# On the survival of quantum depletion of a condensate after release from the trap



<u>theory:</u> **Piotr Deuar** *Institute of Physics, Polish Academy of Sciences, Warsaw, Poland* 

<u>experiment:</u> Jacob Ross



David Shin, Kieran Thomas, Bryce Henson

Andrew Truscott, Sean Hodgman

Australian National University, Canberra, Australia

Ross et al arXiv:2103.15283



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#### Outline

- Palaiseau experiment
- Quantum depletion in a condensate
- Trento theory
- The ANU experiment
- The STAB simulation
- Unexplained issues
- Summary



PRL 117, 235303 (2016) PHYSICAL REVIEW LETTERS

#### Momentum-Resolved Observation of Thermal and Quantum Depletion in a Bose Gas

R. Chang,<sup>1</sup> Q. Bouton,<sup>1</sup> H. Cayla,<sup>1</sup> C. Qu,<sup>2</sup> A. Aspect,<sup>1</sup> C. I. Westbrook,<sup>1</sup> and D. Clément<sup>1,\*</sup> <sup>1</sup>Laboratoire Charles Fabry, Institut d'Optique, CNRS, Univ. Paris Sud, 2 Avenue Augustin Fresnel 91127 PALAISEAU cedex, France <sup>2</sup>INO-CNR BEC Center and Dipartimento di Fisica, Universita di Trento, 38123 Povo, Italy (Received 16 August 2016; published 2 December 2016)

We report on the single-atom-resolved measurement of the distribution of momenta  $\hbar k$  in a weakly interacting Bose gas after a 330 ms time of flight. We investigate it for various temperatures and clearly separate two contributions to the depletion of the condensate by their *k* dependence. The first one is the thermal depletion. The second contribution falls off as  $k^{-4}$ , and its magnitude increases with the in-trap condensate density as predicted by the Bogoliubov theory at zero temperature. These observations suggest associating it with the quantum depletion. How this contribution can survive the expansion of the released interacting condensate is an intriguing open question.



# He\* Experiment



## Momentum resolved density after release



# Scaled by $k^4$





FIG. 4. Contact constant  $C_{\infty}/N_0$  per condensed particle plotted as a function of the condensate density  $n_0$ . The geometric trapping frequency  $\bar{\omega}/2\pi$  and the ratio  $k_B T_a/\mu$  are indicated. The dashed line is the Bogoliubov prediction in the LDA,  $C_{\rm LDA}$ (see text), and the solid line is  $6.5 \times C_{\rm LDA}$ .

#### Quantum depletion in a uniform gas

#### Condensate in k=0

$$\hat{b}_{\mathbf{k}}^{\dagger} = u_{k}\hat{a}_{\mathbf{k}}^{\dagger} + v_{k}\hat{a}_{-\mathbf{k}}$$

$$u_{k} = \cosh\theta_{k}, \quad v_{k} = \sinh\theta_{k}$$

$$\theta_{k} = \frac{1}{2}\log\frac{\hbar^{2}k^{2}/2m}{\varepsilon(k)} < 0$$

$$\varepsilon(k) = \sqrt{\left(\frac{\hbar^{2}k^{2}}{2m}\right)^{2} + gn\frac{\hbar^{2}k^{2}}{m}}$$

$$\rho(\mathbf{k}) = \langle \hat{a}_{\mathbf{k}}^{\dagger}\hat{a}_{\mathbf{k}} \rangle$$

$$= \left(u_{k}^{2} + v_{k}^{2}\right)\langle \hat{b}_{\mathbf{k}}^{\dagger}\hat{b}_{\mathbf{k}} \rangle + v_{k}^{2}$$

$$\langle \hat{b}_{\mathbf{k}}^{\dagger}\hat{b}_{\mathbf{k}} \rangle = (\exp[\varepsilon(k)/k_{B}T] - 1)^{-1}$$



