Bogoliubov quantum dynamics for uncondensed atom clouds at T>>0

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Collaboration
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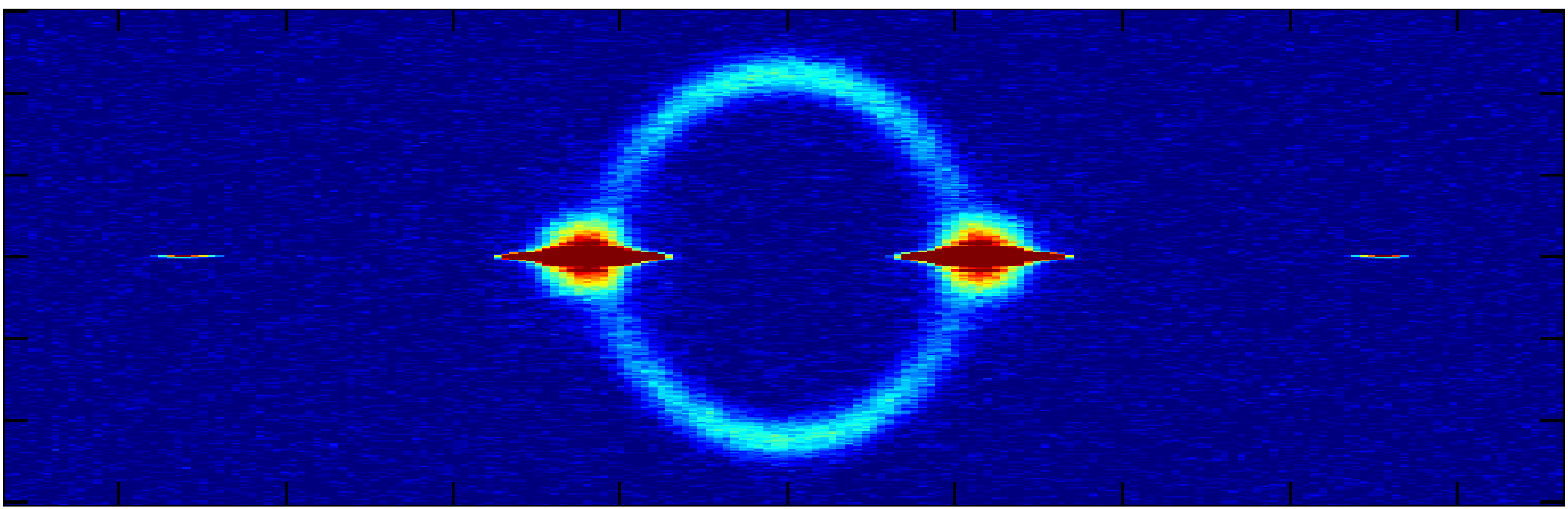
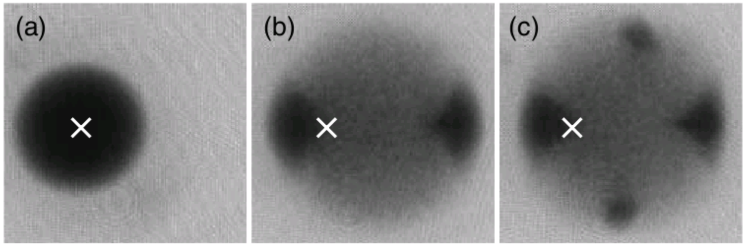


Outline

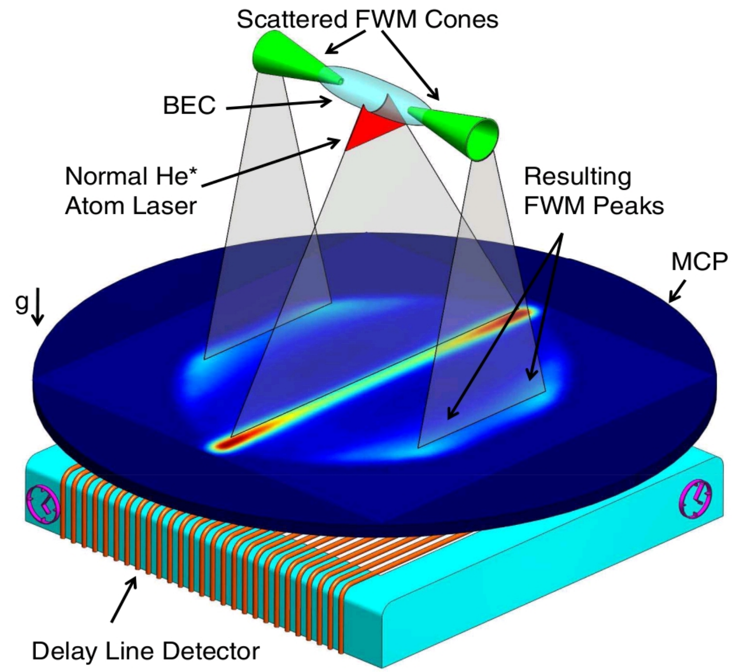
1. Supersonic pair creation
2. Palaiseau BEC collision experiment
3. Simulation of scattered pair dynamics at T=0
4. Quasicondensate 0<T<Tc
5. T > Tc ?

Supersonic pair creation

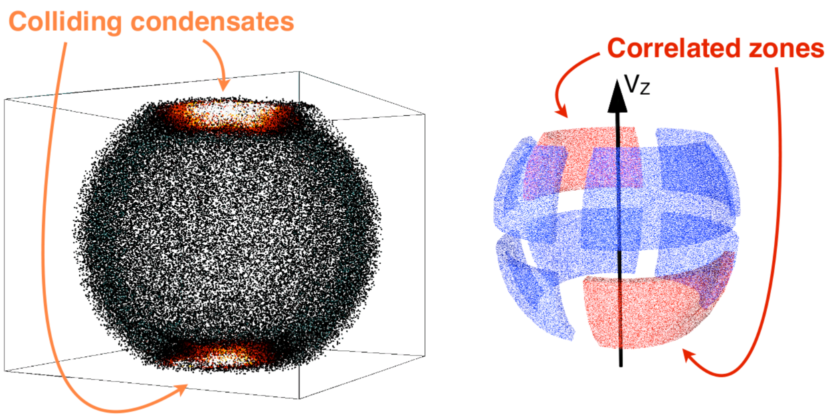


Vogels, Xu, Ketterle, PRL **89,** 020401 (2002)

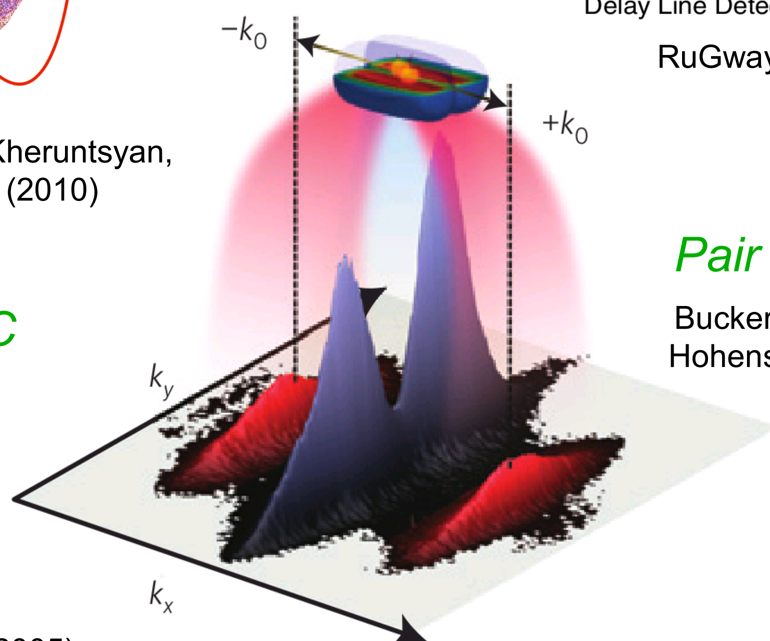
BEC Collisions



RuGway, Hodgman, Dall, Johnsson, Truscott, PRL **107,** 075301 (2011)



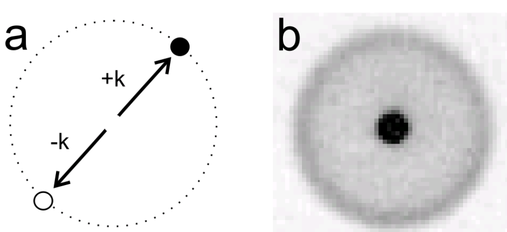
Jaskula, Bonneau, Partridge, Krachmalnicoff, PD, Kheruntsyan, Aspect, Boiron,Westbrook, PRL **105,** 190402 (2010)



