

Soiltons as the early stage of quasicondensate formation during evaporative cooling



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Outline

1. Cooling of a 1D Bose gas
2. Appearance and dynamics of solitons
3. Coherence amidst the solitons
4. Attempts at a local “thermodynamic” description
5. Soliton statistics

trapped 1D Bose gas

$$\hat{H} = \int dx \left\{ \hat{\Psi}^\dagger(x) \left[V(x) - \frac{\hbar^2}{2m} \nabla^2 \right] \hat{\Psi}(x) + \frac{g}{2} \hat{\Psi}^\dagger(x)^2 \hat{\Psi}(x)^2 \right\}$$

$$N \sim 10^3 - 10^4$$

$g > 0 \rightarrow$ repulsive contact interactions

Quasicondensate in a trap

- In the uniform 1D gas, there is no true condensate for $T > 0$
- *However:* finite coherence length l_ϕ

$$g^{(1)}(x, x') \sim \exp\left[-\frac{|x - x'|}{l_\phi}\right] \quad ; \quad l_\phi \sim \frac{N^{2/3}}{T}$$

- → In the trap, BEC occurs when $L < l_\phi$

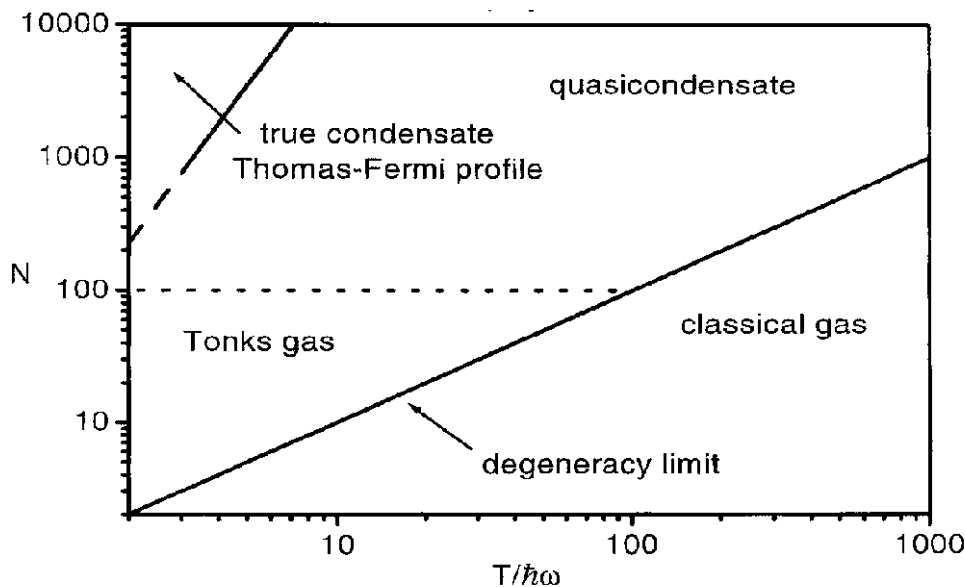
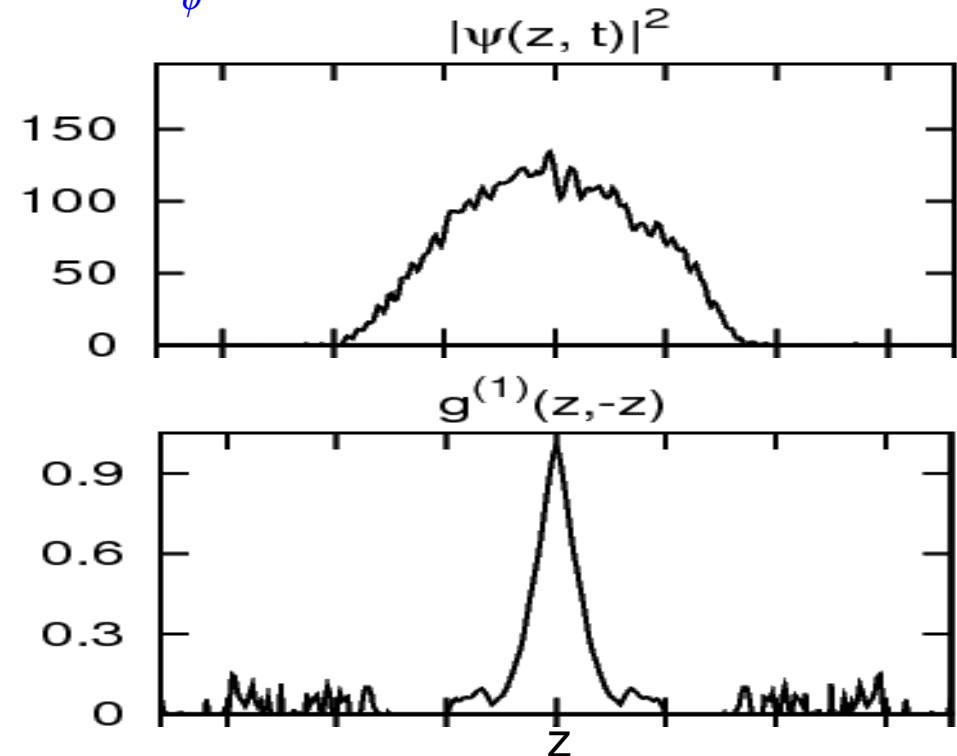


FIG. 1. Diagram of states for a trapped 1D gas.

D. Petrov, G. Shlyapnikov, J. Walraven, PRL **85**, 3745 (2000)



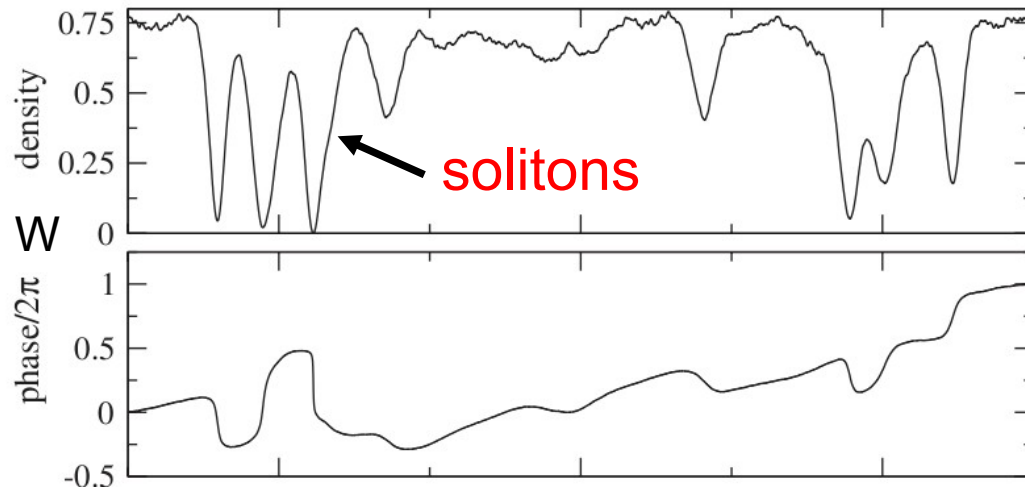
Motivation: What actually goes on during cooling?

View 1:

Solitons formed in a quench via
Kibble-Zurek mechanism

W. Żurek, PRL **102**, 105702 (2008)

B. Damski, W. Żurek, PRL **104**, 160404 (2010)

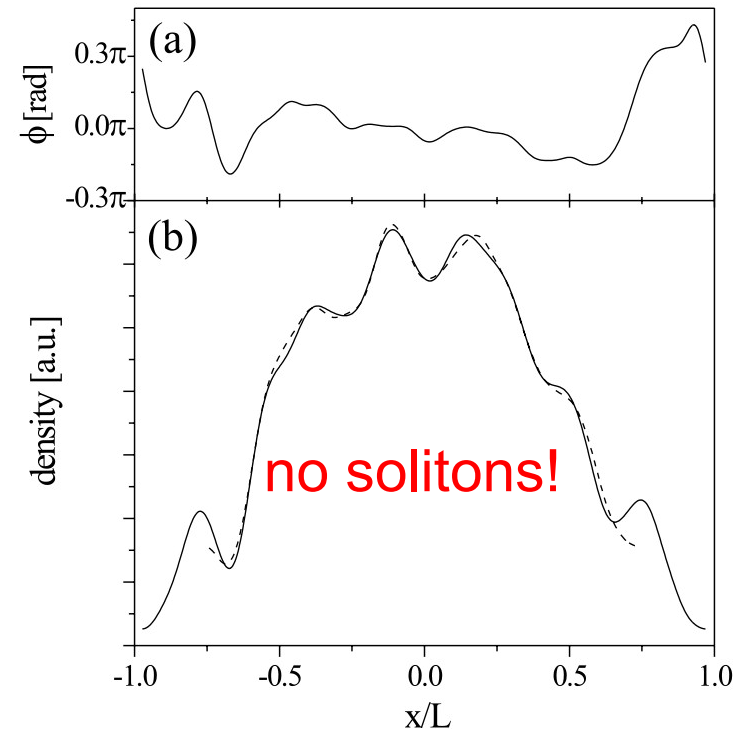


Quench of μ in a thermal bath

What about more realistic cooling?

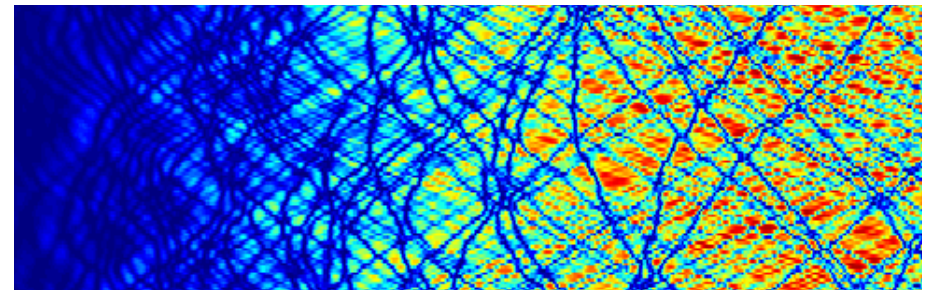
View 2:

Smooth quasicondensate
phase in thermal equilibrium



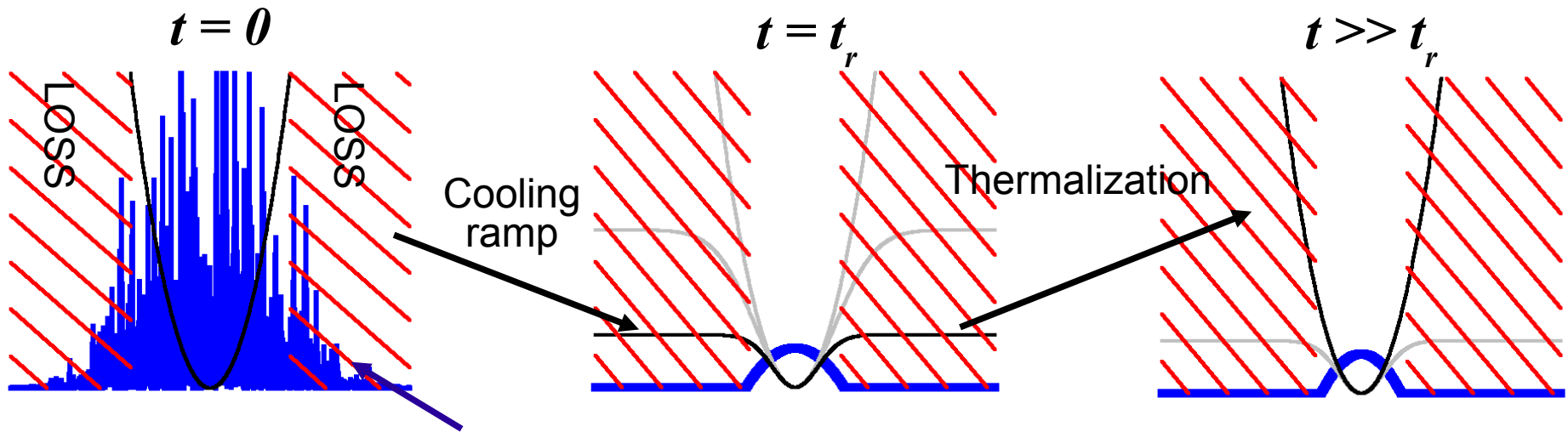
S. Dettmer *et al*, PRL **87**, 160406 (2001)

Also – How does the
quasicondensate actually form?