$$C = \lim_{k \to \infty} k^4 \rho(k)$$

In the non-interacting  $(a \rightarrow 0)$  limit,  $u_k = 1$  and  $v_k = 0$ ,



## Total quantum depletion in a trapped cloud

in situ tails: 
$$\lim_{k \to \infty} n(k) = \frac{\mathscr{C}}{k^4} = \frac{64\pi^2 a^2}{7} \frac{N_0 n_0}{k^4}$$

adiabatic sweep theorem<sup>39</sup>

$$\mathscr{C} = \frac{8\pi ma^2}{\hbar^2} \frac{\partial E}{\partial a}.$$

Tan, S. Energetics of a strongly correlated Fermi gas. *Annals Phys.* **323**, 2952–2970, (2008).

$$\frac{E}{N_0} = \frac{5}{7}\mu = \frac{5}{7}\frac{\hbar\bar{\omega}}{2}\left(\frac{15N_0a}{a_{\rm HO}}\right)^{2/5}$$
$$\mathscr{C} = \frac{8\pi}{7}\left(15^2(aN_0)^7\left(\frac{m\bar{\omega}}{\hbar}\right)^6\right)^{1/5}$$

 $\mathscr{C} = \int C(\mathbf{r}) d^3 \mathbf{r}$ 

 $C = \lim_{k \to \infty} k^4 \rho(k)$ 



FIG. 4. Contact constant  $C_{\infty}/N_0$  per condensed particle plotted as a function of the condensate density  $n_0$ . The geometric trapping frequency  $\bar{\omega}/2\pi$  and the ratio  $k_B T_a/\mu$  are indicated. The dashed line is the Bogoliubov prediction in the LDA,  $C_{\text{LDA}}$ (see text), and the solid line is  $6.5 \times C_{\text{LDA}}$ .



## Theory from the Trento group $\rightarrow$ there should be no survival

#### PHYSICAL REVIEW A 94, 063635 (2016)

#### Expansion of harmonically trapped interacting particles and time dependence of the contact

Chunlei Qu,<sup>1,\*</sup> Lev P. Pitaevskii,<sup>1,2</sup> and Sandro Stringari<sup>1</sup>

<sup>1</sup>INO-CNR BEC Center and Dipartimento di Fisica, Università di Trento, 38123 Povo, Italy <sup>2</sup>Kapitza Institute for Physical Problems RAS, Kosygina 2, 119334 Moscow, Russia (Received 30 August 2016; published 23 December 2016)

We study the expansion of an interacting atomic system at zero temperature, following its release from an isotropic three-dimensional harmonic trap and calculate the time dependence of its density and momentum distribution, with special focus on the behavior of the contact parameter. We consider different quantum systems, including the unitary Fermi gas of infinite scattering length, the weakly interacting Bose gas, and two interacting particles with highly asymmetric mass imbalance. In all cases analytic results can be obtained, which show that the initial value of the contact, fixing the  $1/k^4$  tail of the momentum distribution, disappears for large expansion times. Our results raise the problem of understanding the recent experiment of R. Chang *et al.* [Phys. Rev. Lett. **117**, 235303 (2016)] carried out on a weakly interacting Bose gas of metastable <sup>4</sup>He atoms, where a  $1/r^4$  tail in the density distribution was observed after a large expansion time, implying the existence of the  $1/k^4$  tail in the asymptotic momentum distribution.



FIG. 4. Plot of momentum distribution in the presence of interaction in the expansion. Different lines are the distributions at different expansion time  $\tau = 0$  (solid red), 1 (dashed green), and 5 (dotted blue) and the asymptotic momentum distribution (dot-dashed black). Results are shown in log-log scale plot. At large k, the momentum distribution exhibits a tail of  $1/k^4$ , whereas the asymptotic momentum distribution exhibits a  $1/k^{12}$  tail followed by a  $1/k^{14}$  tail.