Pair emission from a 1D gas

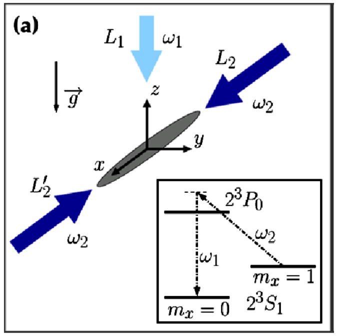
Bucker, Grond, Manz, Berrada, Betz, Koller, Hohenster, Schumm, Perrin, Schmiedmayer, Nature Phys. **7,** 608 (2011)

Dissociation of molecular BEC

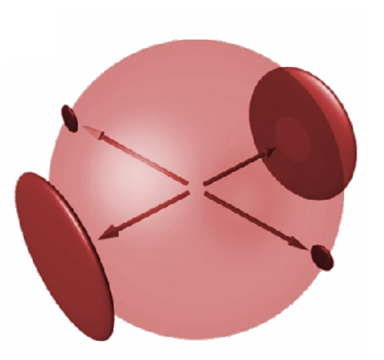


Greiner, Regal, Stewart, Jin, PRL **94,** 110401 (2005)

BEC collision – Palaiseau experiment



t=0 Bragg pulse



t=0.3 s expansion

Experiment:

Chris Westbrook
Denis Boiron
Jean-Christophe Jaskula
Valentina Krachmalnicoff
Marie Bonneau
Vanessa Leung
Guthrie Partridge
Alain Aspect

Theory:

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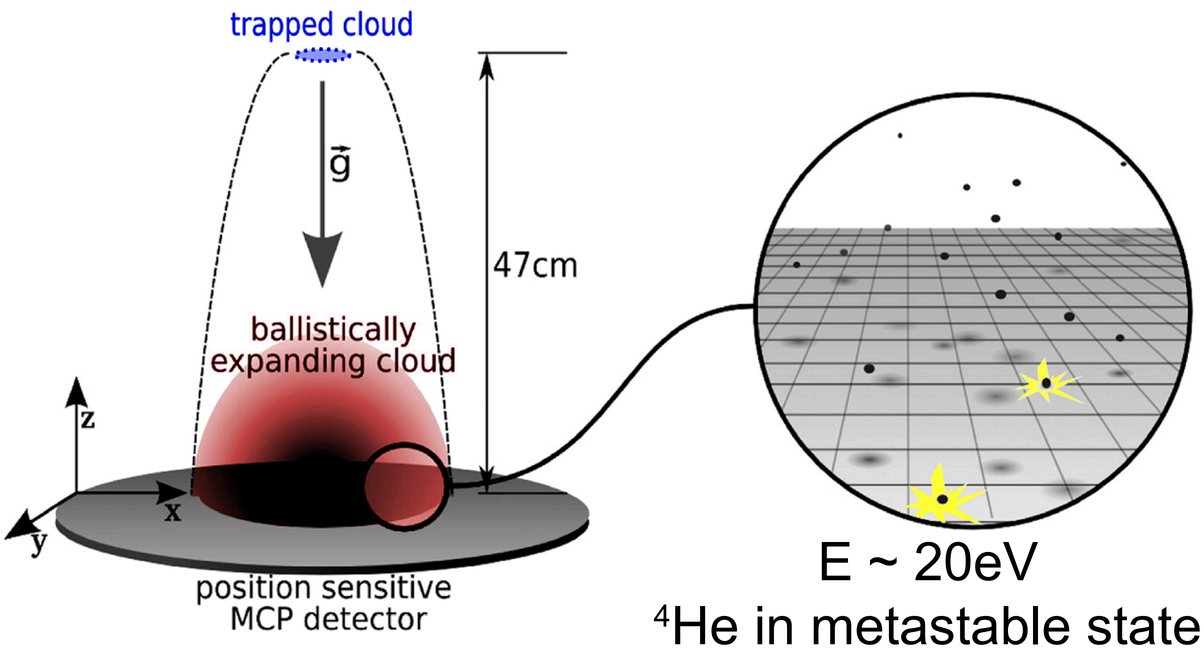
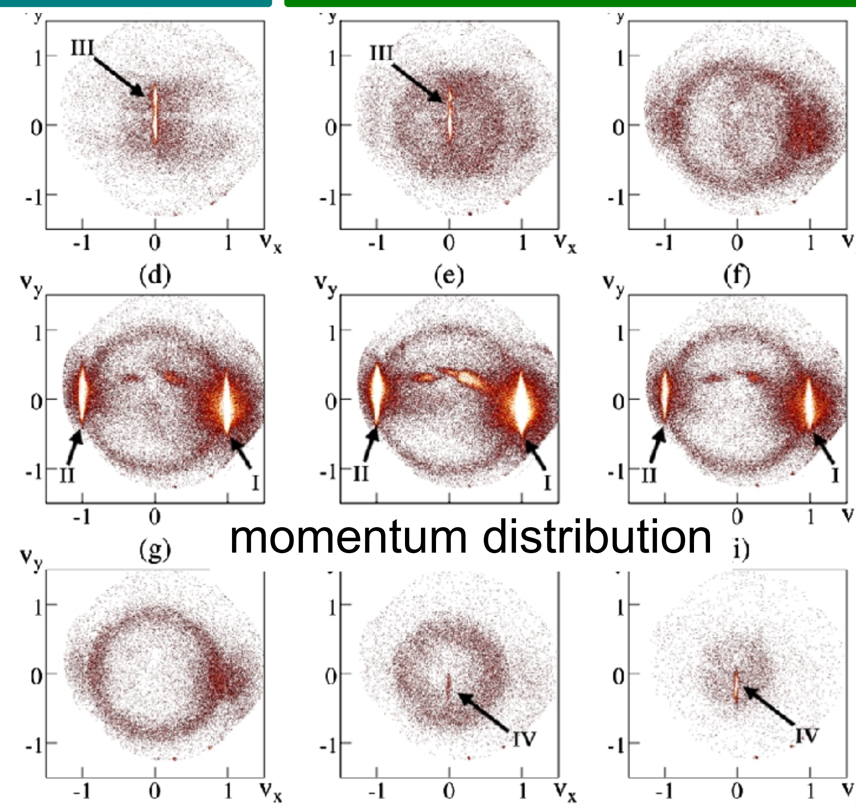
Jan Chwedeńczuk
Tomasz Wasak
Paweł Ziń

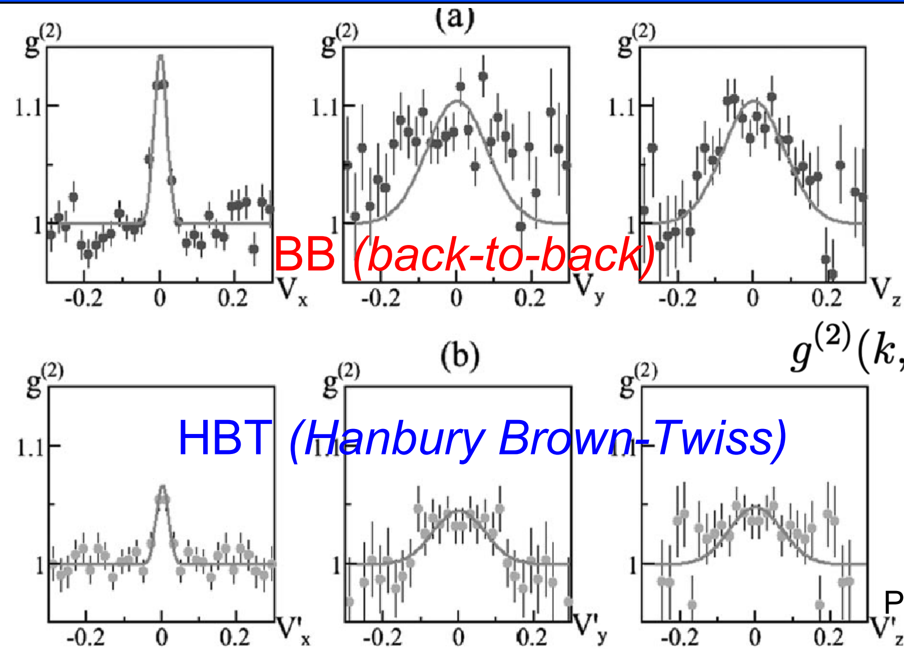




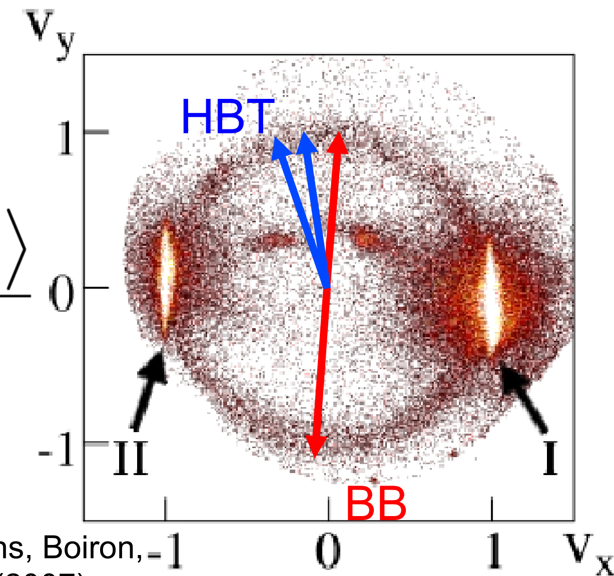
Single atom detection efficiency η ~ 12%

