Naddzwiękowe zderzenia kondensatow i jak oblicza się występujaca tam dynamikę

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Overview

- Supersonic BEC collisions - why is this interesting?

1. The most interesting stuff is not described by GP equations.
2. Experiments can make “precision” measurements.
3. Theory / Experiment agreement could potentially be good

- Why is it “non-trivial”?

1. GP does not suffice.
2. Direct Bogoliubov description is tiresome, tricky and possibly intractable.

- I will explain how to get a relatively “easy” Bogoliubov description

The most interesting stuff is not described by GP equations.

$\Psi(x)$ Obeys the (superfluid) Gröss-Pitaevskii (GP) equation:

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = \left\{-\frac{\hbar^2\nabla^2}{2m} + V(x) + g|\Psi(x,t)|^2\right\}\Psi(x,t)$$

What does the GP description miss?

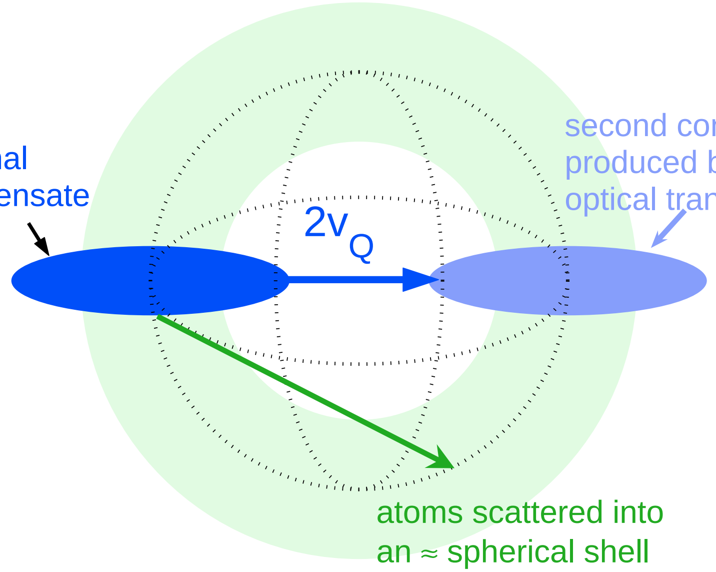
1. Incoherent atoms

2. Supersonic effects
   — above the speed of sound $c(x) = \sqrt{gn(x)/m}$, motion is no longer superfluid.

3. Details on scales smaller than the healing length $\xi(x) = \hbar/mc\sqrt{2}$.

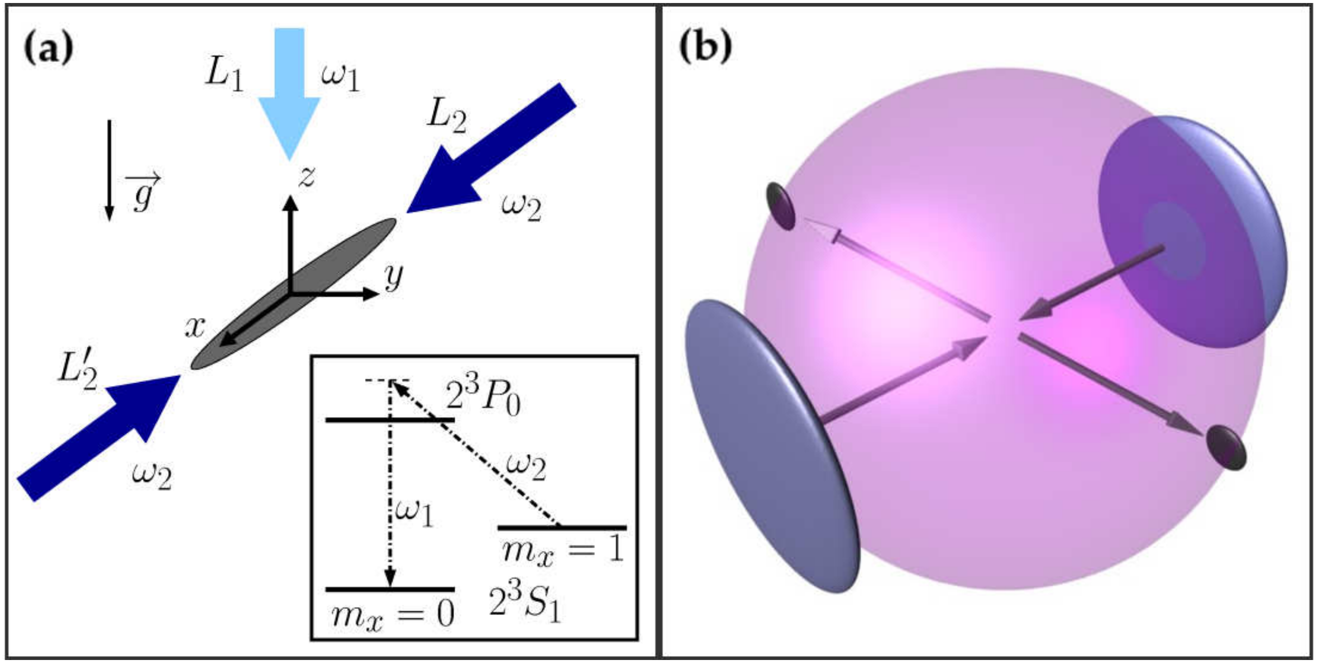
Supersonic BEC collision

Schematic in x-space:



Lasers in x-space

View in k-space



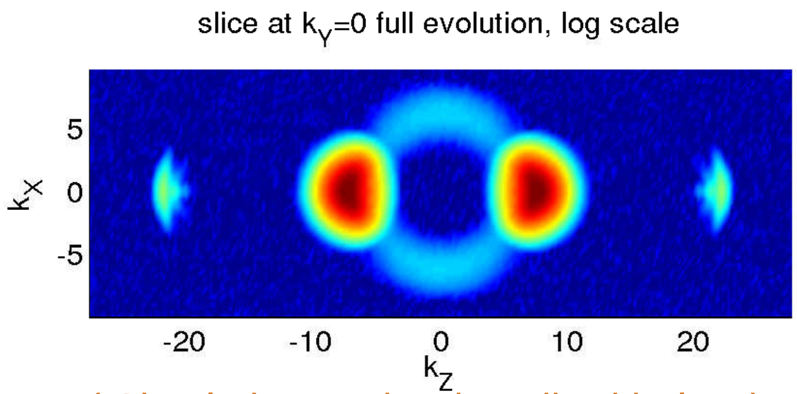
From A. Perrin *et al.*,
PRL **99** 150405 (2007)

Why is mean field no good here?

