Superfluid dipolar Fermi gases and their excitations

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Overview

1. Motivation

   Comparison with standard BCS gas,
   clean realisation of solid-state phases

2. Experimental prospects

   possible realisations, critical temperature T_c

3. Model for the uniform 3D gas

   $\widehat{H}$, assumptions

4. Quasiparticle (pair) excitations

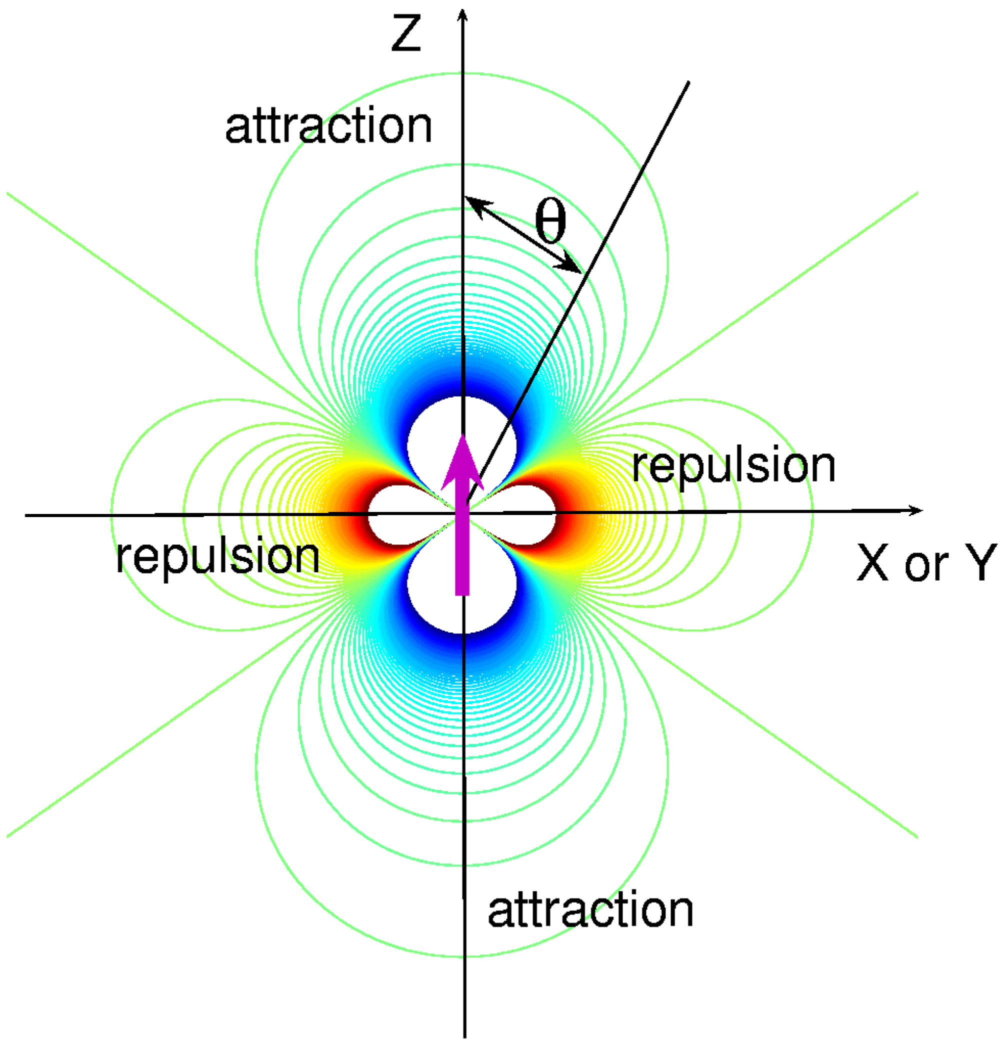
   Anisotropic energy gap for pair breaking, gap nodes

5. Collective excitations & superfluid component

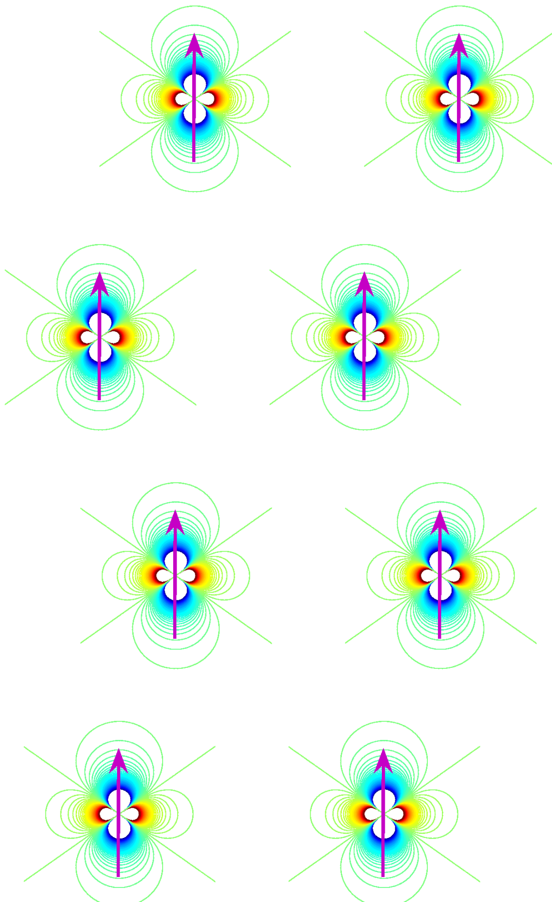
   Hydrodynamics, anisotropic damping,
   unusual superfluid current response

