Two-body correlations

vs

Single experiment snapshots

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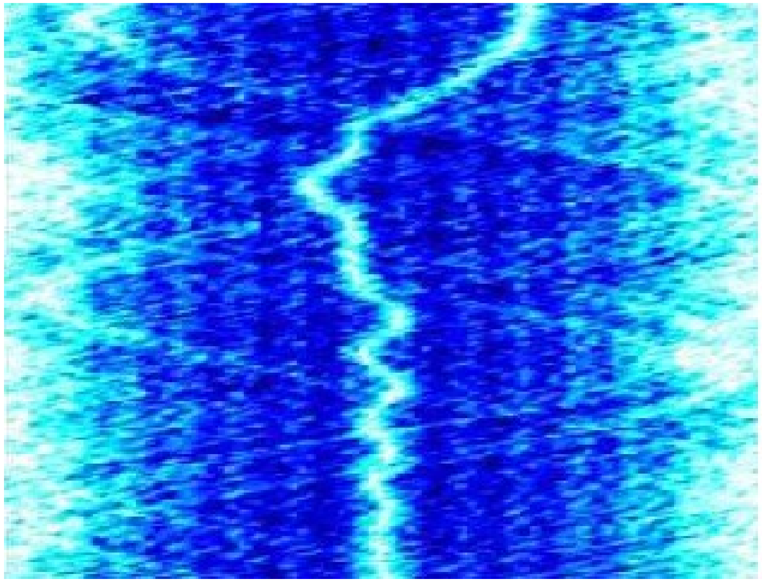
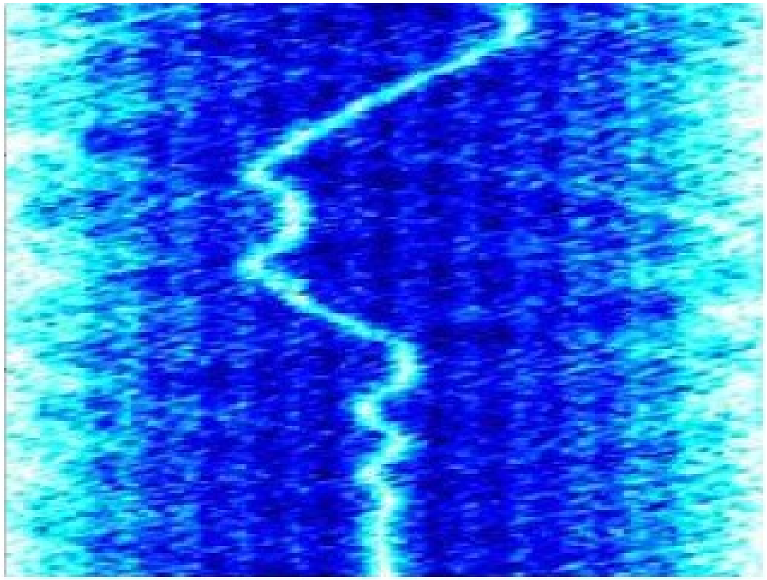
Seminarium BEC – CFT PAN



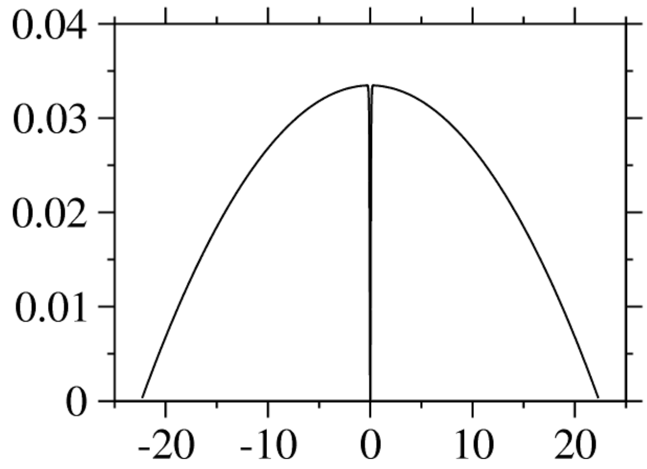


Outline

1. The soliton greying controversy
   - Early UJ work: dark soliton acquires random walk
2. One-particle vs snapshot observables
3. The soliton greying controversy *returns*
   - Colorado 2009 DMRG calculations:
     Saw g2>0 → greying
   - IFPAN/UJ comment:
     But g2 is irrelevant
   - Southampton truncated Wigner calculations:
     Looks like a random walk again
4. Density correlations and soliton greying

Introducing the dark soliton

- Repulsive BEC
  (*in a trap*)

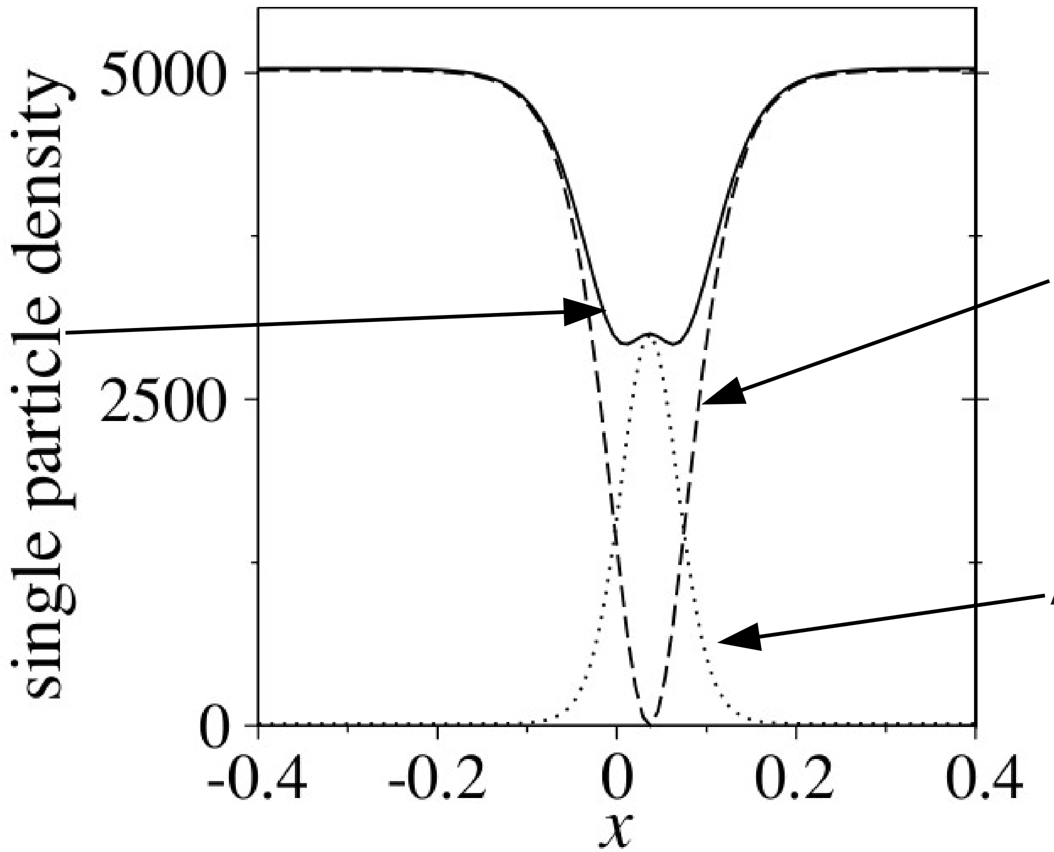


Dziarmaga & Sacha
PRA **66**, 043620 (2002)

- Dark soliton is not the ground state
- Dynamical instability
- Decay dominated by the "anomalous" Bogoliubov mode
  (smaller energy than the condensate mode)

What do dark solitons do when they're alone?