Perrin, Chang, Krachmalnicoff, Schellekens, Boiron, Aspect, Westbrook, PRL **99**, 150405 (2007)

Pairing and density correlations

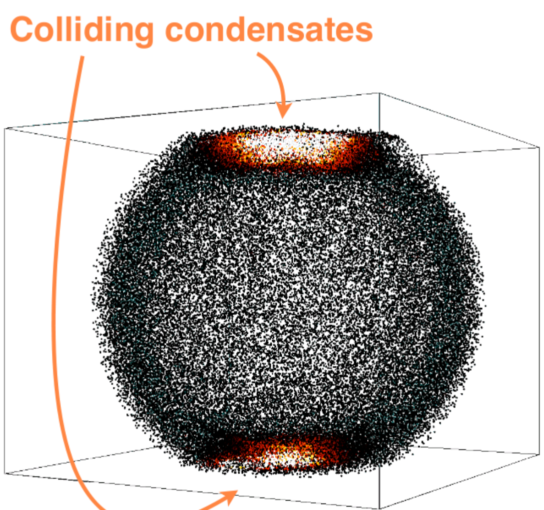


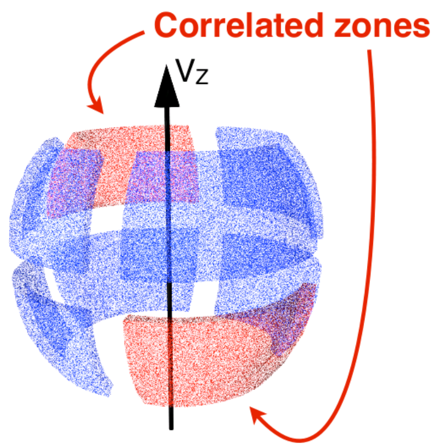
$$g^{(2)}(k,k') = \frac{\left\langle \widehat{\Psi}^\dagger(k)\widehat{\Psi}^\dagger(k')\widehat{\Psi}(k')\widehat{\Psi}(k)\right\rangle}{\langle \widehat{n}(k)\rangle \langle \widehat{n}(k')\rangle}$$



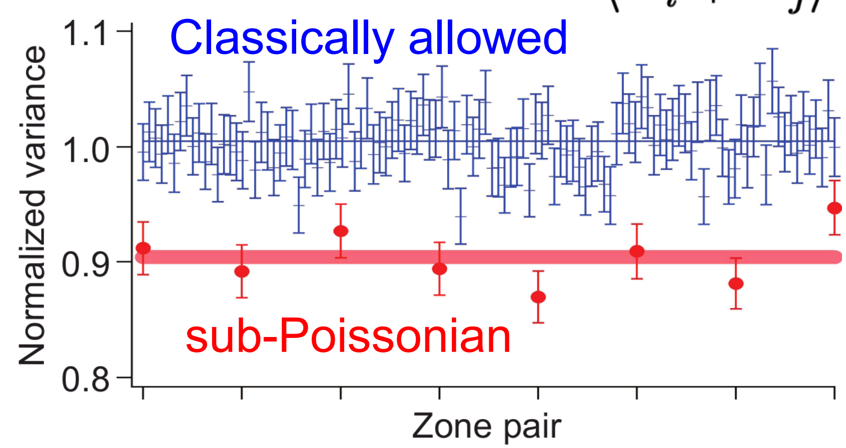
Perrin, Chang, Krachmalnicoff, Schellekens, Boiron, Aspect, Westbrook, PRL **99**, 150405 (2007)

Squeezing of relative particle number between zones





$$V_{ij} = \frac{\mathrm{var}\,[N_i - N_j]}{\langle N_i + N_j\rangle}$$



Jaskula, Bonneau, Partridge, Krachmalnicoff, PD, Kheruntsyan, Aspect, Boiron, Westbrook, PRL **105,** 190402 (2010)

Quantum description at T=0

$$H = \int d^3\mathbf{x} \left\{ \widehat{\Psi}^\dagger(\mathbf{x}) \left[V(x) - \frac{\hbar^2}{2m}\nabla^2 \right] \widehat{\Psi}(x) + \frac{g}{2} \widehat{\Psi}^\dagger(\mathbf{x})^2 \widehat{\Psi}(\mathbf{x})^2 \right\}$$

$H_0(x)$

Boson field

$g > 0$ repulsive contact interactions

Initial state

$$\widehat{\Psi}(x, t=0) = \phi_{GP}(x) \left(\frac{e^{ik_0 z} + e^{-ik_0 z}}{\sqrt{2}} \right)$$

GP ground state in the t<=0 trap

Bragg pulse at t=0 splits into components with momentum $\pm k_0$

The trouble with incoherent scattering

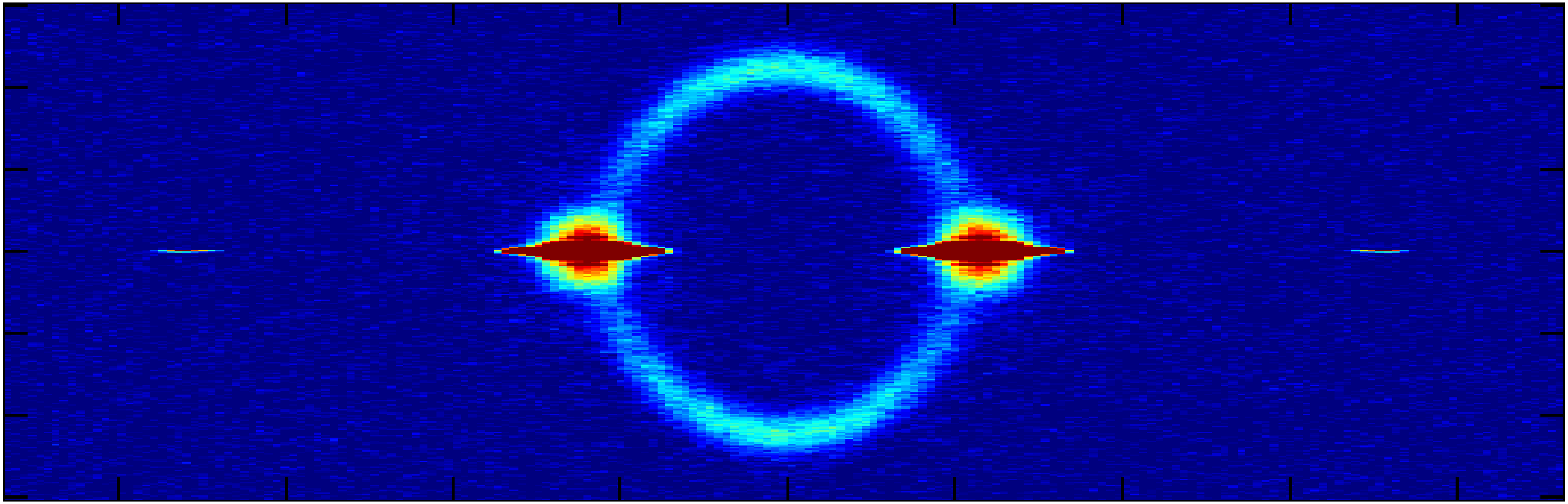
Can't use Gross-Pitaevskii equation

because field is initially zero in the interesting region

Can't use truncated Wigner method

because N=100 000, while we need 10 000 000 lattice sites to describe the physics

(so would have 5 000 000 virtual noise particles (>> N))



First try: positive-P representation

PD, Drummond, PRL, **98**, 120402 (2007)

$$\widehat{\rho} = \int P\left[\psi, \widetilde{\psi}\right] |\psi\rangle \langle\widetilde{\psi}| \, \mathcal{D}^2\psi(\vec{x})\mathcal{D}^2\widetilde{\psi}(\vec{x})$$

Probability distribution of bra & ket coherent fields $\psi(x), \; \widetilde{\psi}(x)$

Complete representation of the full quantum dynamics

Observables

$$\left\langle \widehat{\Psi}^\dagger(x)\widehat{\Psi}(y) \right\rangle = \left\langle \widetilde{\psi}^*(x)\psi(y) \right\rangle_{\text{samples}}$$

dynamics

$$i\hbar\frac{d\psi(x)}{dt} = \left\{ H_0(x) + g\widetilde{\psi}^*(x)\psi(x) + \sqrt{i\hbar g}\,\xi(x,t) \right\} \psi(x)$$

$$i\hbar\frac{d\widetilde{\psi}(x)}{dt} = \left\{ H_0(x) + g\psi^*(x)\widetilde{\psi}(x) + \sqrt{i\hbar g}\,\widetilde{\xi}(x,t) \right\} \widetilde{\psi}(x)$$