Interparticle Potential

$$V_D(R,\theta) = \frac{d^2}{R^3}\left(1 - 3\cos^2\theta\right)$$



Uniform gas



- uniform 3D gas
- static external field (E or B) $\Longrightarrow$ full polarisation
- single-species (spin polarised)
- dilute $\Longrightarrow$ Energy dominated by Fermi sea to leading order
- short-range interaction (e.g. p-wave) negligible (Fermi exclusion)

(1) Motivation

BCS superfluidity

dipole–dipole potential

- *LONG* range interaction
- *ANIsotropic*
- always partly attractive
  BCS pairing if *polarised*
- Needs 1 spin component
- Energy gap has nodes
- Stability conditions nontrivial

(Góral, Brewczyk, Rzążewski)

standard s-wave $\uparrow\downarrow$ potential

- *SHORT* range interaction
- *Isotropic*
- arttractive or repulsive
  BCS pairing only if $a_s < 0$
- Needs 2 spin components
- Energy gap always $\neq 0$

Solid state analogue

- The node structure of the direction-dependent order parameter is similar to that of solid state and He phases, e.g.:
  - Polar phase of $^3\mathrm{He}$.
    (Never experimentally realized)
  - Heavy-fermion superconductors like UPt_3.
    (Difficult to get pure system, many potential phases)

- Qualitatively similar behaviour expected.

- Dipole gas is a much “cleaner” system.
  - $\widehat{H}$ well known
  - spin degrees of freedom can be removed.

- It is potentially better controllable.

(2) Prospects for superfluidity

Possible Physical Realisations

- Heteronuclear polar molecules
  - Several groups actively aiming to cool to ultracold T. e.g. Bigelow (Rochester), Grimm (Innsbruck), Doyle (Harvard), . . .
  - Method 1: Photoassociaton from cold atomic gases
  - Method 2: Buffer gas cooling

- Magnetic atomic dipoles
  - e.g. ^{53}Cr (6 parallel spins in valence electron shell)
  - ultracold gases achieved, but dipole moment too small to be useful for BCS.

- Induce electric dipoles in atoms with strong E fields

Critical Temperature for BCS

standard $\uparrow\downarrow$ gas:

$$T_c = 0.28 E_F \exp\left(-\frac{\pi}{2|a_s|k_F}\right)$$

Dipole gas:

M. Baranov *et al*, PRA **66**, 013606 (2002)

$$T_c = 1.44 E_F \exp\left(-\frac{\pi}{2|a_D|k_F}\right)$$

$\Longrightarrow$ *Effective* scattering length a_D:

$$a_D = -2m\left(\frac{d}{\pi\hbar}\right)^2$$

T_c rises strongly with $a_D \propto md^2$

Candidates for BCS pairing

(large $|a_D|$ desirable)

Short-range interactions

- Two spin components. For example ^{6}Li : $a_s = -114$ nm

Dipoles

- Heteronuclear polar molecules

  ^{15}ND^3 : $a_D = -145$ nm

  HCN : $a_D = -740$ nm

  NaCs : $a_D \gtrsim -500$ nm

- Magnetic atomic dipoles

  ^{52}Cr : $a_D = -0.5$ nm (far too weak)

- Atoms with induced electric dipole

  $a_D \approx -1$ to -10 nm (need $\approx 10^6$ V/cm)

(3) Model

Hamiltonian

$$\widehat{H} = \text{K.E.} + \frac{1}{2}\int d^3x\, d^3y \left\{ \widehat{\Psi}_x^\dagger \widehat{\Psi}_x\, V_D(x-y)\, \widehat{\Psi}_y^\dagger \widehat{\Psi}_y \right\}$$

- $\widehat{\Psi}_x$ is the anihilating Fermi field operator at point x.