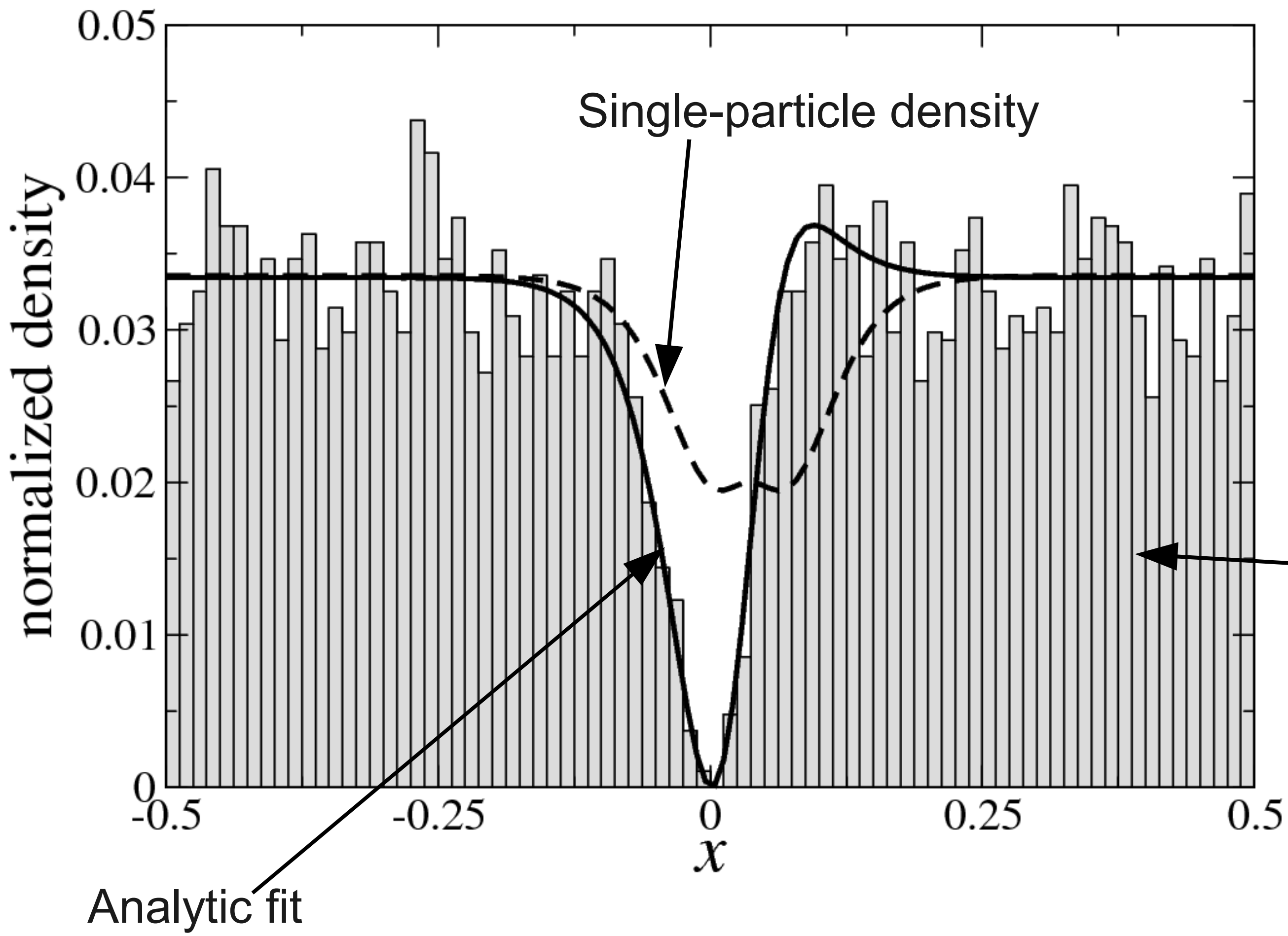
The anomalous mode is localised in the dip



Dziarmaga & Sacha
J. Phys B **39**, 57 (2006)

→ Proposal 1: The soliton GREYS (the dip fills in)

What do dark solitons do when they're alone?



Dziarmaga & Sacha
J. Phys B **39**, 57 (2006)

ONE SHOT
(histogram of
(individual atom positions)

→ It does NOT fill in at all!

Different realisations get random positions

One-particle vs snapshot

- Don't let the GP equation fool you!
- GP wavefunction gives one-particle density

$$
\begin{aligned}
n(x) &= \langle \hat{\Psi}^\dagger(x) \hat{\Psi}(x) \rangle \\
\text{'='} &\ \frac{1}{N} \sum_{j=1}^{N} |\Psi_j^*(x)|^2
\end{aligned}
$$

averaged over *all particles* & *all realisations*

Soliton example $\phi(q) = \tanh(x - q)$

- Pure dark soliton

$$|\Psi(0)\rangle = |\phi(0)\rangle \otimes |\phi(0)\rangle$$

- Condensate superposition

$$|\Psi(-0.5)\rangle + |\Psi(+0.5)\rangle$$

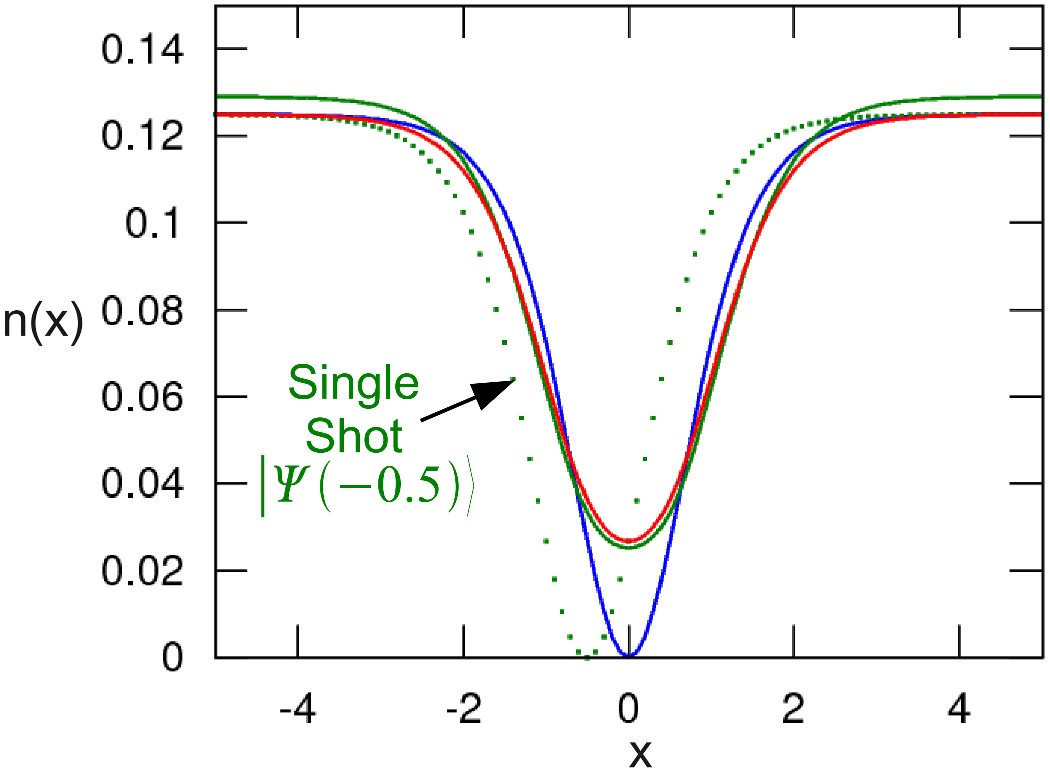
- Non-condensate

$$|\phi(-0.5)\rangle \otimes |\phi(+0.5)\rangle$$

- Mixture

$$\hat{\rho} = \frac{1}{2}|\Psi(-0.5)\rangle\langle\Psi(-0.5)| + \frac{1}{2}|\Psi(+0.5)\rangle\langle\Psi(+0.5)|$$