Evaporative cooling of 1D Bose gas

THE MODEL



- Initial condition: gas at thermal equilibrium, $l_\phi \ll L$

Simulation: *c-field method*

“Quantum field theory, without discretized particles”

$$\hat{\Psi}(x) \rightarrow \psi(x)$$

$$i\partial_t \psi(z, t) = [H(z, t) - i\Gamma(z, t)]\psi(z, t),$$
$$H(z, t) = -\frac{1}{2} \frac{\partial^2}{\partial z^2} + V(z, t) + g_{1D} |\psi(z, t)|^2$$

C-field initial conditions

e.g. in a plane wave basis

Full quantum field c-fields

$$\Psi(\mathbf{r}) = \sum_{\mathbf{k}} a_{\mathbf{k}} \frac{1}{\sqrt{V}} e^{i\mathbf{k}\mathbf{r}} \longrightarrow \Phi(\mathbf{r}) = \sum_{|\mathbf{k}| \leq \mathbf{K}_{\max}} \alpha_{\mathbf{k}} \frac{1}{\sqrt{V}} e^{i\mathbf{k}\mathbf{r}}$$

Replace mode amplitude operators $a_{\mathbf{k}}$
with complex number amplitudes $\alpha_{\mathbf{k}}$

Thermal Initial state

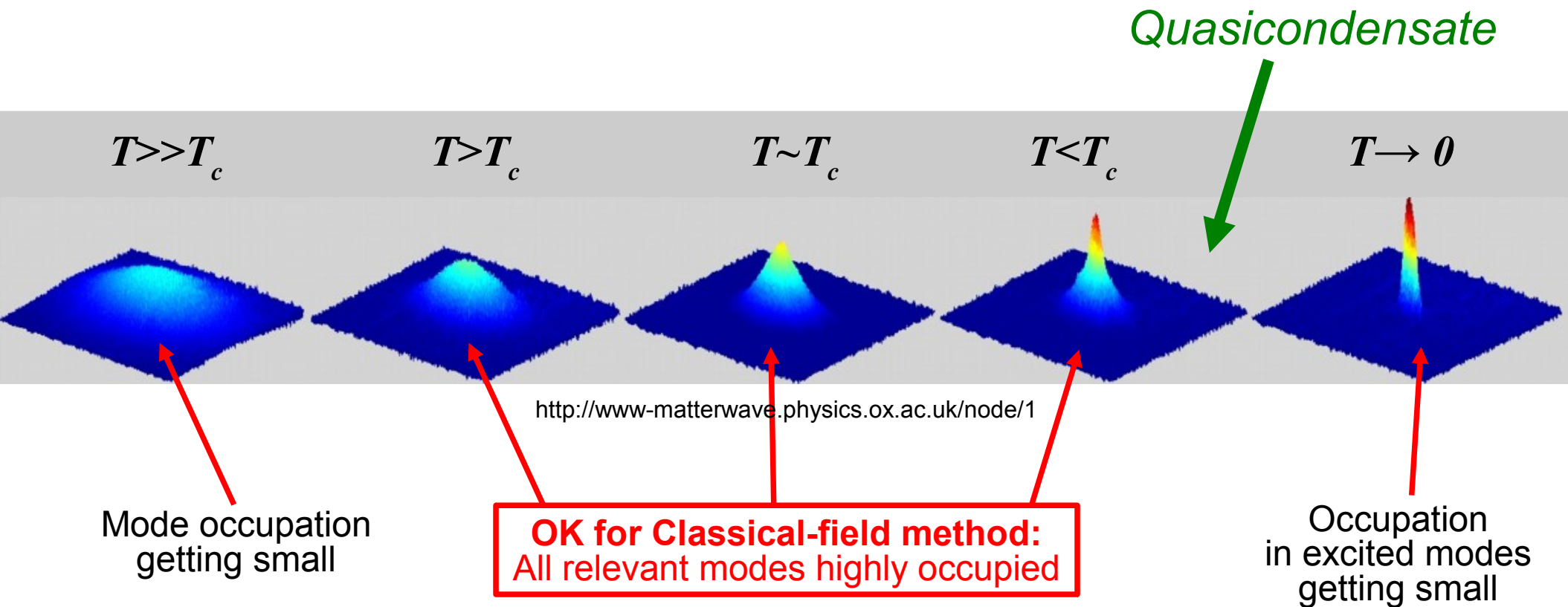
- $|\alpha_{\mathbf{k}}|^2$ Sampled by Metropolis Monte Carlo as per
E. Witkowska, M. Gajda, K. Rzażewski Opt. Commun. **283**, 671 (2010)
- Cutoff matched to give correct distribution of $N_{\text{incoherent}}$ for
an ideal Bose gas at chosen temperature
- Use many realizations to get thermal ensemble

Validity – my rough take on it

$$\left[\hat{\Psi}(x), \hat{\Psi}^\dagger(x') \right] = \delta(x - x') \quad \rightarrow \quad [\psi^*(x), \psi(x')] = 0$$

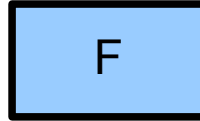
→ it will be fine, ...

*.... as long as there are always many atoms involved
in whatever it is we are studying*