BCS Mean field theory: Postulate the quadratic effective Hamiltonian:

$$\widehat{H}_{\text{eff}} = \frac{1}{2}\int d^3x\, d^3y \Big\{ \frac{\hbar^2}{m} \widehat{\Psi}_x^\dagger\, \nabla^2 \widehat{\Psi}_x\, \delta(x-y) \qquad \textit{Kinetic}$$

$$\Delta^*(x-y)\, \widehat{\Psi}_x \widehat{\Psi}_y - \Delta(x-y)\, \widehat{\Psi}_x^\dagger \widehat{\Psi}_y^\dagger \qquad \textit{BCS}$$

$$+W(x-y) \widehat{\Psi}_x^\dagger \widehat{\Psi}_y \Big\} \qquad \textit{Hartree}$$

- With some "appropriate" $\Delta(x-y)$ and $W(x-y)$

Gap equation

Choose $\Delta(x-y)$ and $W(x-y)$ to minimise the full Free energy

$$F = \langle \widehat{H} \rangle_{\text{eff}} - \mu N - TS$$

when calculated with eigenstates of $\widehat{H}_{\text{eff}}$.

Obtain:

$$\Delta(x-y) = V_D(x-y) \left\langle \widehat{\Psi}_x \widehat{\Psi}_y \right\rangle_{\text{eff}}$$

$$W(x-y) = -V_D(x-y) \left\langle \widehat{\Psi}_x^\dagger \widehat{\Psi}_y \right\rangle_{\text{eff}}$$

Δ, W and Ψ must be self-consistent.

Uniform gas

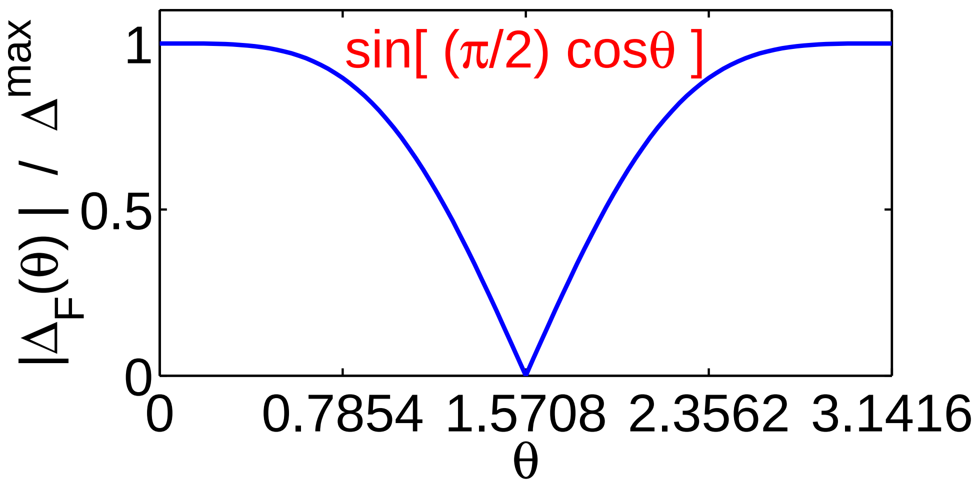
In k–space

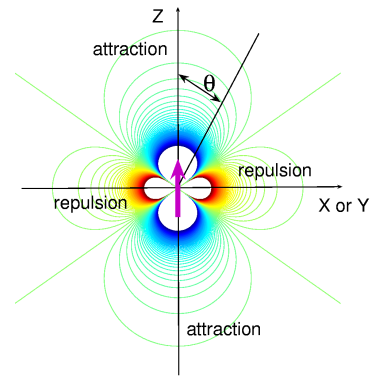
$$\widehat{H}_{\text{eff}} = \frac{1}{2}\int d^3k \left\{ \left(\frac{\hbar^2 k^2}{m} - 2\mu - W(k) \right) \widehat{\Psi}_k^\dagger \widehat{\Psi}_k + \Delta^*(k) \widehat{\Psi}_k \widehat{\Psi}_{-k} - \Delta(k) \widehat{\Psi}_k^\dagger \widehat{\Psi}^\dagger_{-k} \right\}$$

- $W(k)$ is a minor energy shift of Fermi surface $\Longrightarrow$ ignore it
- Order parameter $\Delta(k) \neq 0$ corresponds to BCS pairing of k and $-k$ atoms.
- Important difference to standard $\uparrow\downarrow$gas: $\Delta(k)$ anisotropic and has nodes