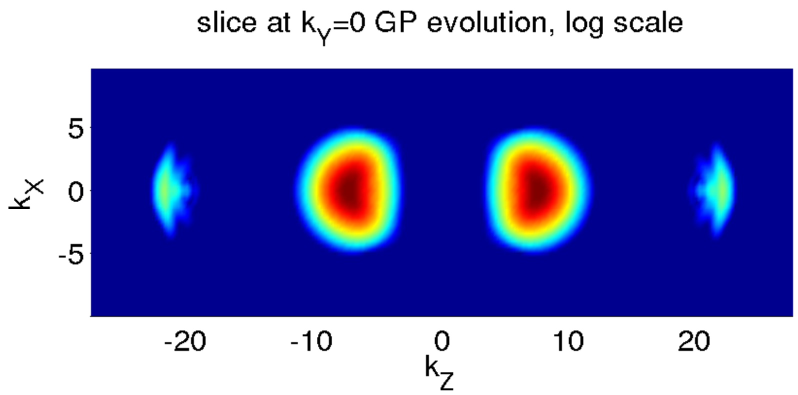
$$i\hbar\frac{\partial\Psi(x,t)}{\partial t}=\left\{-\frac{\hbar^2\nabla^2}{2m}+V(x)+g|\Psi(x,t)|^2\right\}\Psi(x,t)$$

In the halo, initial condensate field $\Psi(x,0)$ is zero, and so stays that way.

(It's a spontaneous process initially)

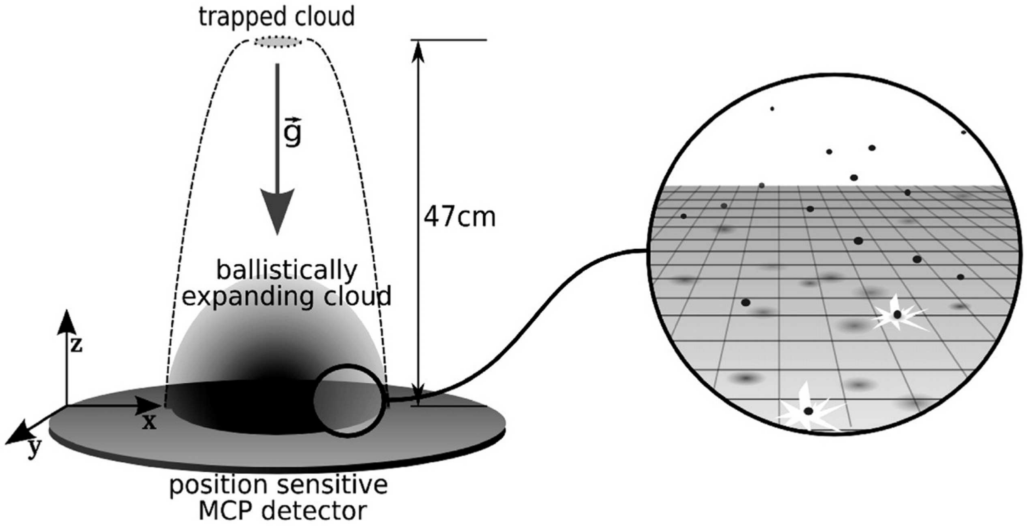


(Simulations to be described below)



Metastable He* experiment

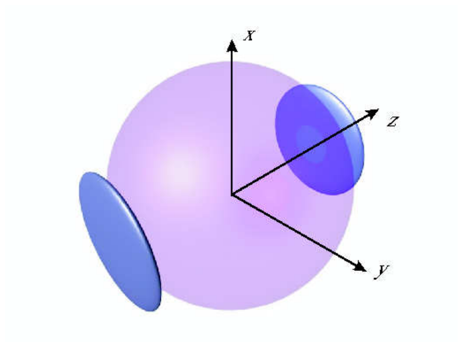
Uses multi-channel plate – Allows mapping of 3D atom distribution

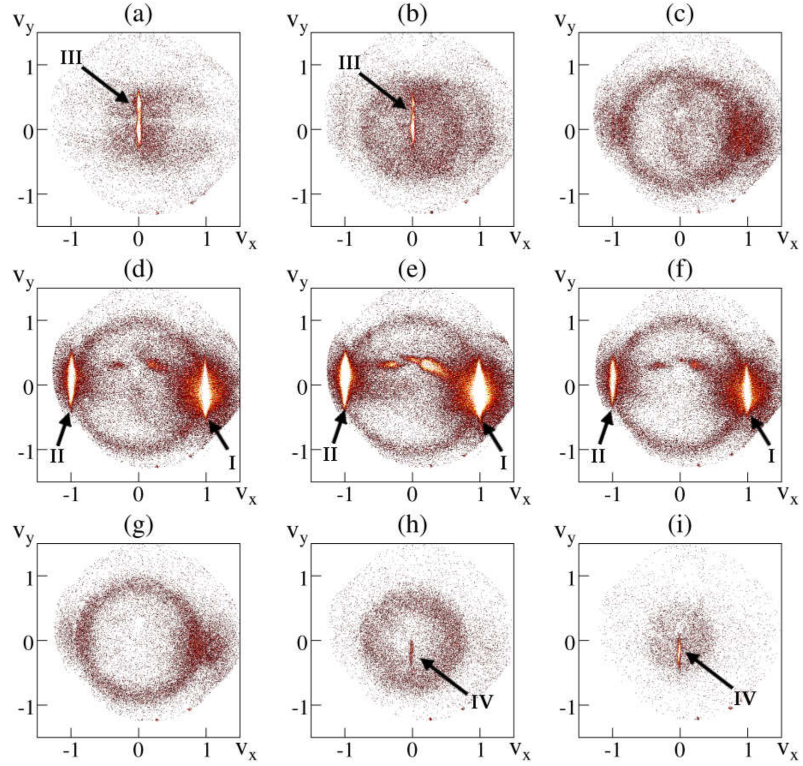


From M. Schellekens *et al.*, Science **310**, 648 (2005).