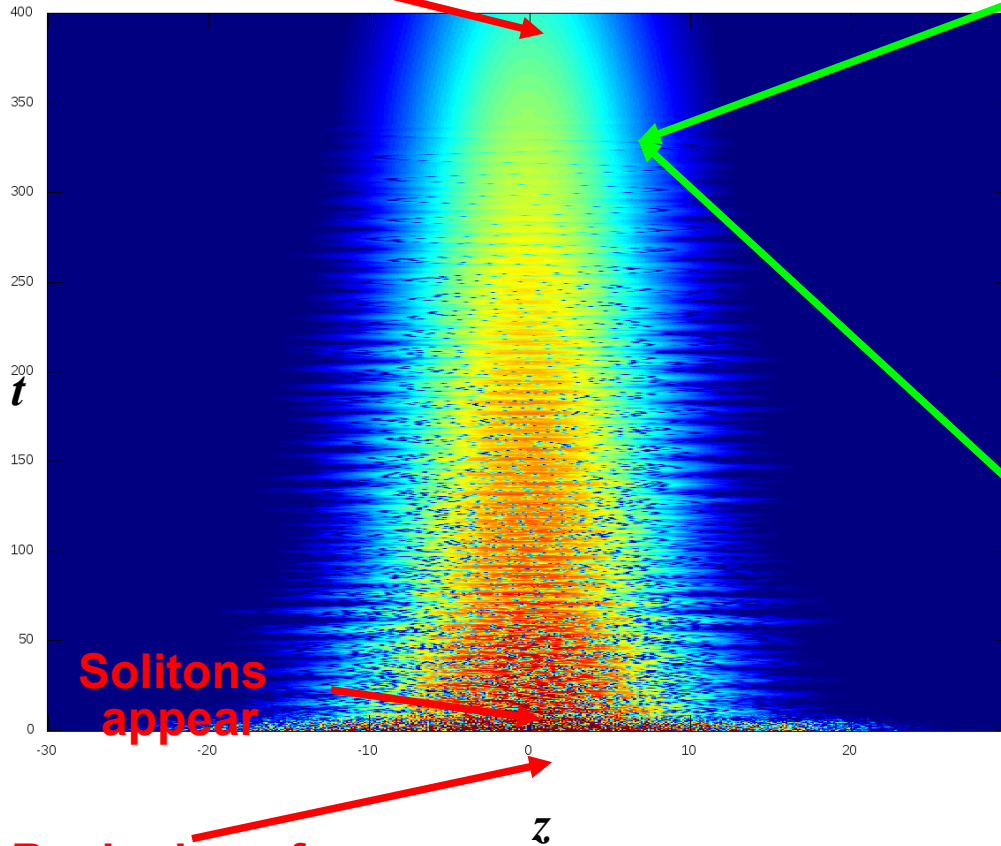


Simulation - slow ramp \rightarrow BEC

Slow ramp $\omega t_r = 400$



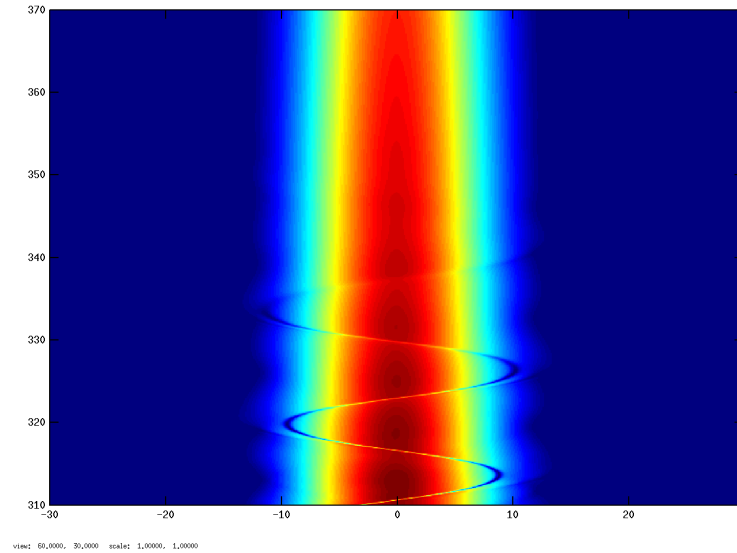
End of evaporation ramp



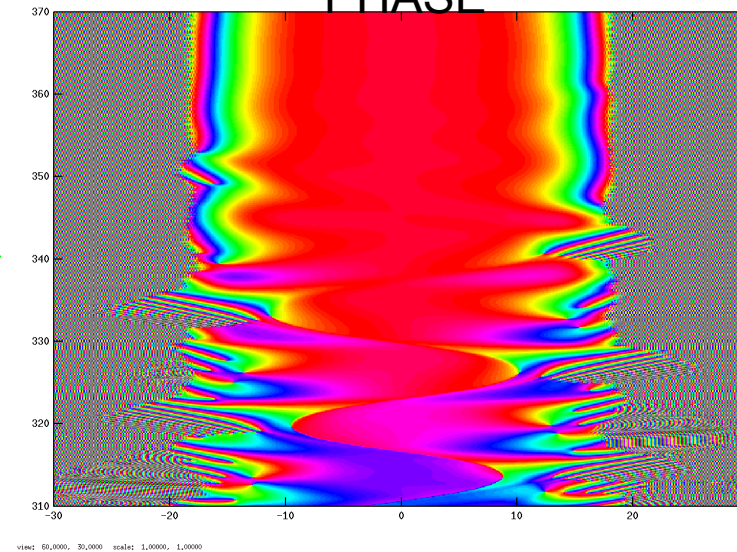
Solitons
appear

Beginning of ramp

DENSITY



PHASE



E. Witkowska, PD, M. Gajda, K. Rzażewski
Phys. Rev. Lett. **106**, 135301 (2011)

Thermalization to a quasicondensate

Fast ramp $\omega t_r = 75$

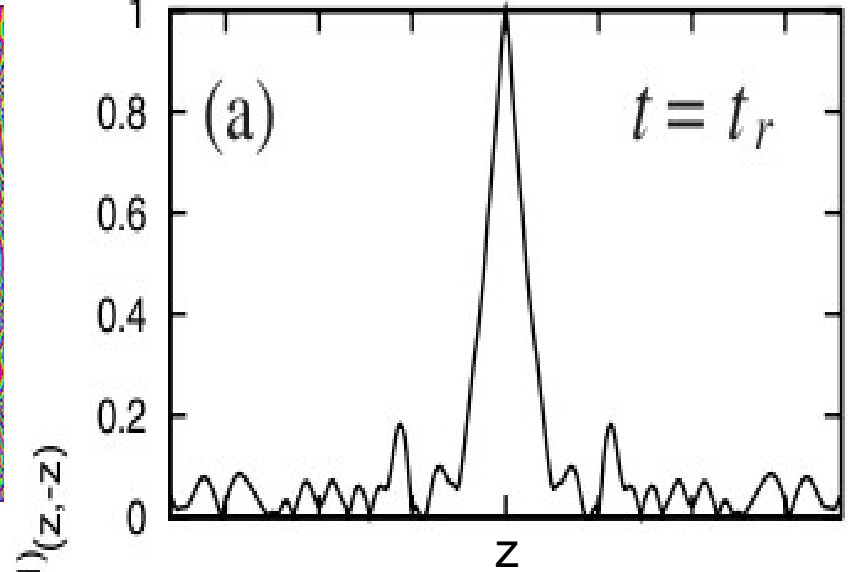
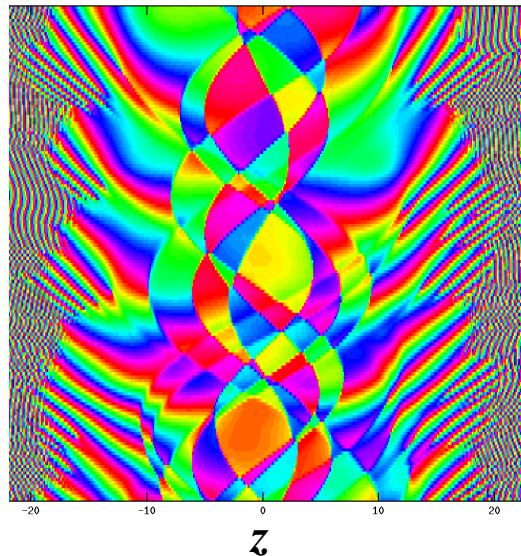
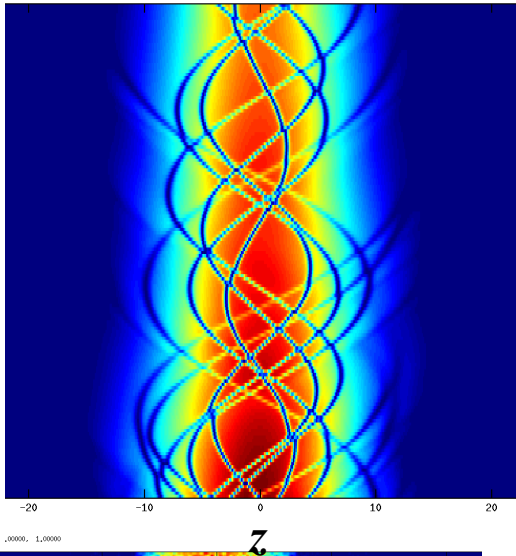
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DENSITY

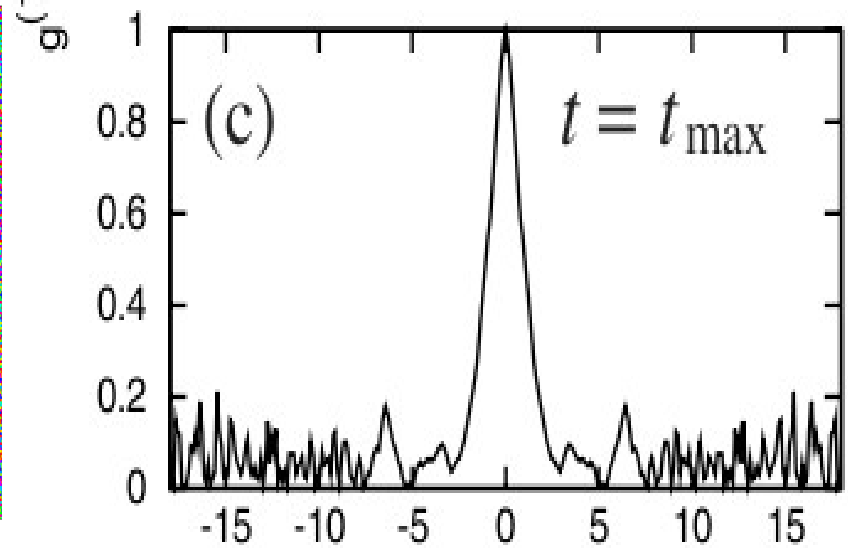
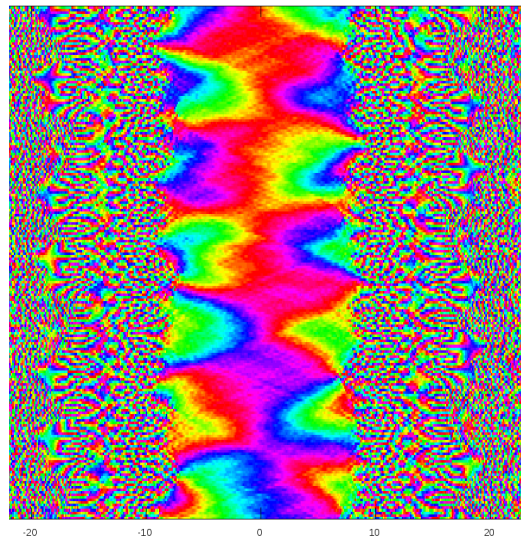
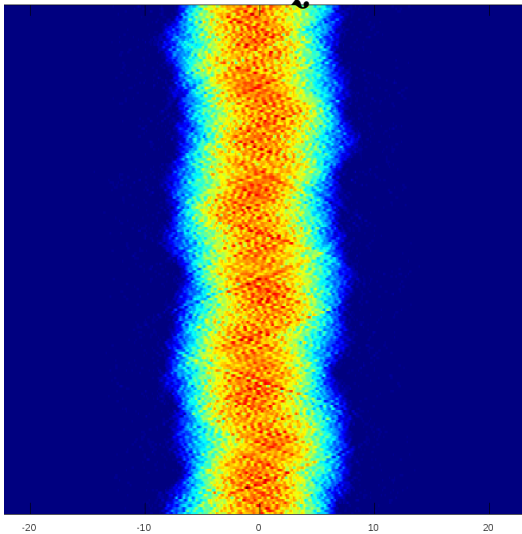
PHASE

COHERENCE

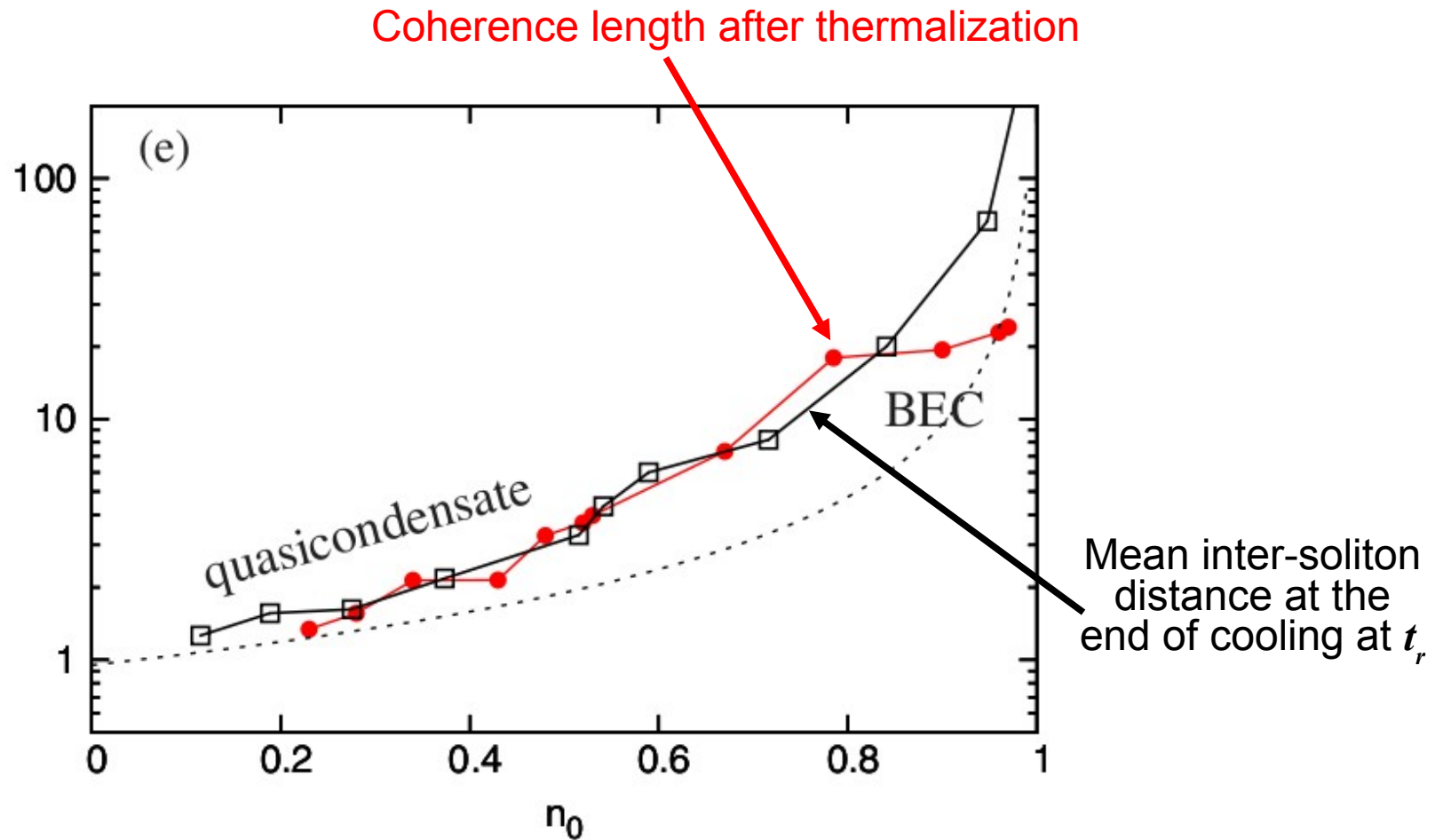
AFTER COOLING RAMP



THERMALIZED



Solitons as the “larval stage” of equilibrium fluctuations



E. Witkowska, PD, M. Gajda, K. Rzażewski
Phys. Rev. Lett. **106**, 135301 (2011)

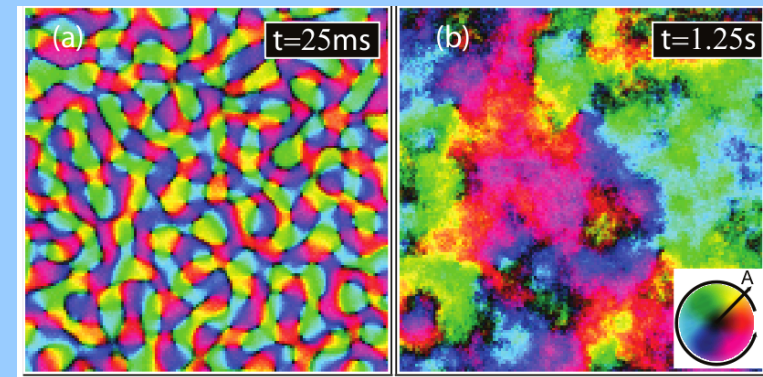
Quasicondensate formation

We did NOT see the usual Kibble-Zurek scenario

*“domain seeds grow with time,
and defects form where they meet”*

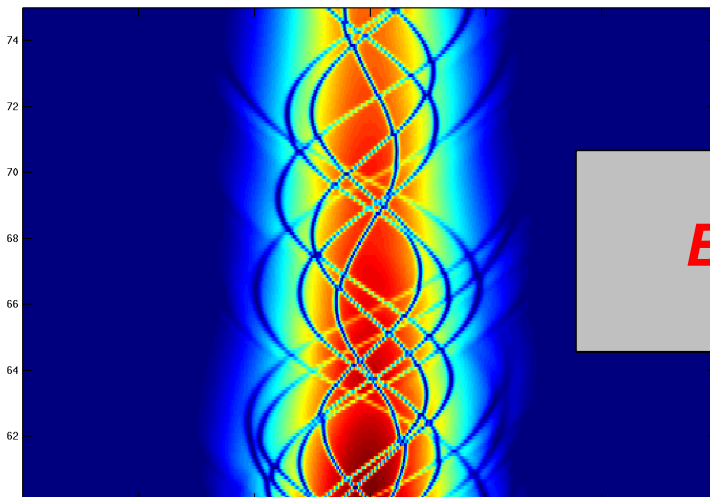
Instead:

- domains are fleeting
- solitons are the stable entities
- “domain size” conserved

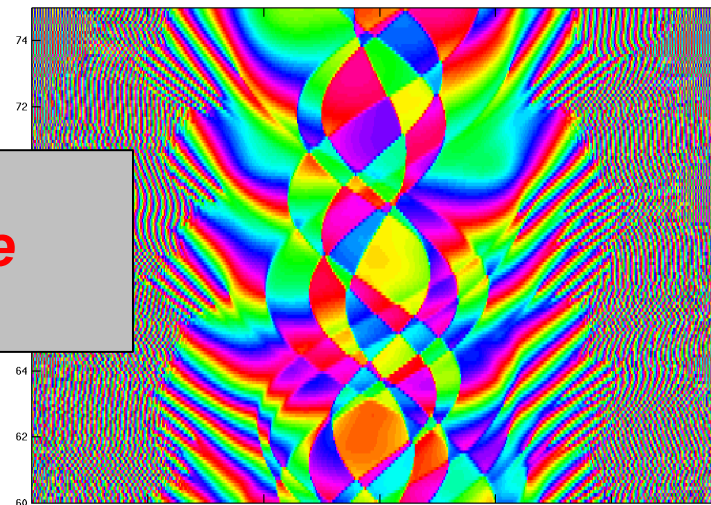


compare e.g. to 2D spin-1 gas:
Barnett, Polkovnikov, Vengalattore,
arXiv:1009.1646

DENSITY



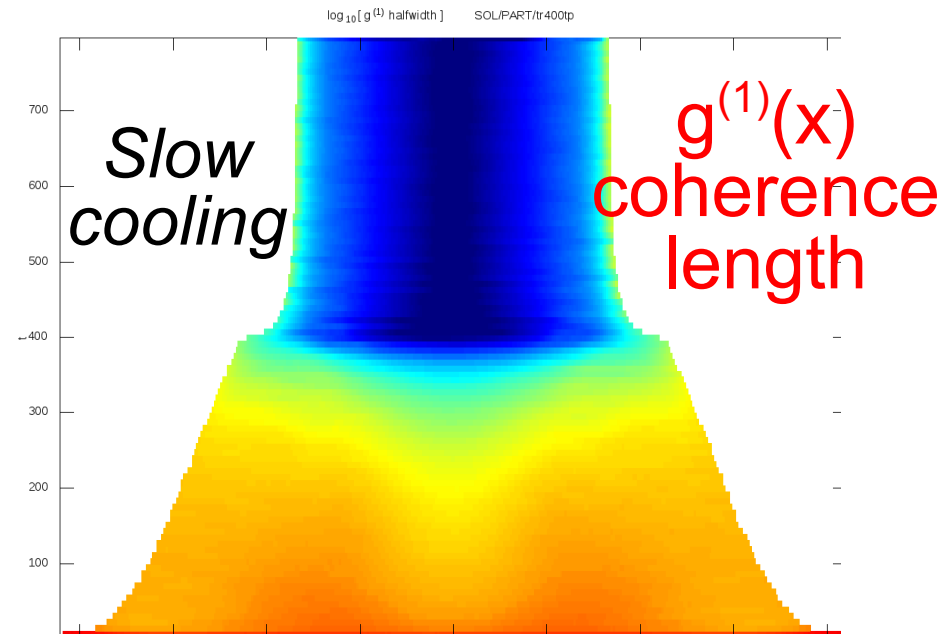
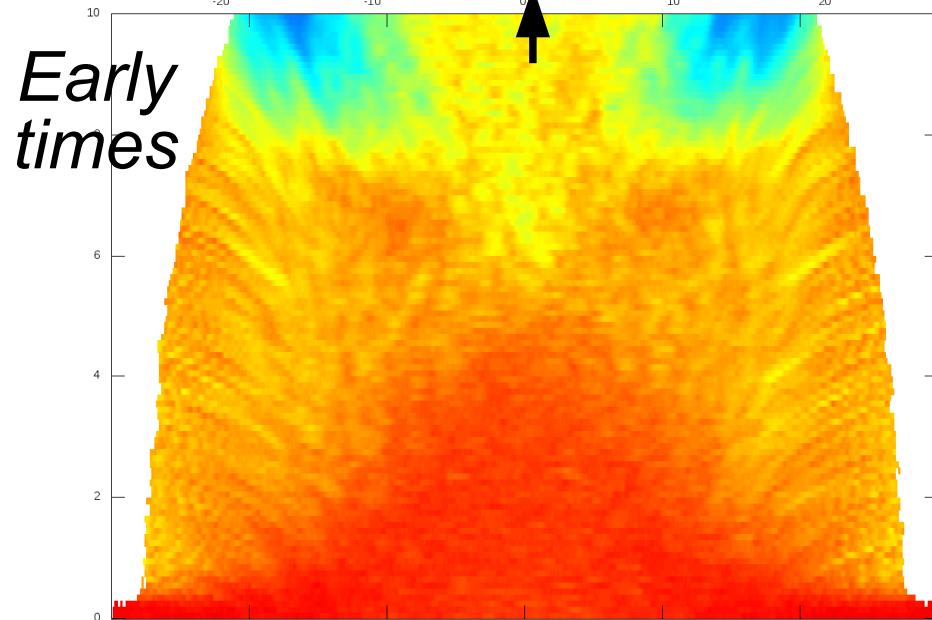
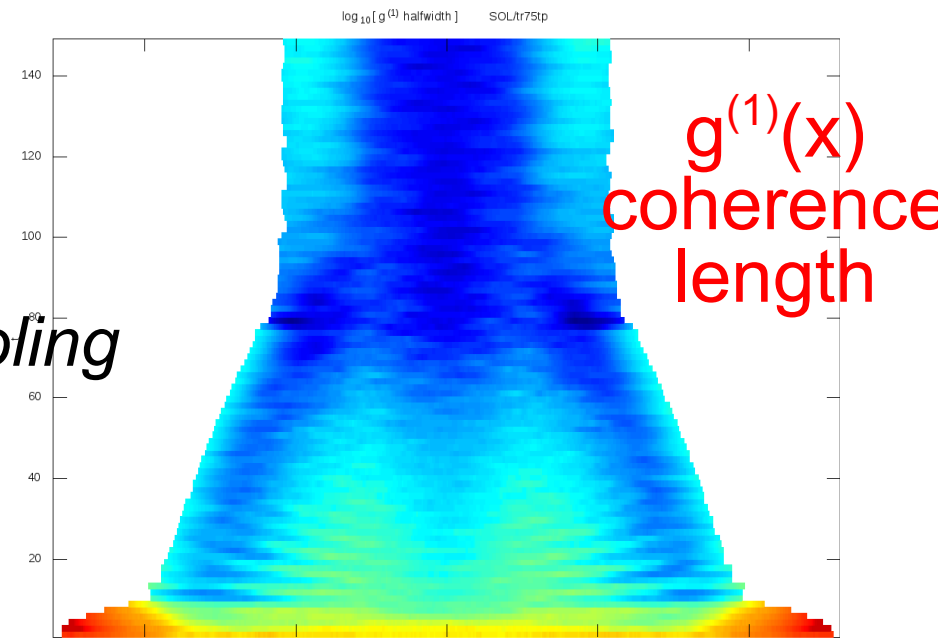
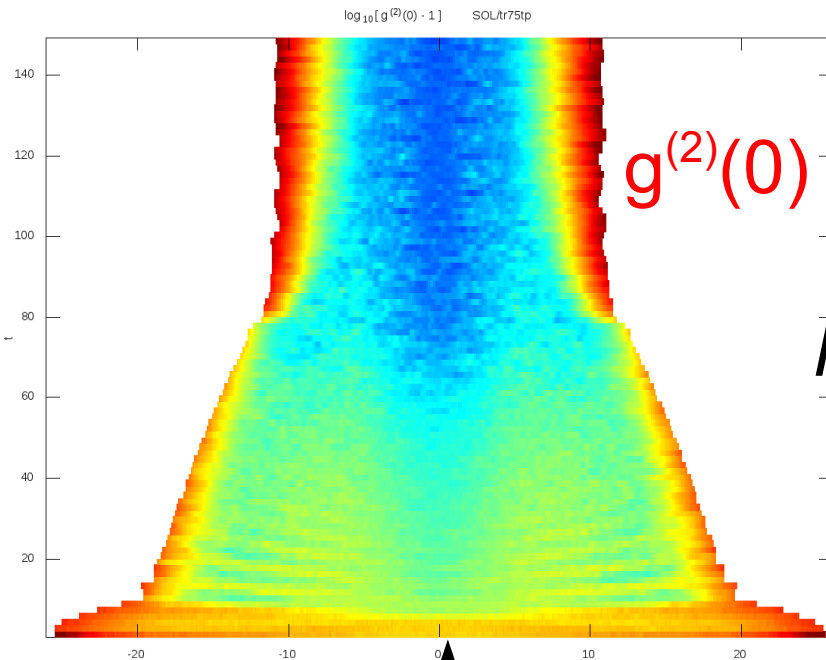
PHASE



