Density and phase correlations in dilute condensates after a quantum quench

PRA, accepted (2019)

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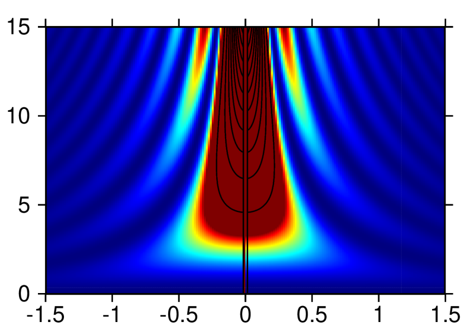
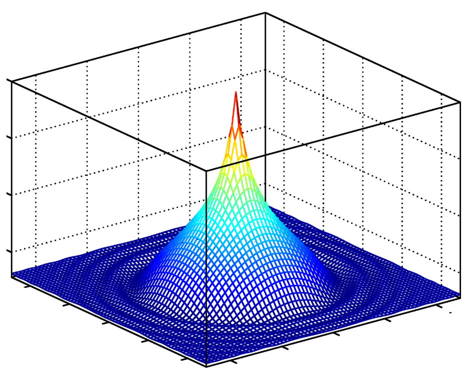
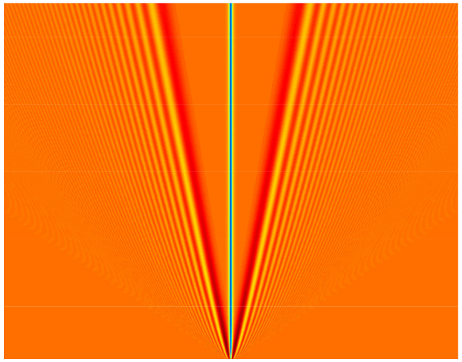






Outline

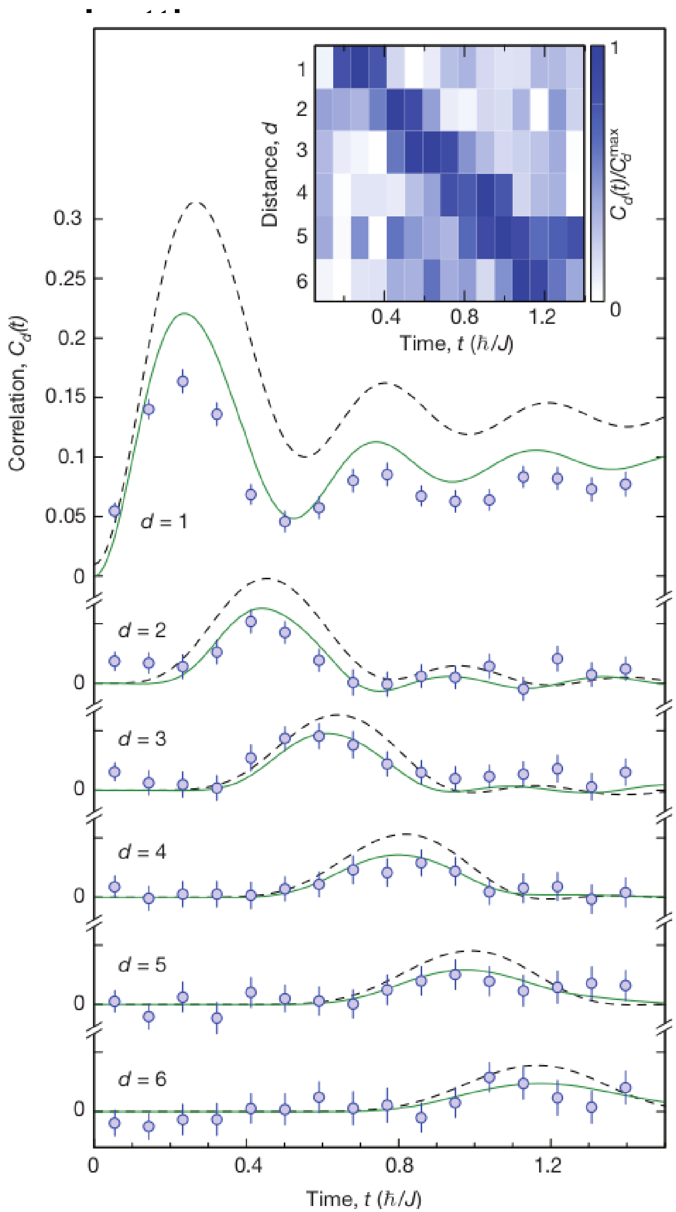
- Motivation
- Outline of the calculation
- Results
- Prospects for experimental observation
- Extension to broader cases

Quantum quench

- Sudden (non-adiabatic) global change in system parameters
- Building block of more complicated dynamics
- Examples in ultracold gases:
  - Rapid jump in *g (Feshbach resonance)*
  - Rapid jump in transverse trap frequency (low-dimensional gases)
  - Rapid jump in magnetic field (spinor gas)
  - Rapid jump in density
  - Rapid appearance of condensate

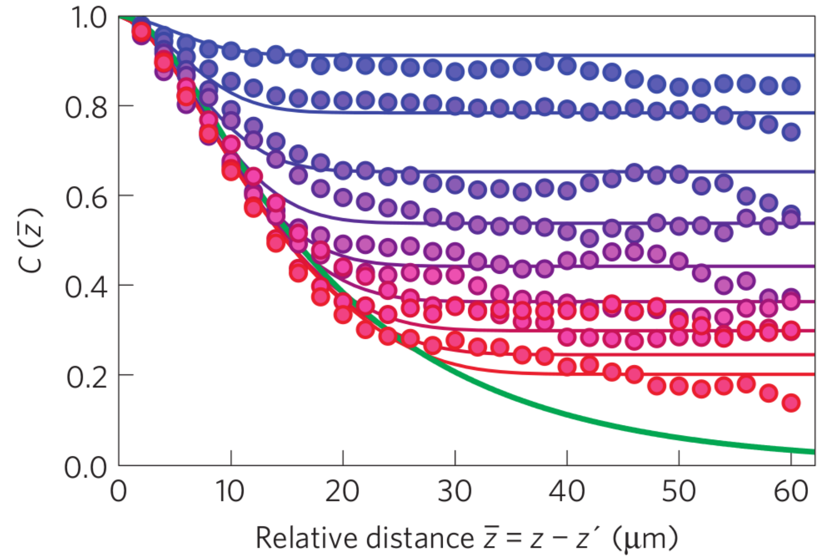
$$g_{1D} \approx \frac{g}{2\pi l_\perp^2} = 2\hbar a_s \omega_\perp$$

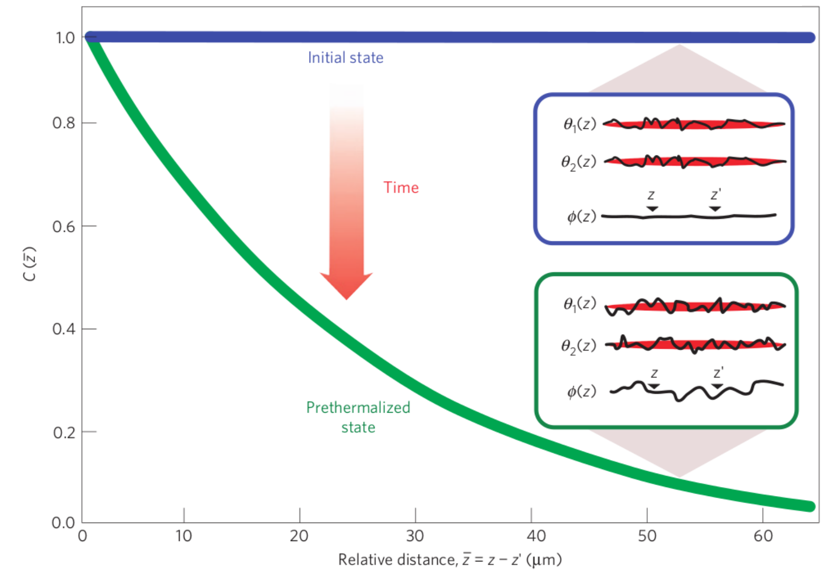
- Produces: waves, correlations, entanglement
- Types
  - Weak
  - Strong
  - Across phase transition



Parity correlations. Mott insulator U/J: **40→9**

Cheneau, Barmettler, Poletti, Endres, Schauss, Fukuhara, Gross, Bloch, Kollath, Khur, Nature **481,** 484 (2012)





