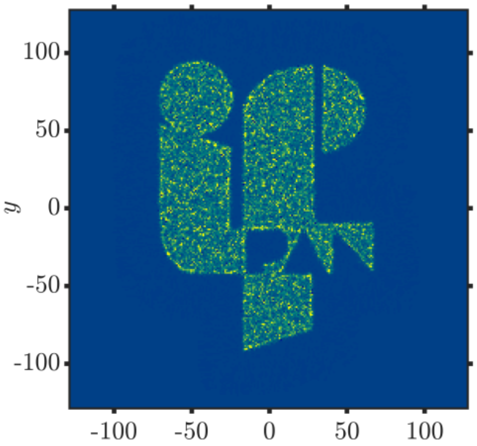
Quantum Noise group – Instytut Fizyki PAN



Piotr Deuar

Maciej Kruk

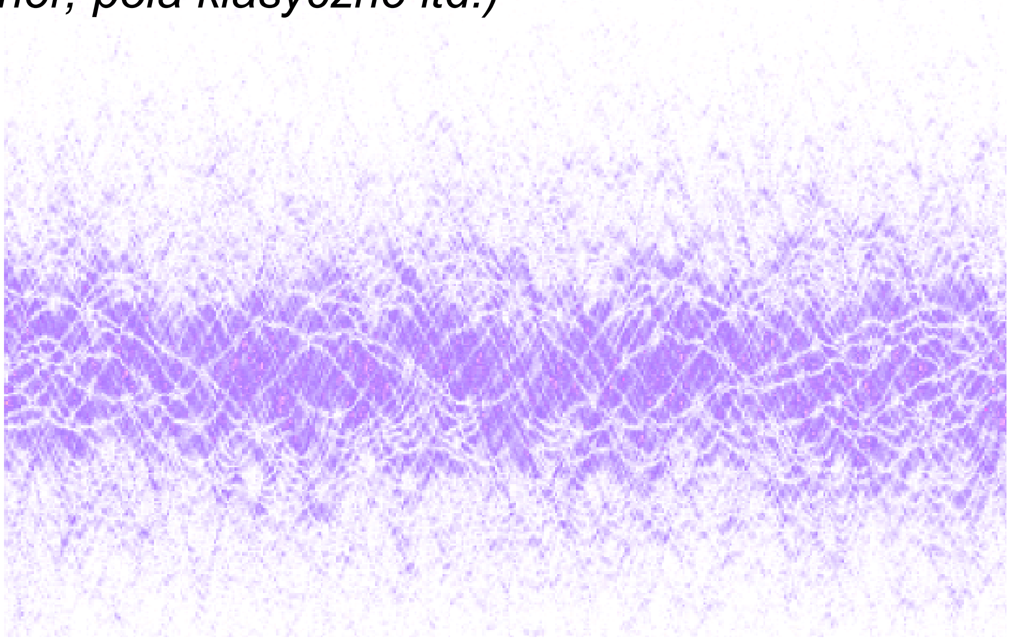
King Lun Ng

NARODOWE CENTRUM NAUKI

Modelujemy bardzo duże układy kwantowe za pomocą równań stochastycznych

(positive-P, truncated Wigner, pola klasyczne itd.)



Skalowalny opis kwantowego układu. Jak?

Przykład: reprezentacja positive-P

M podukładów (mody, oczka, *Δv*) *1, …, j*

"ket" amplitude α_j
"bra" amplitude β_j^*

Baza stanów koherentnych, *lokalna* $|\alpha_j\rangle_j = e^{-|\alpha_j|^2/2}\, e^{\alpha_j \widehat{a}_j^\dagger} |\mathrm{vac}\rangle$

Lokalny operator (*"kernel"*) $\widehat{\Lambda}(\boldsymbol{\lambda}) = \bigotimes_j \frac{|\alpha_j\rangle_j \langle \beta_j^*|_j}{\langle \beta_j^*|_j |\alpha_j\rangle_j}$

Pełna konfiguracja układu $\boldsymbol{\lambda} = \{\alpha_1, \dots, \alpha_M, \beta_1, \dots, \beta_M\}$

Pełna macierz gęstosci $\widehat{\rho} = \int d^{4M}\boldsymbol{\lambda}\, P_+(\boldsymbol{\lambda})\, \widehat{\Lambda}(\boldsymbol{\lambda})$