One-particle density



Grey solitons return

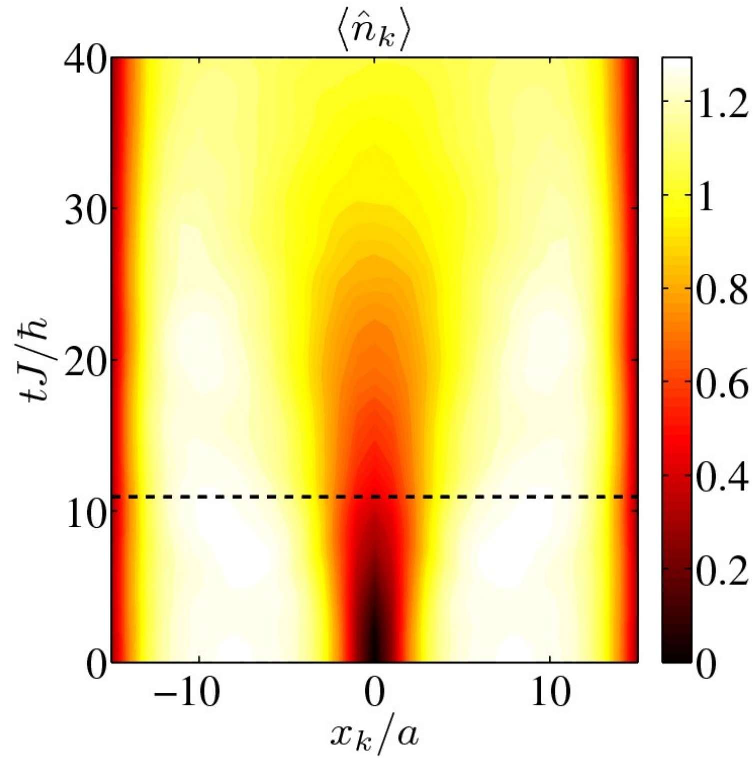
Mishmash & Carr PRL **103**, 140403 (2009); Mishmash *et al.* PRA **80**, 053612 (2009)

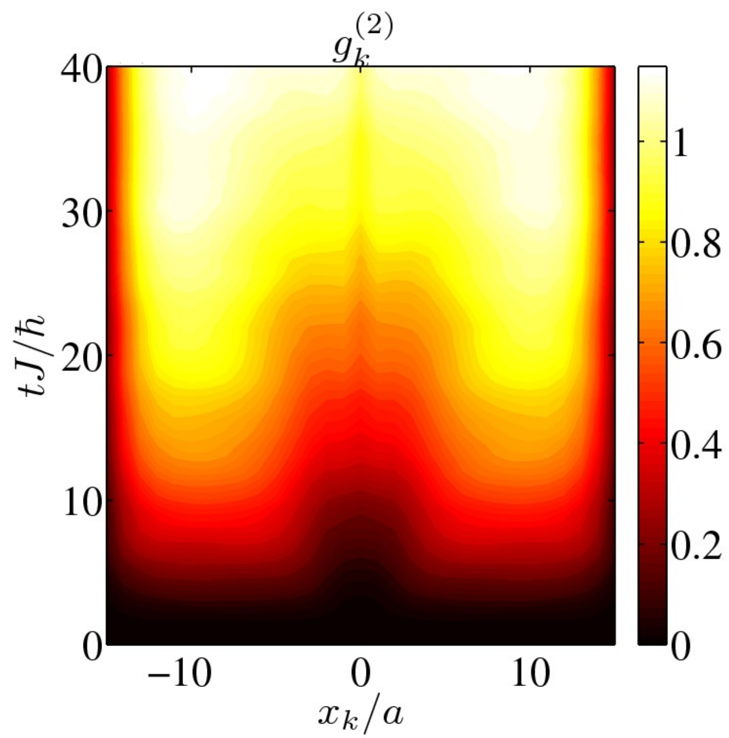
Bose Hubbard model in 1D $\langle n(x) \rangle \approx 1$

DMRG simulations show filling in of two-body correlations

$$g^{(2)} = g^{(2)}(0,x) = \frac{\langle \hat{n}(0)[\hat{n}(x) - \delta_{0x}] \rangle}{n(x)\,n(0)}$$

→ interpreted as greying of solitons

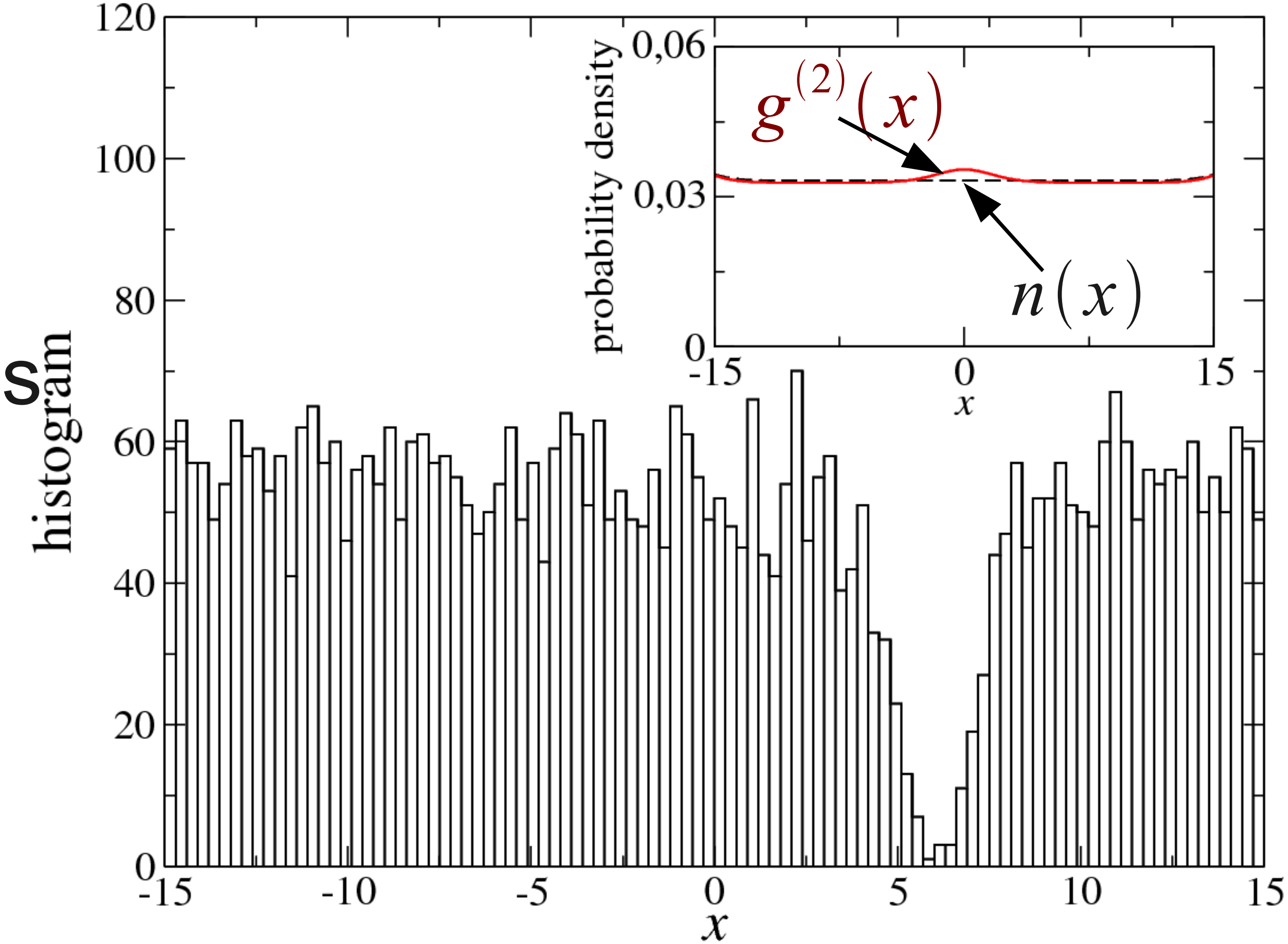




Two body correlations are irrelevant

Dziarmaga PD & Sacha arXiv:1001.1045 (2010)