Phase correlations.
Splitting of two thermal 1d clouds

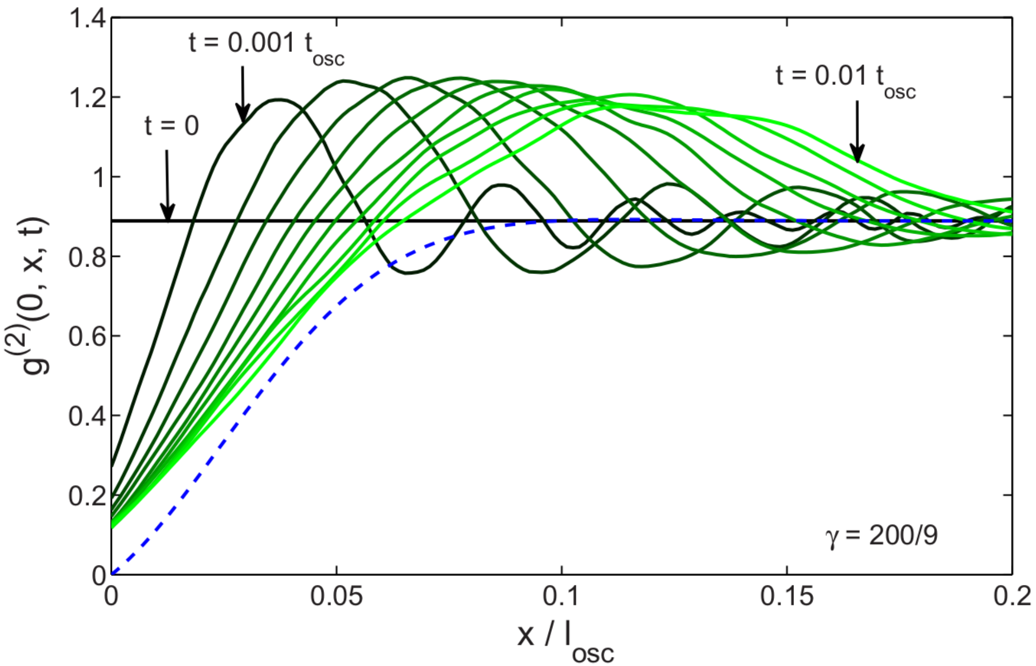
Langen, Gaiger, Kuhnert, Rauer, Schmiedmayer, Nature Phys. **9**, 640 (2013)

Some representative theory examples

Muth, Schmidt, Fleischhauer, NJP **12**, 083065 (2010)

Density correlation

g=0 → *g*=big, TEBD *N=9*

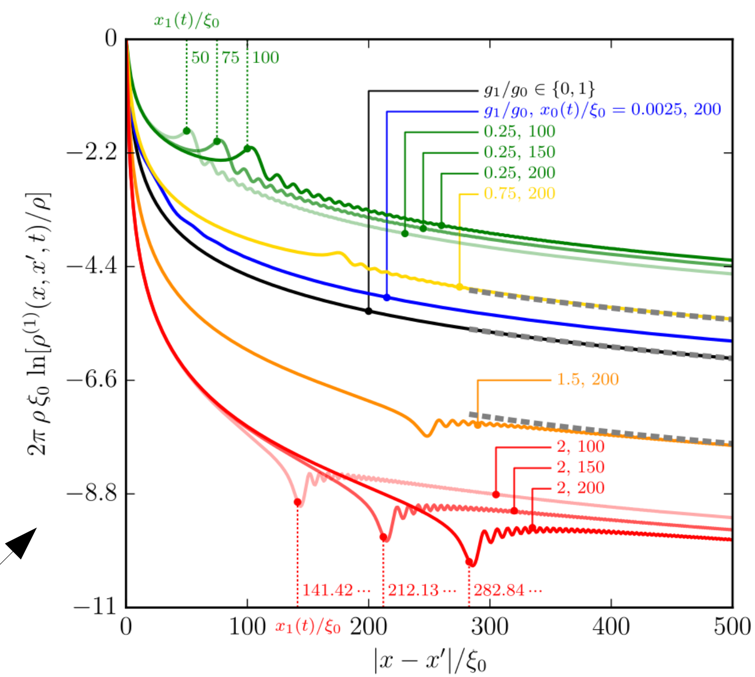


Phase correlation $g^{(1)}(x-x')$

Bogoliubov

Complicated g(t) quench

Martone, Larre, Fabbri, Pavloff, PRA **98**, 063617 (2018)



$$2\pi\,\rho\,\xi_0\,\ln\left[\frac{\rho^{(1)}(x,x',t)}{\rho}\right]$$

$$= -\frac{1}{2}\int_0^{+\infty} dq\,\left|\frac{q-\sqrt{q^2+4}}{(q\sqrt{q^2+4})^{1/2}}\,\frac{{}_2F_1[\alpha_q,\gamma_q-\beta_q,\gamma_q,(1+e^{-t/\tau})^{-1}]}{(1+e^{t/\tau})^{\alpha_q}}\right.$$

$$\left. +2\left(\frac{\sqrt{q^2+4}}{q}\right)^{1/2}\frac{\alpha_q\,\beta_q}{(\alpha_q+\beta_q)\,\gamma_q}\,\frac{{}_2F_1[\alpha_q+1,\gamma_q-\beta_q,\gamma_q+1,(1+e^{-t/\tau})^{-1}]\,e^{t/\tau}}{(1+e^{t/\tau})^{\alpha_q+1}}\right|^2\left[1-\cos\left(q\,\frac{|x-x'|}{\xi_0}\right)\right],$$

Our aim: fill a gap in theory

- Continuum, weak case
  - Not analyzed much, but
  - Should be present in many existing experiments!
  - Generally avoided because not treatable by a smallish lattice or MPS
- Simple expressions
  - Most results to date were numerical or complicated forms
- Compare 1d, 2d, 3d
  - Most studies were in 1d for the usual reasons

Simplest useful model

- Uniform gas
  - Can in principle use LDA
- Dilute
  - e.g. small Lieb-Liniger parameter $\gamma = \frac{g_{1D}}{\rho}$ or gas parameter na^3

$$\widehat{H} = \int d^d\mathbf{x} \left\{ -\frac{\hbar^2}{2m}\widehat{\Psi}^\dagger(\mathbf{x})\nabla^2\widehat{\Psi}(\mathbf{x}) + \frac{g}{2}\widehat{\Psi}^\dagger(\mathbf{x})^2\widehat{\Psi}(\mathbf{x})^2 \right\}$$

- T=0
  - gotta start from something
- Coherent region
  - → can use Bogoliubov $\widehat{\Psi}(x) = \phi_0(x)\,\widehat{a}_0 + \delta\widehat{\Psi}(x)$
- Start from g=0
  - same reason as with T=0 ;-)
  - Can easily write down the initial state

Physically relevant units

$$\widehat{H} = \int d^d\mathbf{x} \left\{ -\frac{\hbar^2}{2m} \widehat{\Psi}^\dagger(\mathbf{x}) \nabla^2 \widehat{\Psi}(\mathbf{x}) + \frac{g}{2} \widehat{\Psi}^\dagger(\mathbf{x})^2 \widehat{\Psi}(\mathbf{x})^2 \right\}$$

Introduce physically relevant units

$$\hbar = 1, \qquad \xi = \hbar/\sqrt{2mg\overline{n}}$$

Obtain time units - "healing time" $t_\xi = \frac{\hbar}{g\overline{n}}$

Only <u>one relevant parameter</u> $\gamma = \left(\frac{m\, g\, \overline{n}}{\hbar^2}\right)^d \frac{1}{\overline{n}^2}$

$$\widehat{H} = -J \sum_{i,j} \widehat{a}_i^\dagger \widehat{a}_j + \frac{U}{2} \sum_i \widehat{a}_i^\dagger \widehat{a}_i^\dagger \widehat{a}_i \widehat{a}_i$$

$$U = \sqrt{\gamma}/(\Delta v)$$

$$J = 1/[2(\Delta v)^{2/d}]$$

Will take $\Delta v \rightarrow 0$

Bogoliubov approximation for $\gamma << 1$

Define $$\widehat{\Psi}(x) = \phi_0(x)\,\widehat{a}_0 + \delta\widehat{\Psi}(x)$$

Assume $$\delta N = \int dx\,\langle \delta\widehat{\Psi}^\dagger(x)\delta\widehat{\Psi}(x)\rangle \quad << \quad N$$

Number-conserving operator $$\widehat{\Lambda}(x) = \frac{\widehat{a}_0}{\sqrt{N_0}}\,\delta\widehat{\Psi}(x) \quad \approx \quad \delta\widehat{\Psi}(x)$$

Replace $$\delta\widehat{\Psi}(x) \to \widehat{\Lambda}(x)$$ In Hamiltonian

Basis $$\widehat{\Lambda}(x,t) = \sum_{k\neq 0}\left[\widehat{b}_k(t)u_k(x) + \widehat{b}_k^\dagger(t)v_k^*(x)\right]$$

Keep only terms O(≤ 2) in $\widehat{\Lambda}(x)$: $$\widehat{H} = \mathrm{const.} + \sum_{k\neq 0}\omega_k \widehat{b}_k^\dagger \widehat{b}_k$$

Bogoliubov solution

So, with
$$\widehat{\Lambda}(x,t) = \sum_{k\neq 0} \left[\widehat{b}_k(t) u_k(x) + \widehat{b}_k^\dagger(t) v_k^*(x)\right]$$

<u>*Solution:*</u>
$$\widehat{b}_k(t) = \widehat{b}_k(0)\, e^{-i\omega_k t}$$

$$u_k(x) = \frac{U_k}{\sqrt{L}}\, e^{ikx}$$
$$v_k(x) = \frac{V_k}{\sqrt{L}}\, e^{ikx}$$
$$\omega_k = \sqrt{\frac{k^2}{2}\left(\frac{k^2}{2}+1\right)}$$
$$2\begin{pmatrix} U_k \\ V_k \end{pmatrix} = \left[\frac{k^2}{k^2+2}\right]^{1/4} \begin{pmatrix} + \\ - \end{pmatrix} \left[\frac{k^2+2}{k^2}\right]^{1/4}$$

To obtain observables

Recall $\delta\widehat{\Psi} \to \widehat{\Lambda}.$ and $$\widehat{\Lambda}(x,t) = \sum_{k\neq 0}\left[\widehat{b}_k(t)u_k(x) + \widehat{b}_k^\dagger(t)v_k^*(x)\right]$$

Now, invert $$\widehat{b}_k(0) = \frac{1}{\sqrt{N_0}}\left[U_k\widehat{a}_k(0)\,\widehat{a}_0^\dagger - V_k\widehat{a}^\dagger_{-k}(0)\,\widehat{a}_0\right]$$