(4) Quasiparticle (pair) excitations

BCS gap $\Delta_F(\theta)$ on Fermi surface



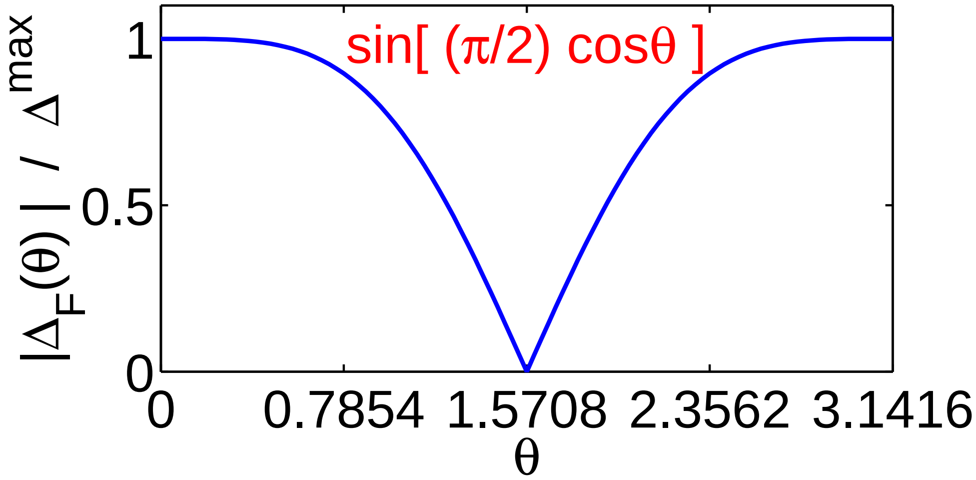


POLE in plane $\perp$ to polarisation
Breaking a pair costs $2 \times E$, where $E(k) = \sqrt{(\text{K.E.} - E_F)^2 + \Delta^2} \geq |\Delta|$.

- Dipoles: Easy to excite a pair in plane $\perp$ to polarisation because energy cost is small.

- $\uparrow\downarrow$gas: Appreciable energy cost of excitations always.

Consequences of pole in Δ



	↑↓ gas	dipoles
dispersion	isotropic	anisotropic
damping of sound at $T = 0$	0	nonzero
Specific heat at low T	$\sim \exp(-\Delta/T)$	$\sim T^2$
normal component at low T	$\sim \exp(-\Delta/T)$	polynomial in T

(5A) Collective excitations

(technical)

Collective excitations (Sound)

Phase perturbations of the ground state order parameter

$$\boxed{\Delta_0(x-y) \quad \longrightarrow \quad \Delta(x,y) = \Delta_0(x-y)\, e^{2i\phi(x,t)}}$$

Assumptions:

- Low energy $\quad (\, \hbar\omega \ll \Delta_0^{\mathrm{max}} \,)$
- Phase perturbations only (amplitude perturbations are higher energy)
- Low ω $\quad \Longrightarrow \quad$ long wavelength ($k \ll k_F$)

  $\Longrightarrow$ insensitive to small-scale of $|x-y|$ $\quad \Longrightarrow \quad$ $\phi \approx \phi(x$ only $)$
- Weak perturbation $\quad \Longrightarrow \quad$ lowest order in ϕ

Consistency equation in k-space

$$
-\frac{\phi_{\mathbf{k}}\Delta^0_{\mathbf{M}}\tau^0_{\mathbf{M}}}{2E^0_{\mathbf{M}}} = \frac{\phi_{\mathbf{k}}\Delta^0_{\mathbf{M}}}{4E^0_{\mathbf{m}}E^0_{\mathbf{n}}}\left\{\left(\frac{\tau^0_{\mathbf{n}}-\tau^0_{\mathbf{m}}}{2}\right)\left[\frac{(E^0_{\mathbf{n}}+\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}-\varepsilon_{\mathbf{m}})+\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega-E^0_{\mathbf{n}}+E^0_{\mathbf{m}}+i0}-\frac{(E^0_{\mathbf{n}}-\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}+\varepsilon_{\mathbf{m}})+\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega+E^0_{\mathbf{n}}-E^0_{\mathbf{m}}+i0}\right]\right.
$$
$$
\left.+\tau^0_{\mathbf{n}}\left[\frac{(E^0_{\mathbf{n}}+\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}+\varepsilon_{\mathbf{m}})-\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega-E^0_{\mathbf{n}}-E^0_{\mathbf{m}}+i0}\right]-\tau^0_{\mathbf{m}}\left[\frac{(E^0_{\mathbf{n}}-\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}-\varepsilon_{\mathbf{m}})-\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega+E^0_{\mathbf{n}}+E^0_{\mathbf{m}}+i0}\right]\right\}.
$$

where $\mathbf{n}=\mathbf{M}+\mathbf{k}/2$, $\mathbf{m}=-\mathbf{M}+\mathbf{k}/2$, $\varepsilon_{\mathbf{k}}=\hbar^2k^2/2m-E_F$, $E^0_{\mathbf{k}}=\sqrt{\varepsilon^2_{\mathbf{k}}+(\Delta^0_{\mathbf{k}})^2}$, and $\tau^0_{\mathbf{k}}=\tanh(E^0_{\mathbf{k}}/2T)$.