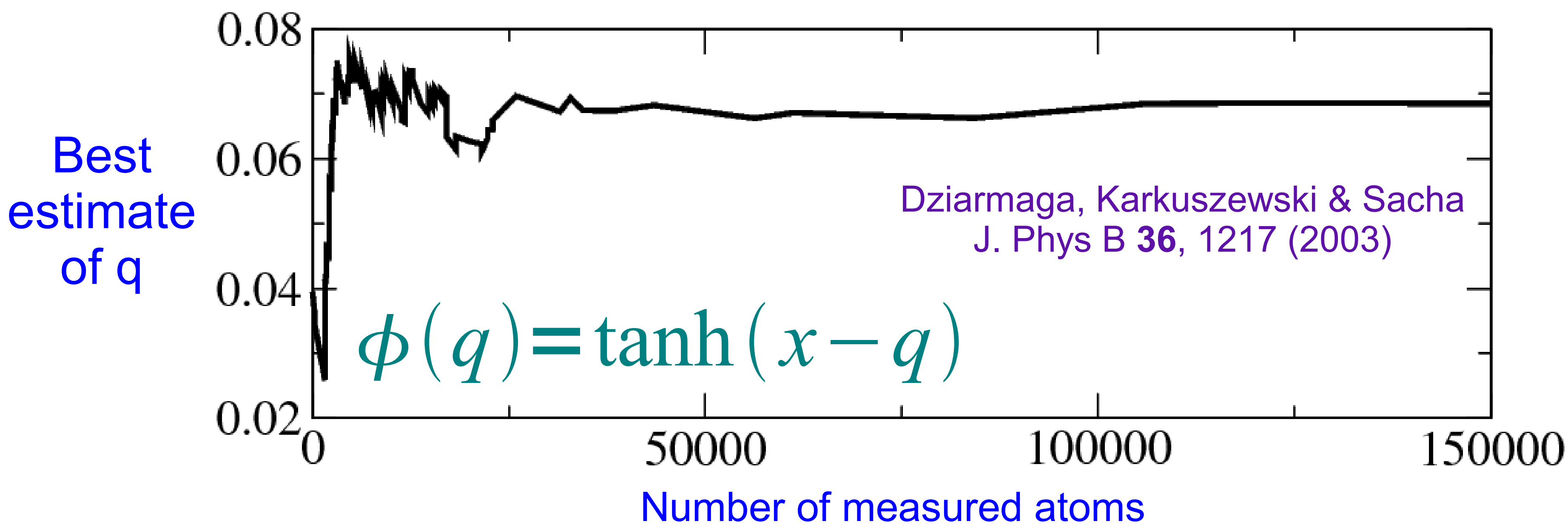
Superposition of 100% dark solitons with random position



→ filled-in g2 does NOT imply filled in solitons

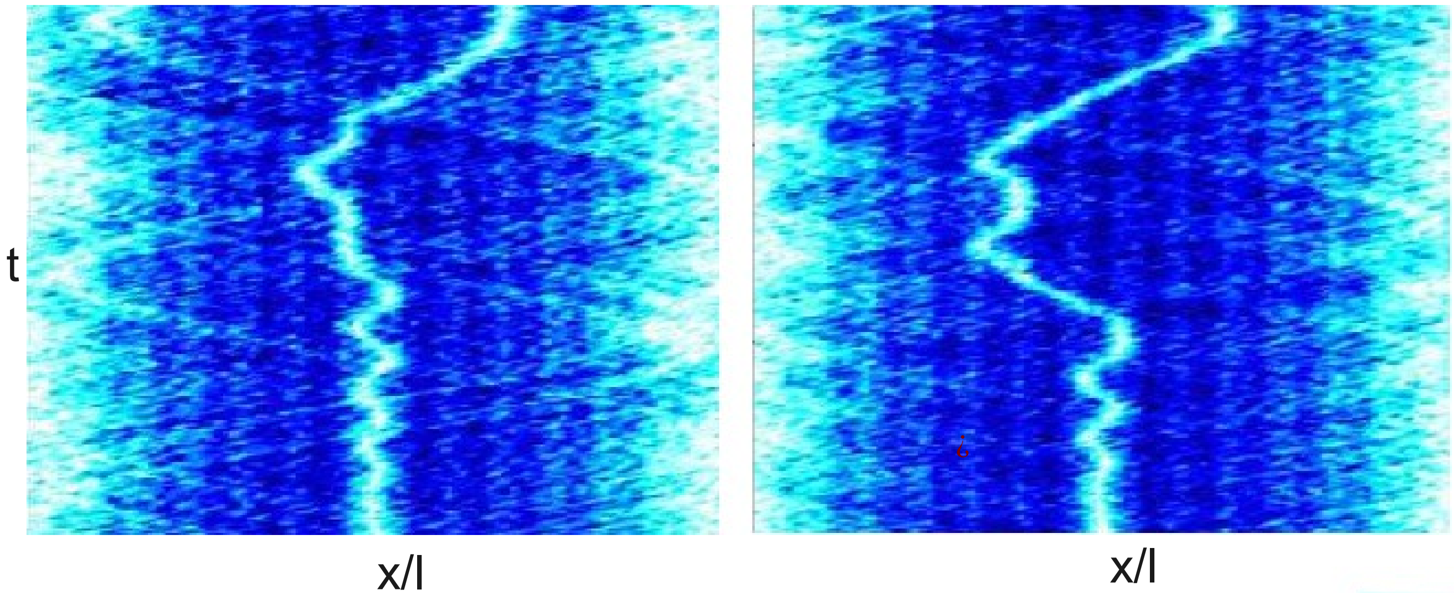
Why is g2 no good for this?

- g2 is a two-particle observable.
- 2 particles may not be enough to collapse a superposition of condensates
- Especially if >2 particles go missing due to the dark soliton

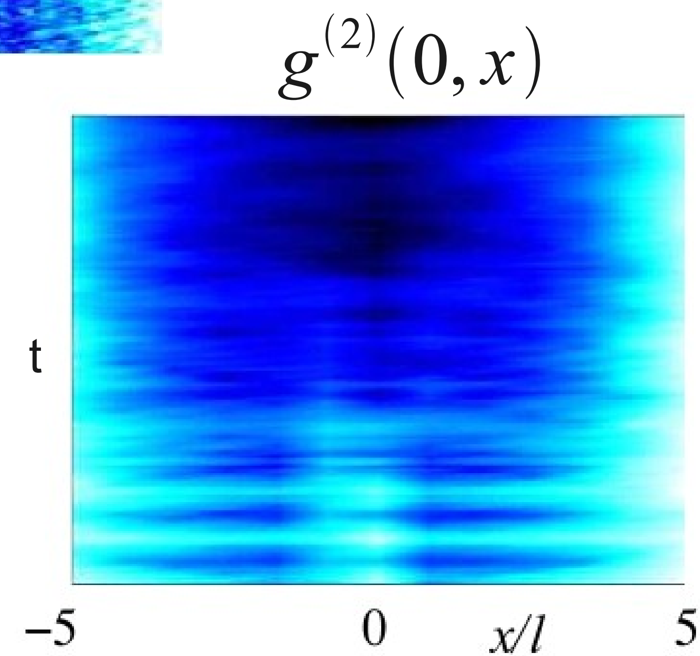


Truncated Wigner calculations

Martin & Ruostekoski arXiv:1001.3385 (2010)



Two single shots of occupation



- g2 fills in
- Single Wigner realisations (≈ 1 experiment) do NOT
- → no greying after all
- BUT, this is a system with much larger N
  → situation not 100% resolved yet

What *IS* the density correlation then?

Common view:
"it represents the typical density structure"

is this true?