In terms of momentum space modes (plane waves) $\widehat{a}_k(0)$

Initial state $|I\rangle$ had vacuum in all excited modes

$$\widehat{a}_k(0)|I\rangle = \langle I|\widehat{a}_k^\dagger(0) = 0 \qquad \forall k \neq 0.$$

Commutation relations $$\left[\widehat{a}_k, \widehat{a}_m^\dagger\right] = \delta_{km}$$

…. now substitute and grind algebra

Spatial correlations

$$g^{(1)}(y,t) = \frac{\langle \widehat{a}^\dagger(\mathbf{r})\widehat{a}(\mathbf{r}')\rangle}{\overline{n}\Delta v}$$

$$= 1 - \frac{1}{2\overline{n}V}\sum_{\mathbf{k}\neq 0}\frac{1}{\omega_k^2}\left[1 - \cos 2\omega_k t - \cos \mathbf{k}\cdot\mathbf{y} + \cos(\mathbf{k}\cdot\mathbf{y} + 2\omega_k t)\right]$$

$$g^{(2)}(y,t) = \frac{\langle \widehat{a}^\dagger(\mathbf{r})\widehat{a}^\dagger(\mathbf{r}')\widehat{a}(\mathbf{r}')\widehat{a}(\mathbf{r})\rangle}{(\overline{n}\Delta v)^2}$$

$$= 1 - \frac{1}{2\overline{n}V}\sum_{\mathbf{k}\neq 0}\frac{k^2}{\omega_k^2}\left[\cos \mathbf{k}\cdot\mathbf{y} - \cos(\mathbf{k}\cdot\mathbf{y} + 2\omega_k t)\right]$$

$$\omega_k = k\sqrt{1 + k^2/4}$$

Continuum limit

$$g^{(1)}(y,t) = 1 - \sqrt{\gamma}\int_0^\infty dk\ \left(\frac{1-\cos 2\omega_k t}{k^2+4}\right)\frac{1-M_d}{a_d\, k^{3-d}}$$

$$g^{(2)}(y,t) = 1 - \sqrt{\gamma}\int_0^\infty dk\ \left(\frac{1-\cos 2\omega_k t}{k^2+4}\right)\frac{M_d\, k^{d-1}}{a_d}$$

$$M_d = \begin{cases} \cos ky & \text{for } d=1 \\ J_0\left[k|y|\right] & \text{for } d=2, \\ \frac{\sin ky}{ky} & \text{for } d=3 \end{cases} \quad a_d = \begin{cases} \pi/2 & \text{for } d=1 \\ \pi & \text{for } d=2 \\ \pi^2 & \text{for } d=3 \end{cases}$$

Convenient single-dimensional integrals

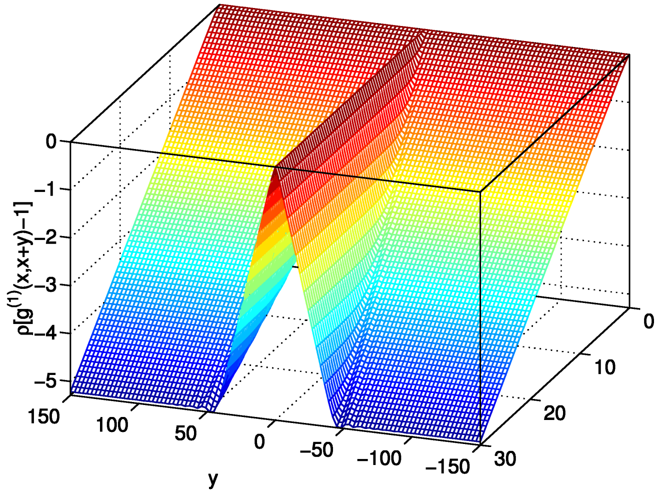
Quantum depletion

$$\delta N \approx \frac{1}{4}\sum_{k\neq 0}\left(\frac{\sin\omega_k t}{\omega_k}\right)^2$$

$$\frac{\delta N(t)}{N} \approx \sqrt{\gamma} \times \begin{cases} \frac{4t-1}{8} & \text{for } d=1 \\ \frac{1}{4\pi}\left(c_1 + \log t \right) & \text{for } d=2 \\ \frac{1}{4\pi}\left(1 + c_2 e^{-c_3 t}\right) & \text{for } d=3 \end{cases}$$

1d, 2d: only correct as long as depletion is small

$$g^{(1)}_{\text{spacelike}}(y,t) \approx 1 - \delta N(t)/N$$



Stationary state is not the ground state

3d

stationary state after quench:

$$\delta N/N = \frac{\sqrt{\gamma}}{4\pi} \approx 0.080\sqrt{\gamma}$$

ground state:

$$\delta N/N = \frac{\sqrt{\gamma}}{3\pi^2} \approx 0.034\sqrt{\gamma}$$

1d

stationary state after quench:

$$g^{(2)}(0) = 1 - \frac{1}{2}\sqrt{\gamma}$$

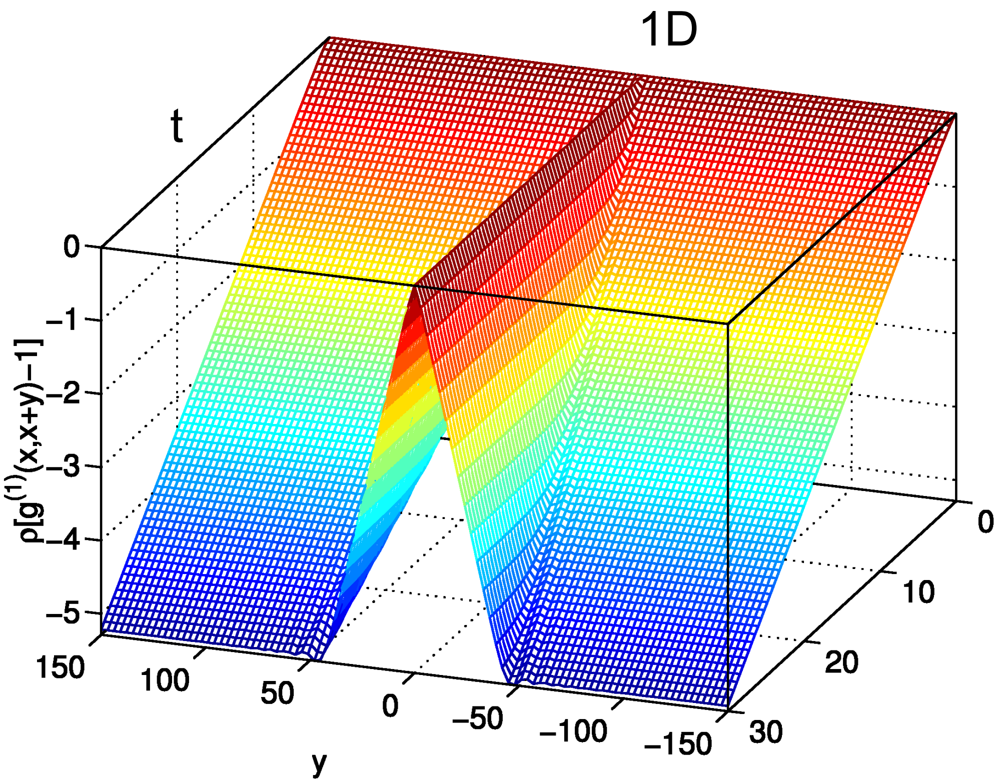
ground state:

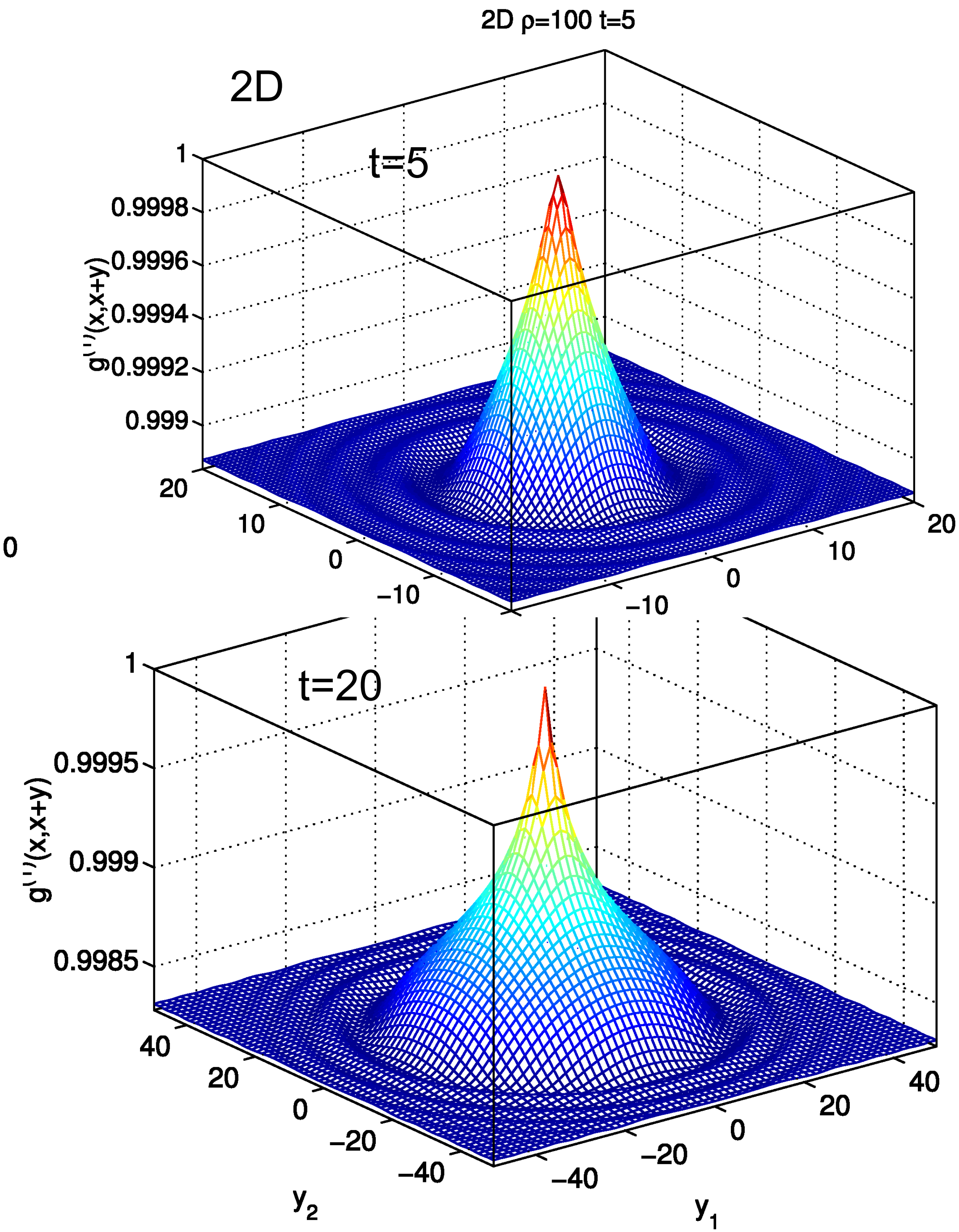
$$g^{(2)}(0) = 1 - \frac{2}{\pi}\sqrt{\gamma} \approx 1 - 0.637\sqrt{\gamma}$$