- Landau processes ($E+\omega\leftrightarrow E'$ — 1st line) and Beliaev processes ($E+E'\leftrightarrow\omega$ — 2nd line).
- Well, it's kind of long :)
- There's also a practical PROBLEM . . .

Practical problem

- For any long wavelength $\mathbf{k}$ of $\phi_{\mathbf{k}}$, there are many solutions with different ω, parametrised by the wavenumber $\mathbf{M} \sim k_F$ from $\Delta^0_{\mathbf{M}}$.

- Experimentalists can control/perturb/see long wavelengths $\mathbf{k}$, but not $\mathbf{M}$

- Presumably, if you perturb system externally with wavenumber $\mathbf{k}$ the result will be some weighted average over all $\mathbf{M}$ solutions.

- But what are the weights?

The solution — an effective Lagrangian

1. In the action integral formulation of quantum mechanics write down an action $S(\Delta, \Psi)$ so that its saddle point $\partial S / \partial\{\Delta, \Psi\} = 0$ gives the full BCS theory.

2. Substitute perturbation $\Delta \longrightarrow \Delta_0 e^{2i\phi}$ to give $S(\Delta_0, \phi, \Psi)$.

3. An effective action S_{eff} for the small perturbation ϕ is obtained by integrating over the irrelevant variables Ψ.

4. get $S_{\text{eff}}(\phi, \Delta_0, \Psi_0)$, where Ψ_0 is the unperturbed ground state wavefunction.

5. Consistency equation for ϕ is given by the saddle-point solution $\partial S_{\text{eff}} / \partial \phi = 0$.

6. Weights turn out to be $\Delta^0_{\mathbf{M}}$.

(5B) Collective excitations

(results)

$T = 0$ Superfluid

Find Bogoliubov sound, same as for the standard $\uparrow\downarrow$BCS gas

$$\boxed{\omega = \left(\frac{v_F}{\sqrt{3}}\right)\, k}$$

To lowest order in $\omega \ll E_F/\hbar$ and $k \ll k_F$.

Not too surprising from hydrodynamics . . .

$T = 0$ Hydrodynamics

Relies on the hydrodynamic Hamiltonian for superfluid velocity v_s

$$H \approx \int d^3x \left\{ \frac{1}{2} m\rho v_s(x)^2 + U(\rho) \right\}$$

and the continuity and current equations

$$\vec{v}_s = \frac{\vec{J}_s(x)}{\rho} = \frac{\hbar}{m} \rho \, \vec{\nabla}\phi(x) \qquad \text{and} \qquad \vec{\nabla} \cdot \vec{J}_s(x) = -\frac{\partial \rho}{\partial t}$$

which are found to be the same for dipoles and short-range gases to order $O(\Delta^{\mathrm{max}}/E_F)$.

Since $U(\rho)$ arises overwhelmingly from the filled Fermi sphere,

$\Longrightarrow$ interaction details have minor effect locally

(Can be significant in a trap, though [Góral, Brewczyk, Rzążewski, Englert])