BKT-like

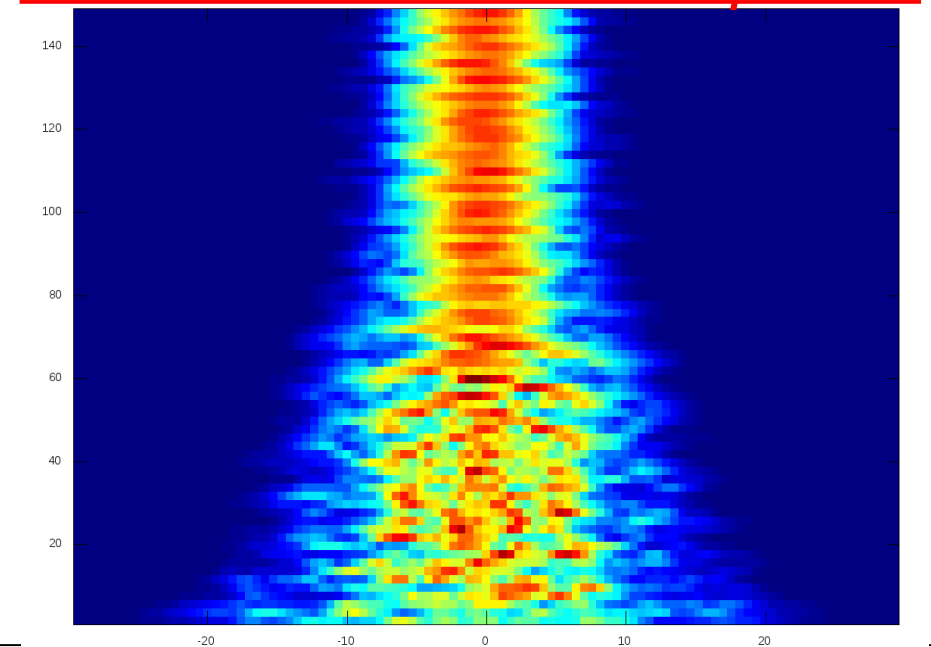
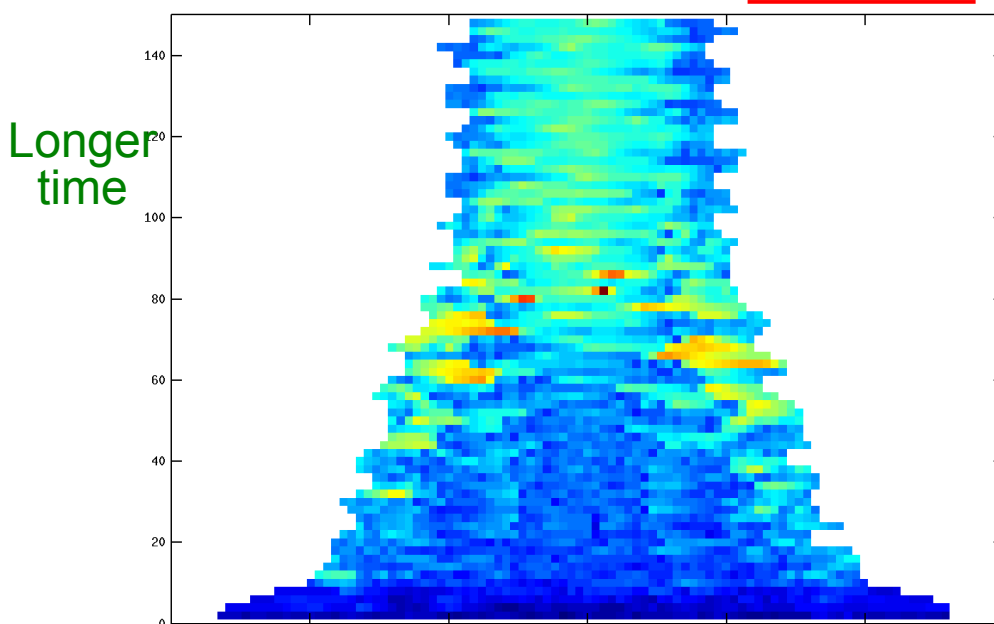
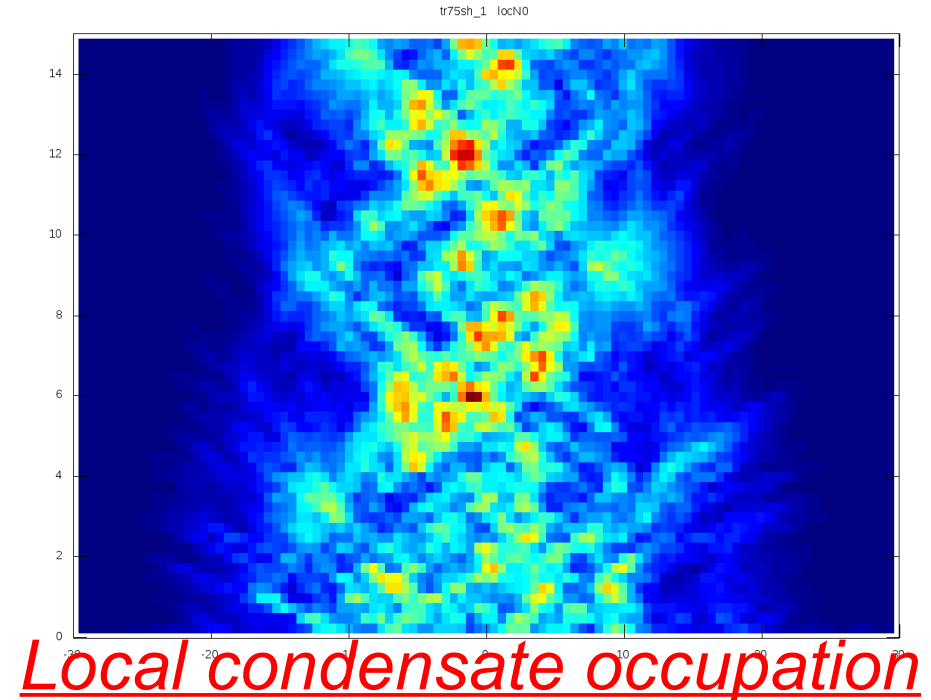
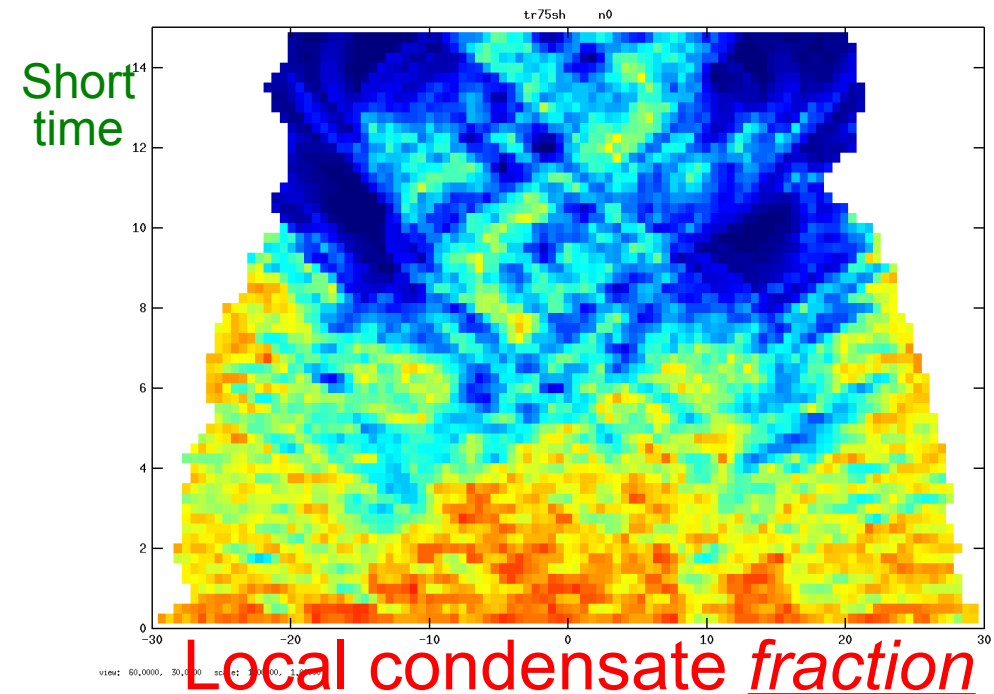
“Local” thermodynamics – ensemble averages

- average over trajectories

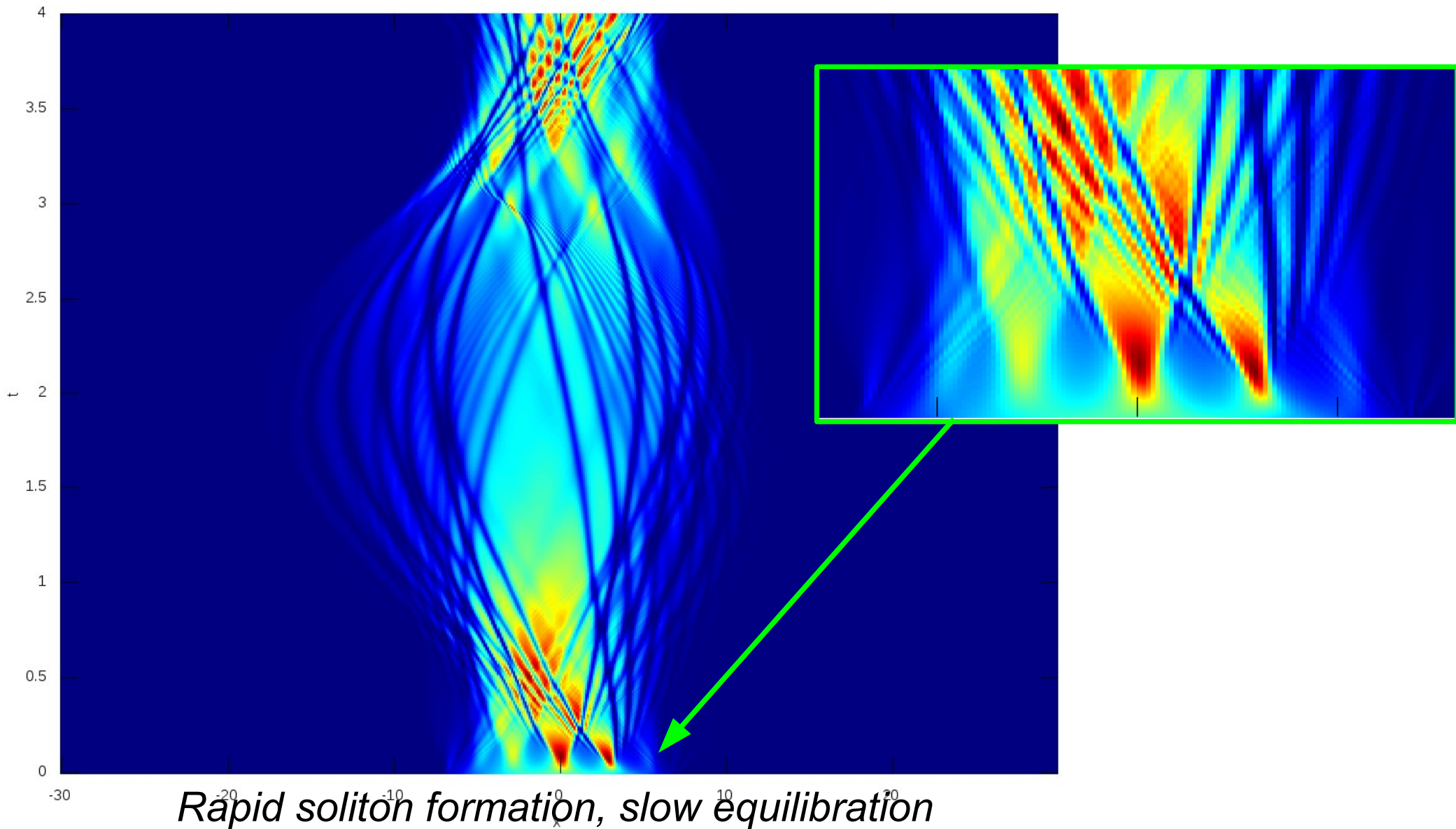


“Local” thermodynamics – 1-trajectory

Chop into small boxes in x-t space, calculate local condensate fraction, coherence length