Korelacje między podukładami są w rozkładzie $P_+(\boldsymbol{\lambda})$

rozkład $P_+(\boldsymbol{\lambda})$ jest dodatni i rzeczywisty ⟶ można próbkować

Density matrix $\hat{\rho}$ ↔ distribution P_+ for the fields ↔ random samples of the fields α β

Master equation:

dissipation γ

$$\frac{\partial \hat{\rho}}{\partial t} = -i\left[\hat{H}, \hat{\rho}\right] + \frac{\gamma}{2}\left(2\hat{a}\hat{\rho}\hat{a}^\dagger - \hat{a}^\dagger\hat{a}\hat{\rho} - \hat{\rho}\hat{a}^\dagger\hat{a}\right)$$

Bose-Hubbard site

$$\hat{H} = \frac{U}{2}\hat{a}^\dagger\hat{a}^\dagger\hat{a}\hat{a} - \Delta\hat{a}^\dagger\hat{a}$$

apply identities

Fokker Planck equation

$$\frac{\partial P_+}{\partial t} = \left\{-\frac{\partial}{\partial\alpha}\left(-iU\alpha\beta + i\Delta - \frac{\gamma}{2}\right)\alpha - \frac{\partial}{\partial\beta}\left(iU\alpha\beta - i\Delta - \frac{\gamma}{2}\right)\beta + \frac{\partial^2}{\partial\alpha^2}\left(\frac{-iU}{2}\right)\alpha^2 + \frac{\partial^2}{\partial\beta^2}\left(\frac{iU}{2}\right)\beta^2\right\}P_+$$

deterministic GPE (ket) — deterministic (bra) — diffusion (quantum noise)

stochastic correspondence

Stochastic (Langevin) equations:

different noises

$$\frac{d\alpha}{dt} = \left(-iU\alpha\beta + i\Delta - \frac{\gamma}{2}\right)\alpha + \sqrt{-iU}\,\alpha\xi(t)$$

$$\frac{d\beta}{dt} = \left(+iU\alpha\beta - i\Delta - \frac{\gamma}{2}\right)\beta + \sqrt{+iU}\,\beta\widetilde{\xi}(t)$$

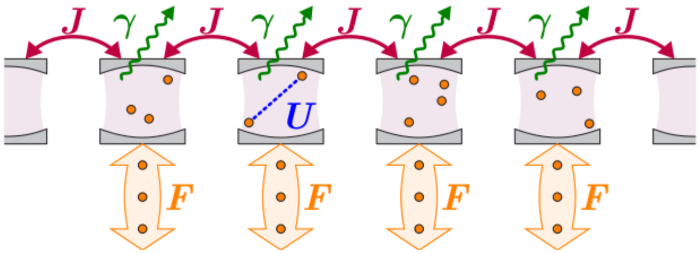
mean field part — quantum noise part

White noise

$$\langle\xi(t)\xi(t')\rangle_{\text{stoch}} = \delta(t-t')$$

Dyssypacyjny model Bosego-Hubbarda

Collab. with Marzena Szymańska & Michał Matuszewski
*PRX Quantum **2**, 010319 (2021); Quantum **5**, 455 (2021)*



$$\widehat{H} = \sum_j \widehat{H}_j - \sum_{\text{connections}\, i,j} \left[J_{ij}\widehat{a}_j^\dagger \widehat{a}_i + J_{ij}^* \widehat{a}_i^\dagger \widehat{a}_j \right]$$

$$\widehat{H}_j = -\Delta_j \widehat{a}_j^\dagger \widehat{a}_j + \frac{U_j}{2} \widehat{a}_j^\dagger \widehat{a}_j^\dagger \widehat{a}_j \widehat{a}_j + F_j \widehat{a}_j^\dagger + F_j^* \widehat{a}_j$$

