordinate X_{imp} is described by the McGuire model,

$$H = P_{imp}^2 + \sum_{j=1}^{N} P_j^2 + g \sum_{j=1}^{N} \delta(x_j - x_{imp}). \tag{12}$$

Periodic boundary conditions + length L: rapidities obey the Bethe equations

$$k_j = \frac{2\pi}{L}\left(n_j - \frac{\delta(k_j)}{\pi}\right), \qquad \delta(k) = \frac{\pi}{2} - \arctan(\Lambda - \alpha k), \qquad j = 1, \dots, N+1, \tag{13}$$

where the quantum numbers n_j are integers and obey the Pauli principle.

- Fredholm determinant:

$$\rho(x; \sigma, \Lambda) = \det\left(1 + \sigma\hat{K} + \sigma\hat{W}\right) - \det\left(1 + \sigma\hat{K}\right). \tag{14}$$

- Effective form-factors theory:

$$\rho(x; \sigma, \Lambda) = \begin{cases} A(\Lambda) \exp\left(-ix \int dk\, k\nu'(k)\right), & |\Lambda| < \Lambda_c, \\ 2J(x) A(\Lambda) \exp\left(-ix \int dk\, k\nu'(k)\right), & |\Lambda| > \Lambda_c. \end{cases} \tag{15}$$

- Critical point:

$$\Lambda_c := \alpha\sqrt{\mu}. \tag{16}$$

Depending on whether we are in $|\Lambda| < \Lambda_c$ or $|\Lambda| > \Lambda_c$ interval, the asymptotic behavior is structurally different [3].

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