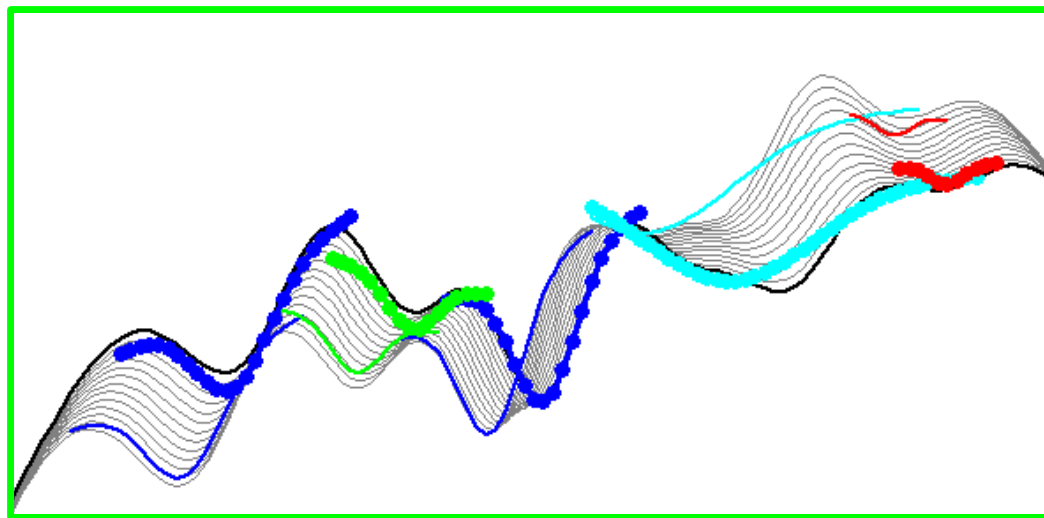
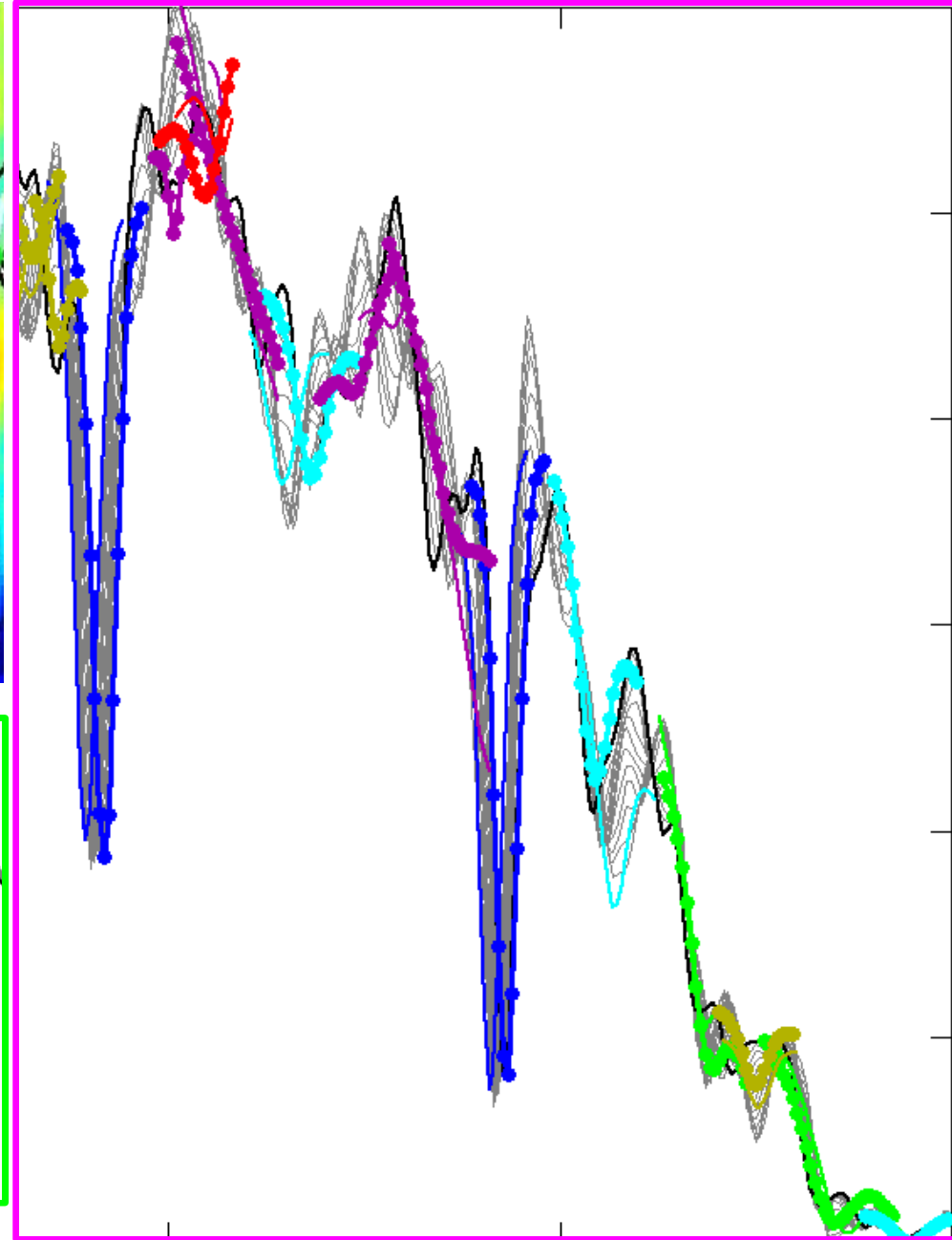
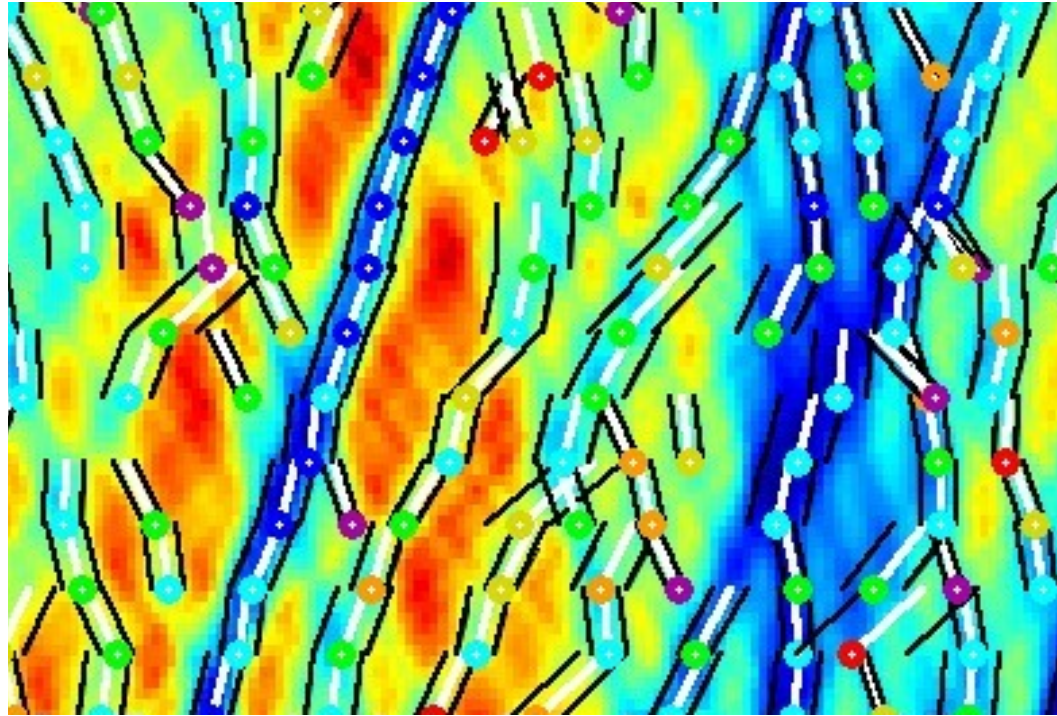
Quasicondensate and density fluctuations



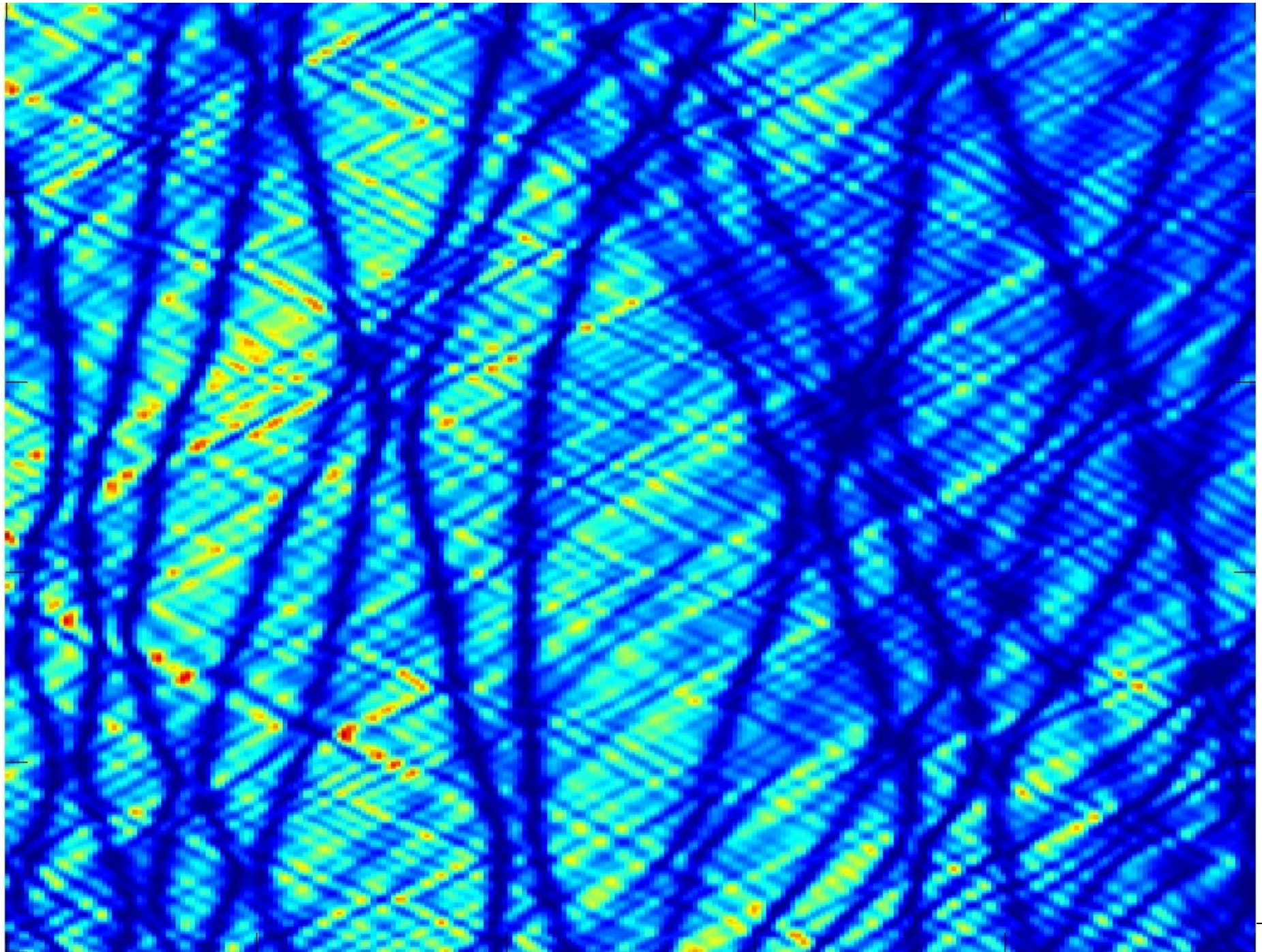
Initial classical field described by model of
Phase-fluctuating 3D condensates in elongated traps
Petrov, Shlyapnikov, Walraven, PRL **87**, 050404 (2001)