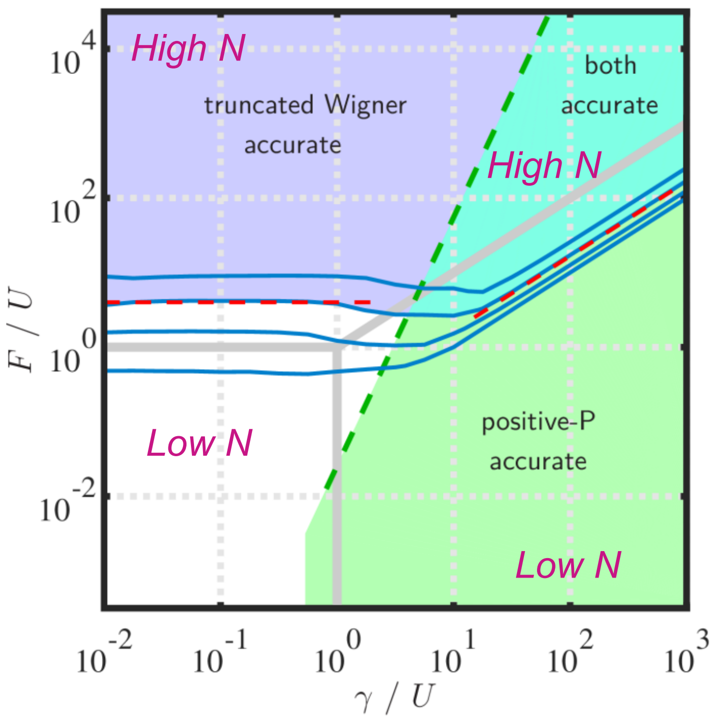
$$\frac{\partial \widehat{\rho}}{\partial t} = -i\left[\widehat{H}, \widehat{\rho}\right] + \sum_j \frac{\gamma_j}{2}\left[2\widehat{a}_j \widehat{\rho}\, \widehat{a}_j^\dagger - \widehat{a}_j^\dagger \widehat{a}_j \widehat{\rho} - \widehat{\rho}\, \widehat{a}_j^\dagger \widehat{a}_j\right]$$

Positive-P equations

$$\frac{\partial \alpha_j}{\partial t} = i\Delta_j \alpha_j - iU_j \alpha_j^2 \widetilde{\alpha}_j^* - iF_j - \tfrac{\gamma_j}{2}\alpha_j + \sqrt{-iU_j}\,\alpha_j\,\xi_j(t) + \textstyle\sum_k iJ_{kj}\alpha_k,$$

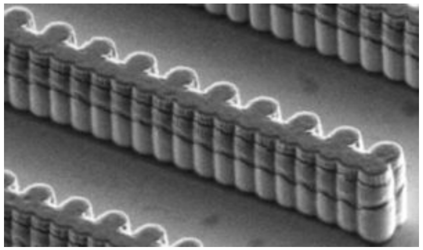
$$\frac{\partial \widetilde{\alpha}_j}{\partial t} = i\Delta_j \widetilde{\alpha}_j - iU_j \widetilde{\alpha}_j^2 \alpha_j^* - iF_j - \tfrac{\gamma_j}{2}\widetilde{\alpha}_j + \sqrt{-iU_j}\,\widetilde{\alpha}_j\,\widetilde{\xi}_j(t) + \textstyle\sum_k iJ_{kj}\widetilde{\alpha}_k$$