After long time of flight,

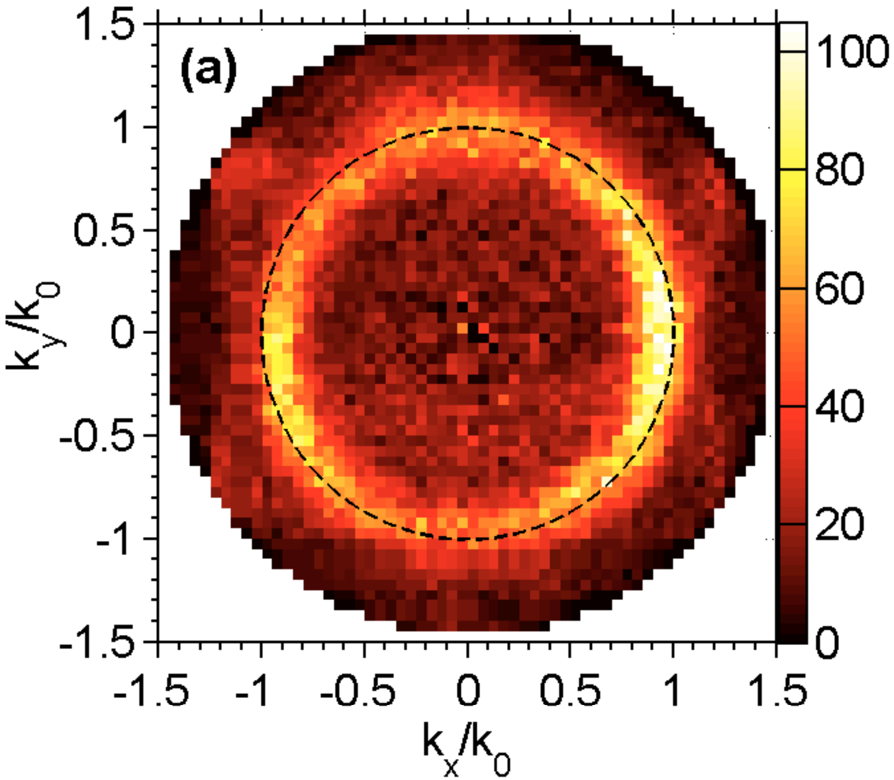
$$n(x,t) \rightarrow n(k,0)$$

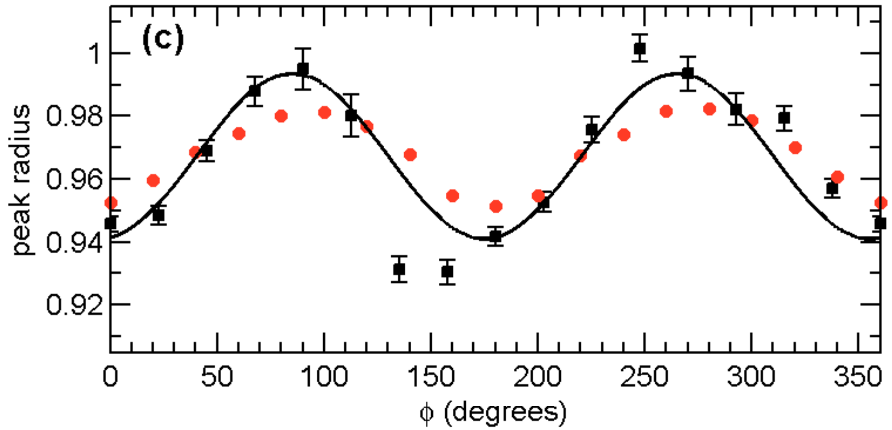




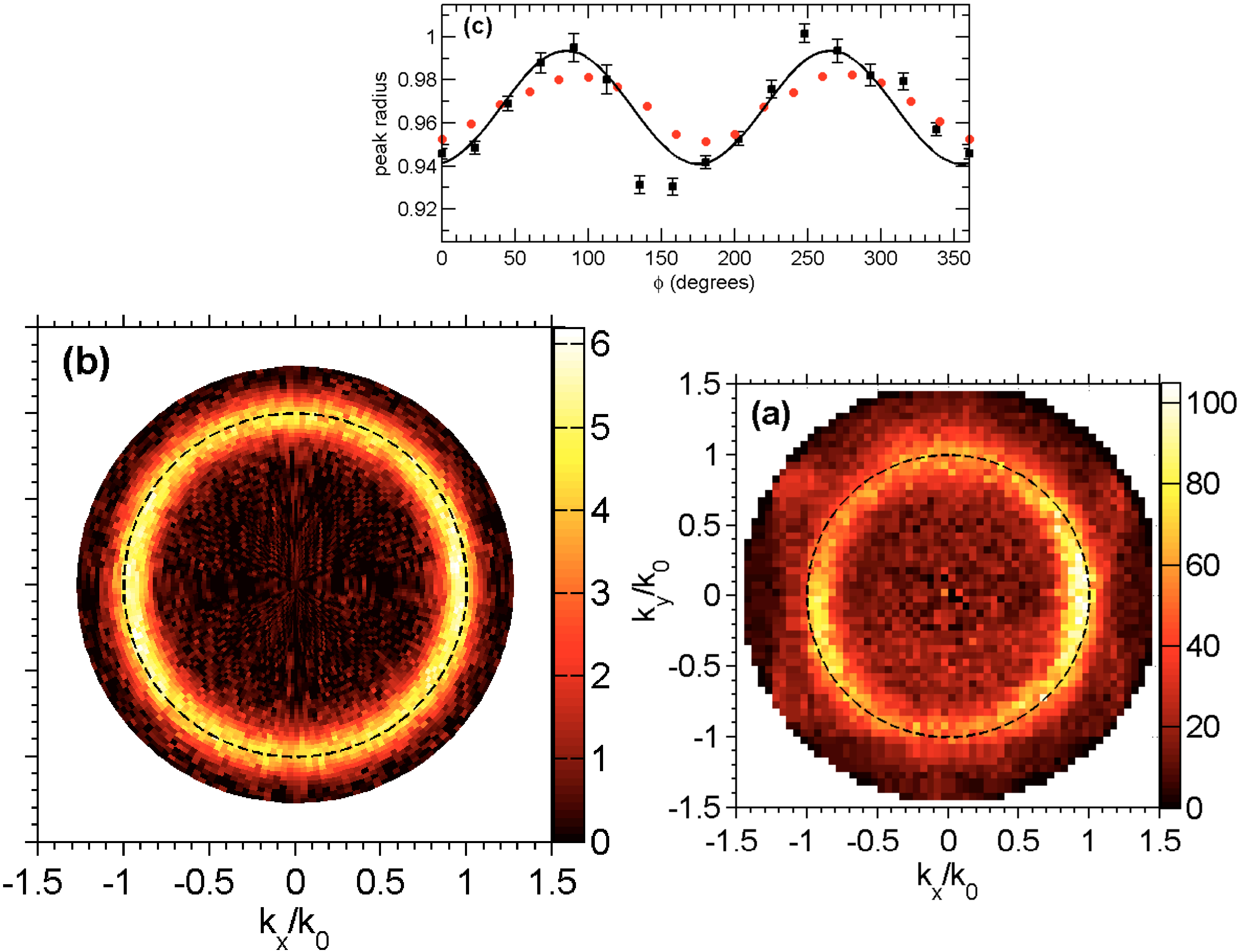
From A. Perrin *et al.*, PRL **99** 150405 (2007)

Experiments can make “precision” measurements





Theorists can also make “precision” predictions



Simulation beyond mean field: Bogoliubov Hamiltonian

1. Write $\widehat{\Psi}(x,t) = \phi(x,t) + \widehat{\psi}_B(x,t)$

2. Substitute into full $\widehat{H} = \int dx\, \widehat{\Psi}^\dagger(x) \left\{ -\frac{\hbar^2 \nabla^2}{2m} + \frac{g}{2}\, \widehat{\Psi}^\dagger(x) \widehat{\Psi}(x) \right\} \widehat{\Psi}(x)$

3. <u>Assume</u> $\widehat{\psi}_B(x,t)$ is orthogonal to $\phi(x,t)$.

4. <u>Assume</u> δN the number of particles contained in $\widehat{\psi}_B$ is $\ll N$, the total number.

5. Remove terms of high order in $\delta N/N$ (quantum depletion) from $\widehat{H}$ to obtain $\widehat{H}_B$

6. For later convenience, separate right- and left-moving condensates (velocities $\approx \pm k_C$) into $\phi(x,t) = \phi_L(x,t) + \phi_R(x,t)$.

