Survival of the quantum depletion of a condensate after release from its trap

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Conclusions: a) our experiment saw survival of the expanded depletion apparently without need for impurities
b) the apparent contact (tail strength) in the experiment was again enhanced compared to in situ values
c) survival and contact strengthening can also be seen in simulations, but less strongly
d) survival relies on an asymmetric trap and fast, adiabatic release, mostly from the edges of the BEC
e) tail strength appears to increase due to acceleration by the mean field during escape from the BEC

Quantum depletion and Tan's contact

Uncondensed atoms in a BEC

Consider a uniform piece of the gas

$$\hat{b}^\dagger_{\mathbf{k}} = u_k \hat{a}^\dagger_{\mathbf{k}} + v_k \hat{a}_{-\mathbf{k}}$$

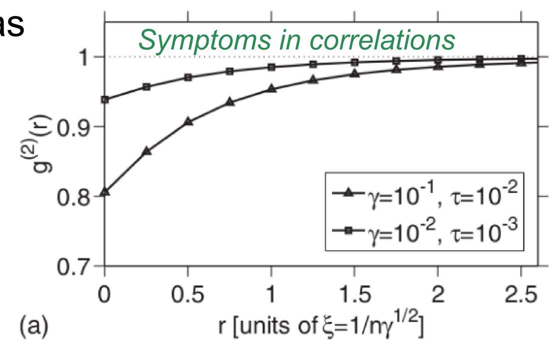
$$u_k = \cosh\theta_k, \qquad v_k = \sinh\theta_k$$

$$\theta_k = \frac{1}{2}\log\frac{\hbar^2k^2/2m}{\varepsilon(k)} < 0$$

$$\varepsilon(k) = \sqrt{\left(\frac{\hbar^2k^2}{2m}\right)^2 + gn\frac{\hbar^2k^2}{m}}$$

$$\rho(\mathbf{k}) = \langle \hat{a}^\dagger_{\mathbf{k}}\hat{a}_{\mathbf{k}}\rangle = (u_k^2+v_k^2)\,\langle \hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}}\rangle + v_k^2$$ *depletion*

$$\langle \hat{b}^\dagger_{\mathbf{k}}\hat{b}_{\mathbf{k}}\rangle = (\exp[\varepsilon(k)/k_BT]-1)^{-1}$$



$$\lim_{k\to\infty}\rho(\mathbf{k}) \propto k^{-4}.$$

Tan's contact

$$C = \lim_{k\to\infty} k^4\rho(k)$$

In the non-interacting ($a \to 0$) limit, $u_k = 1$ and $v_k = 0$,

in situ tails: $$\lim_{k\to\infty} n(k) = \frac{\mathscr{C}}{k^4} = \frac{64\pi^2a^2}{7}\frac{N_0n_0}{k^4}$$