Regimes of applicability

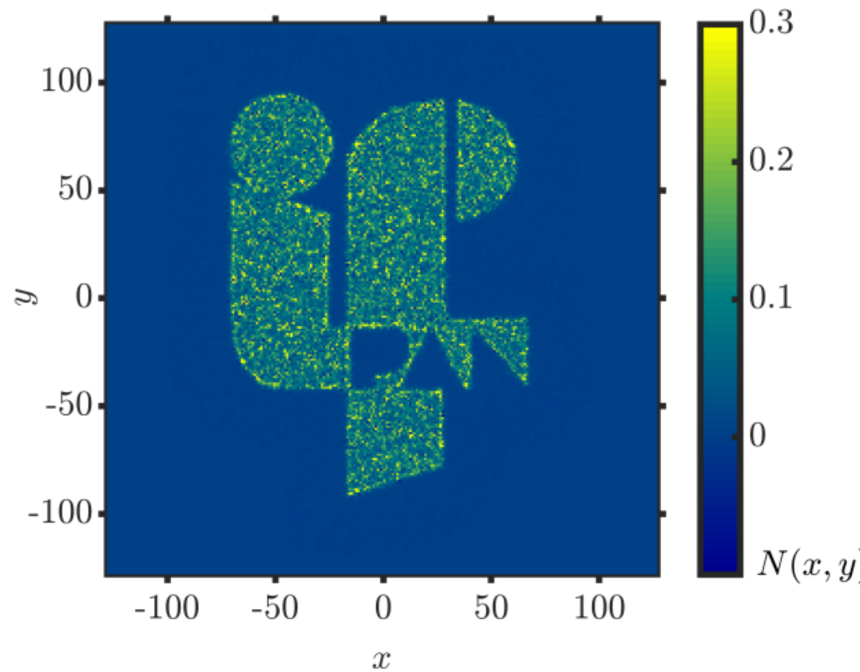


positive-P stability condition:

$$\gamma \gtrsim 3U \max\left[\sqrt{N}, 1\right]$$



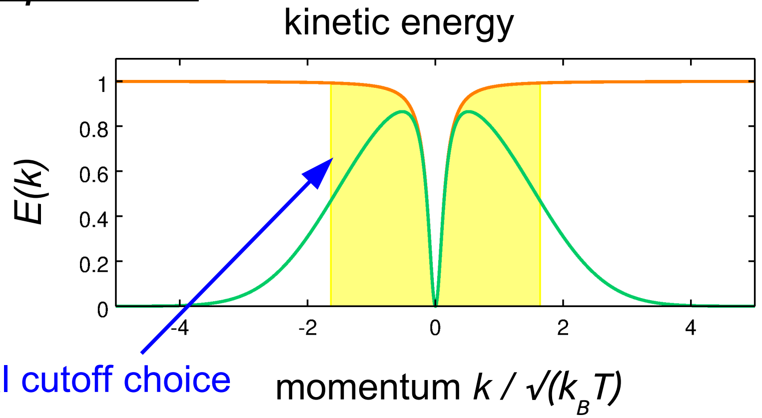
Exact quantum stationary state on a 256 x 256 site lattice

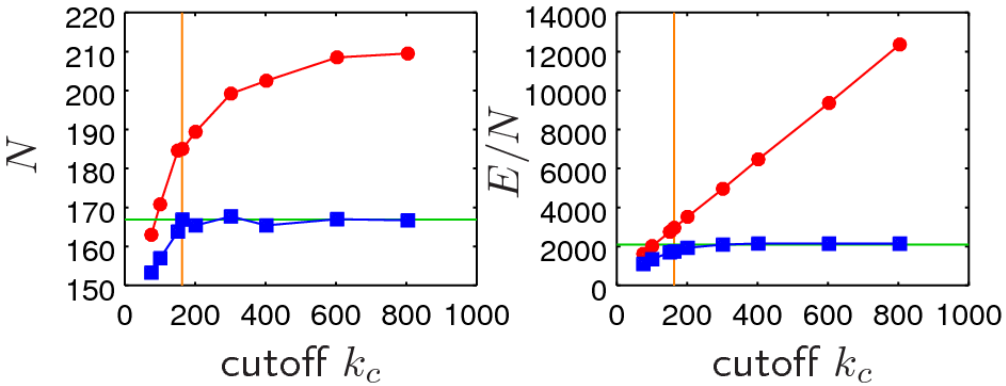


Pola klasyczne – pokonywanie zależności od cutoff-u.