7. Several people now or formerly in this room have investigated this: Rzążewski, Trippenbach, Ziń, Bach, Chwedeńczuk, . . .

time-dependent Bogoliubov Hamiltonian

$$\hat{H}_B = \int dx \Big\{ \hat{\psi}_B^\dagger \left(-\frac{\hbar^2 \nabla^2}{2m} \right) \hat{\psi}_B \qquad \text{K.E.}$$

$$+2g|\phi(t)|^2 \hat{\psi}_B^\dagger \hat{\psi}_B \qquad \text{collective potential}$$

$$+2g\phi_L(t)\phi_R(t)(\hat{\psi}_B^\dagger)^2 + \text{h.c.} \qquad \text{halo pair production}$$

$$+g\left[\phi_L(t)^2 + \phi_R(t)^2\right](\hat{\psi}_B^\dagger)^2 + \text{h.c.} \Big\} \qquad \text{pair production near BECs}$$

GP equations for condensates:

$$i\hbar \frac{d\phi_R(x,t)}{dt} = \left\{ -\frac{\hbar^2}{2m}\nabla^2 + g\left[|\phi_R(x,t)|^2 + 2|\phi_L(x,t)|^2 + \phi_L^*(x,t)\phi_R(x,t)\right] \right\} \phi_R(x,t)$$

$$i\hbar \frac{d\phi_L(x,t)}{dt} = \left\{ -\frac{\hbar^2}{2m}\nabla^2 + g\left[|\phi_L(x,t)|^2 + 2|\phi_R(x,t)|^2 + \phi_R^*(x,t)\phi_L(x,t)\right] \right\} \phi_L(x,t)$$

Cute and useful feature: can remove terms to see what process affects what observation.