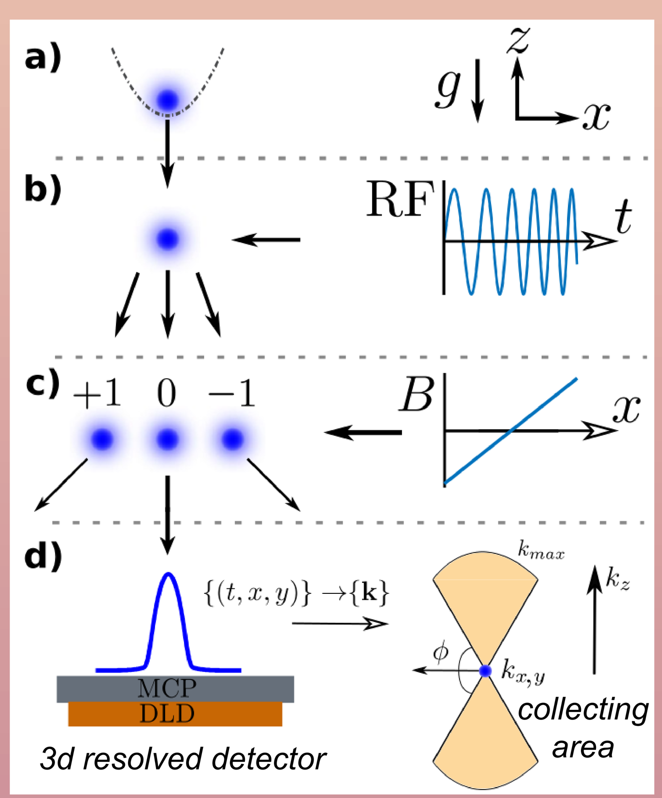
The mystery

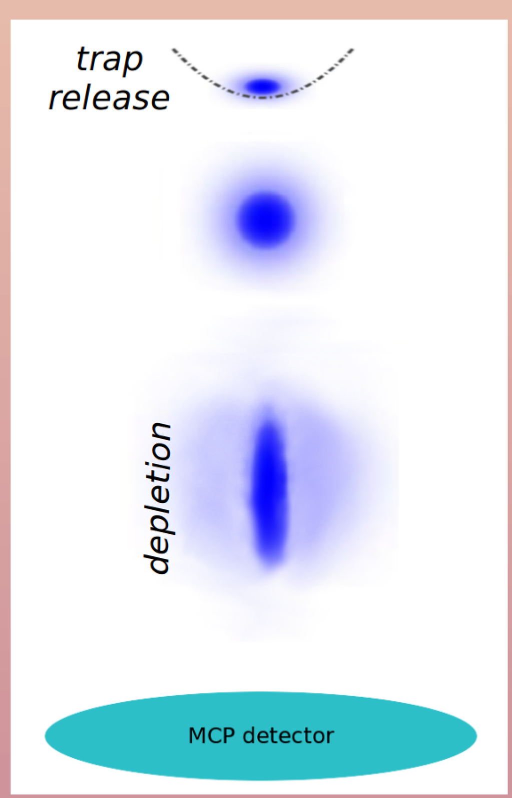
- Palaiseau experiments saw stronger tails than expected *in situ*. *(David Clement's group, PRL 2016)*
- Theory suggests no survival possible
  (Qu, Pitaevskii, Stringari, PRA 94, 063635 (2016))
- Later Palaiseau experiments indicate survival depends on presence of impurities
  (Cayla *et al* arXiv:2204.10697, David Clement's group)

Our aims

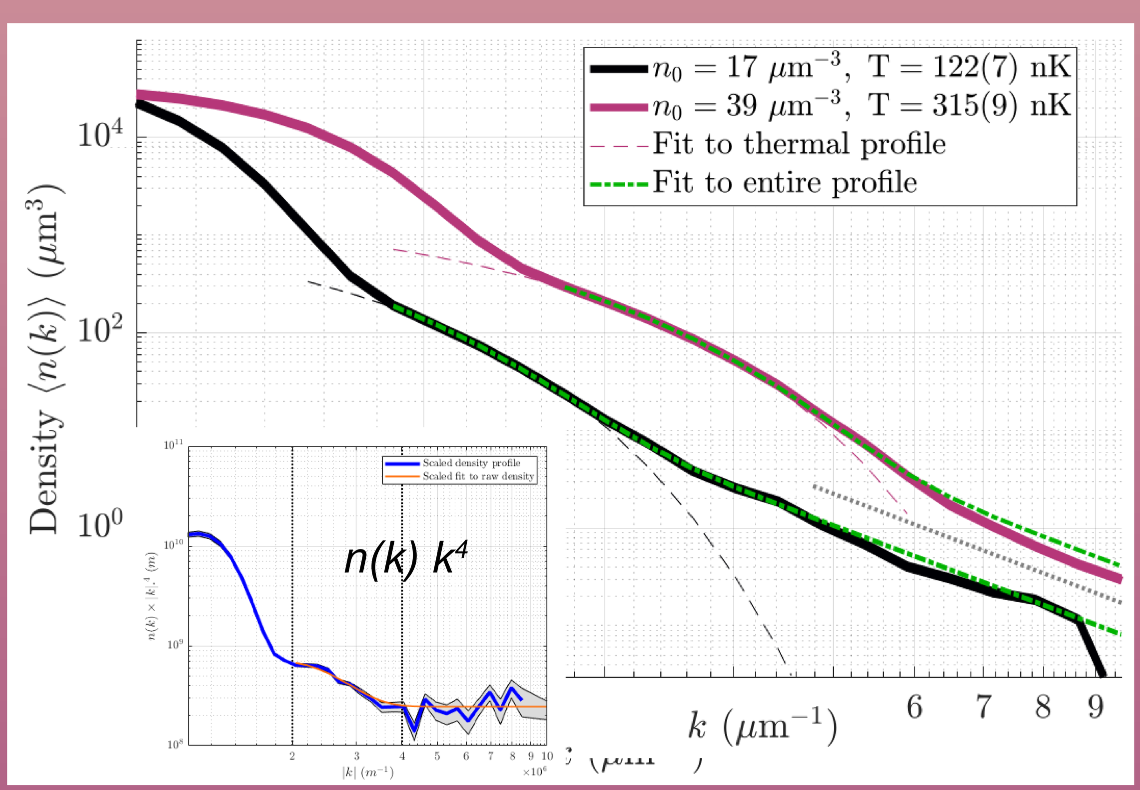
- Canberra expt. has a fully magnetic trap
  → only one component trapped
  → lack of impurities *in situ*. What about tails?
- Theory: can we simulate time evolution and see how survival is possible?