Gaussian real white noise $\xi(x,t), \; \widetilde{\xi}(x,t)$

<u>However:</u>

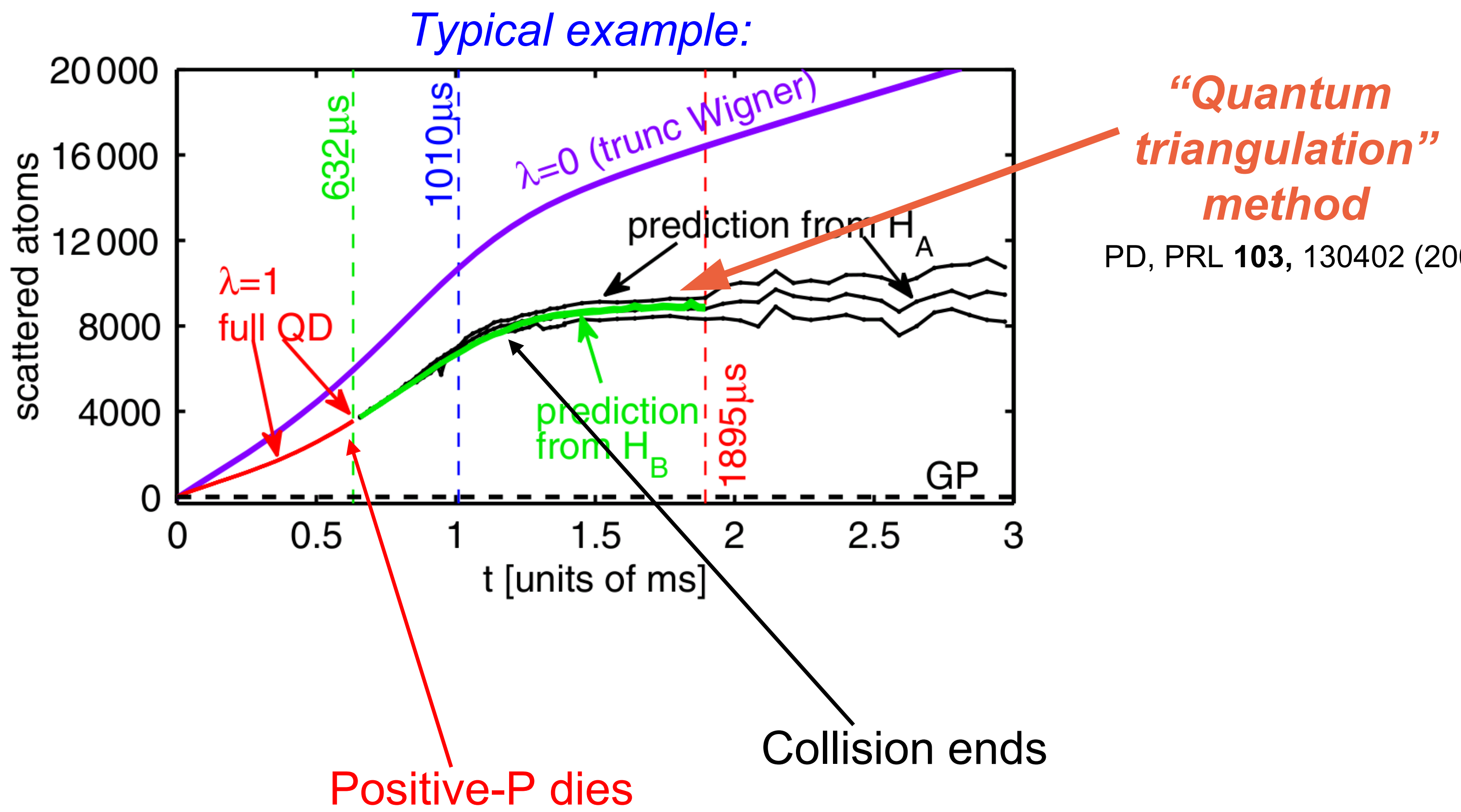
intractable after an inconvenient time

Bad news: instability

The positive-P treatment of the full field becomes intractable after a certain time

Reason: nonlinear amplification of the noise

→ too many trajectories needed



PD, PRL **103,** 130402 (2009)

Second try: Positive-P Bogoliubov

PD, *Chwedenczuk, Trippenbach, Zin*, PRA **83,** 063625 (2011)

$$\widehat{\Psi}(\mathbf{x},t) = \phi(\mathbf{x},t) + \widehat{\delta}(\mathbf{x},t)$$

(symmetry breaking version)

condensate ($\phi(\mathbf{x},t)$)

Bogoliubov fluctuation field ($\widehat{\delta}(\mathbf{x},t)$) – *MUST BE "small"*

Treat only $\widehat{\delta}(\mathbf{x},t)$ using positive-P representation

$$i\hbar\frac{d\phi(x)}{dt} = \left[H_0(x) + g|\phi(x)|^2\right]\phi(x)$$

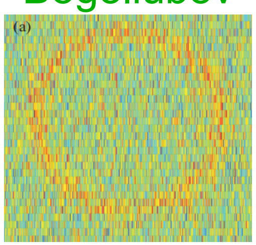
Mean field

$$i\hbar\frac{d\psi(x)}{dt} = \left\{H_0(x) + 2g|\phi(x)|^2\right\}\psi(x) + g\phi(x)^2\widetilde{\psi}(x)^* + \sqrt{i\hbar g}\,\phi(x)\xi(x,t)$$

$$i\hbar\frac{d\widetilde{\psi}(x)}{dt} = \left\{H_0(x) + 2g|\phi(x)|^2\right\}\widetilde{\psi}(x) + g\phi(x)^2\psi(x)^* + \sqrt{i\hbar g}\,\phi(x)\widetilde{\xi}(x,t)$$

Now equations are linear ------> no blow-up of noise :)

Wigner-
-Bogoliubov

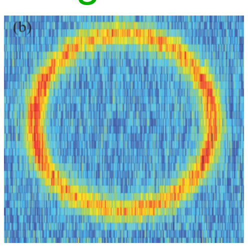


Can use plane wave basis ---> no diagonalizing of 10^6 X 10^6 matrices :)

Signal-to-noise far superior to an earlier Wigner-Bogoliubov treatment

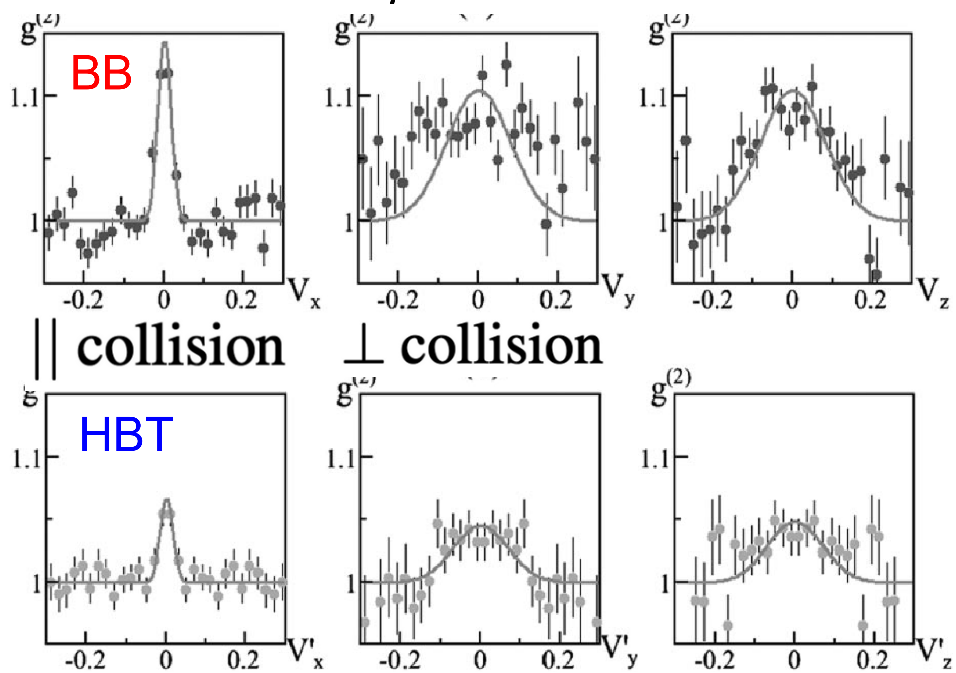
Sinatra, Castin, Lobo, J. Mod. Opt. **47,** 2629 (2000)