A "TECHNICAL" DIFFICULTY **(or two)**

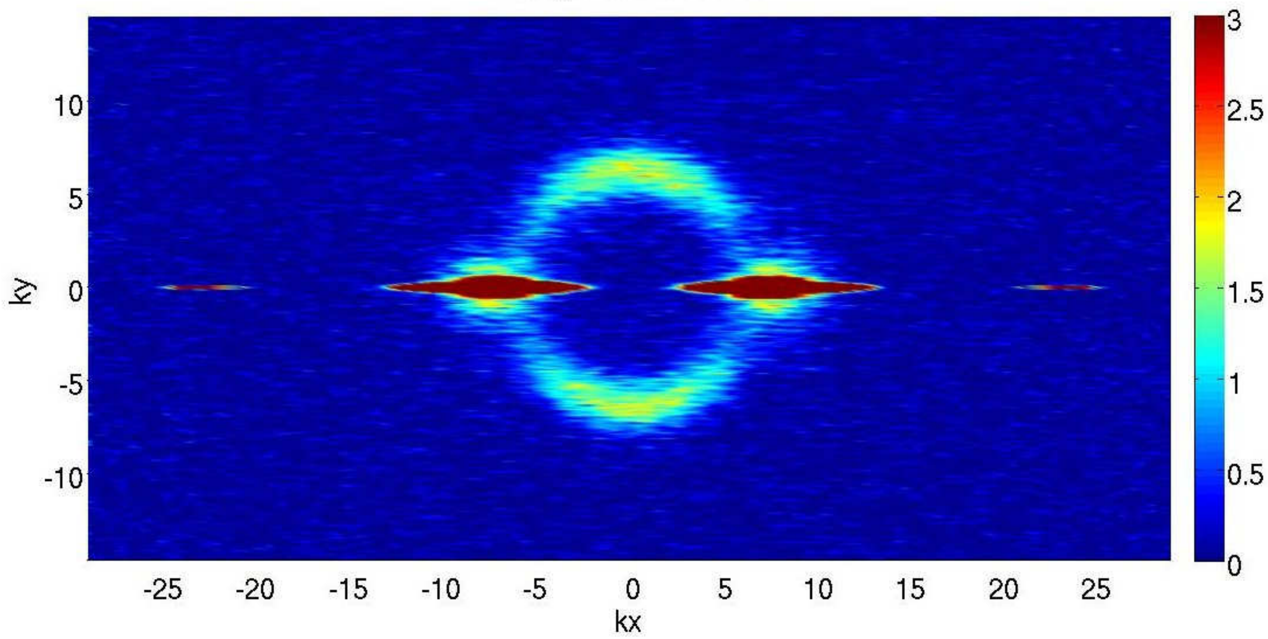
- Experimentally realistic situations require $10^5 - 10^7$ lattice points.

- Standard Bogoliubov quasiparticle evolution procedure requires diagonalization of $\widehat{H}_B$, finding of eigenstates, etc. *This is unlikely given the size of the space!*

- This is a dynamical situation – the coherent background $\phi(x,t)$ changes, so diagonalization would have to be done at each dt step :-(

- Analytic approaches can treat simplified cases, but there are meny terms $\sim\propto gn$, and not all can be done analytically at once.

Processes $\propto \sim gn$



$\phi = \phi_L + \phi_R$
with
$k_L \sim -k_0, k_R \sim k_0$

$\phi_L \phi_R \widehat{\Psi}_B^\dagger(k) \widehat{\Psi}_B^\dagger(-k)$	pair creation
$\phi_R \phi_R \widehat{\Psi}_B^\dagger(k_0 + \delta k) \widehat{\Psi}_B^\dagger(k_0 - \delta k)$	mini-halo and pair depletion near BECs
$\phi_R \phi_R \phi_L^* \phi_R^* (3k_0)$	frequency doubling
$\lvert\phi_L + \phi_R\rvert^2 \widehat{\Psi}_B^\dagger(x) \widehat{\Psi}_B(x)$	potential for $\widehat{\Psi}_B$ atoms
$\lvert\phi_R\rvert^4$	self-potential for right BEC
$\lvert\phi_L\rvert^2 \lvert\phi_R\rvert^2$	repulsion between R and L BECs
$\ldots$	etc.

A SOLUTION

the "STAB" (in the dark?) method

"Stochastic Time-Adaptive Bogoliubov"

Instead of a direct solution of $\widehat{H}_B$, the dynamics of $\widehat{\psi}_B$ can be treated stochastically using phase-space representations (here, the positive-P representation). Obtain:

$$
\begin{aligned}
i\hbar\frac{d\psi_1(x,t)}{dt} &= \left[-\frac{\hbar^2}{2m}\nabla^2 + 2g|\phi(x,t)|^2\right]\psi_1(x,t) \\
&= +g\phi(x,t)^2\psi_2(x,t)^* + i\sqrt{ig}\psi_1(x,t)\,\xi_1(x,t) \\
i\hbar\frac{d\psi_2(x,t)}{dt} &= \left[-\frac{\hbar^2}{2m}\nabla^2 + 2g|\phi(x,t)|^2\right]\psi_2(x,t) \\
&= +g\phi(x,t)^2\psi_1(x,t)^* + i\sqrt{ig}\psi_2(x,t)\,\xi_2(x,t)
\end{aligned}
$$

Here, $\xi_j(x,t)$ are independent Gaussian random variable fields with mean zero and variances $\langle\xi_i(x,t)\xi_j(x',t')\rangle = \delta_{ij}\delta(x-x')\delta(t-t')$. And e.g. $\langle\widehat{\Psi}_B{}^{\dagger}\widehat{\Psi}_B\rangle = \langle\psi_2^*\psi_1\rangle_{\mathrm{stoch}}$

Positive-P method

One writes the density matrix of the system on M lattice points in coherent states $|\psi_j(x)\rangle = e^{\psi_j(x)\widehat{\Psi}_B^\dagger(x)}|0\rangle$ as

$$\widehat{\rho} = |\Psi\rangle\langle\Psi| = \int \mathcal{D}^{2M}\psi_1(x)\mathcal{D}^{2M}\psi_2(x)\, P(\psi_1(x),\psi_2(x),t)\, |\psi_1(x)\rangle\langle\psi_2(x)|$$

– The distribution $P(\ldots)$ can be guaranteed non-negative real

– The complete quantum evolution of the state

$$i\hbar\frac{\partial\widehat{\rho}}{\partial t} = \left[\widehat{H}_B, \widehat{\rho}\right]$$

is equivalent to the random walk of an ensemble of $2M$ random variables $\psi_\nu(x,t)$.

– Expectation values of observables are equivalent to ensemble averages of the variables.

– Most complexity gets shoved into the ensemble, and hopefully averages out for most quantities of interest.