Experiment

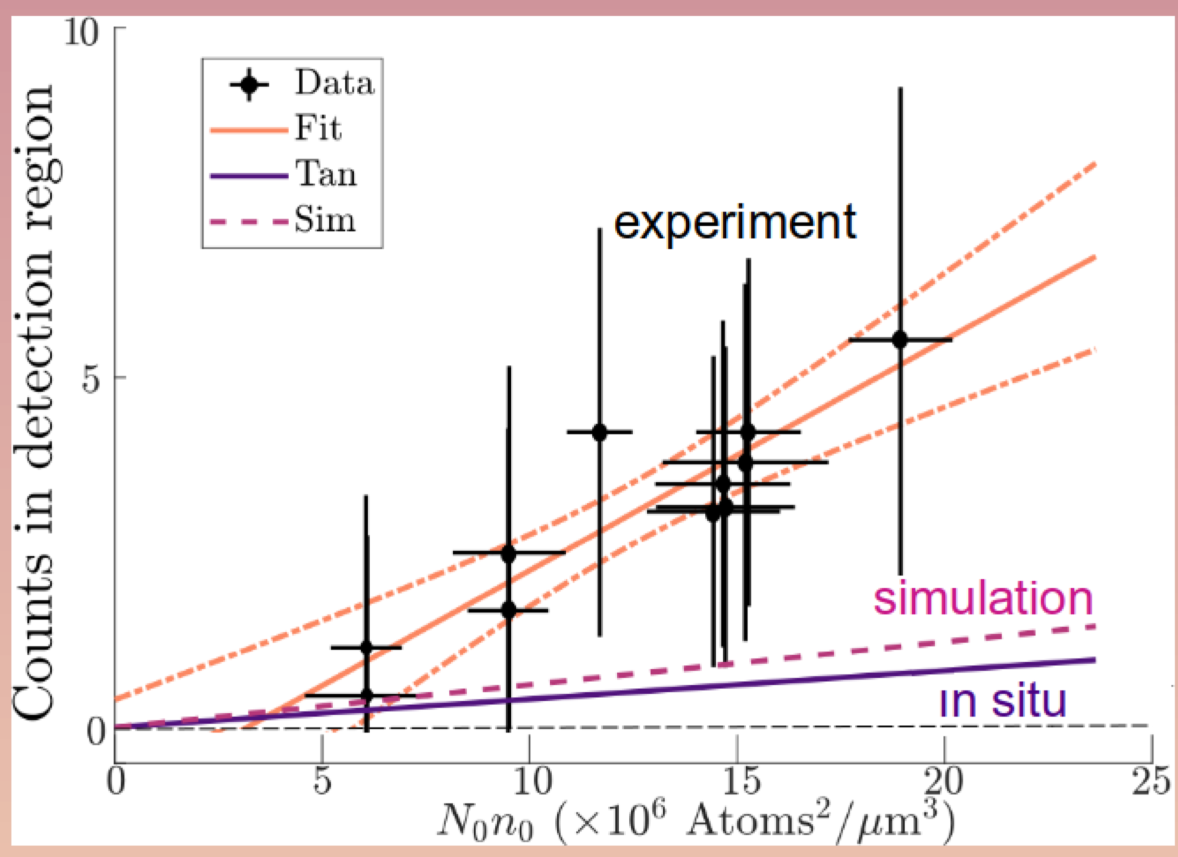




Velocity profile on MCP detector



Experimental tail strength



Simulation method

$$\widehat{\Psi}(\mathbf{x},t) = \phi(\mathbf{x},t) + \widehat{\Psi}_B(\mathbf{x},t)$$ *Bogoliubov*