Beyond hydrodynamics

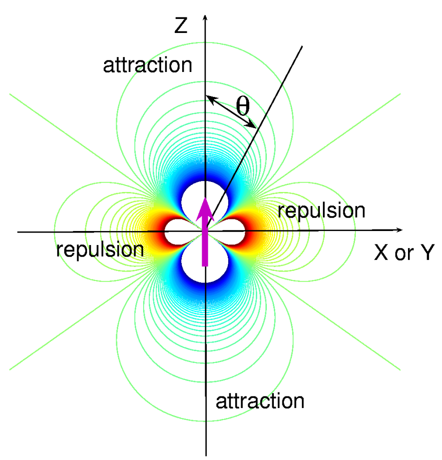
$T = 0$ Anisotropic damping of sound

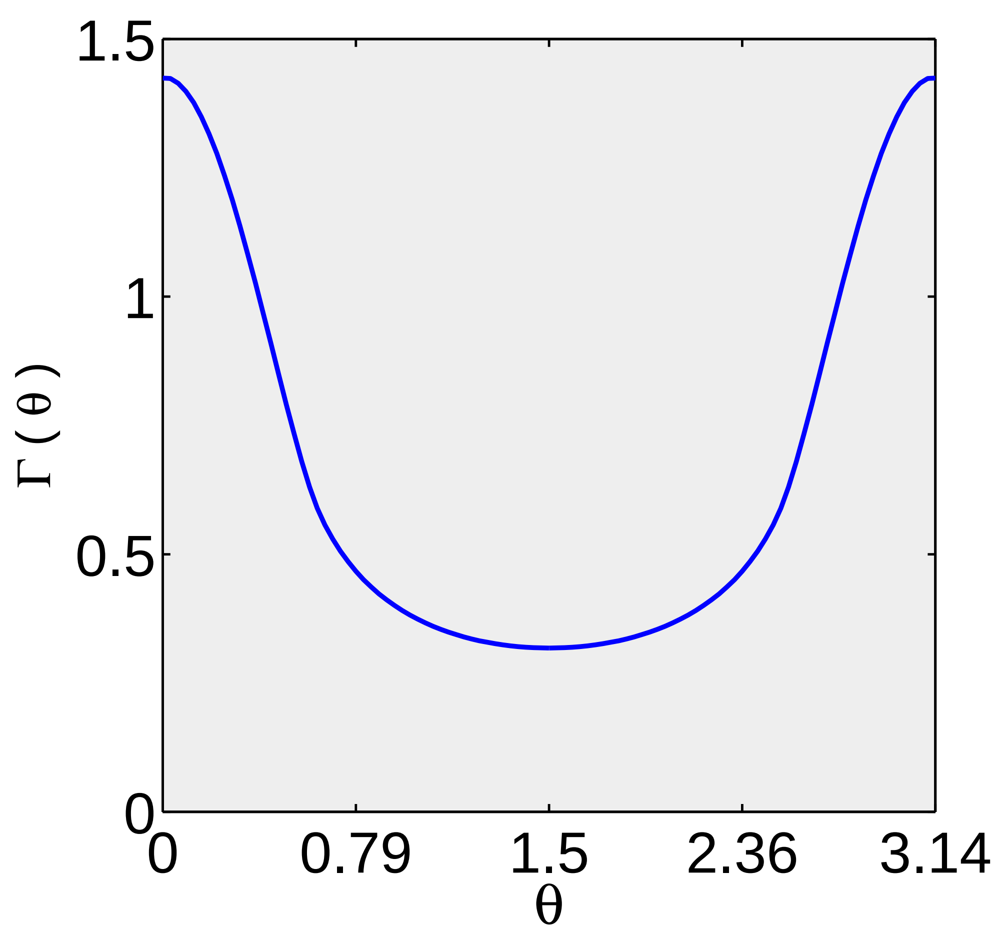
$$\omega = \left(\frac{v_F}{\sqrt{3}}\right) k \left\{1 - i\,k \left(\frac{\hbar v_F}{\sqrt{3}\,\Delta_{\max}}\right) \Gamma(\theta)\right\}$$

absent for standard ↑↓ gas

Beliaev process:

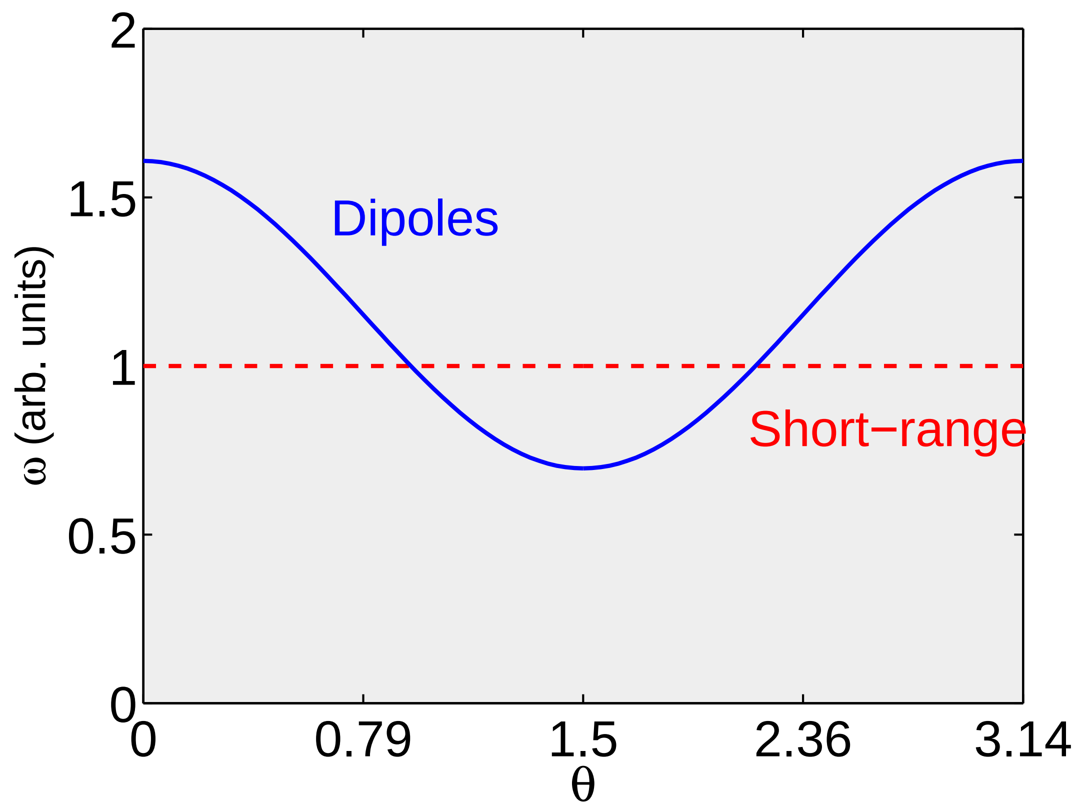
collective $\Longrightarrow$ 2×quasipart.