Dynamics

$$i\hbar\frac{\partial\widehat{\rho}}{\partial t} = \widehat{H}_B(t)\,\widehat{\rho} - \widehat{\rho}\,\widehat{H}_B(t).$$

Where time-dependence comes through the dependence of $\widehat{H}_B$ on $\phi(t)$.

- This is equivalent to a Fokker-Planck equation for the distribution $P(\psi_1,\psi_2)$:

$$i\hbar\frac{\partial P}{\partial t} = \sum_{x\nu}\left[-\frac{\partial}{\partial\psi_\nu(x)}A_\nu(x,t) + \sum_\sigma\frac{\partial^2}{\partial\psi_\nu(x)\psi_\sigma(x)}D_{\nu\sigma}(x,t)\right]P$$

with time-dependent diffusion coefficients $D(x,t)$ and drift rates $A(x,t)$, etc.

- This in turn is is equivalent to Langevin equations for the $2M$ random samples $\psi_1(x)$ and $\psi_2(x)$ such as:

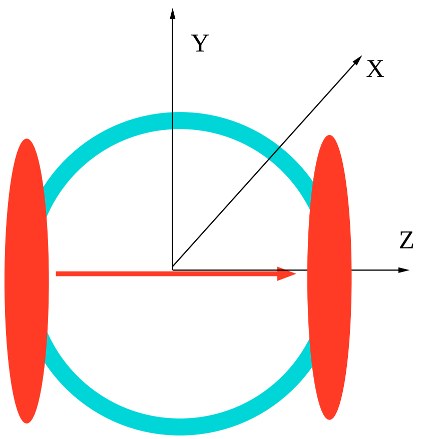
$$\frac{d}{dt}\psi_\nu(x) = A_\nu(\psi_1,\psi_2,t) + \sum_\sigma\sqrt{D_{\nu\sigma}(\psi_1,\psi_2,t)}\;\xi_\sigma(x,t)$$

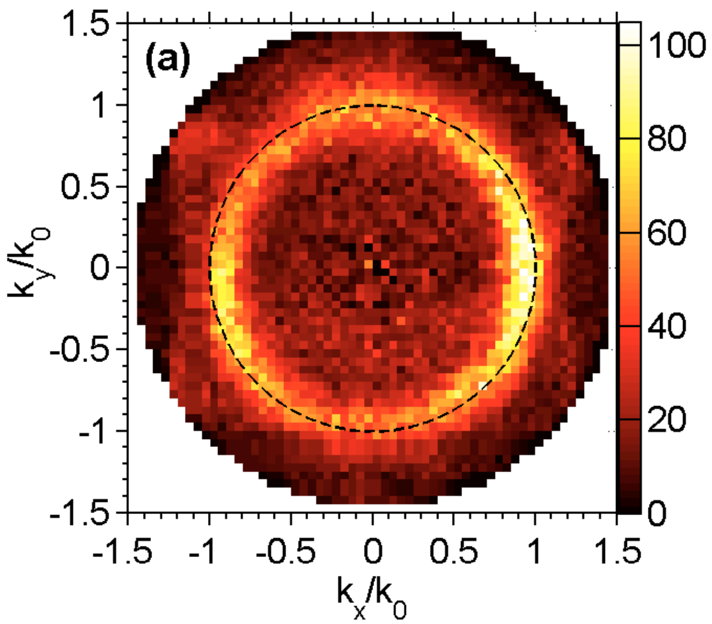
with $\xi_\sigma(x,t)$ being real independent white noise fields, delta-correlated in x and t (and σ).

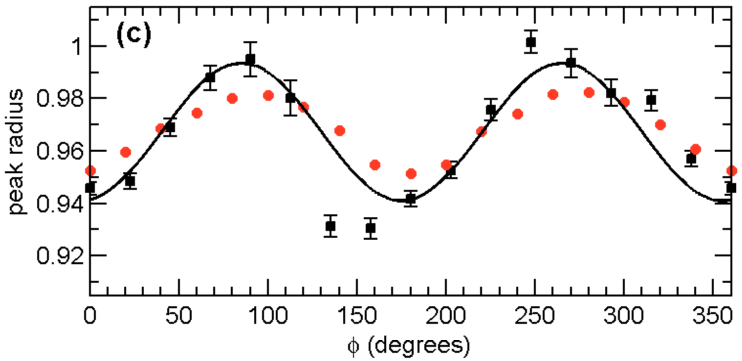
Features

- Good scaling with system size (M= number of lattice sites):
  - Number of variables $\propto\ M$
  - Evolution time $\propto\ M \log M / \Delta t$
- Noisy with precision $\propto 1/\sqrt{S}$ ith S trajectories
- Linear evolution in ψ_ν variables, hence no instability like in full positive-P treatment of the first-principles Hamiltonian.
- Simple basis set (plane waves), despite complicated mean-field evolution.
- Limited to “small” quantum depletion
- No interaction between quasiparticles.
- All spontaneous and stimulated processes included *with no empirical parameters*
- No back-action of quasiparticles on condensate
- full MF evolution included
- Terms with clear physical meaning can be easily added / removed

Halo radius mysteries







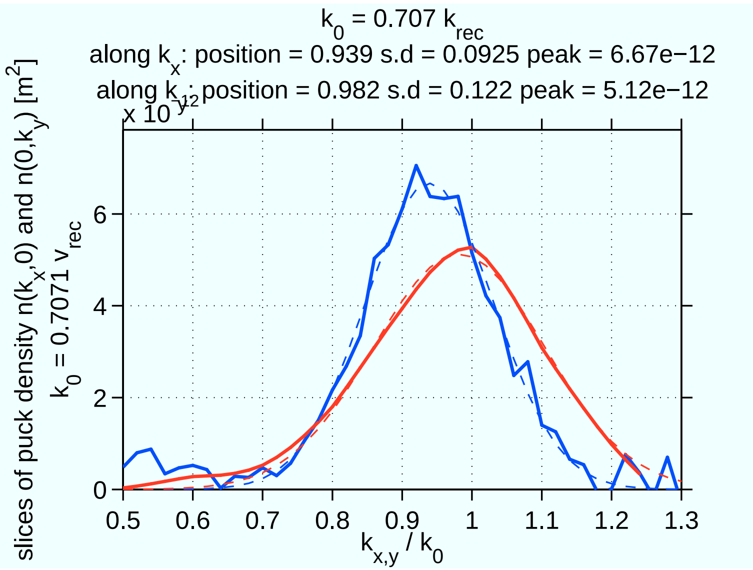
- Halo radius NOT at expected $|v| = v_0$ collision velocity, but smaller
- Halo becomes flattened along collision direction
- Halo becomes ellipsoidal $\perp$ to collision direction if condensates non-spherical