Soliton statistics - automation

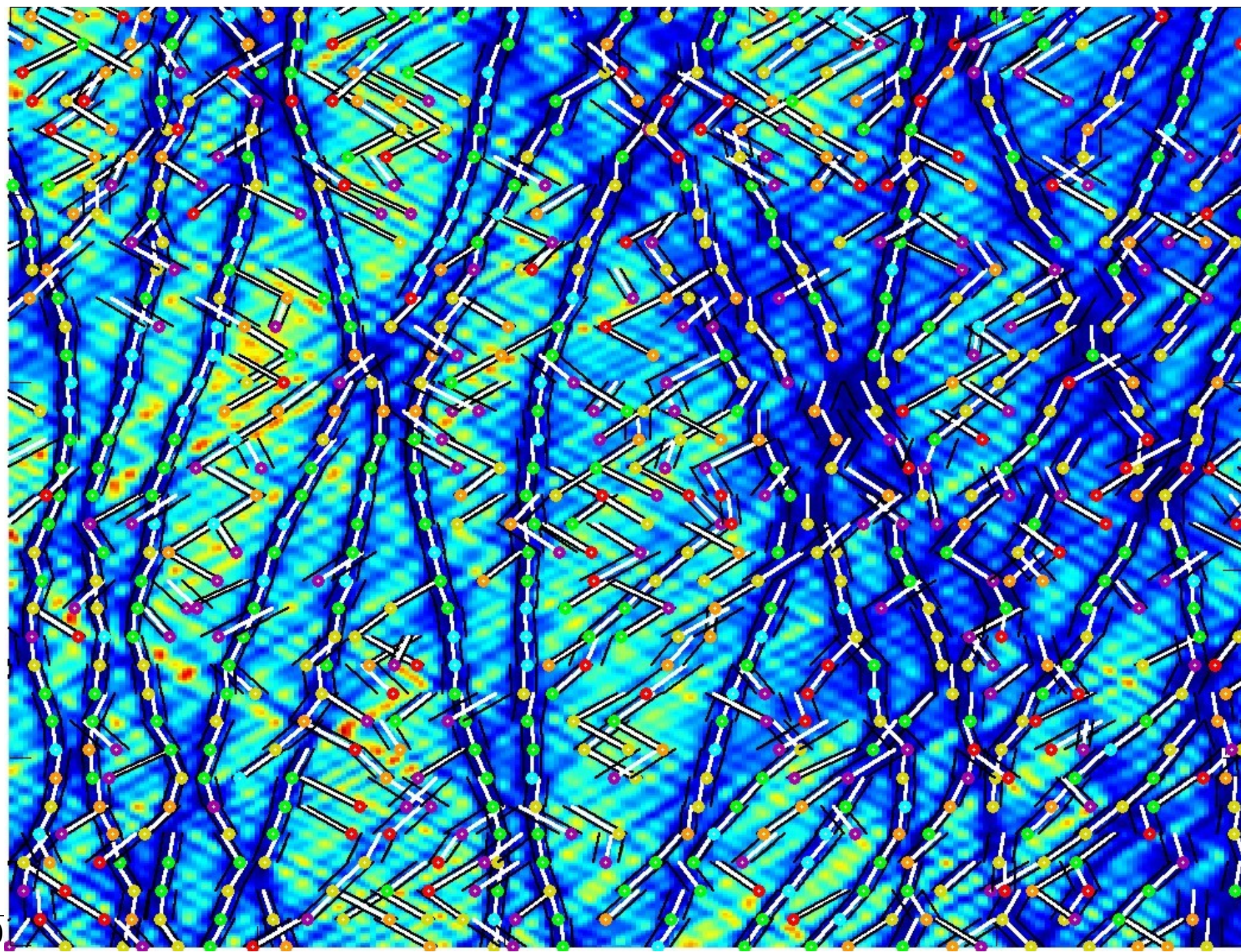
*Fitting of Gaussian dips over a finite box in x **AND** t space*



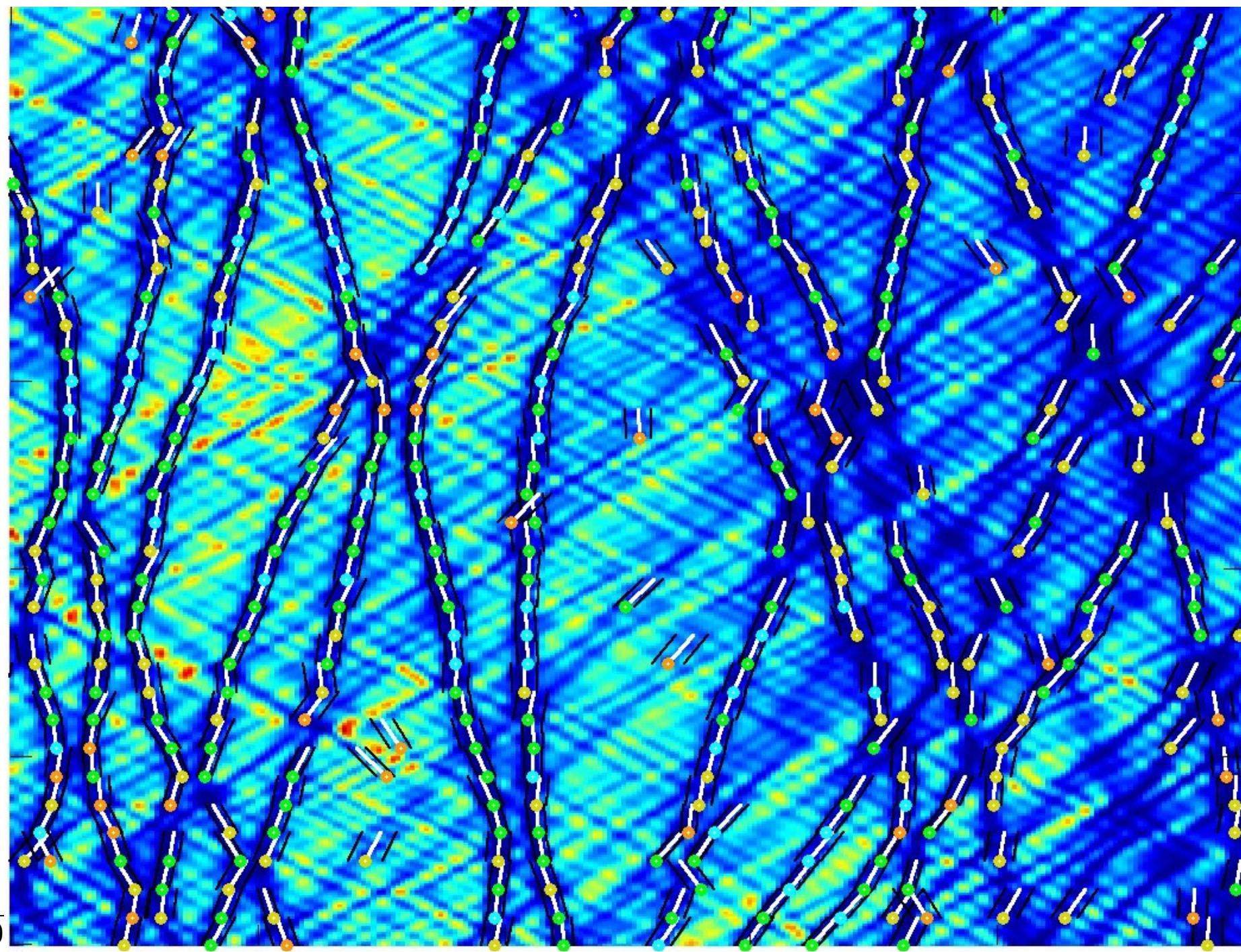
Soliton statistics – before fit



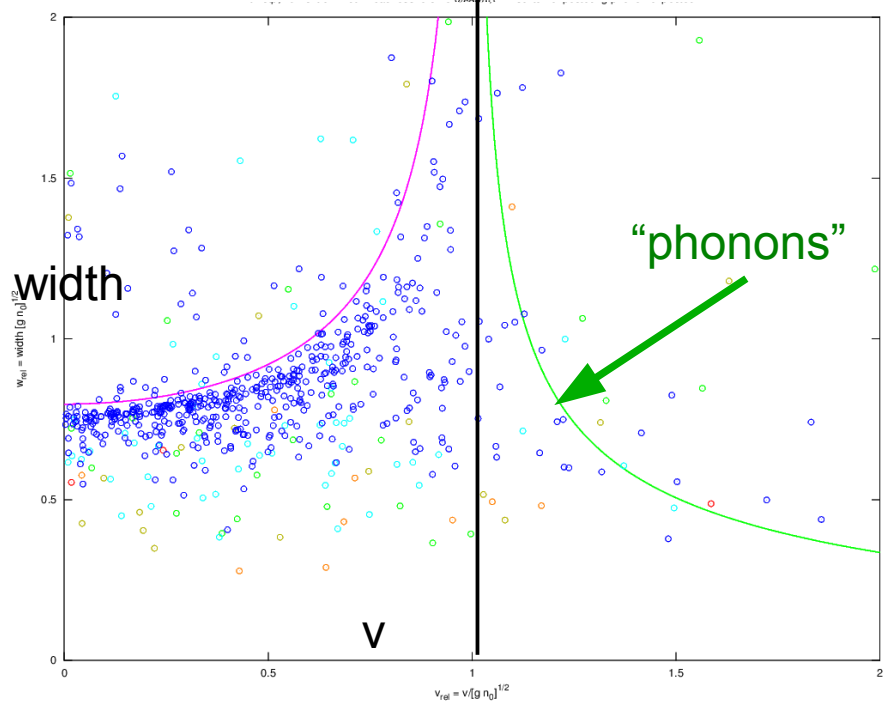
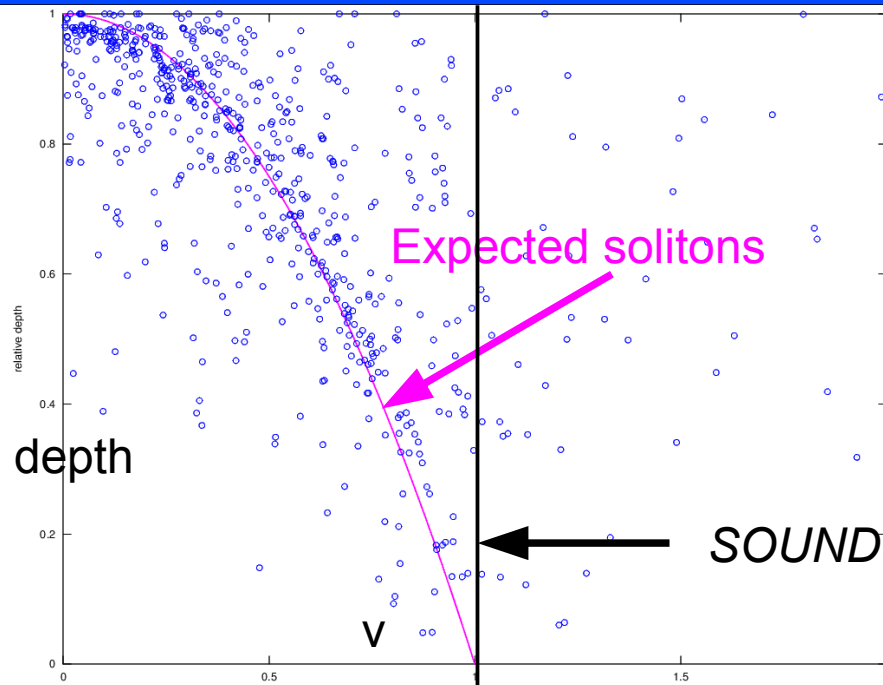
Soliton statistics – fit