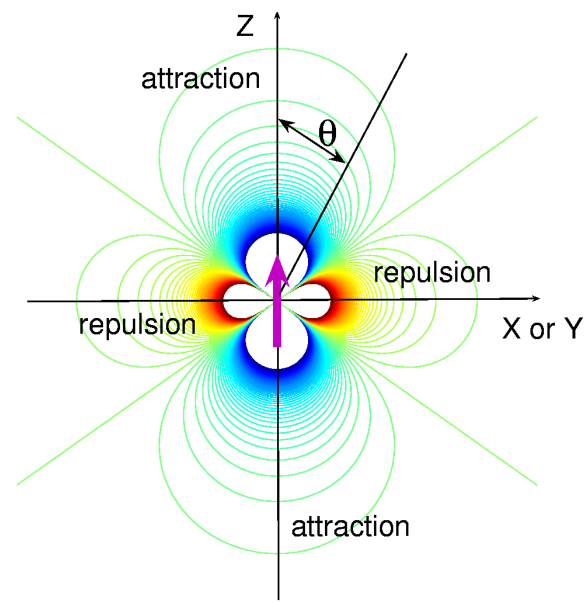




$T \approx T_c$ behaviour

$$\omega = -i \left(\frac{7\zeta(3)}{6\pi^3}\right)\left(\frac{\hbar v_F^2}{T_c}\right) k^2 \left(1 + \frac{3}{2\pi^2}(1+3\cos 2\theta)\right)$$





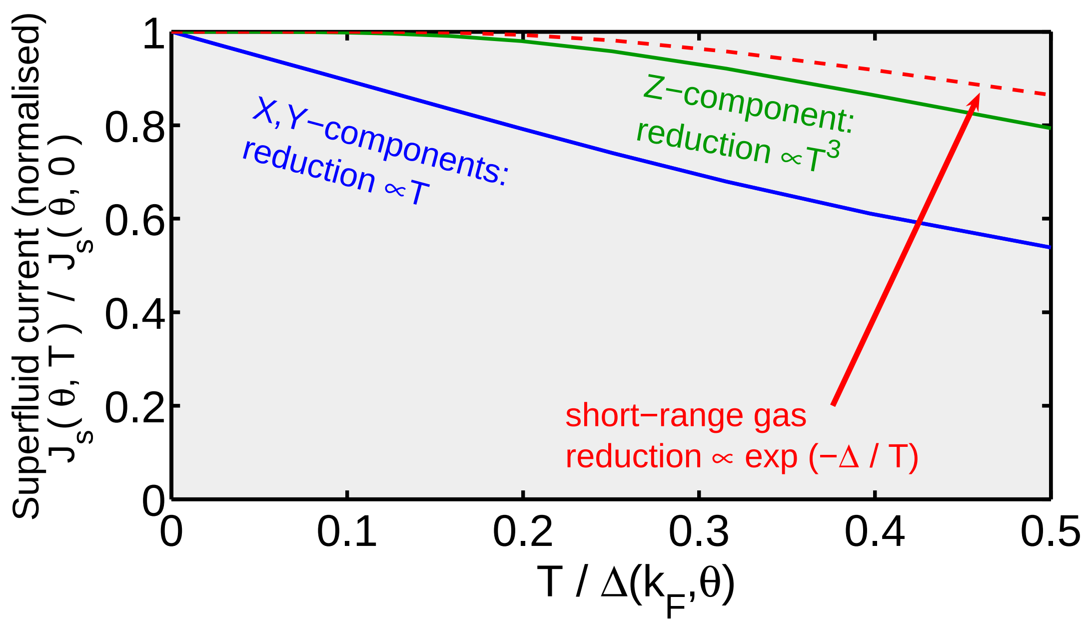
- Purely diffusive (as for standard short-range ↑↓gas)
- Anisotropic (differently to ↑↓gas)