PD, Chwedeńczuk, Ziń, Trippenbach, PRA **83**, 063625 (2011)

plain "STAB" was useful for BEC collisions
when Bogoliubov modes were separated from the condensate in k-space
Now this is no longer true (lots of overlap)
However, need to also impose orthogonality this time

$$\int d^3\mathbf{x}\, \widehat{\Psi}^\dagger_B(\mathbf{x},t)\phi(\mathbf{x},t) = 0$$

Stochastic
Time **A**daptive
Boboliubov

Proper STAB equations that preserve orthogonality

$$i\hbar\frac{d\phi}{dt} = \left[-\frac{\hbar^2}{2m}\nabla^2 + g|\phi|^2 + V(\mathbf{x},t)\right]\phi$$ *GPE mean field* $\mathscr{H}(g,\phi)\psi_B$

$$i\hbar\frac{d\psi_B}{dt} = \mathscr{H}(g,\phi)\psi_B + \mathscr{P}_\perp\left\{g_B|\phi|^2\psi_B + g_B\phi^2\widetilde{\psi}^*_B + \sqrt{-ig_B}\,\phi\,\xi(\mathbf{x},t)\right\}$$

quantum noise

$$i\hbar\frac{d\widetilde{\psi}_B}{dt} = \mathscr{H}(g,\phi)\widetilde{\psi}_B + \mathscr{P}_\perp\left\{g_B|\phi|^2\widetilde{\psi}_B + g_B\phi^2\psi^*_B + \sqrt{-ig_B}\,\phi\,\widetilde{\xi}(\mathbf{x},t)\right\}$$

Thankfully, projection can be done very efficiently

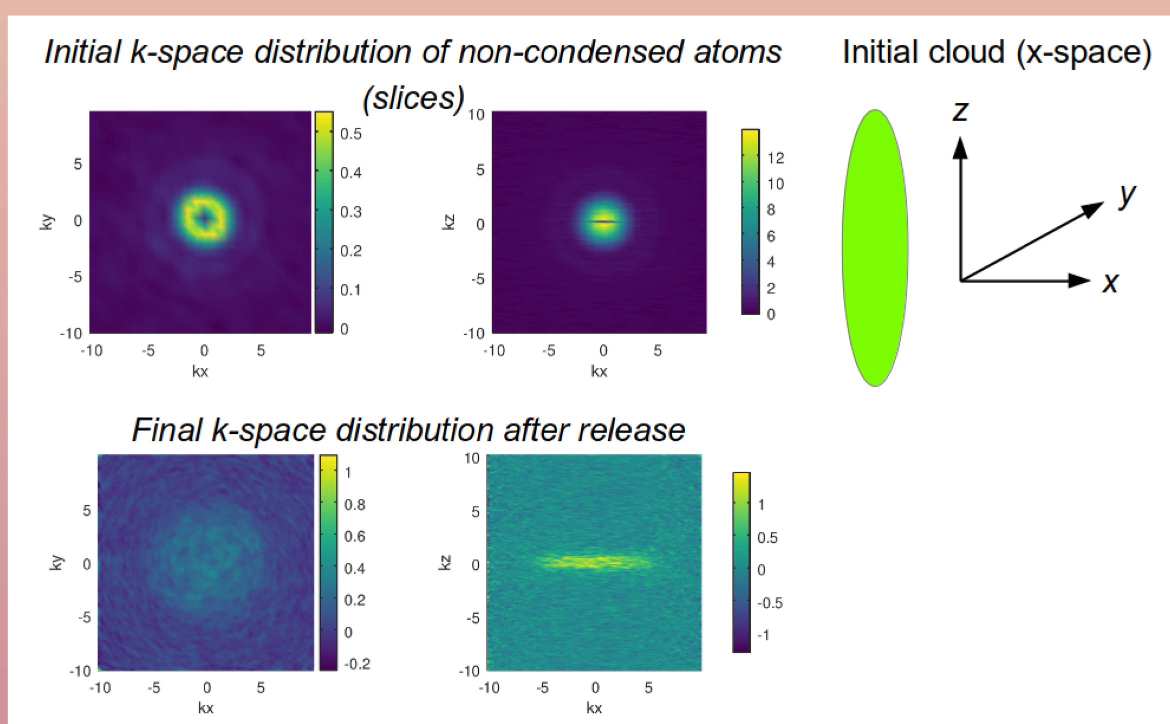
$$\mathscr{P}_\perp f(\mathbf{x}) = f(\mathbf{x}) - \frac{1}{N}\left[\int d^3\mathbf{x}'\,\phi(\mathbf{x}')^*f(\mathbf{x}')\right]\phi(\mathbf{x}).$$