#### Assumes too much adiabaticity or hydrodynamics



#### Our experiment + theory

# On the survival of the quantum depletion of a condensate after release from a magnetic trap

J. A. Ross<sup>1</sup>, P. Deuar<sup>2</sup>, D. K. Shin<sup>1</sup>, K. F. Thomas<sup>1</sup>, B. M. Henson<sup>1</sup>, S. S. Hodgman<sup>1</sup>, and A. G. Truscott<sup>1\*</sup>

<sup>1</sup>Research School of Physics, Australian National University, Canberra 0200, Australia <sup>2</sup>Institute of Physics, Polish Academy of Sciences, Aleja Lotników 32/46, 02-688 Warsaw, Poland \*andrew.truscott@anu.edu.au

#### ABSTRACT

We present observations of the high momentum tail in expanding Bose-Einstein condensates of metastable Helium atoms released from a harmonic trap. The far-field density profile exhibits features that support identification of the tails of the momentum distribution as originating in the in-situ quantum depletion prior to release. Thus, we corroborate recent observations of slowly-decaying tails in the far-field beyond the thermal component. This observation is in conflict with the hydrodynamic theory, which predicts that the in-situ depletion does not survive when atoms are released from a trap. Indeed, the depleted tails even appear stronger in the far-field than expected before release, and we discuss the challenges of interpreting this in terms of the Tan contact in the trapped gas. In complement to these observations, full quantum simulations of the experiment show that, under the right conditions, the depletion can persist into the far field after expansion. Moreover, the simulations provide mechanisms for survival and for the the large-momentum tails to appear stronger after expansion due to an acceleration of the depleted atoms by the mean-field potential. However, while in qualitative agreement, the final depletion observed in the experiment is much larger than in the simulation.

# $\begin{array}{l} \textit{Magnetic trap instead of optical} \\ \rightarrow \textit{no impurities in situ} \end{array}$

*Ross et al arXiv:2103.15283* 



#### ANU experiment



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## Fun and games with calibrating



**Figure S1.** Determining the RF transfer efficiency. The time-of-flight profiles of each pulse are resolved (a) by applying a weak Stern-Gerlach pulse during the time of flight. The pulses are aligned with respect to their centre-of-mass (b) and used to determine the pointwise fraction ((c), dotted line). Detector saturation is evident in the peaks (dashed lines), but not in the thermal tails (solid lines), which are used to compute the transfer efficiency. Because of its lower flux, the  $m_J = -1$  pulse does not show any clear evidence of saturation (d) and is used to determine the thermal fraction and hence  $N_0$ .



## Tail strengths (ANU+IFPAN)



FIG. 4. Contact constant  $C_{\infty}/N_0$  per condensed particle plotted as a function of the condensate density  $n_0$ . The geometric trapping frequency  $\bar{\omega}/2\pi$  and the ratio  $k_B T_a/\mu$  are indicated. The dashed line is the Bogoliubov prediction in the LDA,  $C_{\text{LDA}}$ (see text), and the solid line is  $6.5 \times C_{\text{LDA}}$ .



# Theory: Bogoliubov equations for the scattered field

$$\widehat{\Psi}(\mathbf{x},t) = \phi(\mathbf{x},t) + \widehat{\delta}(\mathbf{x},t)$$

#### **Evolution equations:**

$$i\hbar \partial_t \hat{\delta}(\mathbf{x},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + 2g|\phi(\mathbf{x},t)|^2\right]\hat{\delta}(\mathbf{x},t) + g\phi^2(\mathbf{x},t)\hat{\delta}^{\dagger}(\mathbf{x},t)$$

$$i\hbar \partial_t \phi(\mathbf{x},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + g|\phi(\mathbf{x},t)|^2\right]\phi(\mathbf{x},t).$$

#### Initial condition: more on that later



Bogoliubov hurdles

$$i\hbar \partial_t \hat{\delta}(\mathbf{x},t) = \left[-\frac{\hbar^2}{2m}\nabla^2 + 2g|\phi(\mathbf{x},t)|^2\right]\hat{\delta}(\mathbf{x},t) + g\phi^2(\mathbf{x},t)\hat{\delta}^{\dagger}(\mathbf{x},t)$$

- Looks like a linear problem, so why not just diagonalize  $\hat{H}_{\rm eff}~$  and have everything, but.....
  - 1. The numerical lattice might be too large  $(10^6 10^7 \text{ points in a})$

3D calculation) (note also the *"human time"* bottleneck!)