Veering superfluid current $0 < T < T_c$

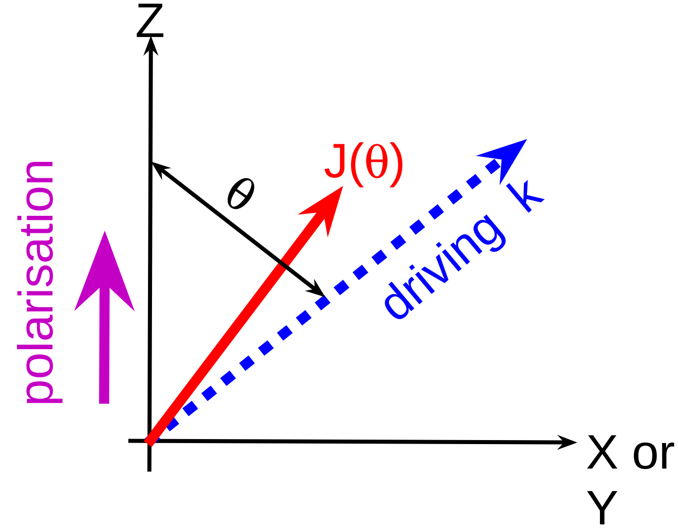
- Current response J_s to an external phase perturbation of the gap

$$\Delta(x,y,t) = \Delta_0(x-y)e^{2i\phi(x,t)}$$

- Driving frequency ω, wave-vector k, in direction θ.



Veering current



Direction-dependent superfluid

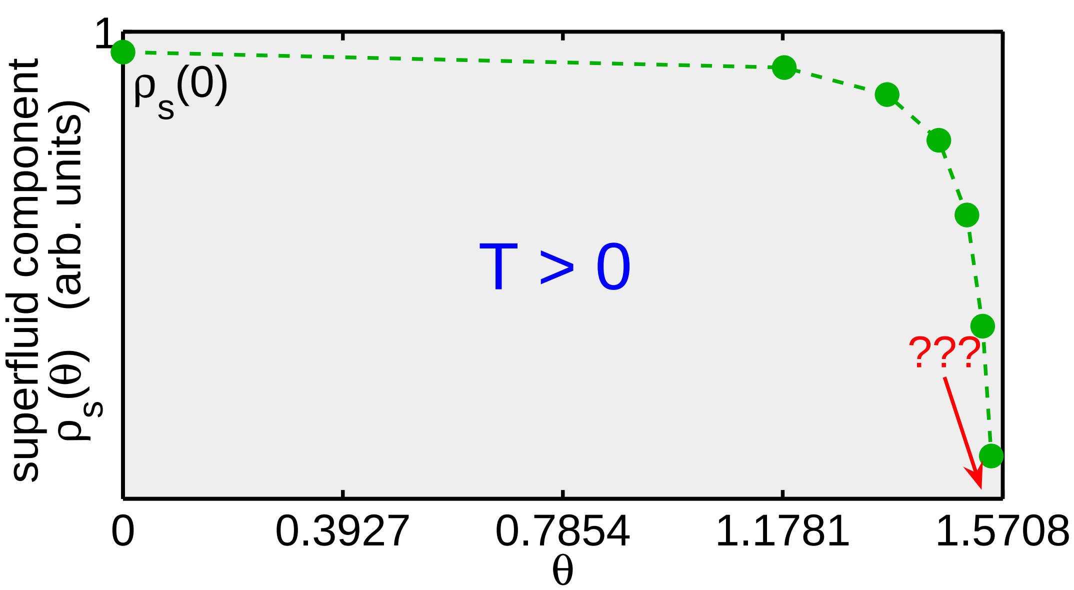
(preliminary and tentative)

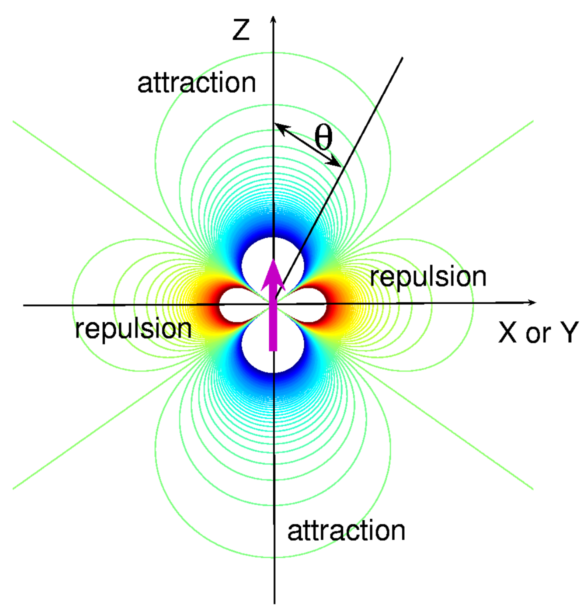
Can define direction-dependent "normal" and "superfluid" components

$$\rho = \rho_n(\theta) + \rho_s(\theta)$$

so that the usual current equation applies:

$$\vec{J}_s = \frac{\hbar}{m} \rho_s \vec{\nabla} \phi$$





Related avenues of research

- Other low energy modes - e.g. perturbation of the polarisation axis.
- What's going on with the current near $\theta = \pi/2$.
- Are the Δ-amplitude modulation modes low-energy near $\theta = \pi/2$?
- Are there interesting low energy perturbations of the discarded Hartree field $W(x,y)$?