Mystery 1: Why is Halo radius smaller

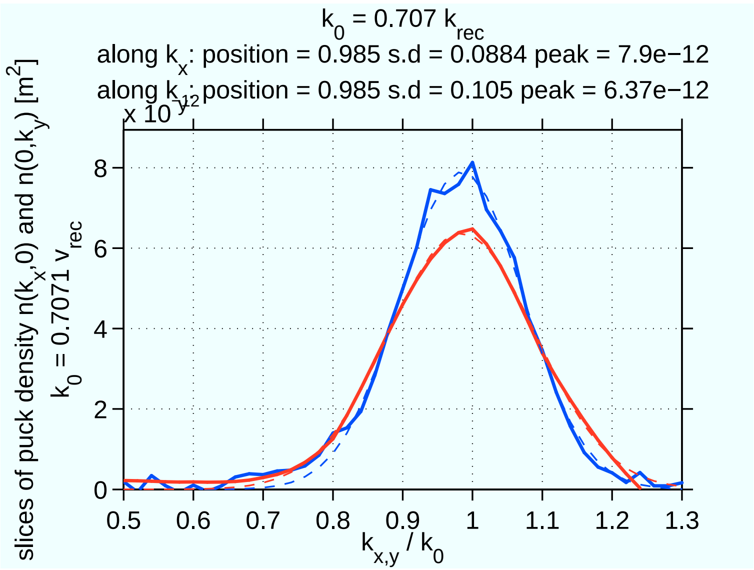
It costs $\mu = \frac{3}{2}gn(x)$ to remove a particle from the condensate (the mean-field energy from the repluslion of the remaining particles), but $2gn(x)$ to place one in a non-condensate mode. The energy balance is

$$\frac{\hbar^2 k_0^2}{2m} + \frac{3}{2}gn = \frac{\hbar^2 k^2}{2m} + 2gn$$

When the mean-field energy is removed from the Hamiltonian $\widehat{H}_B$, the radius reverts to v_0 (and ellipticity disappears)



Full calculation



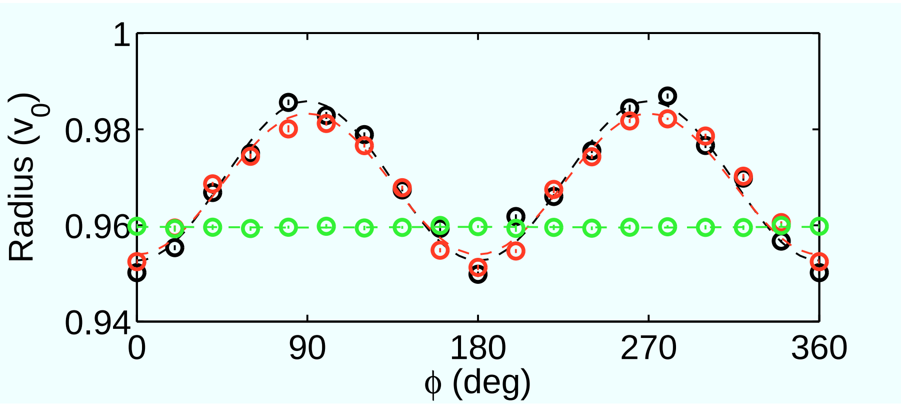
No MF energy $g|\phi|^2$ terms

Mystery 2: Why is the Halo an ellipse?

Particles can roll off the condensate to recover some or all of the lost mean-field energy $\propto gn(x)$.

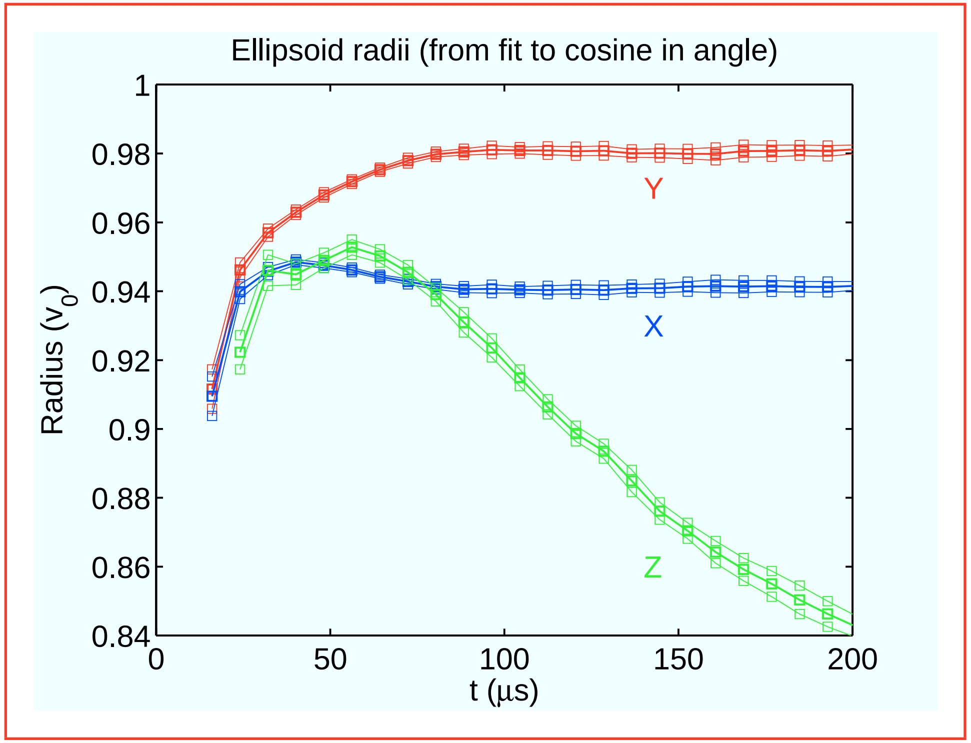
BUT - in the long condensate direction, this does not happen because the density "falls out" from under the particles as the condensates move away before the particle can roll far.