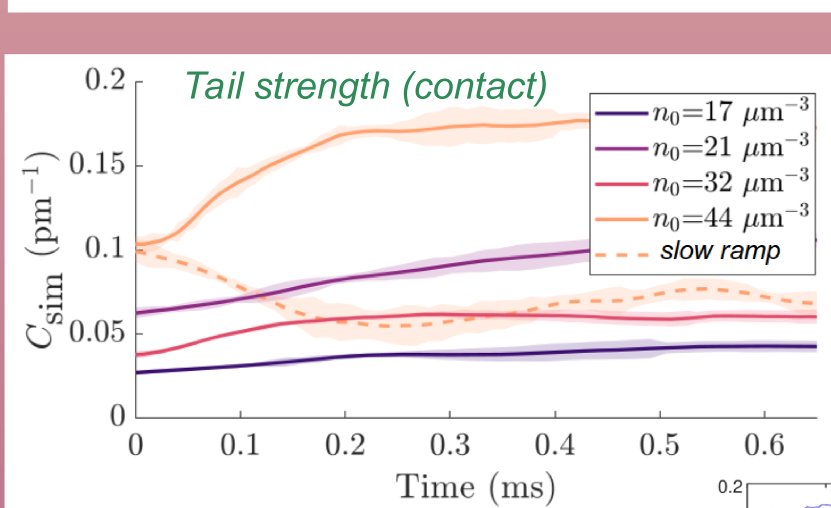
Gaussian real white noise $$\langle\xi(\mathbf{x},t)\xi(\mathbf{y},t')\rangle = \delta^3(\mathbf{x}-\mathbf{y})\delta(t-t')$$

Depletion density

$$n_B(\mathbf{k}) = \mathrm{Re}\,\langle\widetilde{\psi}^*_B(\mathbf{k},t)\psi_B(\mathbf{k},t)\rangle_{\text{stoch}}$$