Positive-P-
-Bogoliubov



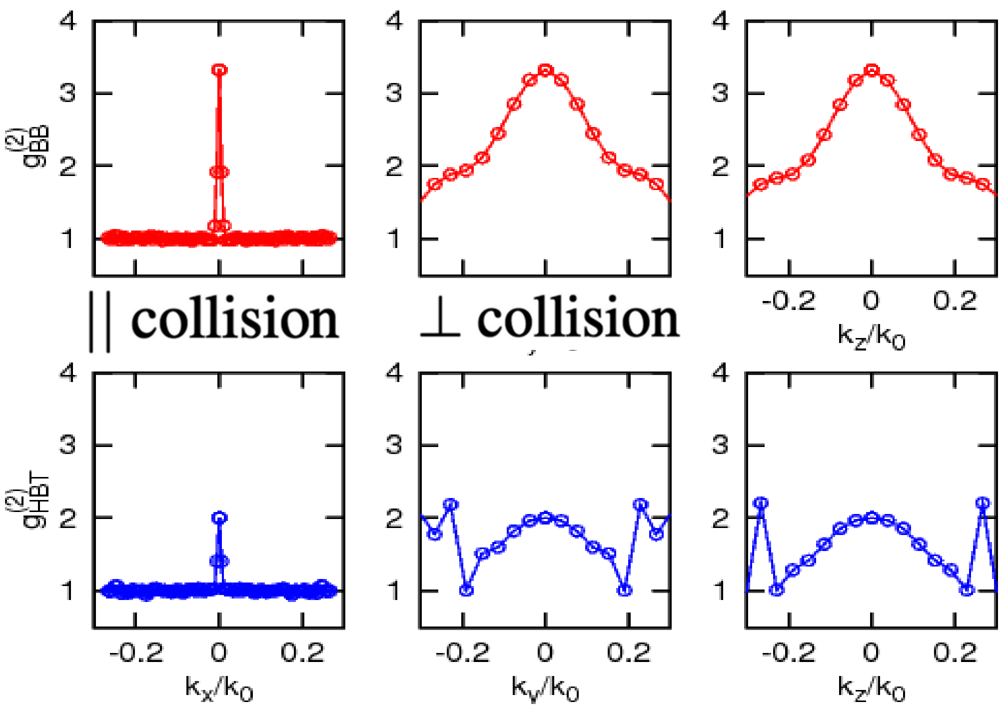
1st generation experiment: fair agreement

experiment



Perrin, Chang, Krachmalnicoff, Schellekens, Boiron, Aspect, Westbrook, PRL **99**, 150405 (2007)

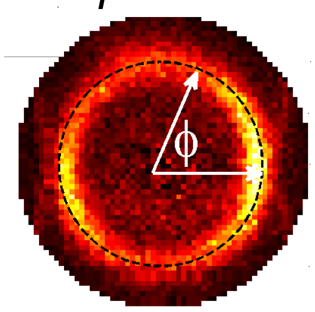
simulation



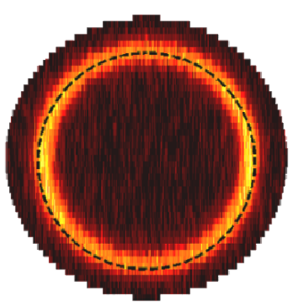
Pair correlations along collision

experiment			numerics		
BB width	BB width / HBT width	BB height / HBT height	BB width	BB width / HBT width	BB height / HBT height
$0.017\ k_0$	1.1	2.1	$0.004\ k_0$	1	2.2

experiment

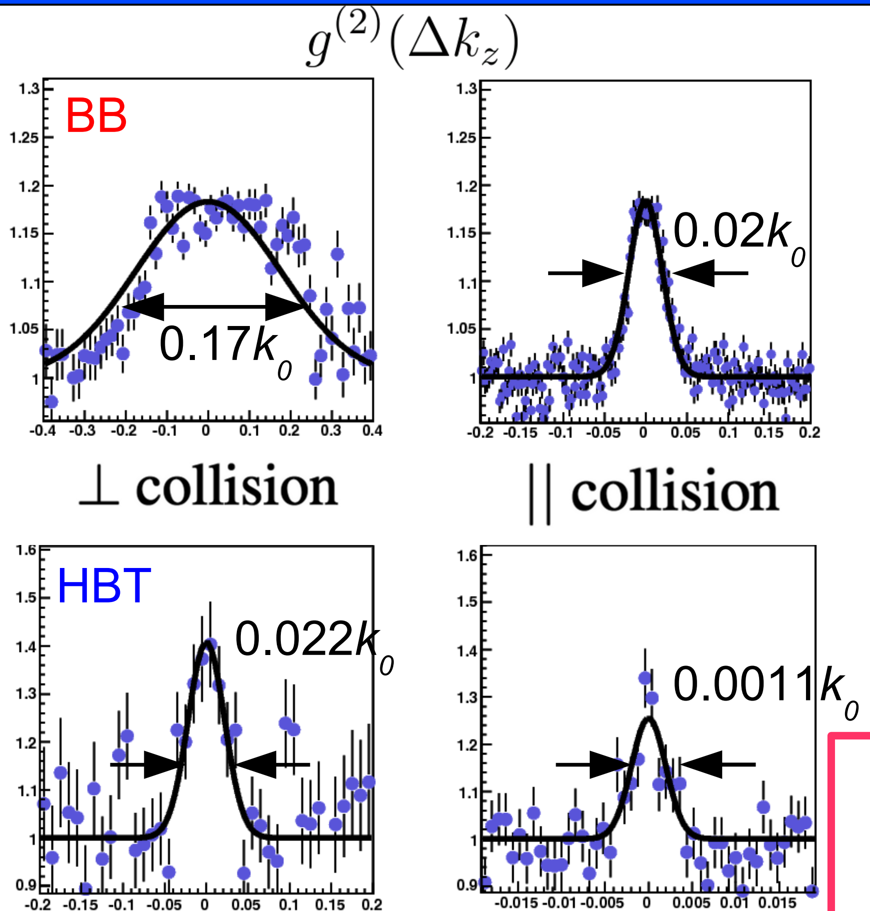


simulation

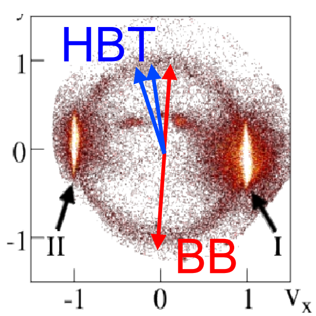


Krachmalnicoff, Jaskula, Bonneau, Leung, Partridge, Boiron, Westbrook, PD, Zin, Trippenbach, Kheruntsyan, PRL **104,** 150402 (2010)

2^{nd} generation experiment: something suspicious

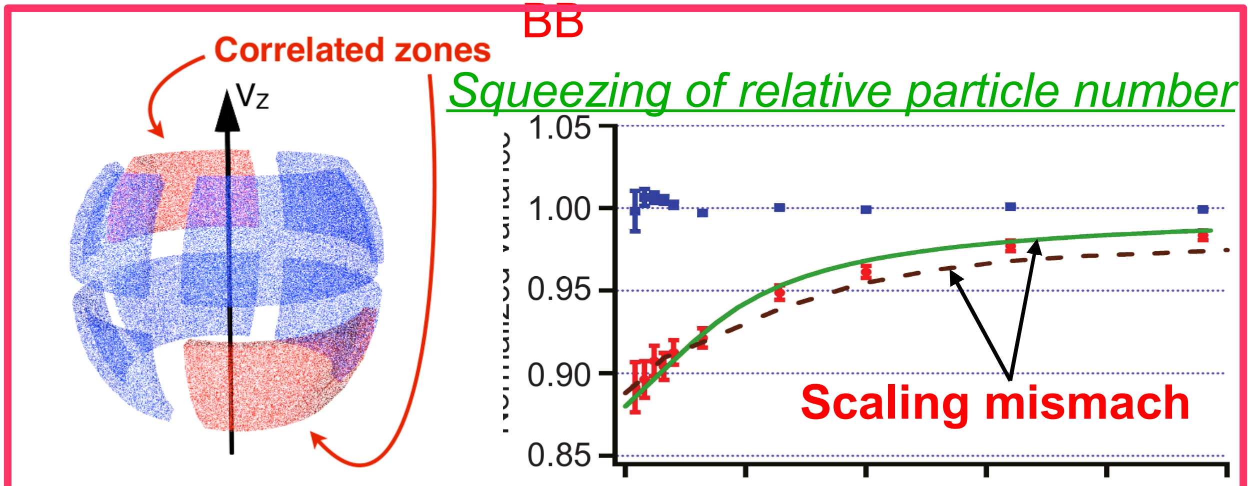


J-C. Jaskula, PhD Thesis (2010)



Pair correlations along long axis

	experiment		numerics	
	BB width / HBT width	BB height / HBT height	BB width / HBT width	BB height / HBT height
1st generation	1.1	2.1	1	2.2
2nd generation	18	0.5	1	2.2



Jaskula *et al*, PRL **105,** 190402 (2010)

The suspect: a quasicondensate

experiment	ω_z	ω_r	aspect
1st generation	47	1150	24.5
2nd generation	7.5	1500	200

