STATIONARY SOLUTIONS OF STOCHASTIC EQUATIONS WITH QUANTUM FLUCTUATIONS FOR ULTRACOLD GASES

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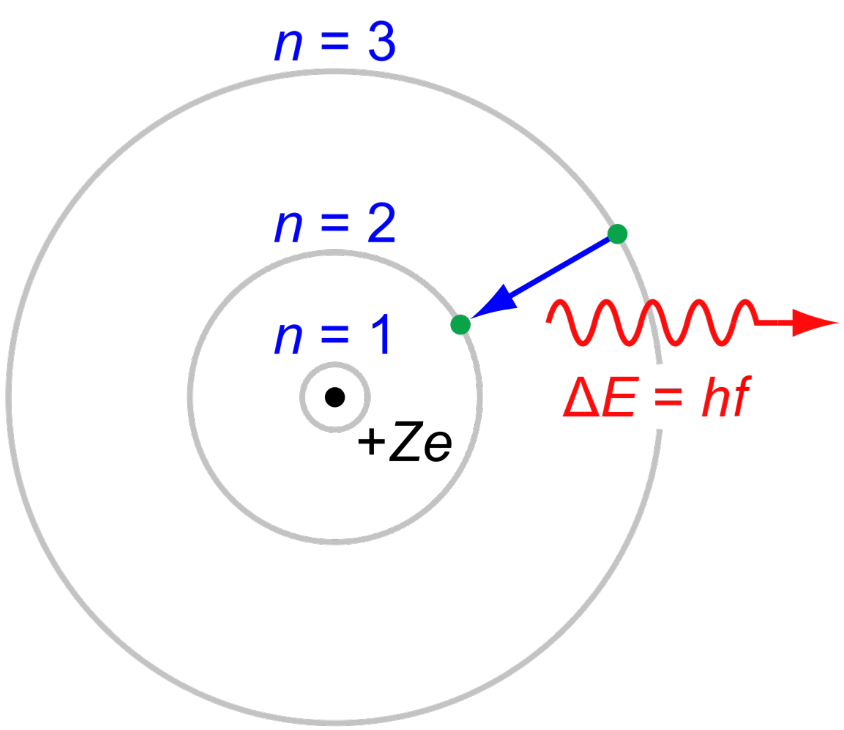
The quantum atom



Matter wavelength:

$$\lambda = \frac{h}{mv}$$

In the quantum atom, the electrons follow quantum rules.

In quantum gas, the atoms follow quantum rules.

This requires that $\lambda \sim d$

d - the typical distance between identical particles.