Simulation: distribution, time evolution

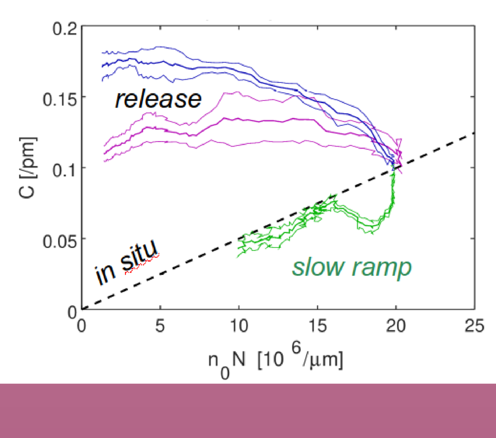




central densities n_0

Slow ramp Reduces survival

Trajectory in "C-N" space



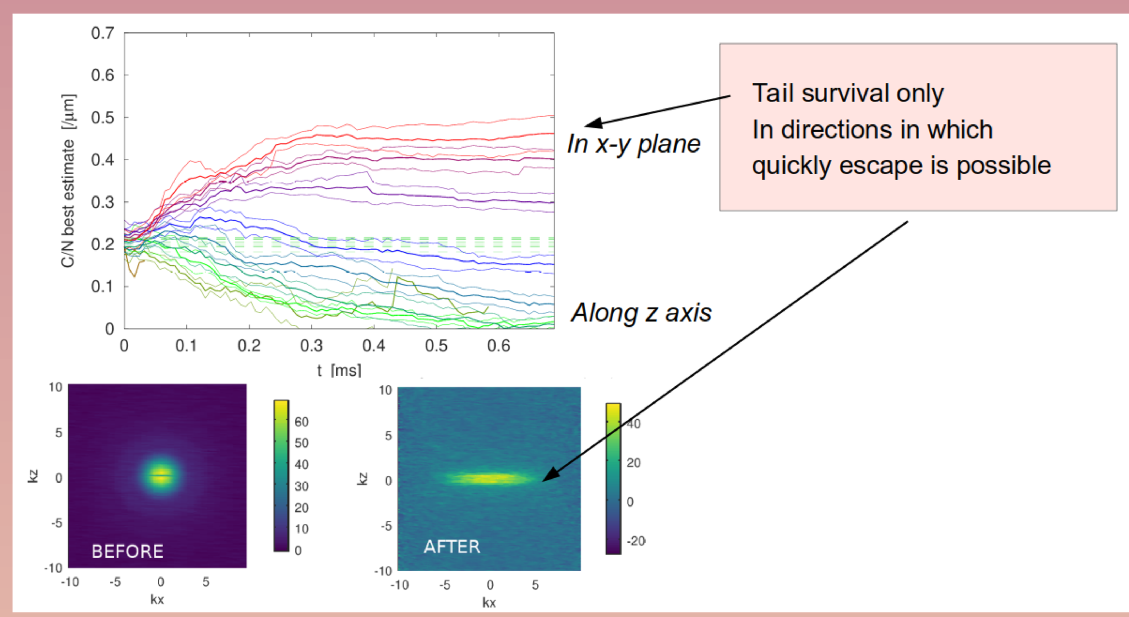
Trap release like in experiment

$$V(\mathbf{x}) = \frac{m}{2}\left(\omega_x^2x^2 + \omega_y^2y^2 + \omega_z^2z^2\right)e^{-t/\tau_{\text{release}}}$$

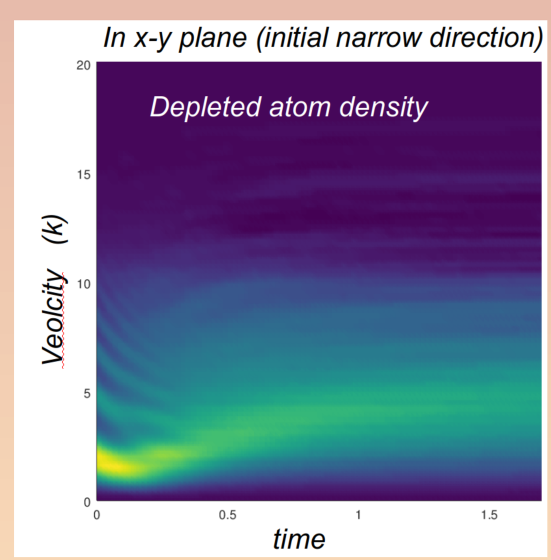
Slow ramp to compare to Trento theory

$$V(\mathbf{x}) = \frac{m}{2}\left[\left(\omega_x^2x^2 + \omega_y^2y^2\right)\left(1-\frac{t}{2t_{\text{ramp}}}\right)^2 + \omega_z^2z^2\right]$$

Anisotropy - simulation



Apparent growth of tails



Atoms accelerated by BEC background

In a power law distribution shift of density to higher velocity is equivalent to an increase in prefactor

Total depletion actually reduces

Toy model – survival process

Single pair of modes in local density *n(t)*

$$\frac{d}{dt}\begin{bmatrix}\widehat{a}_k(t)\\ \widehat{a}^\dagger_{-k}(t)\end{bmatrix} = -\frac{i}{\hbar}\begin{bmatrix}\hbar^2k^2/2m + 2gn(t) - \mu(t) & gn(t)\\ -gn(t) & -\hbar^2k^2/2m - 2gn(t) + \mu(t)\end{bmatrix}\begin{bmatrix}\widehat{a}_k(t)\\ \widehat{a}^\dagger_{-k}(t)\end{bmatrix}$$

$$R_0 = \frac{\sqrt{y^2+z^2}}{R_\perp} = \frac{\omega_\perp\sqrt{y^2+z^2}}{\sqrt{2gn_0/m}}$$ Initial radial position