Elongated 3D cloud (→ 24.5)

VERY elongated 3D cloud (→ 200)

Effectively a quasicondensate at our temperatures

condensate fraction ~5%

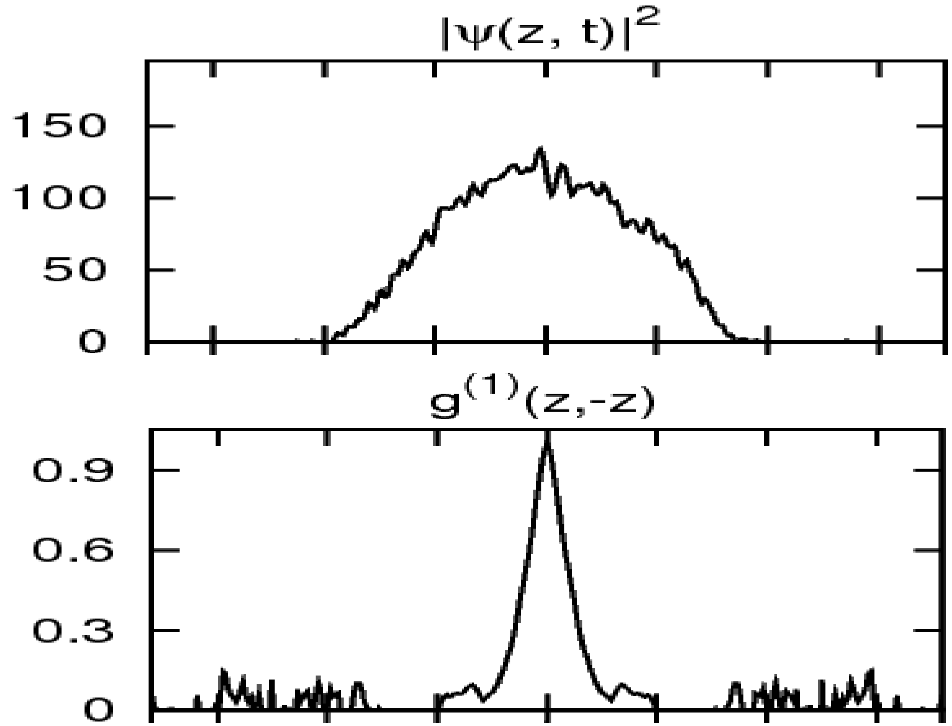
Described by model of

Phase-fluctuating 3D condensates in elongated traps

Petrov, Shlyapnikov, Walraven, PRL **87**, 050404 (2001)

Quasicondensate

Single experimental run



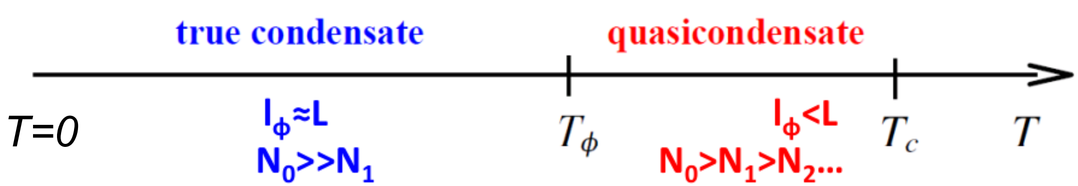
Density fluctuations small (like a BEC)

Phase coherence lost at long range

Witkowska, PD, Gajda, Rzazewski, PRL **106,** 135301 (2011)

1D Bose Gas in a Trap

At thermal equilibrium: for weakly interacting gas there are two characteristic temperatures



Corroborating evidence

In a quasicondensate:

$$g^{(1)}(x,x') \sim \exp\left[-\frac{|x-x'|}{l_\phi}\right] \quad ; \quad l_\phi \sim \frac{N^{2/3}}{T}$$

This is the Fourier transform of *n(k)*

So, narrow $g^{(1)}(x,x')$ ===> wide momentum distribution n(k) *(quasicondensate)*

Consistent with This observation

	experiment		numerics	
	$\frac{\text{BB width}}{\text{HBT width}}$	$\frac{\text{BB height}}{\text{HBT height}}$	$\frac{\text{BB width}}{\text{HBT width}}$	$\frac{\text{BB height}}{\text{HBT height}}$
1st generation	1.1	2.1	1	2.2
2nd generation	18	0.5	1	2.2

Dealing with a quasicondensate

Let's see what the Petrov description looks like

Petrov model of very elongated 3D cloud

Phase-fluctuating 3D condensates in elongated traps
Petrov, Shlyapnikov, Walraven, PRL **87**, 050404 (2001)

Thomas-fermi density profile

$$\widehat{\Psi}(\vec{x}) \approx \sqrt{n(\vec{x})}e^{i\widehat{\theta}(x)}$$

Fluctuating phase

$$\widehat{\theta}(x) = \sum_{j=1}^{\infty} \left[\frac{(j+2)(2j+3)g}{4\pi R^2 L \epsilon_j (j+1)}\right]^{1/2} P_j^{(1,1)}\left(\frac{x}{L}\right) \frac{(\alpha_j + \alpha_j^*)}{2}$$

$$\epsilon_j = \hbar\omega_x \sqrt{j(j+3)/4}$$

Random amplitudes
Bose distributed

Dettmer, Hellweg, Ryytty, Arlt, Ertmer, Sengstock, Petrov, Shlyapnikov, Kreutzmann, Santos, Lewenstein, PRL **87**, 160406 (2001)

$$\langle|\alpha_j|^2\rangle = N_j, \text{ where } N_j = [\exp(\epsilon_j/k_B T) - 1]^{-1}$$

This is a "classical fields" description
(something we know about here in Warsaw ;)

Classical field /PGPE/SGPE/... model

e.g. free space : plane wave basis

Full quantum field

$$\Psi(\mathbf{r}) = \sum_{\mathbf{k}} a_{\mathbf{k}} \frac{1}{\sqrt{V}} e^{i\mathbf{kr}}$$

c-fields

$$\Phi(\mathbf{r}) = \sum_{|\mathbf{k}|\leqslant \mathbf{K}_{max}} \alpha_{\mathbf{k}} \frac{1}{\sqrt{V}} e^{i\mathbf{kr}}$$

Replace mode amplitude operators $a_{\mathbf{k}}$
with complex number amplitudes $\alpha_{\mathbf{k}}$

Thermal initial state:

- $|\alpha_{\mathbf{k}}|^2$ Distributed according to Bose-Einstein distribution
- Phase of $\alpha_{\mathbf{k}}$ is random
- Use many realizations to get thermal ensemble

Useful papers:

Brewczyk, Gajda, Rzazewski, J. Phys B **40,** R1 (2007)
Blakie, Bradley, Davis, Ballagh, Gardiner, Adv. Phys. **57**, 363 (2008)
Proukakis, Jackson, J. Phys A **41,** 203002 (2008)
Brewczyk, Borowski, Gajda, Rzazewski, J Phys B **37**, 2725 (2004)

Validity

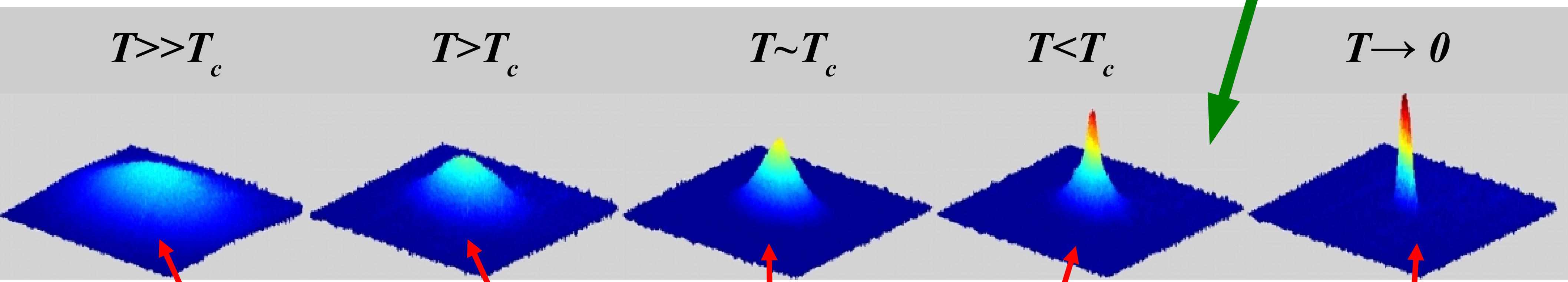
$$\left[\hat{\Psi}(x), \hat{\Psi}^\dagger(x')\right] = \delta(x - x') \quad \rightarrow \quad [\psi^*(x), \psi(x')] = 0$$

