1. Motivation

- Interested in mechanisms of pair production of particles for quantum-atom optics
  (pair correlated photons have been pivotal to quantum optics; e.g., demonstrations of EPR paradox and violations of Bell’s inequalities)
- Pair-correlated atoms in BEC collisions have been detected in [1]
- Modified experiments are underway; the new geometry gives better detection access
- First-principles simulations are now possible for the entire collision duration (in contrast to [2])

2. Hamiltonian and positive-P equations

\[
\dot{\phi} = \left[ \frac{\hbar}{2m} \nabla^2 \phi + \frac{\hbar}{2m} \nabla \nabla \phi + \frac{\hbar}{2m} \nabla \nabla \phi + \frac{\hbar}{2m} \nabla \nabla \phi \right] \quad (1)
\]

Positive-P (phase-space) method: the quantum many-body dynamics, governed by the Hamiltonian (1), is simulated exactly via stochastic differential equations for c-fields [2]:

\[
\frac{\partial \phi(x,t)}{\partial t} = \frac{\hbar}{2m} \nabla^2 \phi(x,t) + \frac{\hbar}{2m} \nabla \nabla \phi(x,t) + \frac{\hbar}{2m} \nabla \nabla \phi(x,t) + \frac{\hbar}{2m} \nabla \nabla \phi(x,t)
\]

- \( \phi(x,t) \) - noise terms, with \( \left( \phi(x,t) \phi(x',t) \right) = \delta(x-x')\delta(t-t) \)
- \( U_{\phi} = 4\hbar^2/\alpha m - s\)-wave scattering interaction

3. Simulation parameters

- Total No. of He* atoms in the initial BEC: 10^2
- Initial condition: pure BEC in a coherent state, split into two counter-propagating halves
- Harmonic trap: 47/1150 Hz
- Relative collision velocity: \( v_c = 7.36 \text{ cm/s} \)
- Numerical lattice: 1024 x 48 x 112 lattice points
- Number of modes simulated: 5505024
- Positive-P simulations run for 70 \( \mu \text{s} \)
- Number of stochastic trajectory averages: 2000
- Computing time: 6-7 days on 10 CPUs

Figure 1. Schematic diagram of the collision geometry. The two disks represent the colliding condensates; the sphere represents the scattered atoms. The cigar shaped initial condensate is shown in the middle.

Figure 2. Atomic momentum distribution (positive-P simulation, average of 2000 stochastic trajectories): Three orthogonal cuts through the origin after 70 \( \mu \text{s} \) collision duration, by which time the collision has essentially ceased. The (nearly) spherical shell of scattered atoms is clearly seen, with the darker regions corresponding to the colliding condensates. A more careful quantitative analysis reveals a surprise: the scattering shell renders itself as an ellipsoid (see Fig. 4); the experiment also shows this!

Figure 3. Real-space density of the atomic cloud after 70 \( \mu \text{s} \), showing the spatial separation of the two colliding condensates.

Figure 4. Slice of the atomic momentum distribution through the origin in the \( k_x, k_y \) plane in polar coordinates (left panel); the respective radial densities along \( k_x \) and \( k_y \) axis (right panel) show the anisotropy of the distribution and its ellipticity (\( x = 0.03 \)). The axis are in units of \( k_B/eV \).

### Table: Comparison of theoretical and experimental results

<table>
<thead>
<tr>
<th>Comparison</th>
<th>Peak position (units of ( k_B ))</th>
<th>Peak width (units of ( k_B ))</th>
<th>Peak density (arb. units)</th>
<th>( N_{\text{scattered}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Theory</strong></td>
<td>( k_{B1} )</td>
<td>( k_{B2} )</td>
<td>( k_{B3} )</td>
<td>( k_{B4} )</td>
</tr>
<tr>
<td>0.951001</td>
<td>0.991001</td>
<td>0.80120005</td>
<td>0.12610055</td>
<td>1.4010051</td>
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<tr>
<td><strong>Experiment</strong></td>
<td>0.831002</td>
<td>0.941002</td>
<td>0.991001</td>
<td>0.11010005</td>
</tr>
</tbody>
</table>

Stochastic Hartree-Fock-Bogoliubov approach

- Apply the HFB scheme: \( \left( \phi(x,t) \phi(x,\tau) \right) = \delta(x-x')\delta(t-\tau) \)
- Re-formulate the Heisenberg equations of motion for \( \delta \phi(x,t) \)
  \[ \frac{\partial \delta \phi(x,t)}{\partial t} = \frac{\hbar^2}{2m} \nabla^2 \delta \phi(x,t) + 2U \left( \phi(x,t) \right) \frac{\partial \phi(x,t)}{\partial t} \]

According to the positive-P method:
- Easier to solve than the standard Bogoliubov method in 3D
- Can incorporate the mean field dynamics of the colliding condensates \( \phi(x,t) \) as the solution to the GP equation
- Agreement with the full positive-P simulation!

- Scattered atoms move in a complicated mean-field potential landscape of the separating and expanding condensates
- Shift \( \delta k_x \rightarrow \delta k_x \) in the radius of the sphere can be estimated as
  \[ k_x^2 + \delta k_x^2 = k_x^2 + 2U \phi(x,t) = k_x^2 + 2U \phi(x,t) = \delta k_x^2 \]
- Ellipticity: atoms moving along \( y \) slide down a steeper hill of the mean-field potential and regain the energy shift: \( U_{\phi}(\phi) \) as kinetic energy (radius is back to \( k_x \), unlike the atoms along \( x \))

REFERENCES