DeNardis, Wouters, Brockman, Caux, PRA **89**, 033601 (2014)

Found state after quench
That was neither thermal nor
Generalized Gibbs Ensemble (GGE)

$g^{(1)}(y,t)$ phase correlations

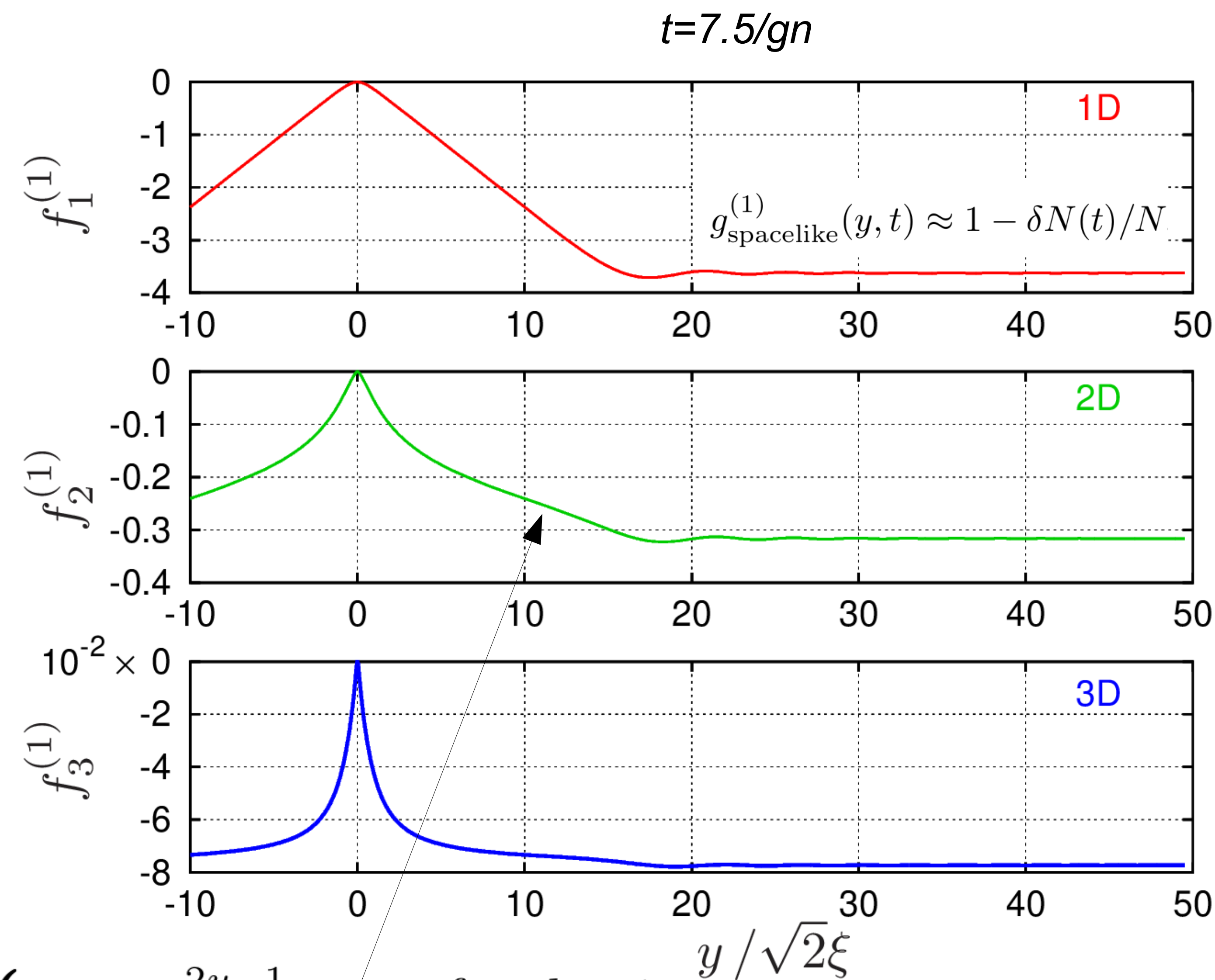




Spatial $g^{(1)}(y,t)$

Universal function

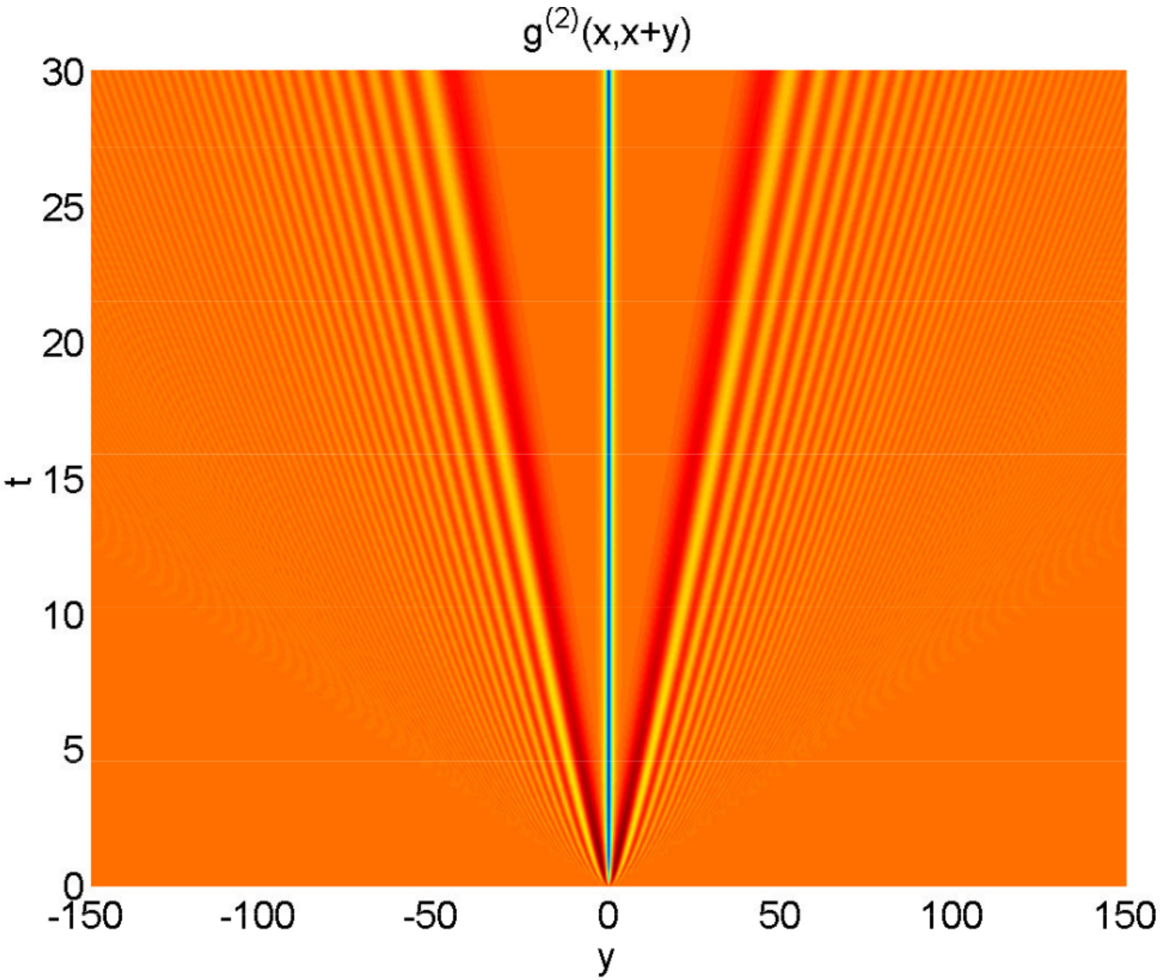
$$f_d^{(1)} = [g^{(1)}(y,t) - 1]/\sqrt{\gamma}$$