→ *it will be fine, ...*

.... as long as there are always many atoms involved

in whatever it is we are studying

Quasicondensate



http://www-matterwave.physics.ox.ac.uk/node/1

Mode occupation getting small

OK:
All relevant modes highly occupied

Occupation in excited modes getting small

Dealing with a quasicondensate

Each classical field realization is :

1. Independent of the others
2. Evolves via the Gross-Pitaevskii equation

$$i\hbar\frac{d\phi(x)}{dt} = \left[H_0(x) + g|\phi(x)|^2\right]\phi(x)$$

3. ==> suppose we treat them like condensate wavefunctions

   on which extra Bogoliubov fluctuations will form

4. Treat these <u>extra</u> fluctuations with the positive-P method

1^{st} Trick: each CF realization is independent

INITIAL STATE t=0

Treat $\alpha_{CF}(x)$ as condensates

$$\phi(x) = \alpha_{CF}(x)$$

t > 0

$\alpha_{CF}(x)$ Classical field realization 1

- $\psi(x),\ \widetilde{\psi}(x)$ Bogoliubov realization 1A
- $\psi(x),\ \widetilde{\psi}(x)$ Bogoliubov realization 1B
- $\psi(x),\ \widetilde{\psi}(x)$ Bogoliubov realization 1C

$\alpha_{CF}(x)$ Classical field realization 2

- $\psi(x),\ \widetilde{\psi}(x)$ Bogoliubov realization 2A
- $\psi(x),\ \widetilde{\psi}(x)$ Bogoliubov realization 2B
- $\psi(x),\ \widetilde{\psi}(x)$ Bogoliubov realization 2C

$\alpha_{CF}(x)$ Classical field realization 3

- $\psi(x),\ \widetilde{\psi}(x)$ Bogoliubov realization 2A
- $\psi(x),\ \widetilde{\psi}(x)$ Bogoliubov realization 2B
- $\psi(x),\ \widetilde{\psi}(x)$ Bogoliubov realization 2C

2^{nd} Trick: each Bogoliubov trajectory is independent

INITIAL STATE t=0

Treat $\alpha_{CF}(x)$ as condensates

$$\phi(x) = \alpha_{CF}(x)$$

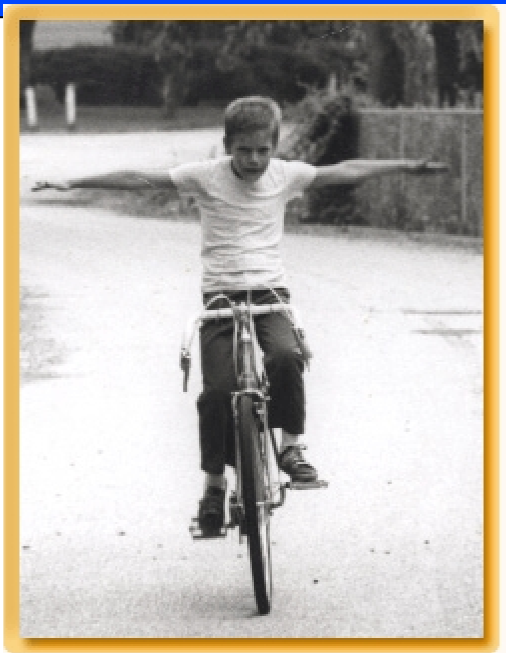
t > 0

$\alpha_{CF}(x)$ Classical field realization 1 → $\psi(x),\ \widetilde{\psi}(x)$ Bogoliubov realization 1

$\alpha_{CF}(x)$ Classical field realization 2 → $\psi(x),\ \widetilde{\psi}(x)$ Bogoliubov realization 2

$\alpha_{CF}(x)$ Classical field realization 3 → $\psi(x),\ \widetilde{\psi}(x)$ Bogoliubov realization 2

Look mum, no condensate! ($n_0 \sim 0.05$)

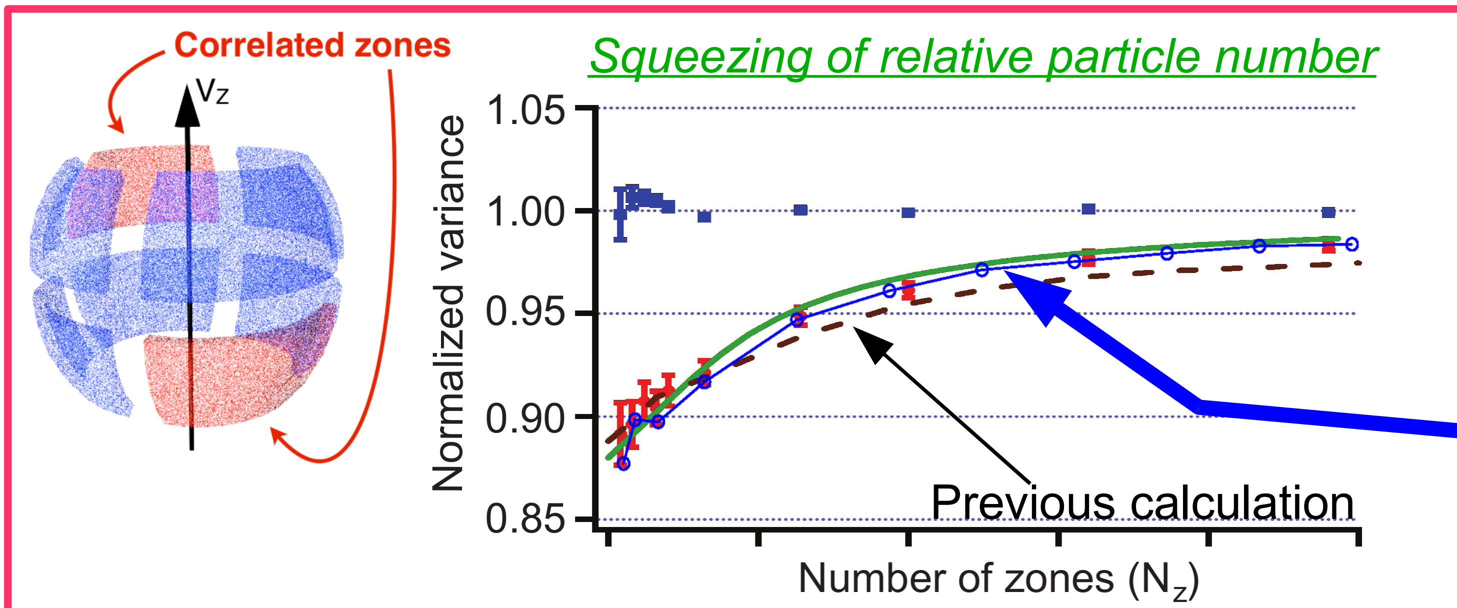


Look mum, no hands!

Pair correlations $g^{(2)}(\Delta k_z)$

	experiment		numerics	
	$\frac{\text{BB width}}{\text{HBT width}}$	$\frac{\text{BB height}}{\text{HBT height}}$	$\frac{\text{BB width}}{\text{HBT width}}$	$\frac{\text{BB height}}{\text{HBT height}}$
1st generation	1.1	2.1	1	2.2
2nd generation	18	0.5	~~1~~ 16	~~2.2~~ 0.3

Visible effect of Quasicondensate on pairing



Surprise! improved match

Caveats

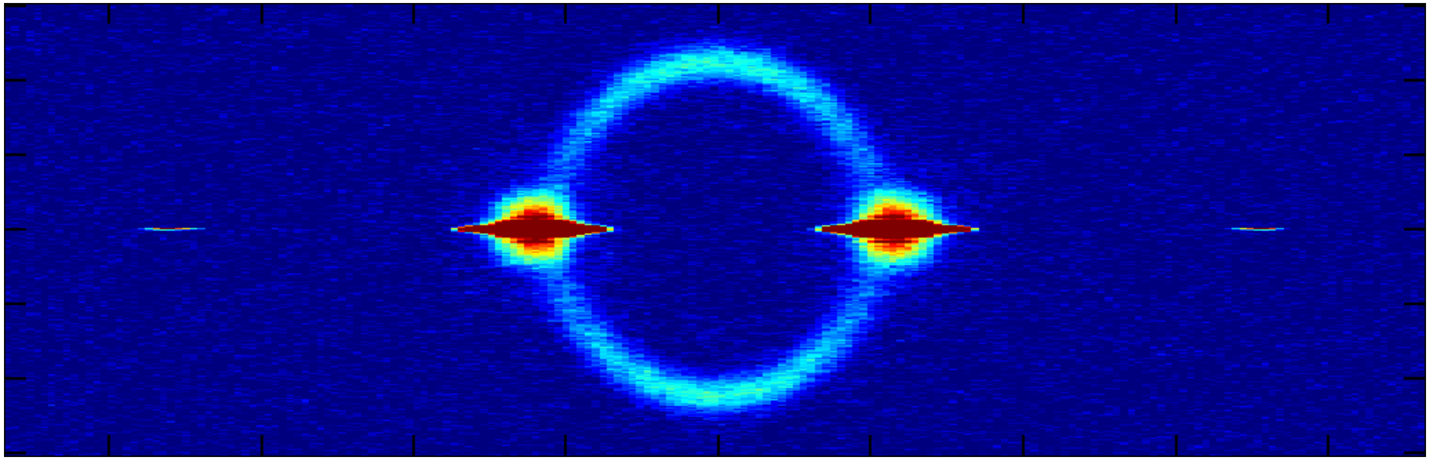
Caveat 1: need *additional t>0* depletion to be small