WHILE - in the short directions, a halo particle moves fast and rolls off before the condensate can change much.



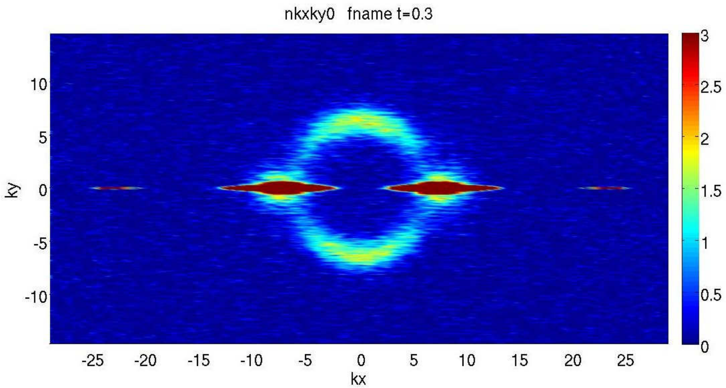
Black: full calculation

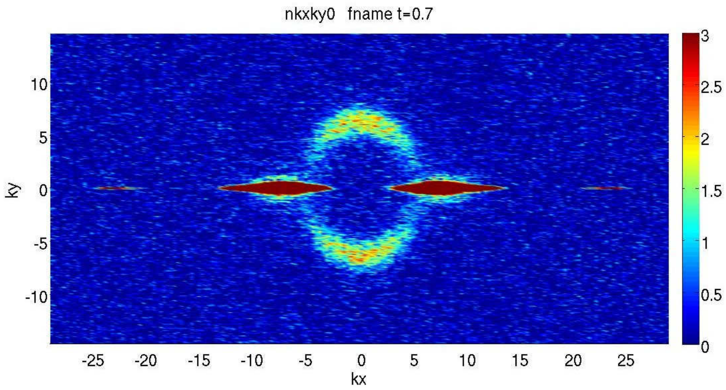
Green: no MF potential for Ψ_B

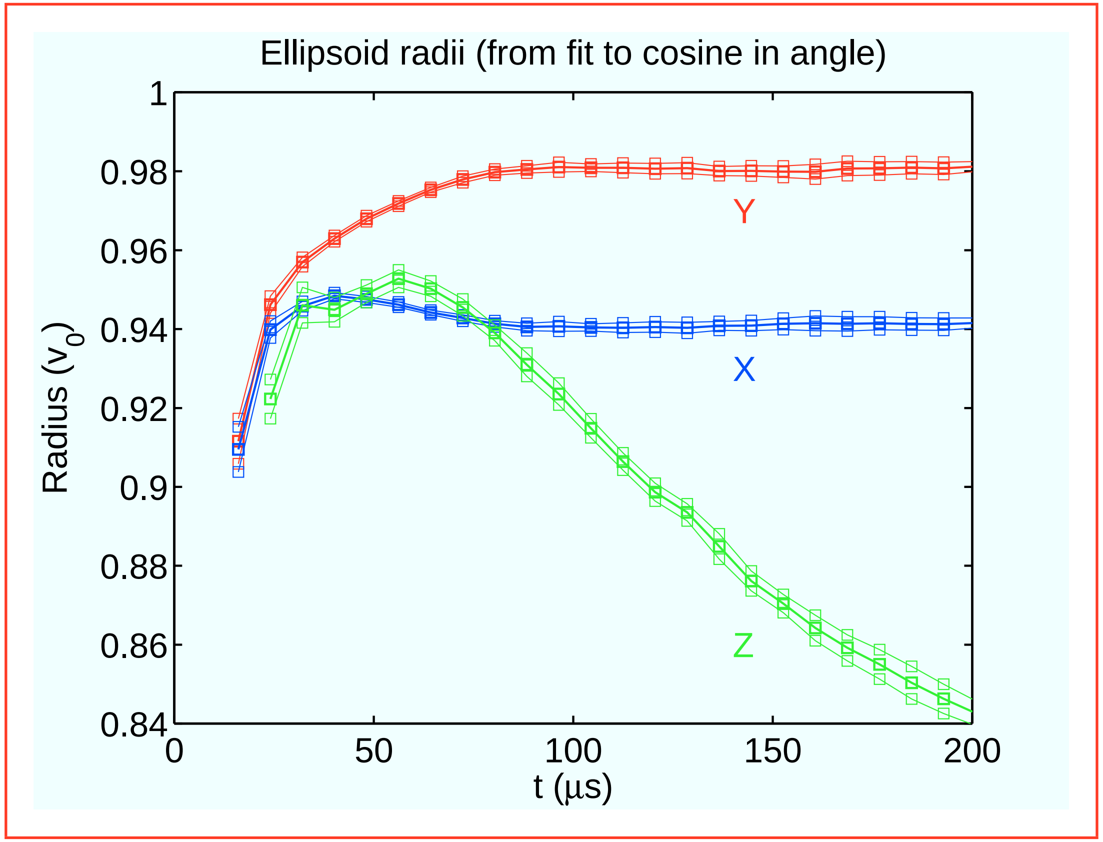


Why is the Halo flattened along collision direction?

Along the collision direction, halo particles become bogged in the potential valley forming between the condensates (because they are slightly slower due to the halo radius shift mentioned before), and become *deccelerated* in this valley.







Thank you :)