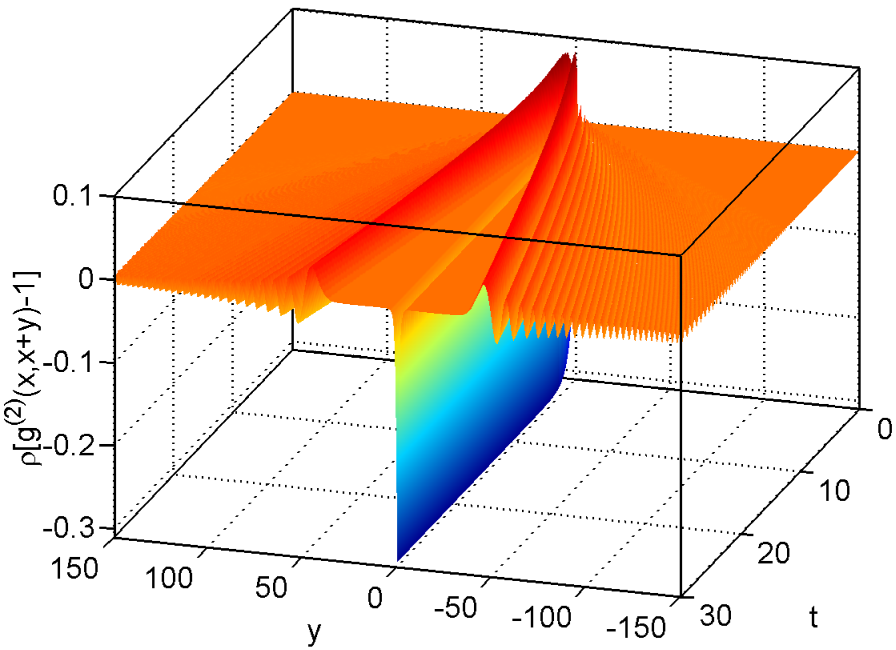


$$g^{(1)}_{\text{timelike}}(y) \approx 1 - \sqrt{\gamma} \times \begin{cases} \frac{2y-1}{8} & \text{for } d=1 \\ \frac{1}{4\pi}\left(\gamma_E + \log y\right) & \text{for } d=2 \\ \frac{1}{4\pi}\left(1 - \frac{1}{2y}\right) & \text{for } d=3 \end{cases}$$

NARODOWE CENTRUM NAUKI

$g^{(2)}(y,t)$ density correlations





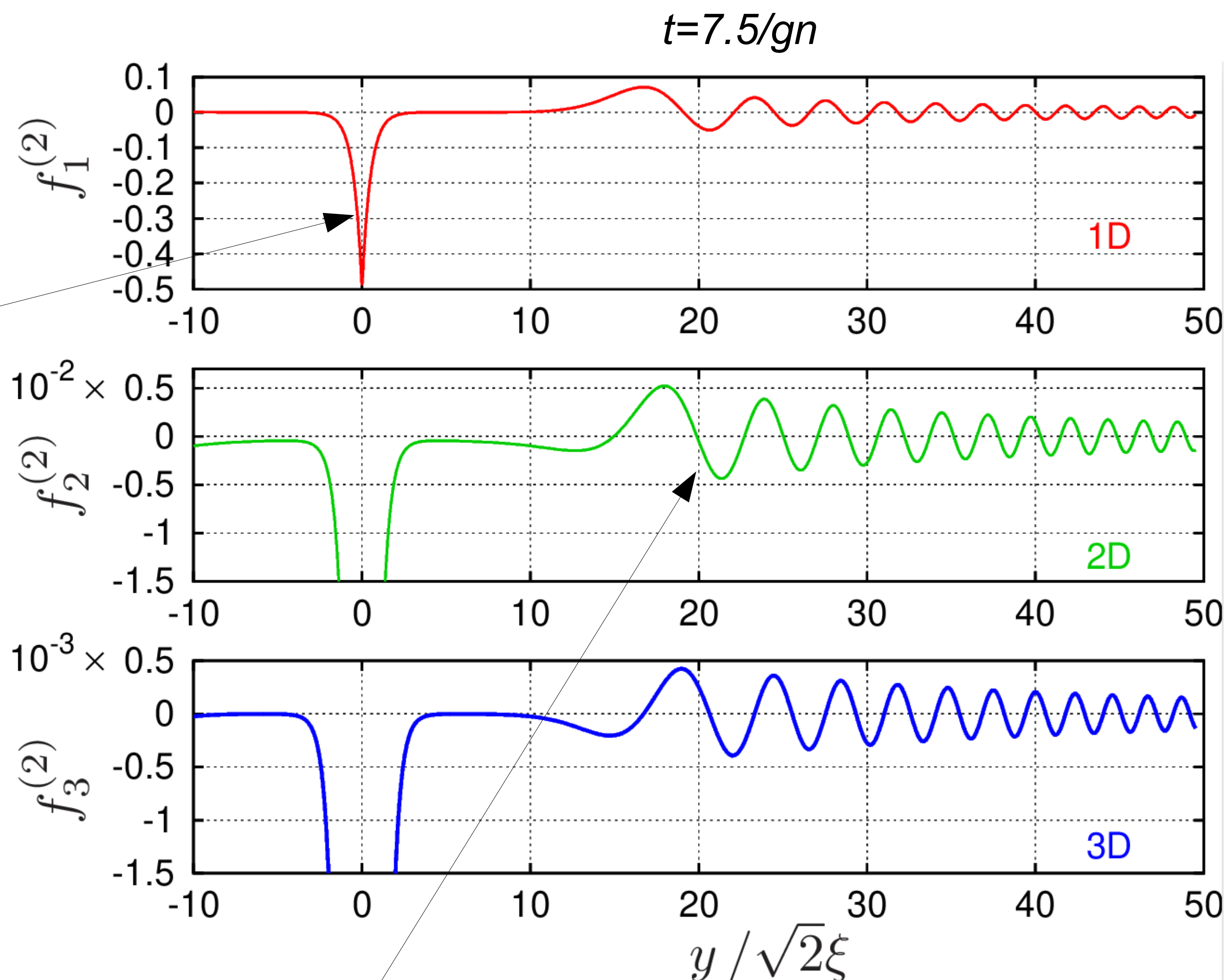
Spatial $g^{(2)}(y,t)$

Universal function

$$f_d^{(2)} = [g^{(2)}(y,t) - 1]/\sqrt{\gamma}$$

$$g^{(2)}_{\mathrm{timelike}}(y) \approx$$

$$1 - \sqrt{\gamma} \times \begin{cases} \frac{e^{-2y}}{2} & \text{for } d=1 \\ \frac{K_0[2y]}{\pi} & \text{for } d=2 \\ \frac{e^{-2y}}{2\pi y} & \text{for } d=3 \end{cases}$$



$$F_d[x] = \frac{1}{\pi}\int_0^\infty du\; u^{\frac{d-1}{2}} \cos\left[xu + \frac{u^3}{3} + \frac{\pi}{4}(d-1)\right]$$

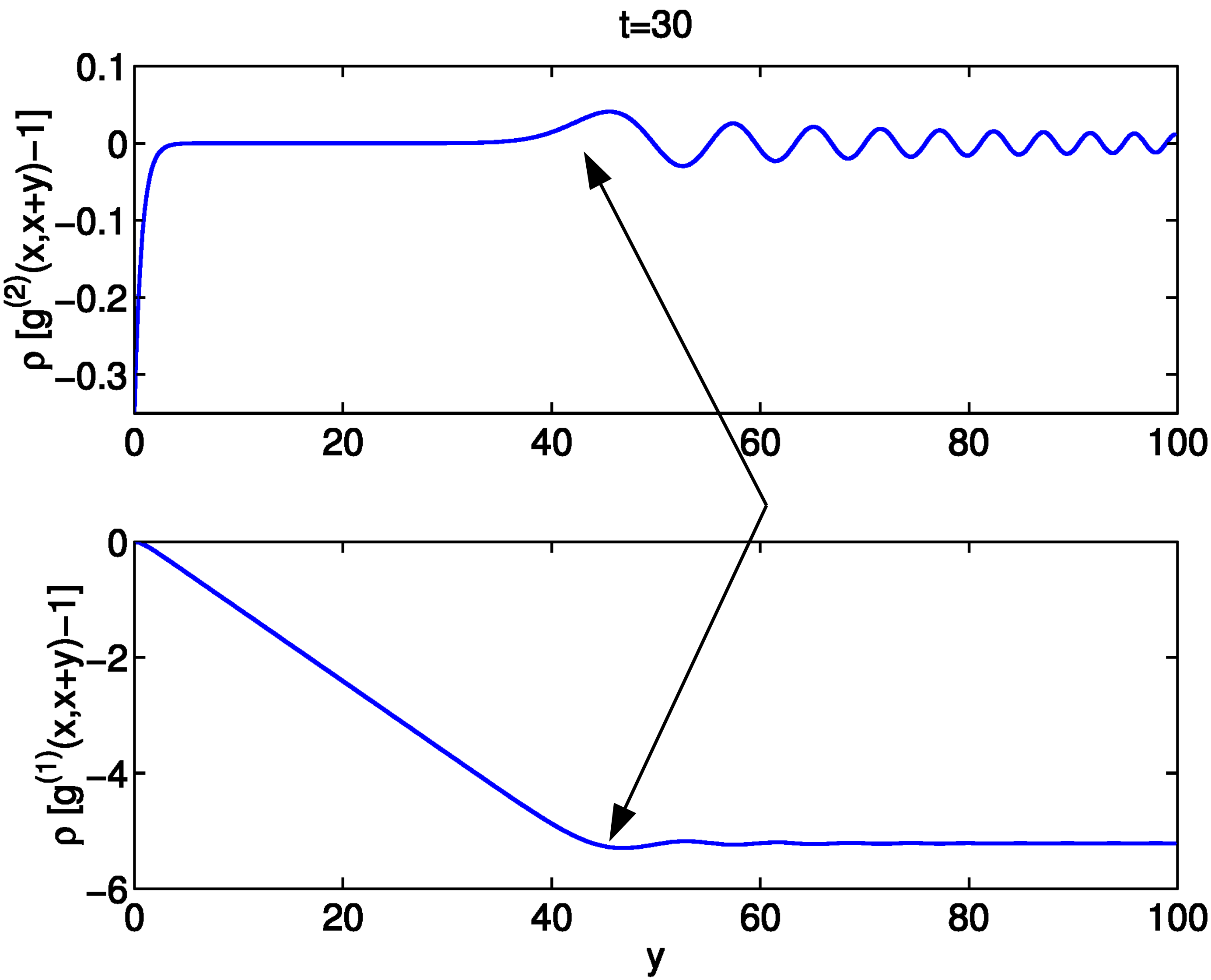
$$x = \left(\frac{4}{3t}\right)^{1/3}[y - 2t]$$