(initial depletion is apparently irrelevant)

Caveat 2: don't look at the condensate regions

(plane waves are not orthogonal to the condensate)

-----> mix-up of Bogoliubov modes and condensate there



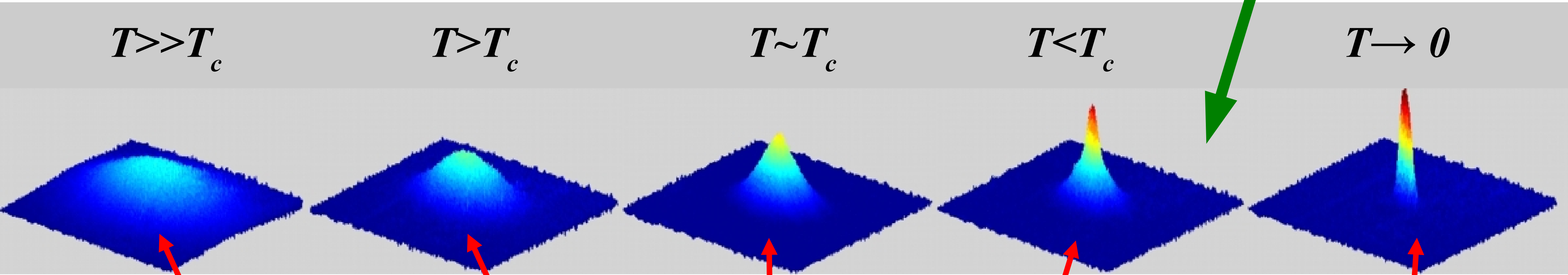
What about higher temperatures?

$$\left[\hat{\Psi}(x), \hat{\Psi}^\dagger(x')\right] = \delta(x - x') \qquad \rightarrow \qquad [\psi^*(x), \psi(x')] = 0$$

→ it will be fine, ...

.... as long as there are always many atoms involved in whatever it is we are studying

Quasicondensate



http://www-matterwave.physics.ox.ac.uk/node/1

Mode occupation getting small

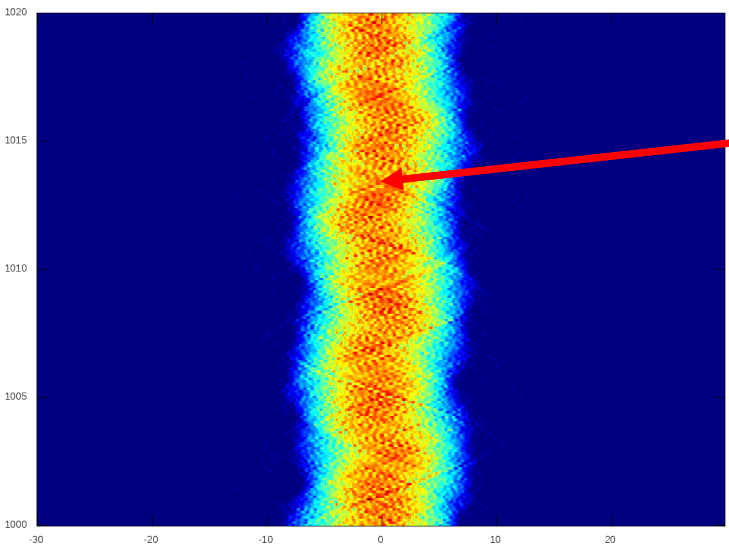
OK for Classical-field method:
All relevant modes highly occupied

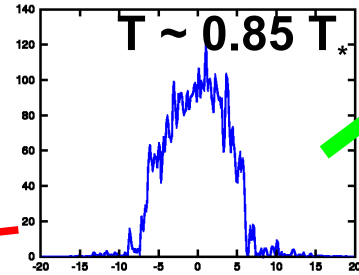
Occupation in excited modes getting small

Even less condensate?

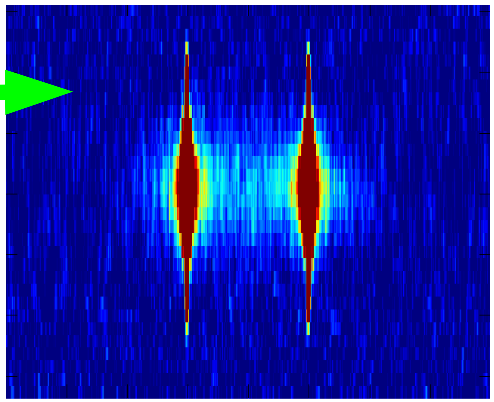
Classical field simulation of 1D evaporative cooling

Witkowska, PD, Gajda, Rzazewski, PRL **106,** 135301 (2011)

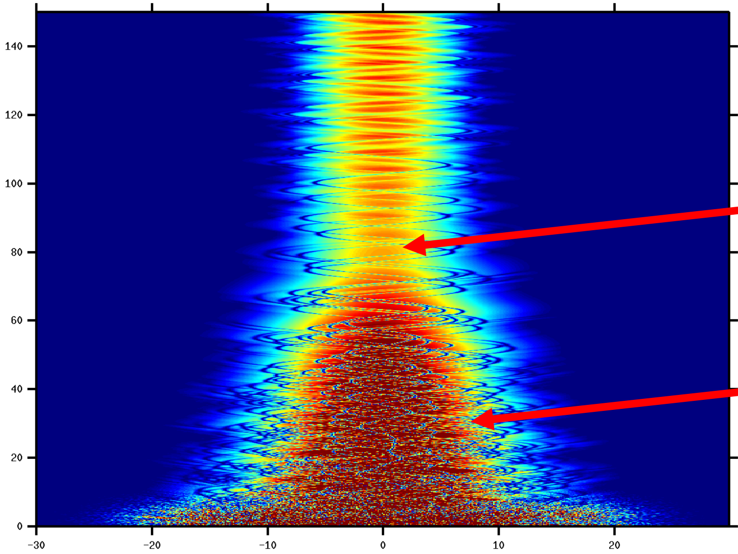


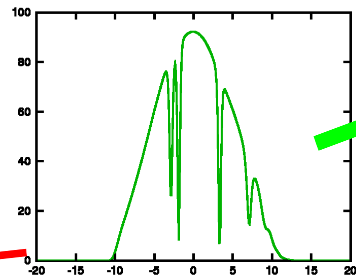


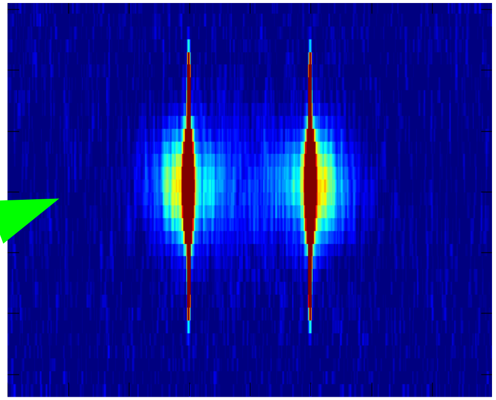
$n_0 \sim 0.25$

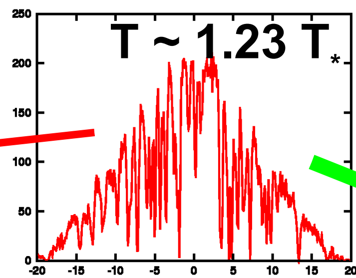


RELEASE 1D CONFINEMENT AND COLLIDE

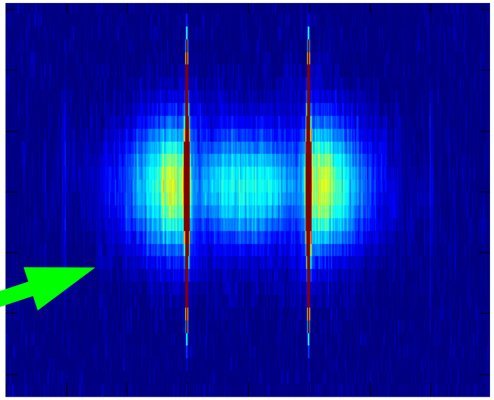








$n_0 \sim 0.01$



Summary

- Quantitative simulation of dynamics of pair scattering

  With positive-P treatment of Bogoliubov field

- Bogoliubov dynamics made tractable for large systems

  By use of plane wave modes + stochastic noise

- Treat classical field realizations as condensates

  Apparently works even with no true condensate

- Need to work on number-conserving Bogoliubov version