Toy "classical" calculation

$$G^{(2)}(x,y)=\langle n(x)n(y)\rangle$$

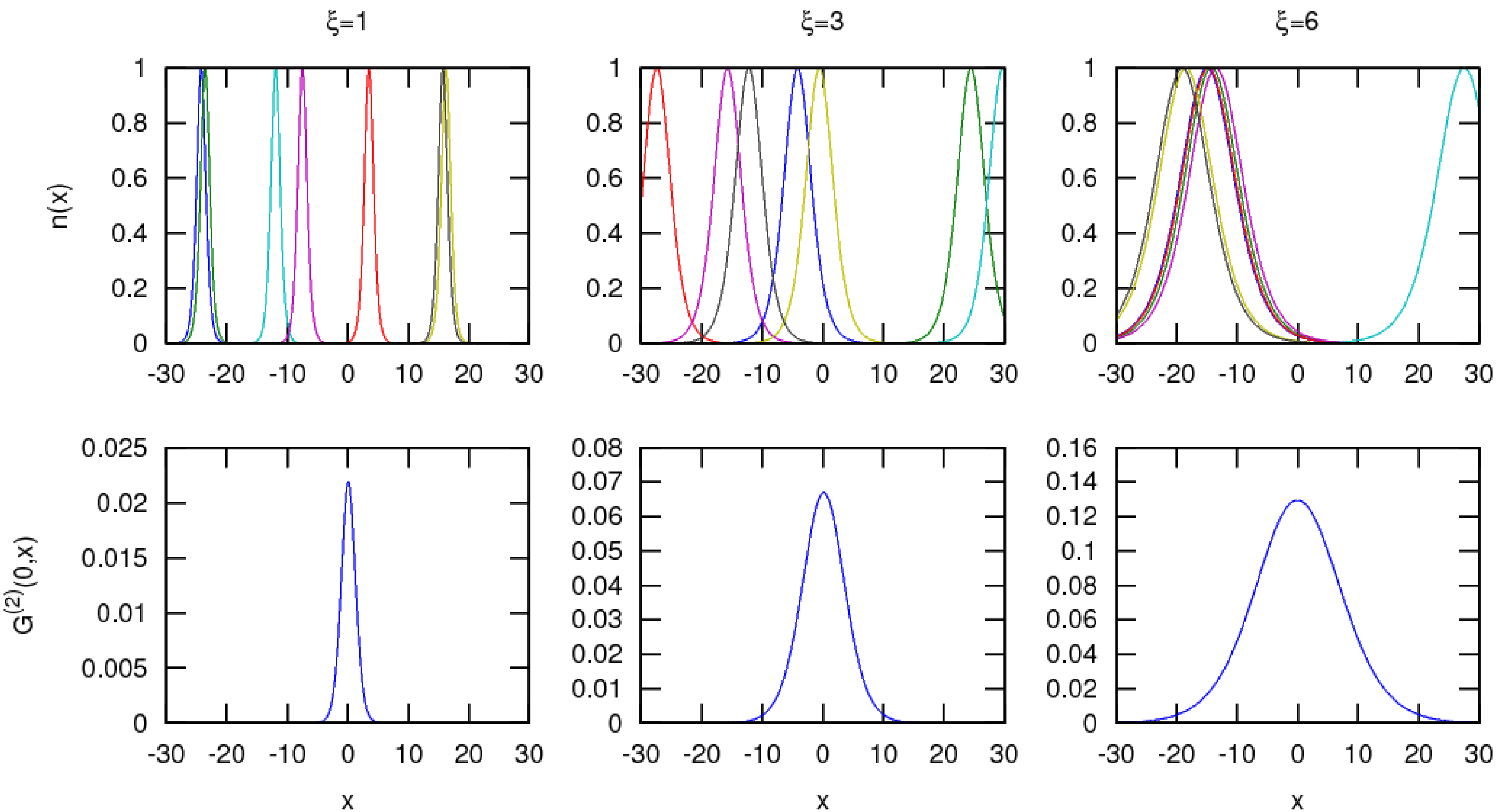
$$|\phi(x)|^2=n_0+\lambda[1-\tanh((x-q)/\xi)^2]$$
$$P(q)\sim\exp(-q^2/2\sigma^2)$$