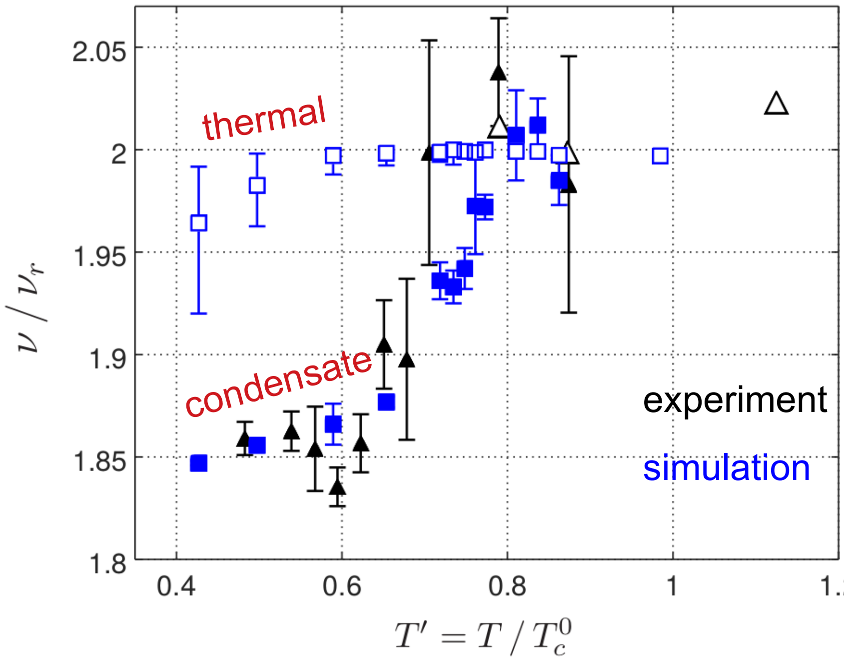
PD + Joanna Pietraszewicz, arXiv:1904.06266

<u>*The problem*</u>





Famous experiment
T-dependent collective mode frequency:
Jin, Matthews, Ensher, Wieman, Cornell, PRL 78, 764 (1997)



Classical fields (cutoff problem)

$$i\hbar \frac{d\phi(\mathbf{x})}{dt} = \mathcal{E}(\mathbf{x})\phi(\mathbf{x}) \qquad \mathcal{E}(\mathbf{x}) = -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) + g|\phi(\mathbf{x})|^2$$

SGPE (stochastoc GPE) [still a cutoff problem]

$$i\hbar \frac{d\phi(\mathbf{x})}{dt} = (1 - i\gamma)\left[\mathcal{E}(\mathbf{x}) - \mu\right]\phi(\mathbf{x}) + \sqrt{2\hbar\gamma k_B T}\,\eta(\mathbf{x},t)$$

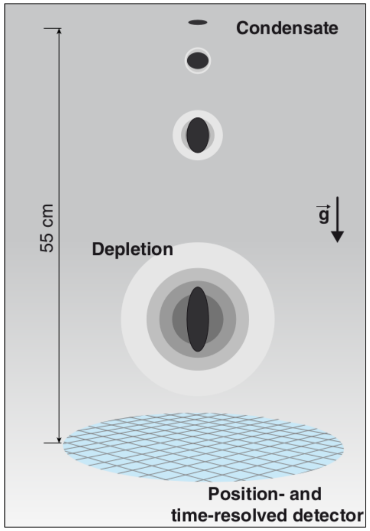
Regularised SGPE {cutoff problem is pacified}

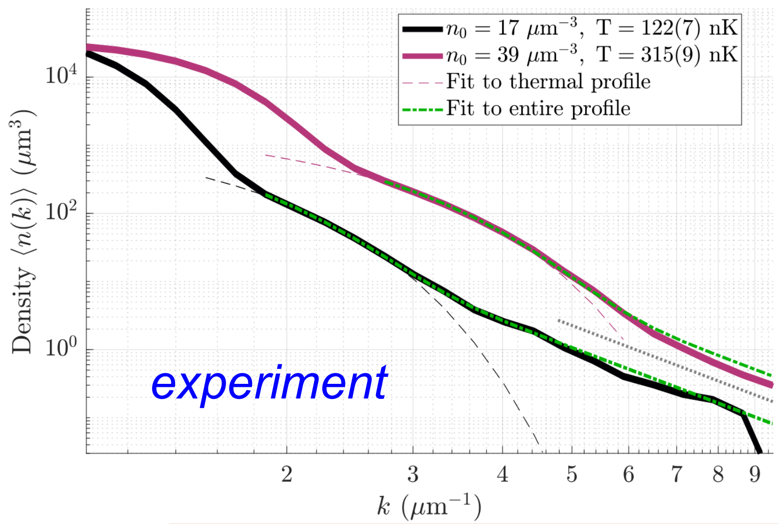
$$i\hbar \frac{d\phi(\mathbf{x})}{dt} = \mathcal{E}(\mathbf{x})\phi(\mathbf{x}) - i\gamma k_B T\left[e^{\frac{\mathcal{E}(\mathbf{x})-\mu}{k_B T}} - 1\right]\phi(\mathbf{x}) + \sqrt{2\hbar\gamma k_B T}\,\eta(\mathbf{x},t)$$