$$g^{(2)}_{\mathrm{wave}}(y) \approx 1 + \sqrt{\gamma} \times \begin{cases} \frac{1}{2\,(6t)^{1/3}}\,\mathrm{F}_1[-x] & \text{for } d=1 \\ \frac{1}{2\sqrt{\pi y}\,(6t)^{1/2}}\,F_2[-x] & \text{for } d=2 \\ \frac{1}{2\pi y\,(6t)^{2/3}}\,F_3[-x] & \text{for } d=3 \end{cases}$$

Match between features in density and phase

$g^{(2)}(y,t=30)$

$g^{(1)}(y,t=30)$



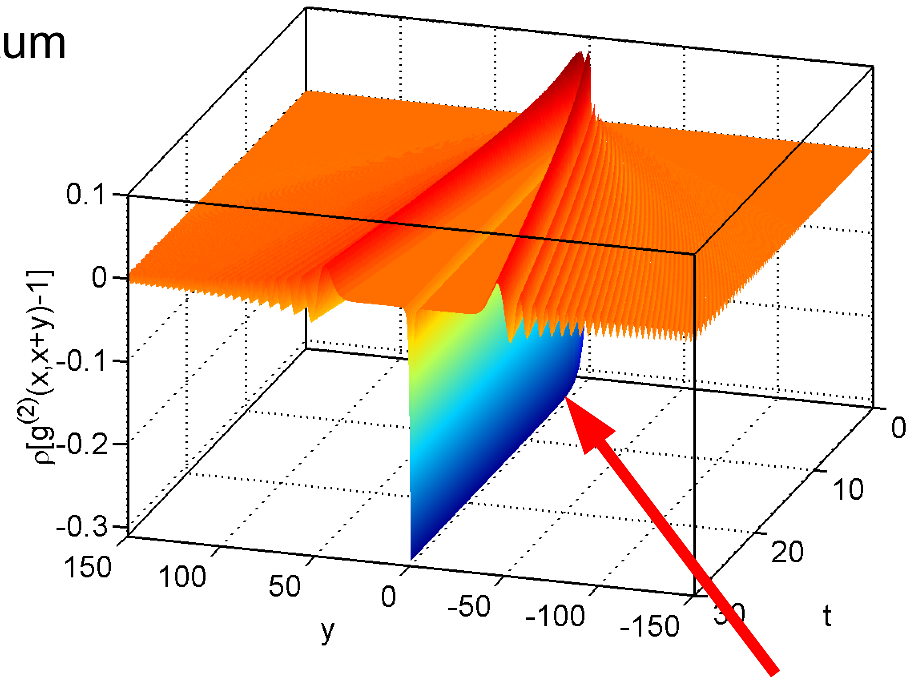
NARODOWE CENTRUM NAUKI

Local $g^{(2)}(0,t)$ density fluctuations

At long times, depends strongly on allowable momentum

$$g^{(2)}_{\text{timelike}}(0) =$$

$$1-\sqrt{\gamma}\times\begin{cases} \frac{1}{\pi}\tan^{-1}\frac{k_{\max}}{2} & \text{for } d=1 \\ \frac{1}{2\pi}\log\left(1+\frac{k_{\max}^2}{4}\right) & \text{for } d=2 \\ \frac{1}{\pi^2}\left(k_{\max}-2\tan^{-1}\frac{k_{\max}}{2}\right) & \text{for } d=3 \end{cases}$$



Time dependence of quench onset in 1d

$$g^{(2)}(0) \approx 1-\sqrt{\gamma}\times\begin{cases} \frac{3}{2}\sqrt{t}-\frac{1}{2}t^{3/2} & \text{for } t\lesssim\frac{1}{4} \\ \frac{1}{\pi}\tan^{-1}\frac{k_{\max}}{2}-\frac{c_2 e^{-c_3 t}}{2t^{c_4}} & \text{for } t\gtrsim\frac{1}{4} \end{cases}$$

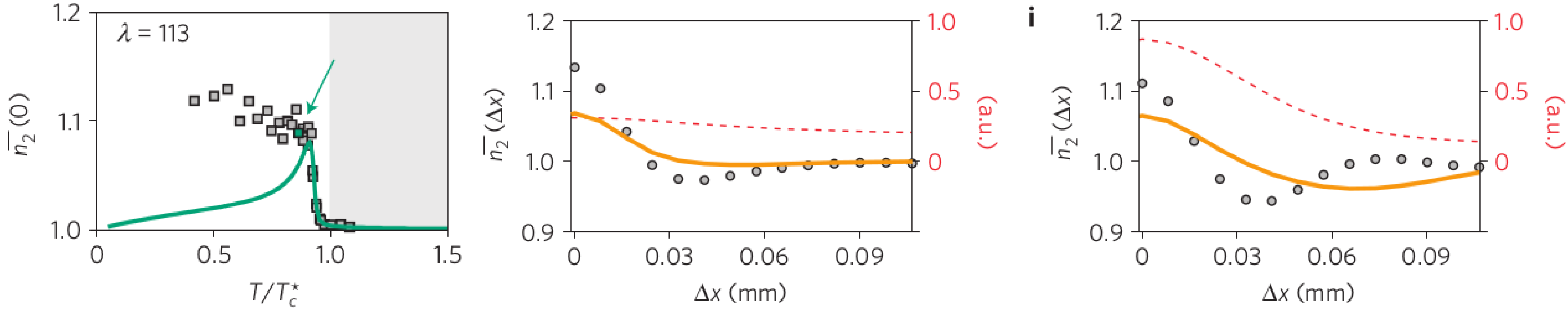
$$c_2 = 0.35(1),\ c_3 = 2.05(1) \text{ and } c_4 = 0.33(2)$$

Observability of density correlations *in situ*

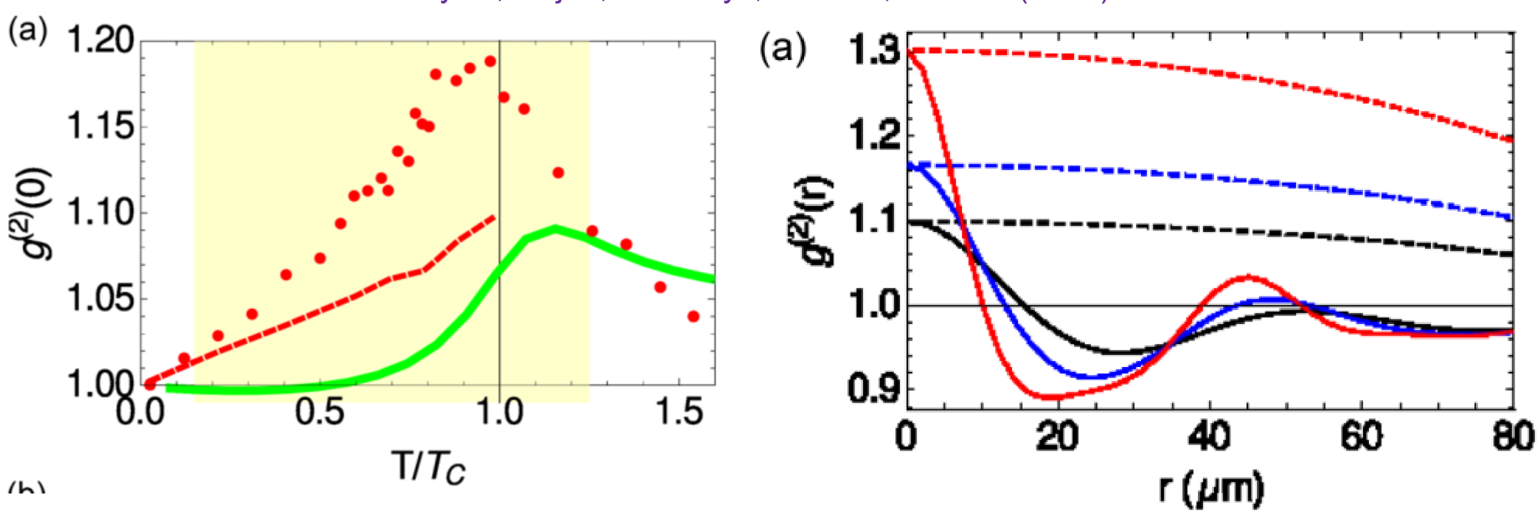
- Would be nicer than after expansion..
- e.g. would not have so many interpretation problems…

example: density correlations after expansion from an elongated gas

Perrin, Bucker, Manz, Betz, Coller, Plisson, Schumm, Schmiedmayer, Nat. Phys. **8**, 195 (2012)