The bright soliton 1/3

Variation with width ξ

$$|\phi(x)|^2 = n_0 + \lambda[1 - \tanh((x-q)/\xi)^2]$$

$\sigma = 100 \quad n_0 = 0 \quad \lambda = 1$



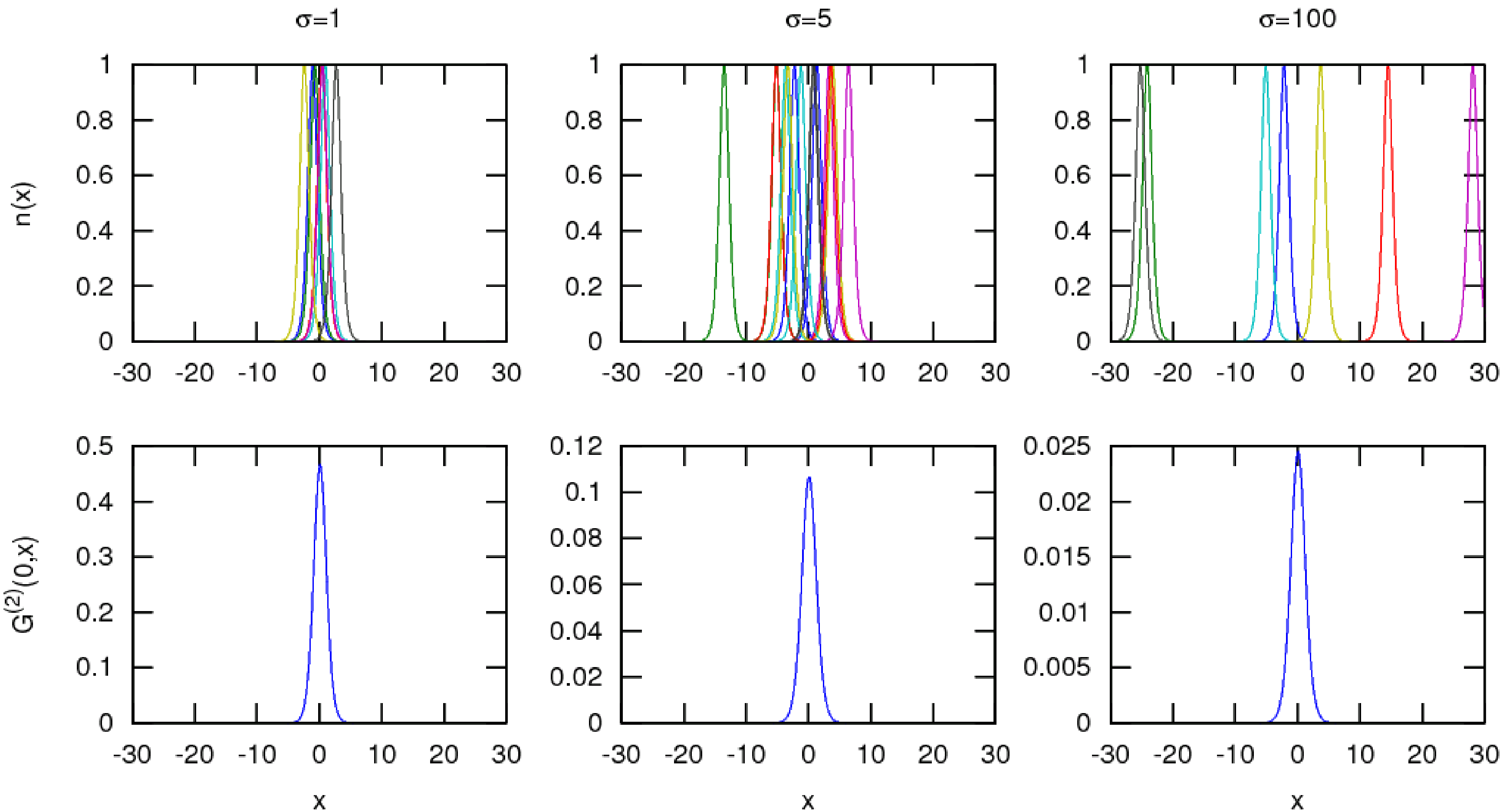
G2 is a good guide to width

The bright soliton 2/3

Variation with spread σ

$$|\phi(x)|^2 = n_0 + \lambda[1 - \tanh((x-q)/\xi)^2]$$

$$\xi = 1 \quad n_0 = 0 \quad \lambda = 1$$



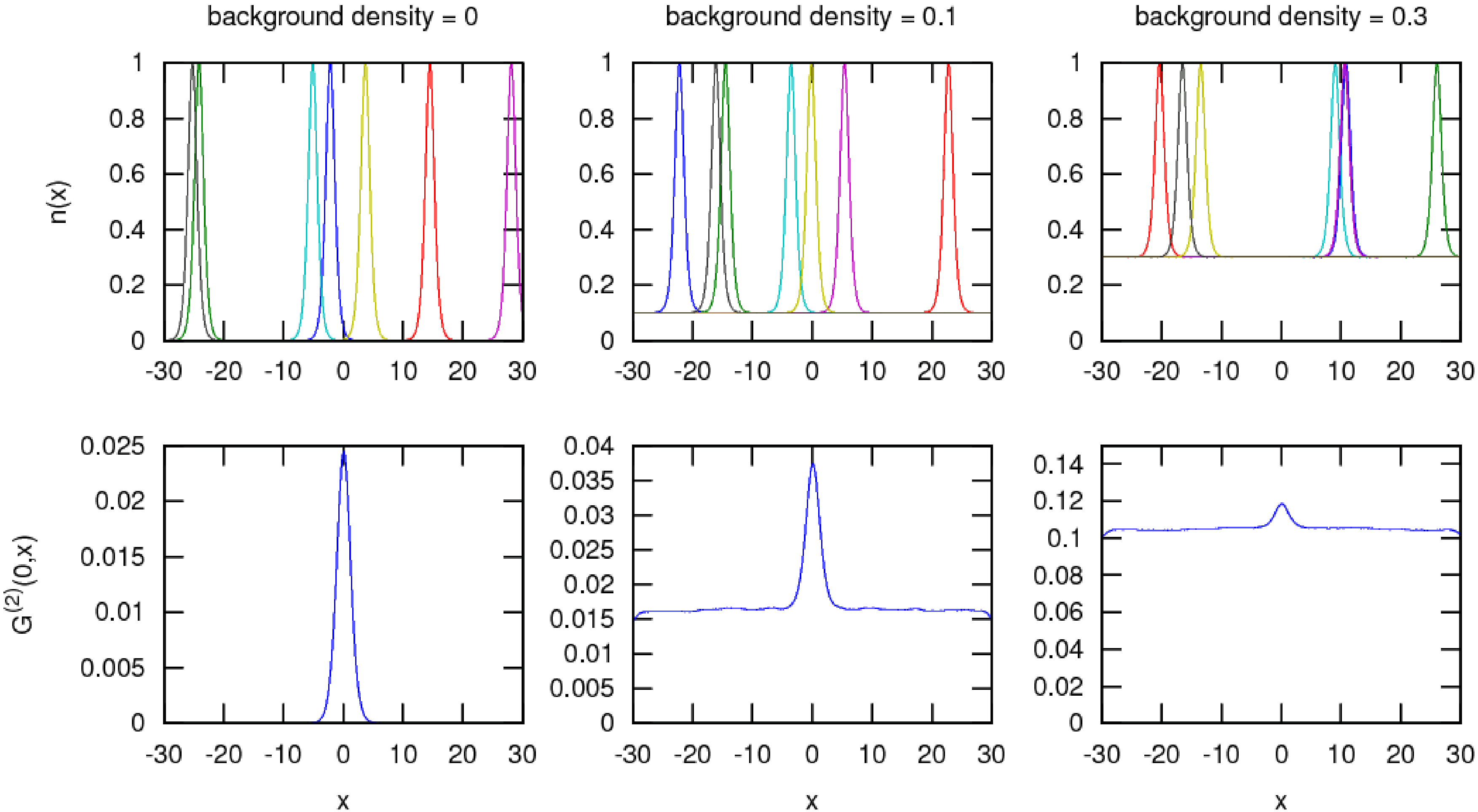
G2 shape is insensitive to spread

The bright soliton 3/3

Variation with background

$$|\phi(x)|^2 = n_0 + \lambda[1 - \tanh((x-q)/\xi)^2]$$

$$\sigma = 100 \quad \xi = 1 \quad \lambda = 1 - n_0$$



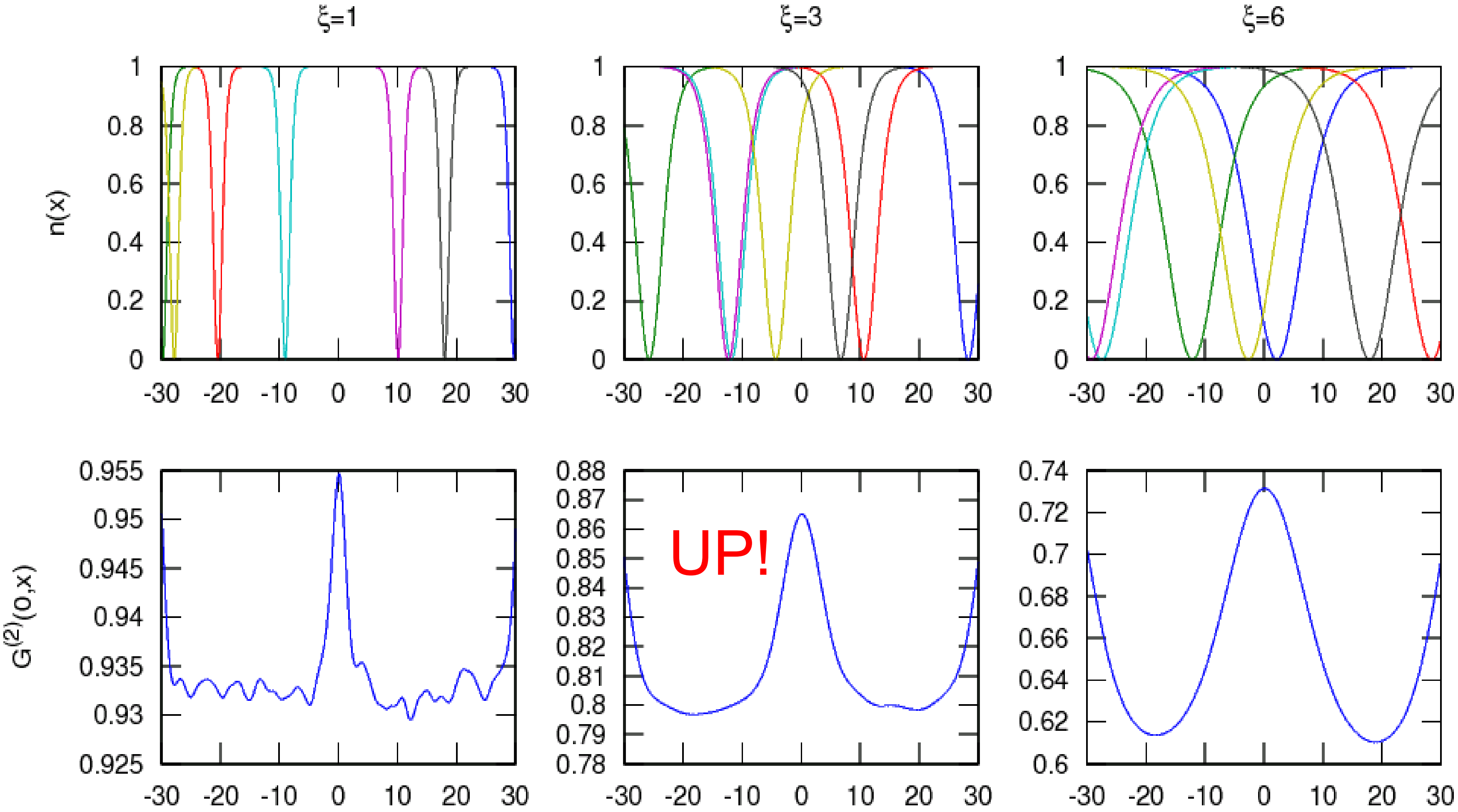
G2 is a good guide to width and shape, maybe even height