No dynamical theory has been successful in replicating this – over 20 years

Opróżnienie kwantowe poza kondensatem

Collaboration with Andrew Truscott (ANU, Canberra) arXiv:2103.15283





$$\textit{in situ}\ \text{tails:}\quad \lim_{k\to\infty} n(k) = \frac{\mathscr{C}}{k^4} = \frac{64\pi^2 a^2}{7}\frac{N_0 n_0}{k^4}$$

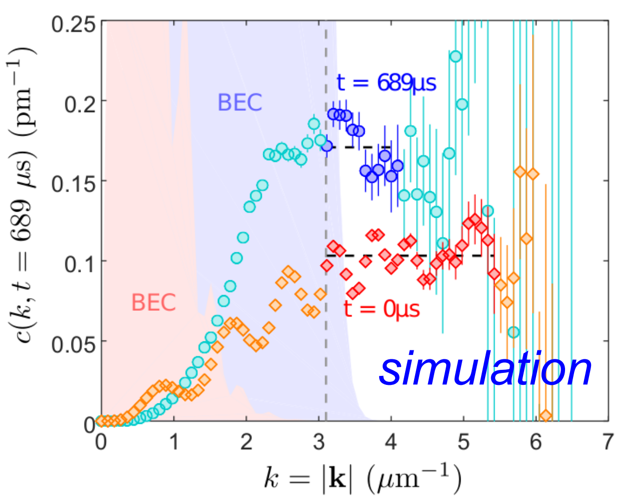
	Released tail strength compared to *in situ* value	
Chang, Bouton, Cayla, Qu, Aspect, Westbrook, Clement, PRL 117, 235303 (2016)	x 6 ± 1	Eksperyment (Palaiseau)
Qu, Pitaevskii, Stringari PRA 94, 063635 (2016)	x 0	Teoria (Trento)
Ross, Deuar, Shin, Thomas, Henson, Hodgman, Truscott, ArXiv: 2103.15283	x 2 ± 0.2	Symulacje (Warszawa)
Ross, Deuar, Shin, Thomas, Henson, Hodgman, Truscott, ArXiv: 2103.15283	x 5 ± 3	Eksperyment (Canberra)
Cayla, Massignan, Giamarchi, Aspect, Westbrook, Clement to appear (2022)	x 0 - 6	Eksperyment2 (Palaiseau)