Estimated radial position at time t (flight + riding condensate expansion)

$$r(t) = R_0R_\perp\sqrt{1+\omega_\perp^2t^2} + \frac{\hbar kt}{m}$$

Radial position relative to condensate

$$r_{c0}(t) = \frac{r(t)}{\sqrt{1+\omega_\perp^2t^2}}$$

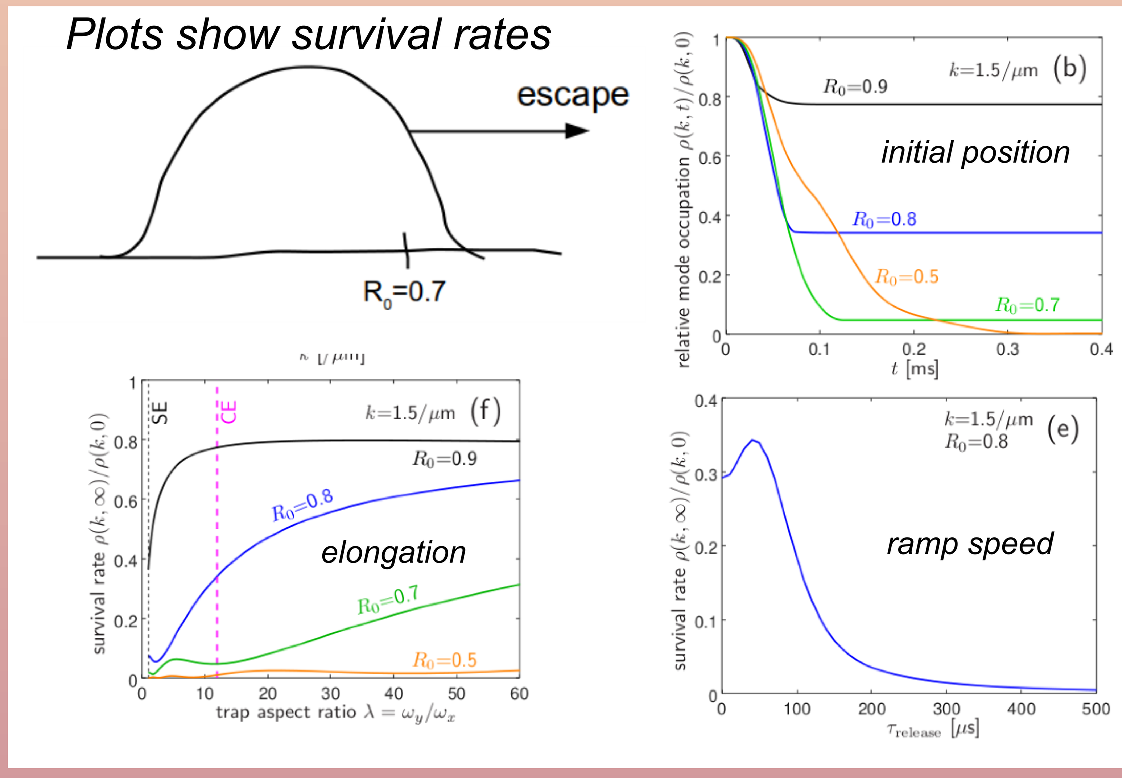
Working backwards to initial radial position estimate

$$n_c(r,t) = \frac{n_c(r,0)}{(1+\omega_\perp^2t^2)\sqrt{1+\omega_x^2t^2}}, \quad \text{where} \quad n_c(r,0) = \begin{cases} n_0\left[1-(r/R_\perp)^2\right] & \text{if } r < R_\perp \\ 0 & \text{if } r \geq R_\perp\end{cases}$$

Ramped BEC density

$$n(t) = n(0)e^{-3t/\tau_{\text{release}}} + (1-e^{-3t/\tau_{\text{release}}})\,n_c(r_{c0}(t),t)$$

Plots show survival rates



Final comparison of tail strengths

	tail strength vs value *in situ*	
Chang, Bouton, Cayla, Qu, Aspect, Westbrook, Clement, PRL 117, 235303 (2016)	x 6 ± 1	Experiment (Palaiseau)
Qu, Pitaevskii, Stringari PRA 94, 063635 (2016)	0	Theory (Trento)
Ross, Deuar, Shin, Thomas, Henson, Hodgman, Truscott, Sci Rep 12, 13178 (2022)	x 2 ± 0.2	Simulations (Warsaw)
Ross, Deuar, Shin, Thomas, Henson, Hodgman, Truscott, Sci Rep 12, 13178 (2022)	x 5 ± 3	Experiment (Canberra)
Cayla, Massignan, Giamarchi, Aspect, Westbrook, Clement to appear (2022)	x 0 – 6 depending on impurities	Experiment 2 (Palaiseau)

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