2. BEC evolves parallel to the Bogoliubov field

#### $\rightarrow$ would have to re-diagonalize

at each time step (boooo.....)  $_{i\hbar\partial}$ 

$$\phi_t \phi(\mathbf{x},t) = \left[ -\frac{\hbar^2}{2m} \nabla^2 + g |\phi(\mathbf{x},t)|^2 \right] \phi(\mathbf{x},t).$$

Diagonalization can be avoided by using the positive-P representation



Simulation - "STAB" method

$$\widehat{\Psi}(\mathbf{x},t) = \phi(\mathbf{x},t) + \widehat{\delta}(\mathbf{x},t)$$

11/ )

Bogoliubov

Stochastic Time Adaptive Boboliubov

Can use plane wave basis ---> no diagonalizing of  $10^7 \times 10^7$  matrices :)

Gaussian real white noise  $\langle \xi (z) \rangle$ 

$$\langle \xi(\mathbf{x},t)\xi(\mathbf{y},t')\rangle = \delta^3(\mathbf{x}-\mathbf{y})\delta(t-t')$$

$$\rho_1(\mathbf{x}, \mathbf{x}') = \left\langle \hat{\Psi}^{\dagger}(\mathbf{x}) \hat{\Psi}(\mathbf{x}') \right\rangle = \operatorname{Re} \langle \tilde{\psi}(\mathbf{x})^* \psi(\mathbf{x}') \rangle_{st}$$



#### Extra fun this time:

"plain STAB" was useful for BEC collisions

$$\widehat{\Psi}(\mathbf{x},t) = \boldsymbol{\phi}(\mathbf{x},t) + \widehat{\Psi}_B(\mathbf{x},t)$$

when Bogoliubov modes were separated from the condensate in k-space Now this is no longer true (lots of overlap)

Need to impose orthogonality for real this time:

$$\int d^3 \mathbf{x} \,\widehat{\Psi}_B^{\dagger}(\mathbf{x},t) \phi(\mathbf{x},t) = 0$$

Proper STAB equations that preserve orthogonality

$$\begin{split} i\hbar \frac{d\phi}{dt} &= \left[ -\frac{\hbar^2}{2m} \nabla^2 + g |\phi|^2 + V(\mathbf{x}, t) \right] \phi \\ i\hbar \frac{d\psi_B}{dt} &= \left[ -\frac{\hbar^2}{2m} \nabla^2 + g |\phi|^2 + V(\mathbf{x}, t) \right] \psi_B + \mathscr{P}_\perp \left\{ g |\phi|^2 \psi_B + g \phi^2 \widetilde{\psi}_B^* + \sqrt{-ig} \phi \, \xi(\mathbf{x}, t) \right\} \\ i\hbar \frac{d\widetilde{\psi}_B}{dt} &= \left[ -\frac{\hbar^2}{2m} \nabla^2 + g |\phi|^2 + V(\mathbf{x}, t) \right] \widetilde{\psi}_B + \mathscr{P}_\perp \left\{ g |\phi|^2 \widetilde{\psi}_B + g \phi^2 \psi_B^* + \sqrt{-ig} \phi \, \widetilde{\xi}(\mathbf{x}, t) \right\} \end{split}$$

Thankfully, projection can be done very efficiently  $\mathscr{P}_{\perp}f(\mathbf{x}) = f(\mathbf{x}) - \frac{1}{N} \left[ \int d^3 \mathbf{x}' \ \phi(\mathbf{x}')^* f(\mathbf{x}') \right] \phi(\mathbf{x}).$ 



#### Serious extra fun – initial conditions with depletion





#### Depletion atoms distribution

Initial k-space distribution (slices)



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## Dependence of Tan's contact on cloud size (simulation)



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#### Evolution of contact during expansion