"STAB" method

Stochastic

Time Adaptive

Boboliubov



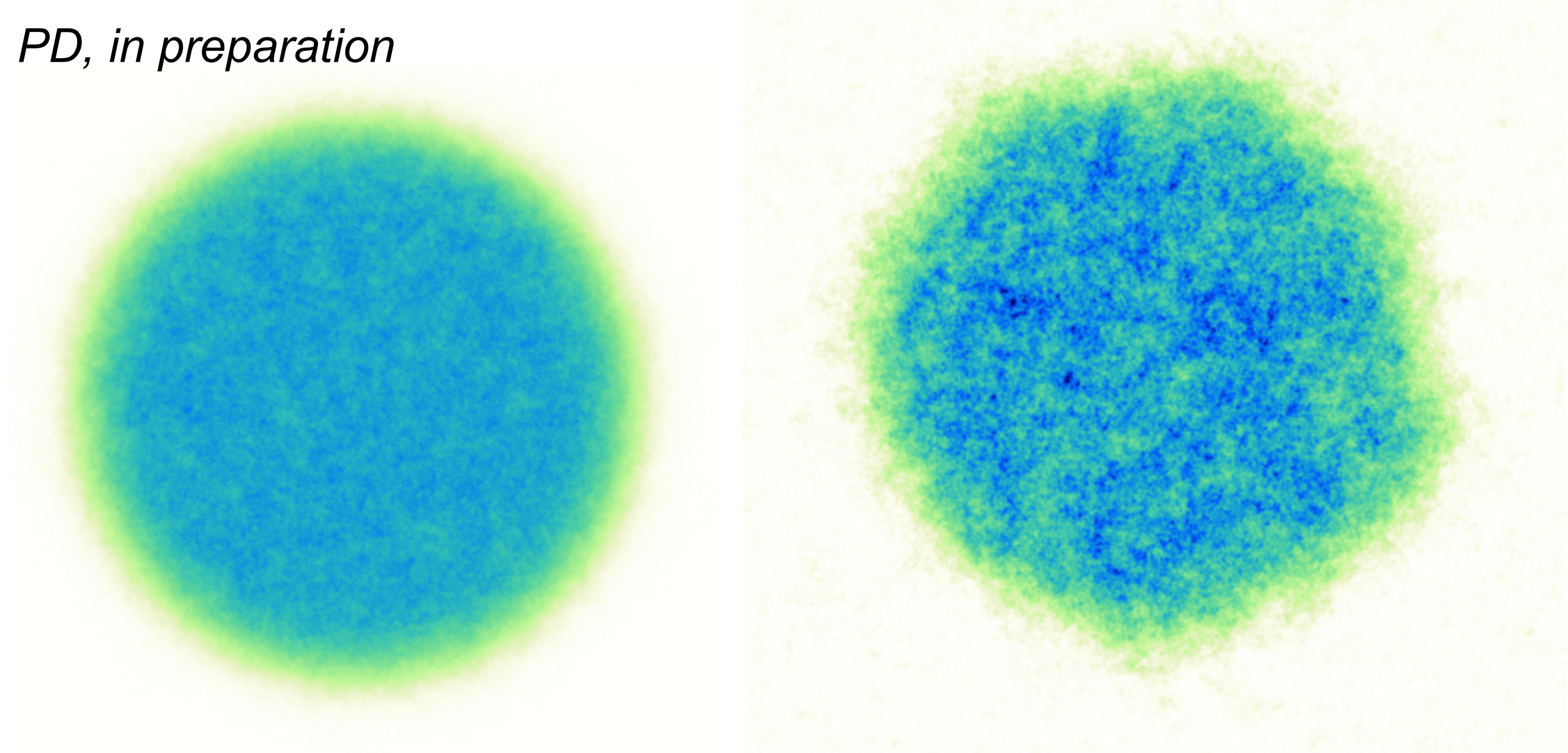
$$\widehat{\Psi}(\mathbf{x},t) = \phi(\mathbf{x},t) + \widehat{\Psi}_B(\mathbf{x},t)$$

$$i\hbar\frac{d\psi_B}{dt} = \left[-\frac{\hbar^2}{2m}\nabla^2 + g|\phi|^2 + V(\mathbf{x},t)\right]\psi_B + \mathscr{P}_\perp\left\{g|\phi|^2\psi_B + g\phi^2\widetilde{\psi}_B^* + \sqrt{-ig}\ \phi\,\xi(\mathbf{x},t)\right\}$$

$$i\hbar\frac{d\widetilde{\psi}_B}{dt} = \left[-\frac{\hbar^2}{2m}\nabla^2 + g|\phi|^2 + V(\mathbf{x},t)\right]\widetilde{\psi}_B + \mathscr{P}_\perp\left\{g|\phi|^2\widetilde{\psi}_B + g\phi^2\psi_B^* + \sqrt{-ig}\ \phi\,\widetilde{\xi}(\mathbf{x},t)\right\}$$

Krople kwantowe w niezerowej temperaturze

Kruk, King, PD, in preparation



Quasi-2D droplets in contact with thermal reservoir: low temperature (left) and high temperature (right).

$$i\hbar \frac{d\phi(\mathbf{x})}{dt} = \left[-\frac{\hbar^2}{2m}\nabla^2 - |\delta g|\,|\phi(\mathbf{x})|^2 + \alpha|\phi(\mathbf{x})|^3 \right] \phi(\mathbf{x})$$

Extended GPE
(standard T=0 theory)
Petrov 2015

Our stochastic EGPE: T > 0

$$i\hbar \frac{d\phi(\mathbf{x})}{dt} = (1 - i\gamma) \left[-\frac{\hbar^2}{2m}\nabla^2 - \mu - |\delta g|\,|\phi(\mathbf{x})|^2 + \alpha|\phi(\mathbf{x})|^3 \right] \phi(\mathbf{x}) + \sqrt{2\gamma k_B T}\, \eta(\mathbf{x}, t)$$