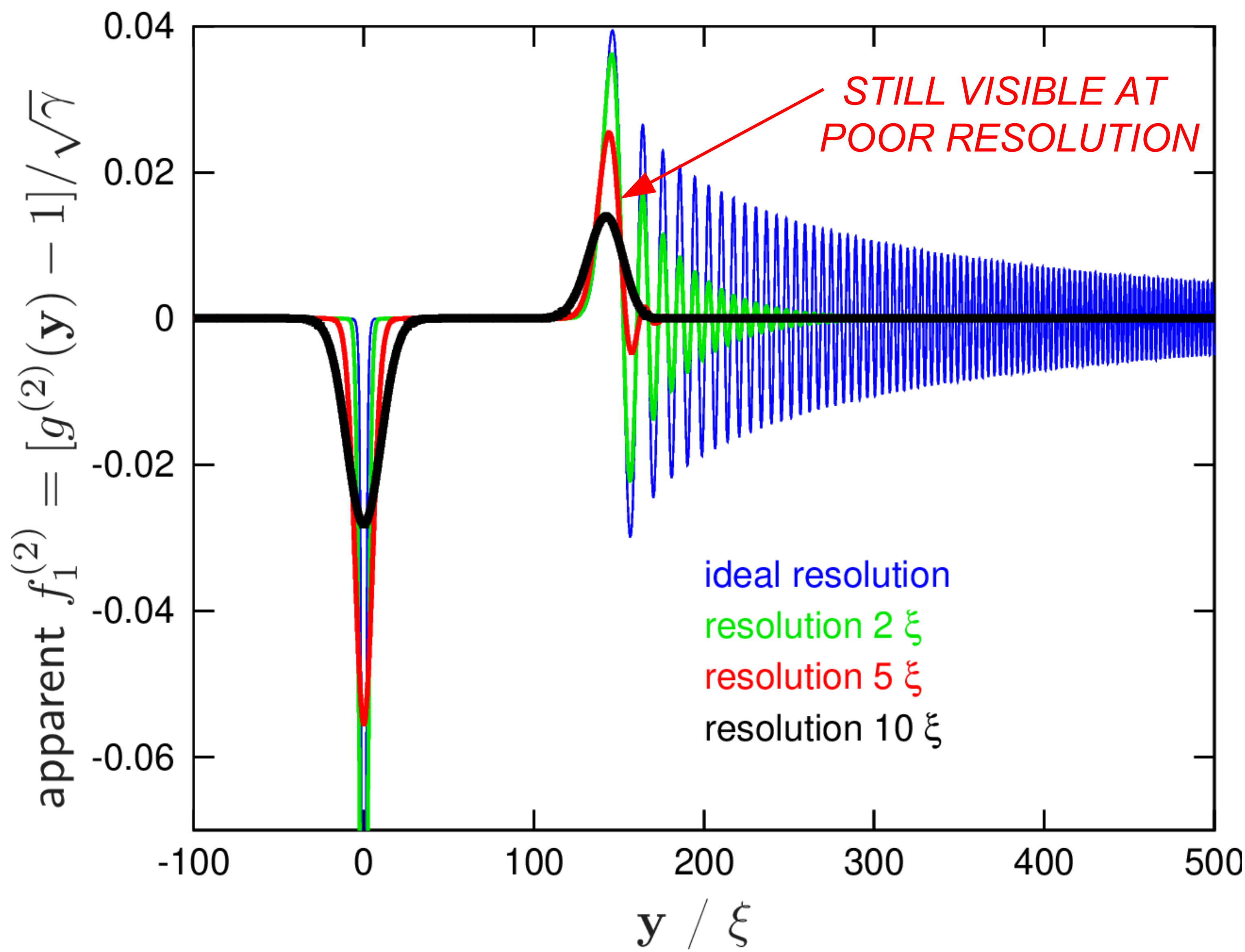


Gawryluk, Gajda, Brewczyk, PRA **92**, 043607 (2015)



Resolution and detectability of wavefront

The issue: most structures are of healing length ξ size, but experiments do not resolve this



Some estimates

- Example experiment (Palaiseau) Jacqmin, Armijo, Berrada, Kheruntsyan, Bouchoule, PRL **106**, 230405 (2011) Schemmer, Bouchoule, Doyon, Dubail, arXiv:1810.07170 (2018)
  - 87Rb
  - 1200 atoms
  - 4.5um resolution
  - Trap 4 x 3900 x 3900 Hz
- Best visibility:
  - Correlation wave travels a distance of RTF, In ~ peak density
    - t=28ms (80 healing time units)
    - Peak width 3.9um
    - Height 0.006
    - Need to average ~ 2500 realizations
  - More density is not necessarily better
- Other cases, e.g. Langen, Gaiger, Kuhnert, Rauer, Schmiedmayer, Nature Phys. **9**, 640 (2013) are roughly similar, needing 1400 – 4000 realizations

$$w_{\mathrm{rms}} = 1.802 \times t^{1/3}$$

$$h_{\mathrm{peak}} = 0.1474 \times \sqrt{\gamma}/t^{1/3}$$

More realistic quasicondensate

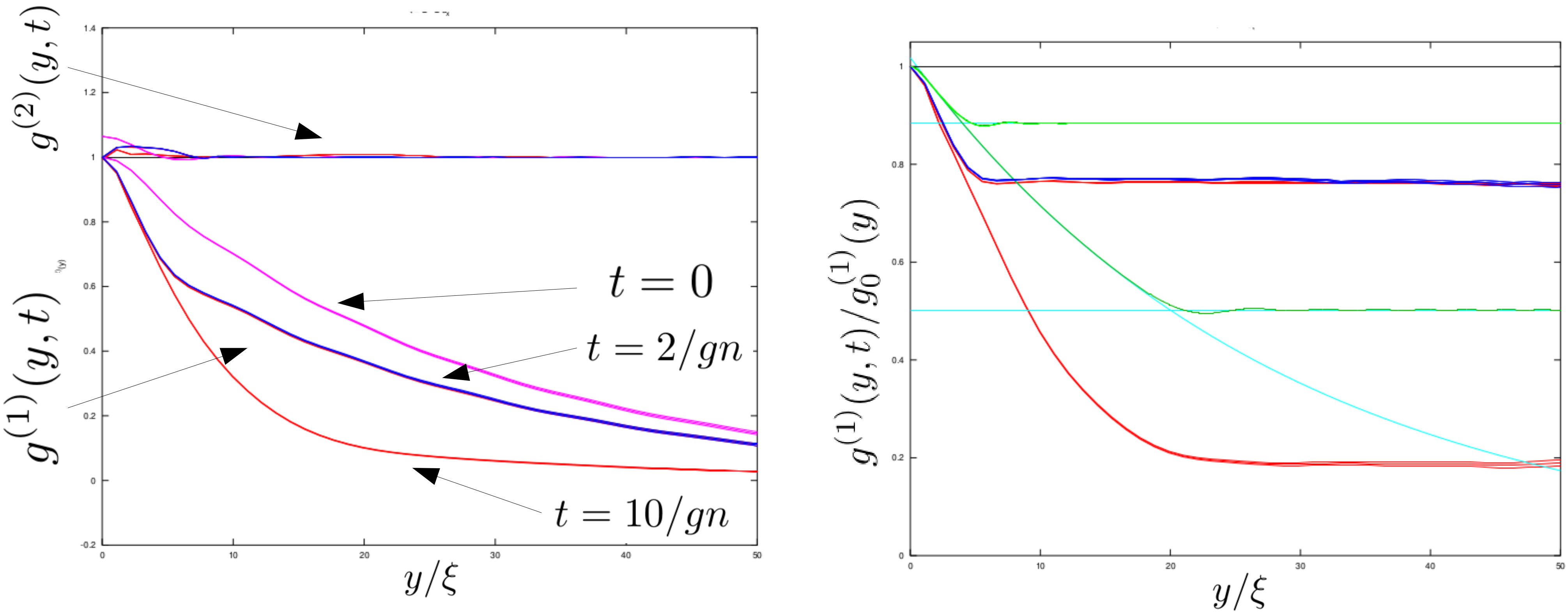
- Real quasicondensates do not form in an ideal gas, but with g > 0
- Real quasicondensates have thermal-dominated phase fluctuations
- Testing of this is in progress…
  - Start with classical field ensemble with g > 0 T > 0
  - Evolve using positive-P and truncated Wigner to
    - Get true quench dynamics including thermal and quantum fluct
    - Check their accuracy against each other
  - Compare to predictions of this work
- Note that as distance grows one generically expects:

$$g^{(1)}(y) = \exp\left[g^{(1)}_{\text{Bogoliubov}}(y) - 1\right]$$

Mora, Castin, PRA **67**, 053615 (2003)