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## Tail "growth" mechanism







40

20

0

-20

kх

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kх



#### Tail escape mechanism and anisotropy







#### Toy two-mode model of escape

In a uniform gas in the Bogoliubov approximation, the  $\hat{a}_k$  mode is coupled only to  $\hat{a}_{-k}^{\dagger}$  and the condensate. The Bogoliubov-de Gennes equations for these modes can then be written<sup>80</sup>

$$\frac{d}{dt} \begin{bmatrix} \hat{a}_k(t) \\ \hat{a}_{-k}^{\dagger}(t) \end{bmatrix} = -\frac{i}{\hbar} \begin{bmatrix} \hbar^2 k^2 / 2m + 2gn(t) - \mu(t) & gn(t) \\ -gn(t) & -\hbar^2 k^2 / 2m - 2gn(t) + \mu(t) \end{bmatrix} \begin{bmatrix} \hat{a}_k(t) \\ \hat{a}_{-k}^{\dagger}(t) \end{bmatrix}.$$
(27)

Notice the provision for a time-dependent background condensate density n(t). We stay in the real-particle basis rather than the Bogoliubov quasiparticles  $\hat{b}_k$  which allows to avoid calculating time-dependent changes of the coefficients  $u_k$ ,  $v_k$ . The chemical potential is  $\mu(t) = gn(t)^{80}$ .

The equations (27) can be used as input for equations of motion of the low order moments  $\rho(k) = \langle \hat{a}_{\pm k}^{\dagger} \hat{a}_{\pm k} \rangle$ ,  $A(k) = \langle \hat{a}_{k} \hat{a}_{-k} \rangle$ , and  $A^{*}(k) = \langle \hat{a}_{k}^{\dagger} \hat{a}_{-k}^{\dagger} \rangle$ , for example  $d\rho(k)/dt = \langle (d\hat{a}_{k}^{\dagger}/dt)\hat{a}_{k} \rangle + \langle \hat{a}_{k}^{\dagger}(d\hat{a}_{k}/dt) \rangle$ . Assuming equal initial occupations  $\rho(k, 0) = \rho(-k, 0)$ , one obtains an evolution equation for two coupled quantities  $\rho(k, t)$  and  $A(k, t) = A_{r}(k) + iA_{i}(k)$ . It is

$$\frac{d}{dt} \begin{bmatrix} \rho(k,t) \\ A_r(k,t) \\ A_i(k,t) \end{bmatrix} = \frac{1}{\hbar} \begin{bmatrix} 0 & 0 & -2gn(t) \\ 0 & 0 & 2(\hbar^2 k^2 / 2m - gn(t)) \\ -2gn(t) & -2(\hbar^2 k^2 / 2m - gn(t)) & 0 \end{bmatrix} \begin{bmatrix} \rho(k,t) \\ A_r(k,t) \\ A_i(k,t) \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -gn(t) \end{bmatrix}.$$
(28)

The initial conditions are  $\rho(k,0) = v_k^2$ ,  $A(k,0) = v_k u_k$ , corresponding to the Bogoliubov ground state. Taking n(t) constant one obtains the solution (16).

$$\rho(k,t) = \rho(k,0) - \frac{gn[\varepsilon_0(k)^2 - \varepsilon(k)^2]}{4\varepsilon(k)^2\varepsilon_0(k)} [1 - \cos 2\varepsilon(k)t]$$
 Most naive toy model:  $n(t)$  = step function

A more realistic model is obtained by estimating the n(t) that particles in the k mode experience as the trap is released





#### Recent repeat experiment in Palaiaseau

#### Observation of $1/k^4$ -tails in the asymptotic momentum distribution of Bose polarons

Hugo Cayla,<sup>1</sup> Pietro Massignan,<sup>2</sup> Thierry Giamarchi,<sup>3</sup> Alain Aspect,<sup>1</sup> Christoph I. Westbrook,<sup>1</sup> and David Clément<sup>1</sup>