The dark soliton 1/3

Variation with width ξ

$$|\phi(x)|^2 = n_0 + \lambda[1 - \tanh((x-q)/\xi)^2]$$

$$\sigma = 100 \quad n_0 = 1 \quad \lambda = -1$$



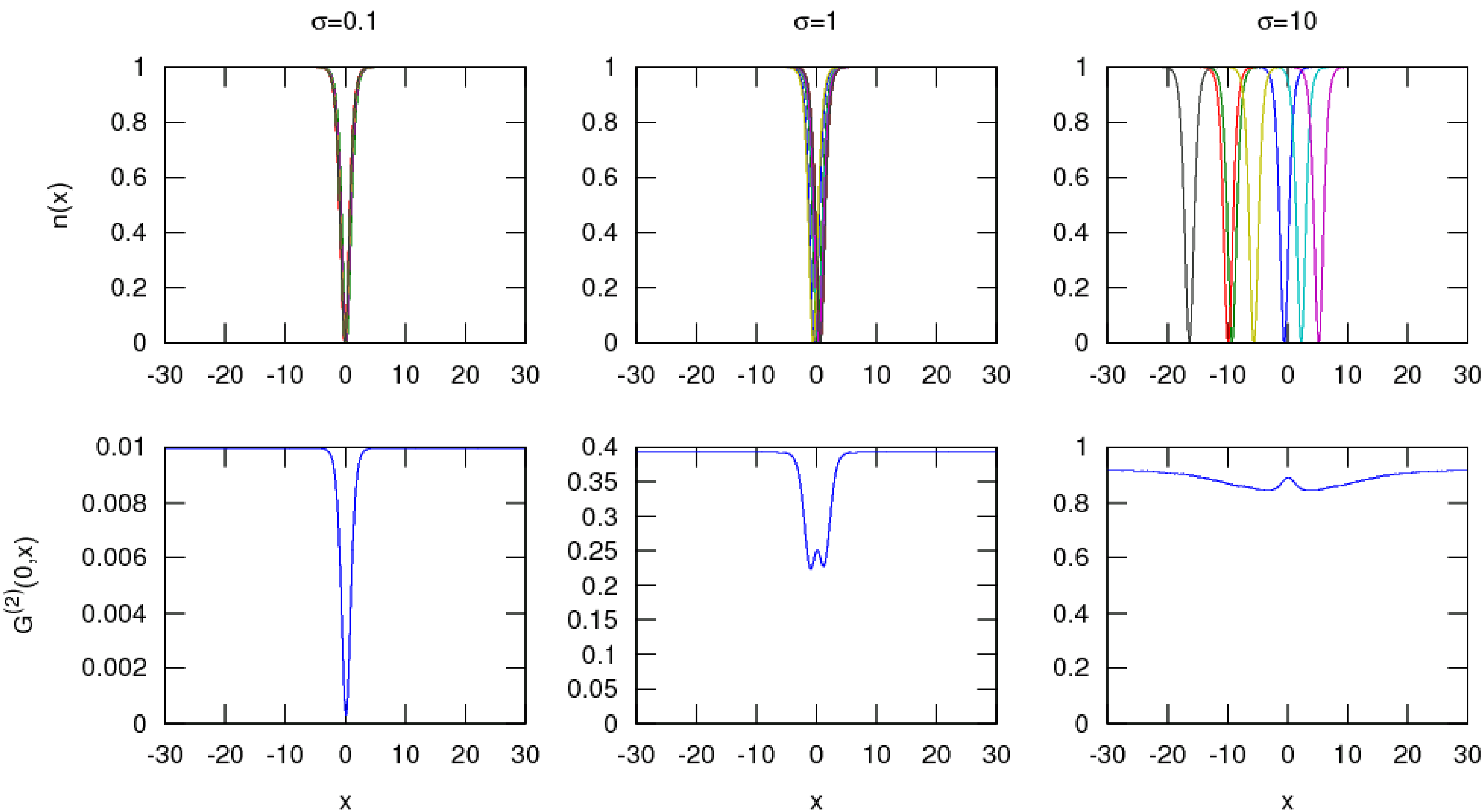
G2 does not look like the soliton at all!

The dark soliton 2/3

Variation with spread σ

$$|\phi(x)|^2 = n_0 + \lambda[1 - \tanh((x-q)/\xi)^2]$$

$$\xi = 1 \quad n_0 = 1 \quad \lambda = -1$$



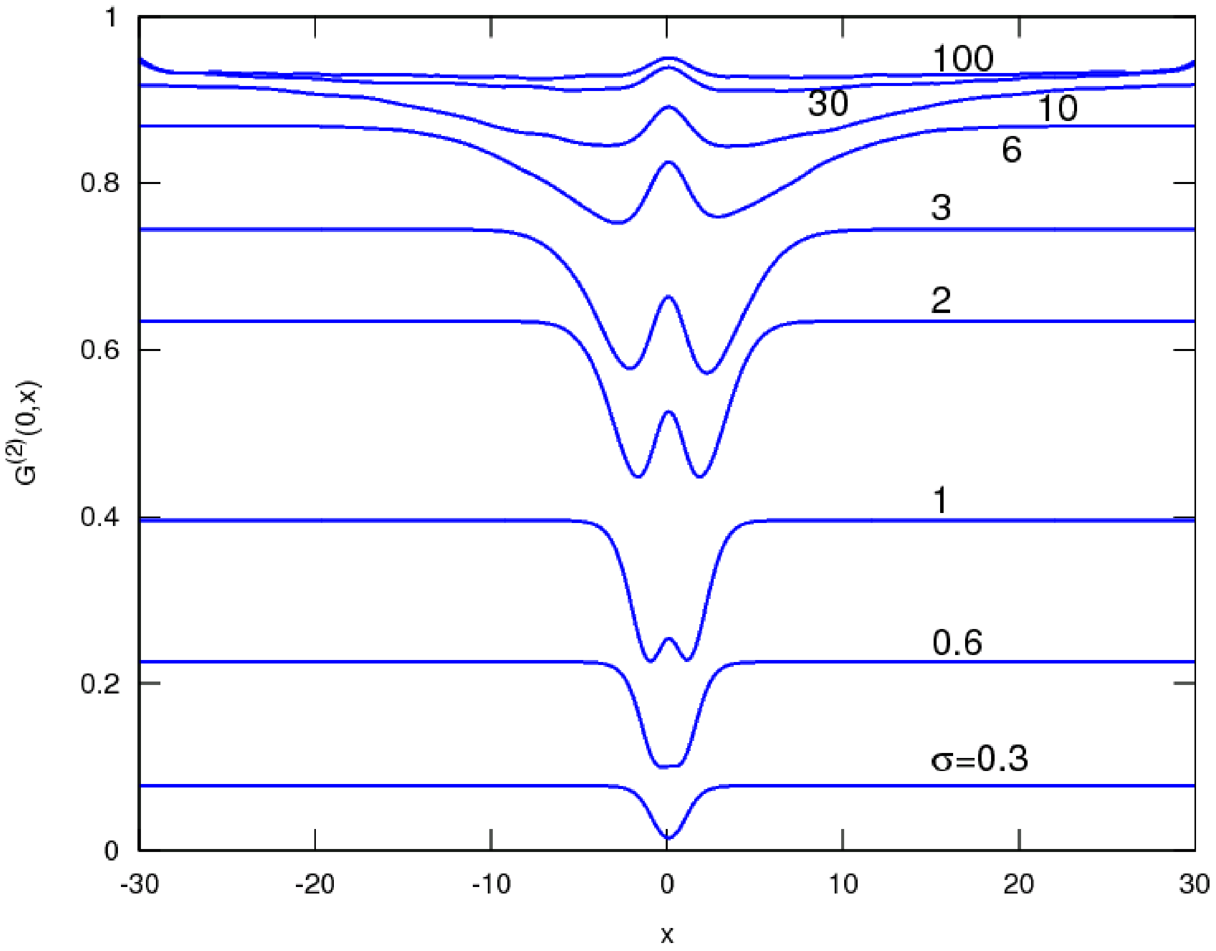
G2 shape is not even necessarily related to the soliton