Naïve comparison

- Quench from $g_0 \rightarrow 4g_0$



Positive-P simulation

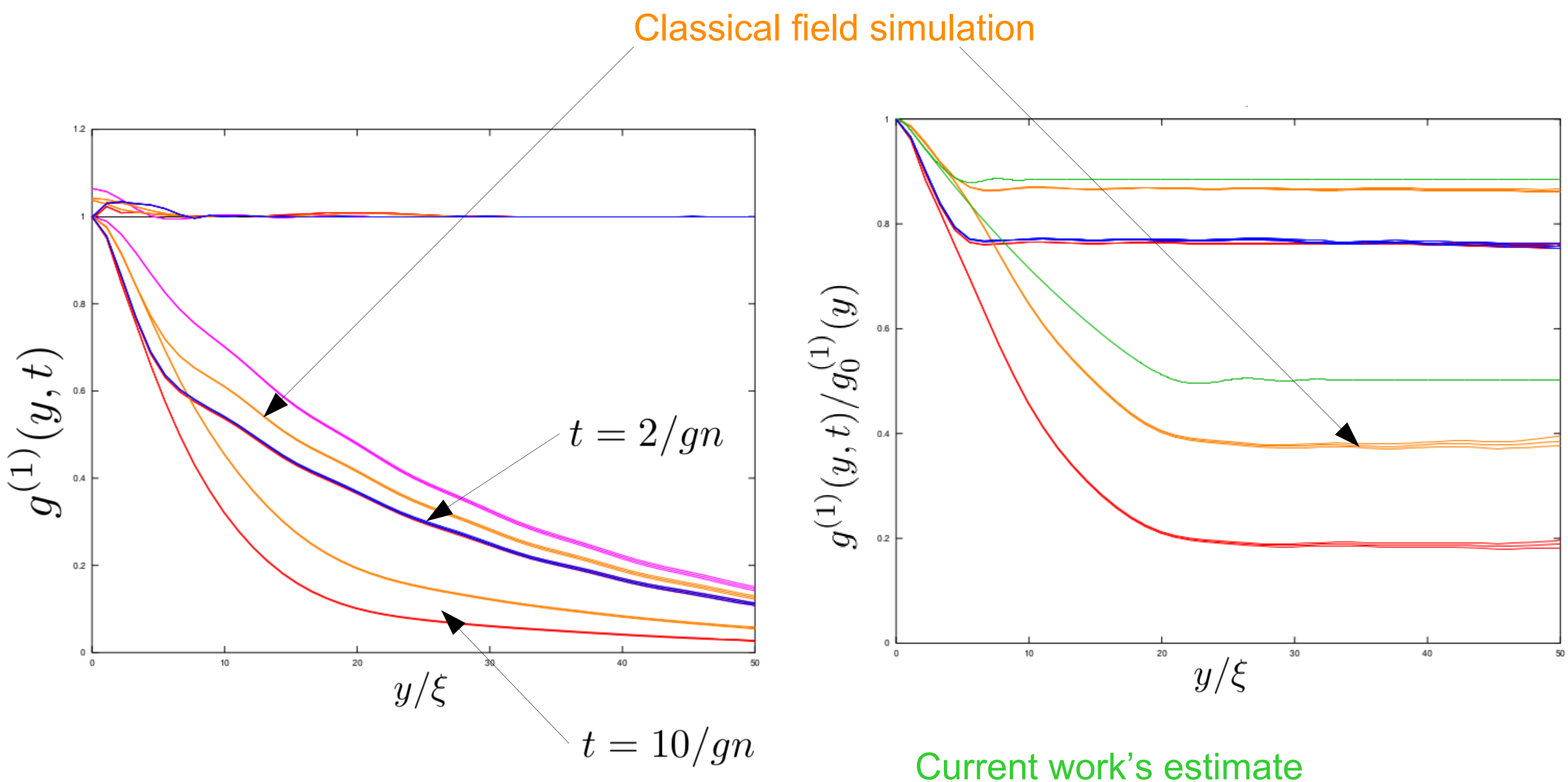
truncated Wigner simulation

initial quasicondensate state

Current work's estimate

simplified expressions

Classical correlations induced by quench?



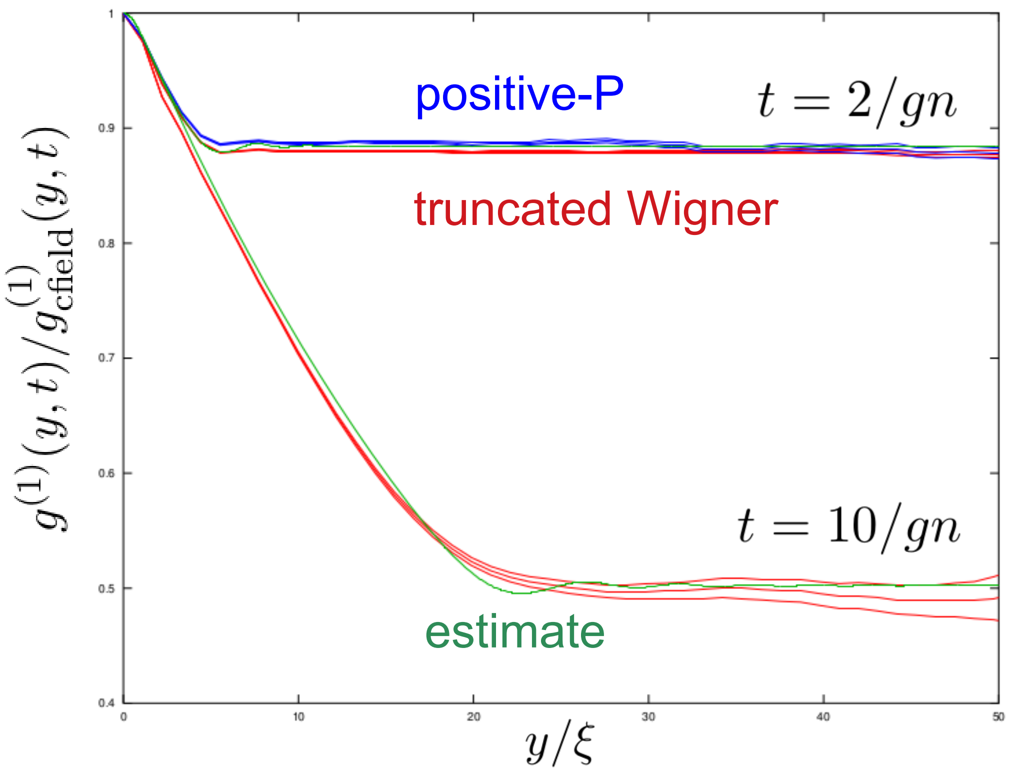
Positive-P simulation

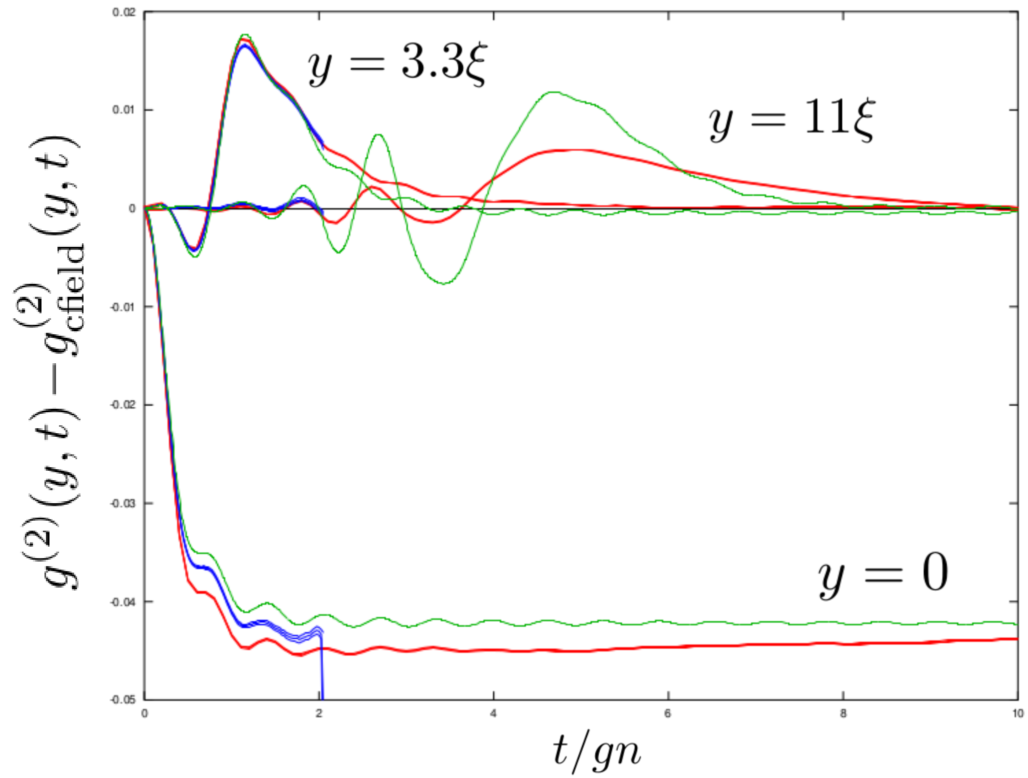
truncated Wigner simulation

initial quasicondensate state

Proper relationship

$$g^{(n)}(y,t) = \exp\left[g^{(n)}_{\text{cfield}}(y,t) + \delta g^{(n)}_{\text{thiswork}}(y,t)\right]$$





Conclusions / Outlook

- MAIN RESULT:
  Simple expressions for phase and density correlations after a quench
  (those due to quantum fluctuations)

Phys. Rev. A **99**, … (2019)
arXiv:1310.1301v2

- NEXT:
  Interplay with existing and/or classical correlations could be quantified
  - existing thermal correlations
  - sound wave excitations
  - correlations due to nonzero starting g

- Prospects for observability of density correlations *in situ*
  probably only in 1d for a really dilute gas

- Related systems that could be treated
  spinor, g < 0, ...?

Additional thanks :

Pasquale Calabrese   Tomasz Świsłocki   Jean-Sebastien Caux