 <sup>1</sup>Laboratoire Charles Fabry, Institut d'Optique, CNRS, Univ. Paris Sud, 2 Avenue Augustin Fresnel 91127 PALAISEAU cedex, France
 <sup>2</sup>Departament de Física, Universitat Politècnica de Catalunya, Campus Nord B4-B5, E-08034 Barcelona, Spain
 <sup>3</sup>Department of Quantum Matter Physics, University of Geneva, 24 quai Ernest-Ansermet, 1211 Geneva, Switzerland (Dated: April 11, 2022)

#### arXiv: 2204.10697

We measure the asymptotic momentum density, after an expansion in the presence of interactions, of dilute spin impurities in a Bose-Einstein condensate. In the absence of impurities, we confirm the theoretical scenario of C. Qu *et al.* [Phys. Rev. A **94**, 063635 (2016)] according to which signatures of the quantum depletion vanish during an expansion of an interacting Bose-Einstein condensate. When impurities are present, we observe tails decaying as  $1/k^4$  at large momentum k. These results highlight the key role played by impurities when present, a possibility that had not been considered in our previous work R. Chang *et al.* [Phys. Rev. Lett. **117**, 235303 (2016)]. We show that the algebraic tails originate from the impurity-BEC interaction, but that (their amplitudes greatly exceed those expected for the in-trap contact of weakly-interacting Bose polarons). We attribute this discrepancy to the non-trivial dynamics of the expansion in the presence of bath-impurity interactions.



FIG. 1. (a) To detect only spin impurities  $(m_J = 0)$ , a magnetic gradient is applied during the expansion to push the BEC atoms  $(m_J = +1)$  away from the detector. (b) To probe simultaneously BEC atoms and impurities, a radio-frequency (RF) pulse couples the atomic states  $m_J = 0$  and  $m_J = +1$  during the expansion, before applying the magnetic gradient, producing a known mixture of  $m_J = 0$  and  $m_J = \pm 1$ .





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#### Current status of explanations

	Released tail strength compared to <i>in situ</i> value		1.0 1.0 $\bigcirc \overline{0}^{2}\pi=220 \text{ Hz } ; k_{B}T_{a}/\mu = 1.2$ $\bigcirc \overline{0}^{2}\pi=255 \text{ Hz } ; k_{B}T_{a}/\mu = 0.75$ $\bigcirc \overline{0}^{2}\pi=360 \text{ Hz } ; k_{B}T_{a}/\mu = 0.9$ $\bigcirc \overline{0}^{2}\pi=360 \text{ Hz } ; k_{B}T_{a}/\mu = 0.9$
Chang, Bouton, Cayla, Qu, Aspect, Westbrook, Clement, PRL 117, 235303 (2016)	x 6 ± 1	Eksperyment (Palaiseau)	Initial result: Tails survive but too strongly for a simple explanation
Qu, Pitaevskii, Stringari PRA 94, 063635 (2016)	x 0	Teoria (Trento)	Claims no survival can occur Seems relevant only when process is sufficiently slow
Ross, Deuar, Shin, Thomas, Henson, Hodgman, Truscott, ArXiv: 2103.15283	x 2 ± 0.2	Symulacje (Warszawa)	STAB simulation explains how depletion can survive, and gives a mechanism for amplification, but too weak
Ross, Deuar, Shin, Thomas, Henson, Hodgman, Truscott, ArXiv: 2103.15283	x 5 ± 3	Eksperyment (Canberra)	Corroborates initial Palaiseau result but amplification mechanism unclear
Cayla, Massignan, Giamarchi, Aspect, Westbrook, Clement to appear (2022)	x 0 - 6	Eksperyment2 (Palaiseau)	Claims to explain Palaiaseau result due to impurities, but explanation seems inapplicable to Canberra results (Canberra magnetic trap cannot hold impurities)

#### Ideas towards better explanation:

- If impurity effect is very strong, might occur during RF pulse splitting after release in Canberra
- Will try to simulate Palaiseau experiment with impurities using STAB to confirm conjecture
- Tails may be due to "Oort cloud" of trapped excited atoms (vaguely suggested by toy model)

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