A ultracold gas of atoms

Room temperature ~ 300K, $\lambda \approx 0,3a_0$

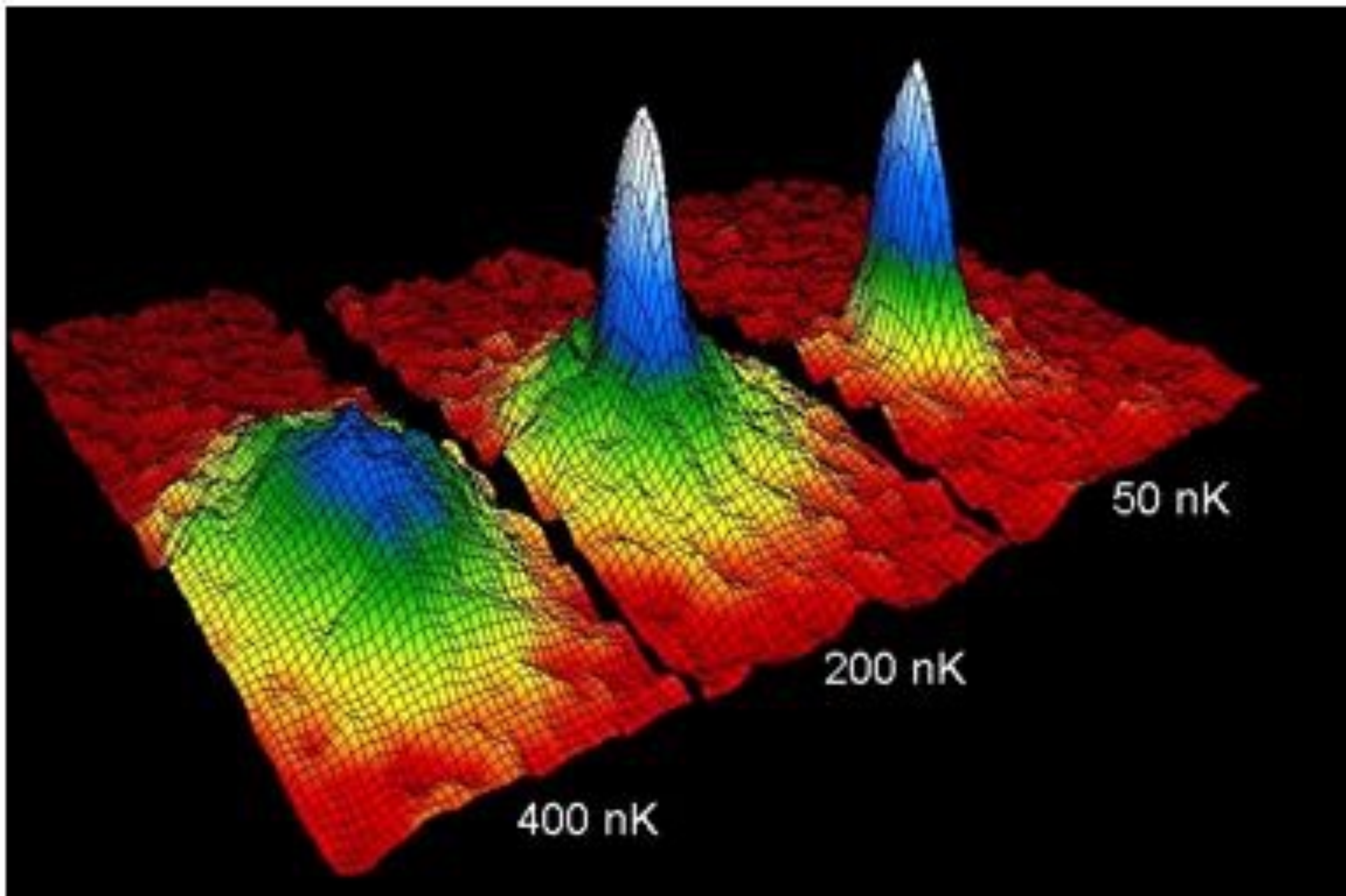
At 100nK, atoms move very slowly,

$$\lambda \approx 20,000a_0$$

So that's the condition we need for a quantum gas, where the atoms themselves behave quantum mechanically.

$$N = 10^6$$

$$n = 10^{13}cm^{-3}$$

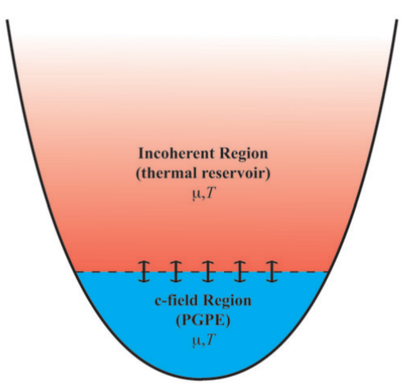

50 nK
200 nK
400 nK

Stochastic Gros- Pitaevskii Equation (SGPE).

Ultracold gases at non-zero temperatures are fairly well described by the Stochastic Gross Pitaevskii Equation:

$$\hat{H} = \int dx \left\{ \hat{\Psi}^\dagger(x) \left[V(x) - \frac{\hbar^2}{2m}\nabla^2 \right] \hat{\Psi}(x) + \frac{g}{2} \hat{\Psi}^\dagger(x)^2 \hat{\Psi}(x)^2 \right\}$$

$$\hbar \frac{\partial \psi(\mathbf{x})}{\partial t} = -i(1 - i\gamma)\mathcal{L}_{GP}\psi(\mathbf{x}) + \sqrt{2\gamma\hbar k_B T}\, \eta(\mathbf{x}, t)$$



$$\mathcal{L}_{GP} = -\frac{\hbar^2}{2m}\nabla^2 + V(\mathbf{x}) - \mu + g\,|\psi(\mathbf{x})|^2$$

- does not include quantum fluctuations which become important at lower temperatures :(

WSGPE evolution equation

$$\frac{\partial \varphi(x,t)}{\partial t} = -i(1-i\gamma)\left(-\frac{1}{2}\frac{\partial^2}{\partial x^2} + V(x) - \mu + g\left(|\varphi|^2 - \frac{1}{\Delta x}\right)\right)\varphi + \sqrt{\gamma(2T + E - \mu + \frac{g}{\Delta x}\left(|\varphi|^2 - \frac{1}{\Delta x}\right))}\ \eta(t).$$