The dark soliton 2a/3

- Variation with spread

$$|\phi(x)|^2 = n_0 + \lambda[1 - \tanh((x-q)/\xi)^2]$$

$$\xi = 1 \quad n_0 = 1 \quad \lambda = -1$$

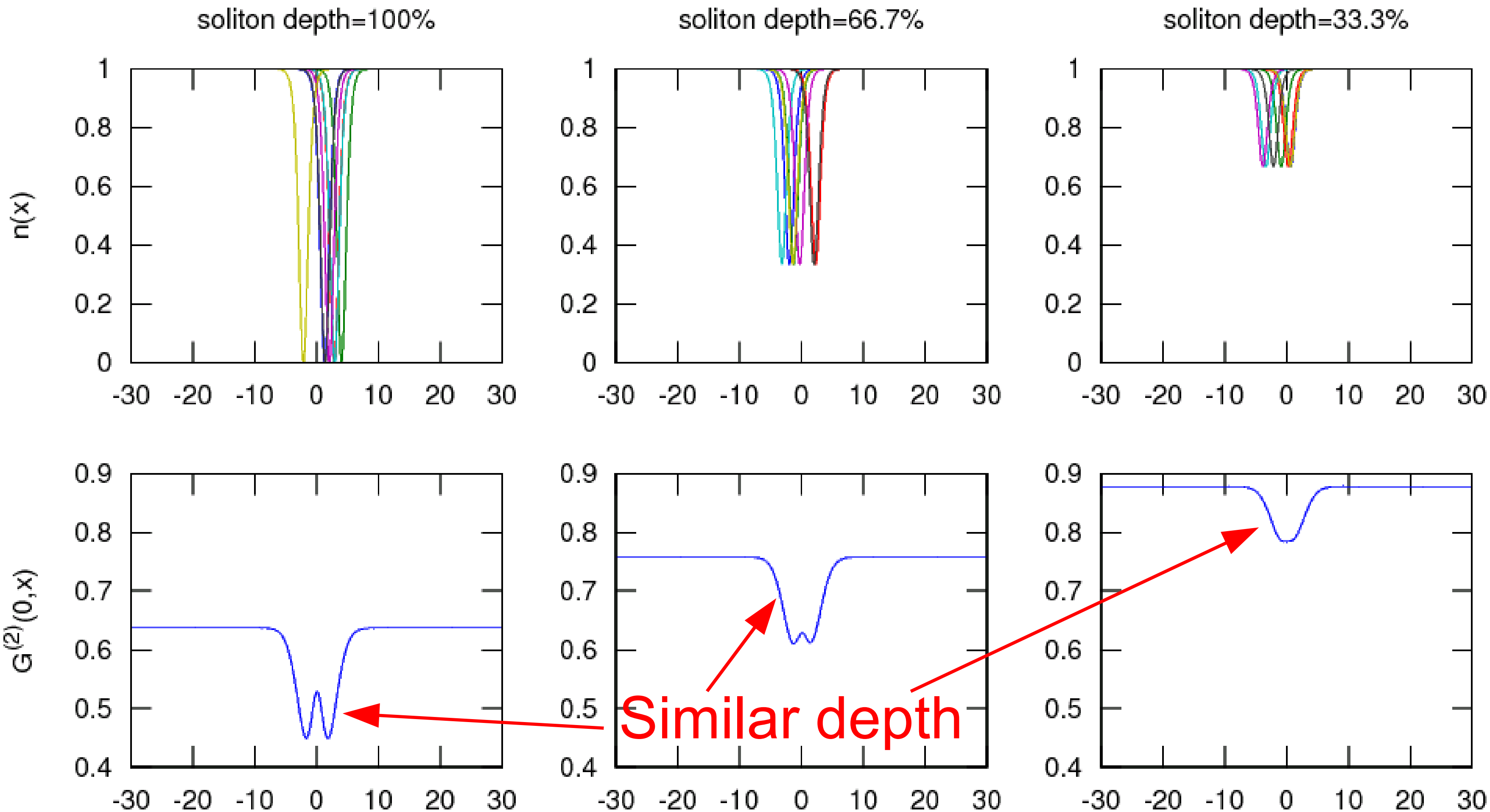


The dark soliton 3/3

Variation with greyness

$$|\phi(x)|^2 = n_0 + \lambda[1 - \tanh((x-q)/\xi)^2]$$

$$\sigma = 100 \quad \xi = 1 \quad n_0 = 1$$



G2 is a surprisingly bad guide to greyness

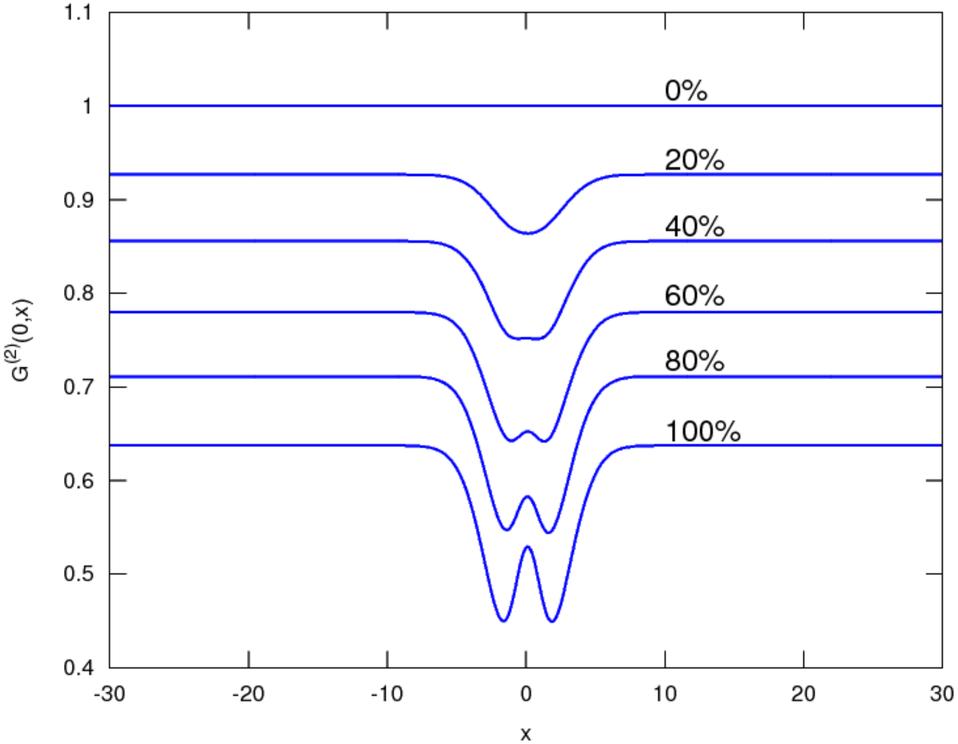
The dark soliton 3a/3

- Variation with greyness

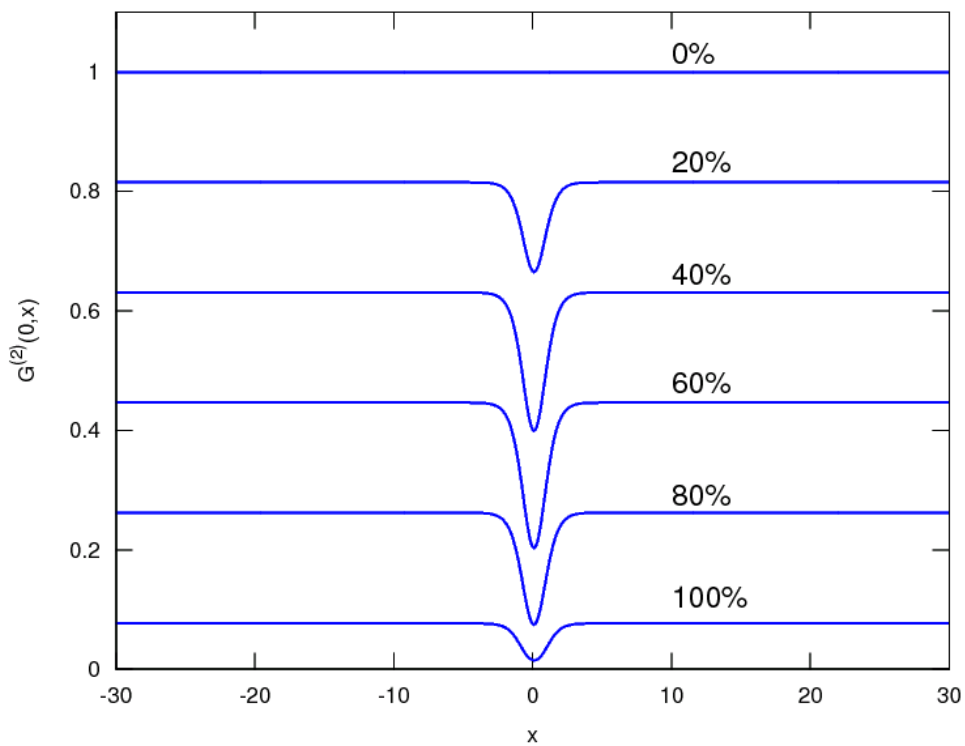
$$|\phi(x)|^2 = n_0 + \lambda[1 - \tanh((x-q)/\xi)^2]$$

$$\sigma = 100 \quad \xi = 1 \quad n_0 = 1$$

$\sigma = 2$



$\sigma = 0.3$



Conclusions: What do dark solitons do when they're alone?

1. They randomly walk
2. No evidence of appreciable greying beyond static quantum depletion.
3. There may be a loophole for greying at low particle numbers
4. Low-order correlations are not reliable indicators of single shot experiments