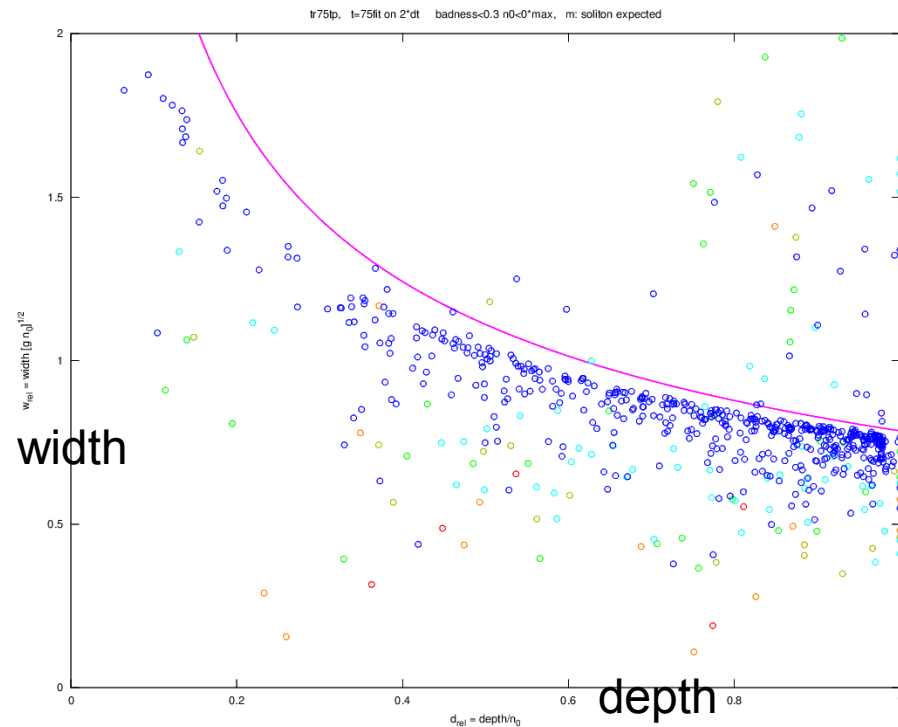
Soliton statistics – filtering



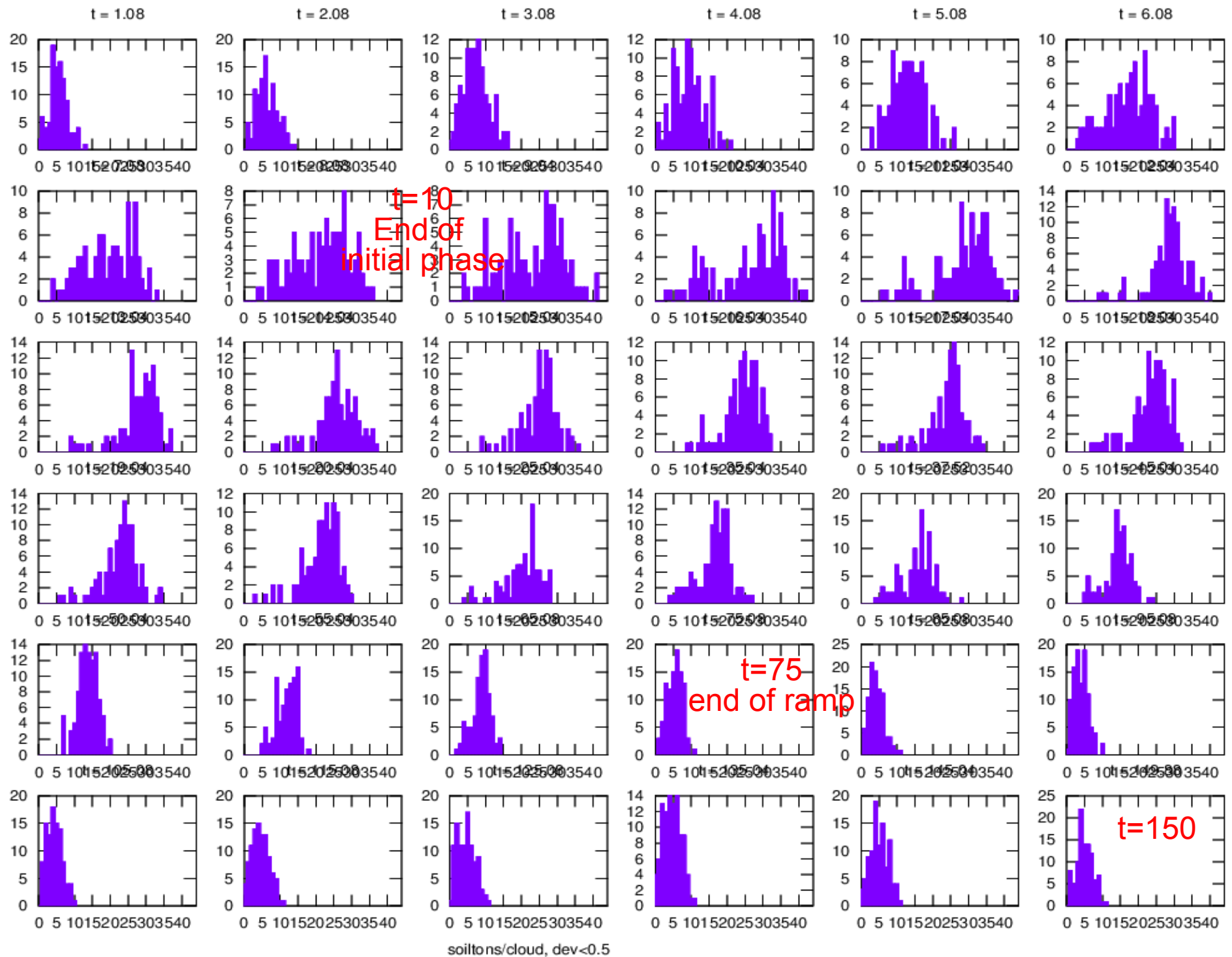
Filtering



Remove fits far from “soliton” lines from consideration



Soliton statistics - evolution



Conclusions: what of the original questions?

- Kibble-Zurek mechanism?

Confusion has been sown

- Defects \leftrightarrow quasicondensate?

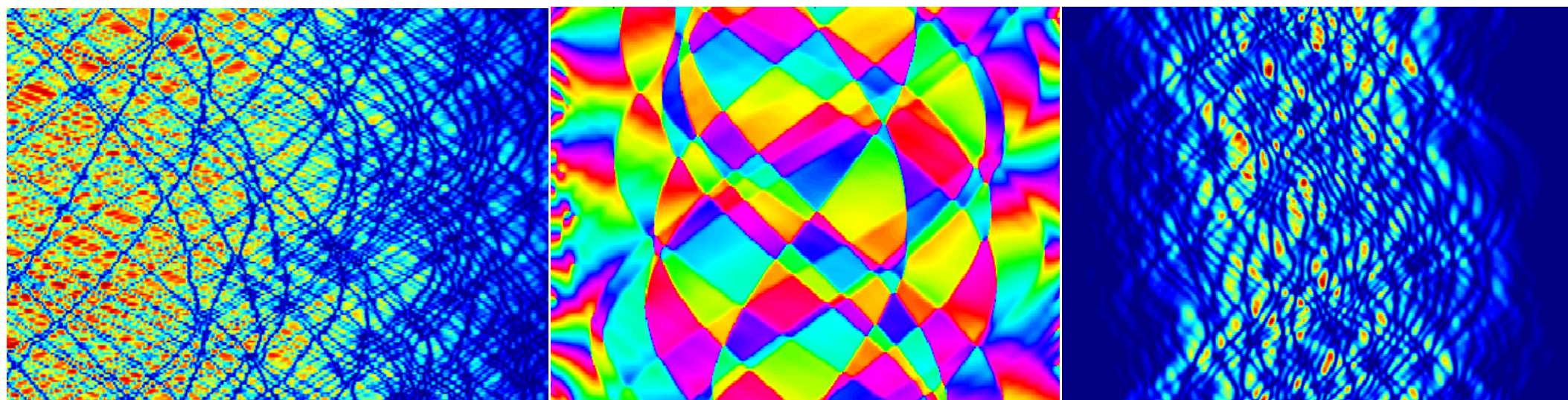
Solitons = "larval stage" of long time phase fluctuations

- Time scales

Rapid soliton formation, slow equilibration

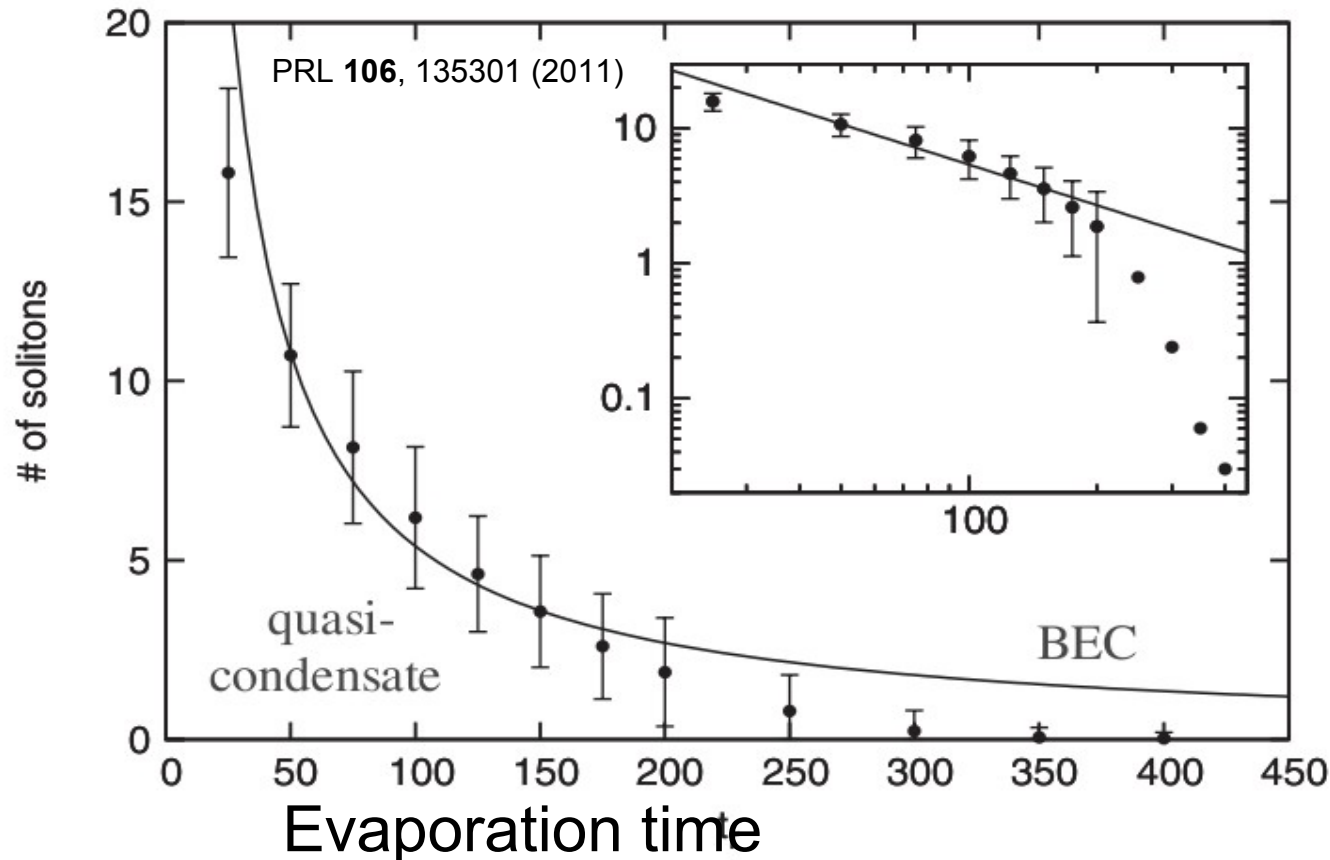
- Analysis

Local-thermodynamics and automated soliton counting possible



Kibble-Zurek defect formation

Soliton number at the end of the ramp



prediction:

W. Zurek, PRL **102**, 105702 (2009)

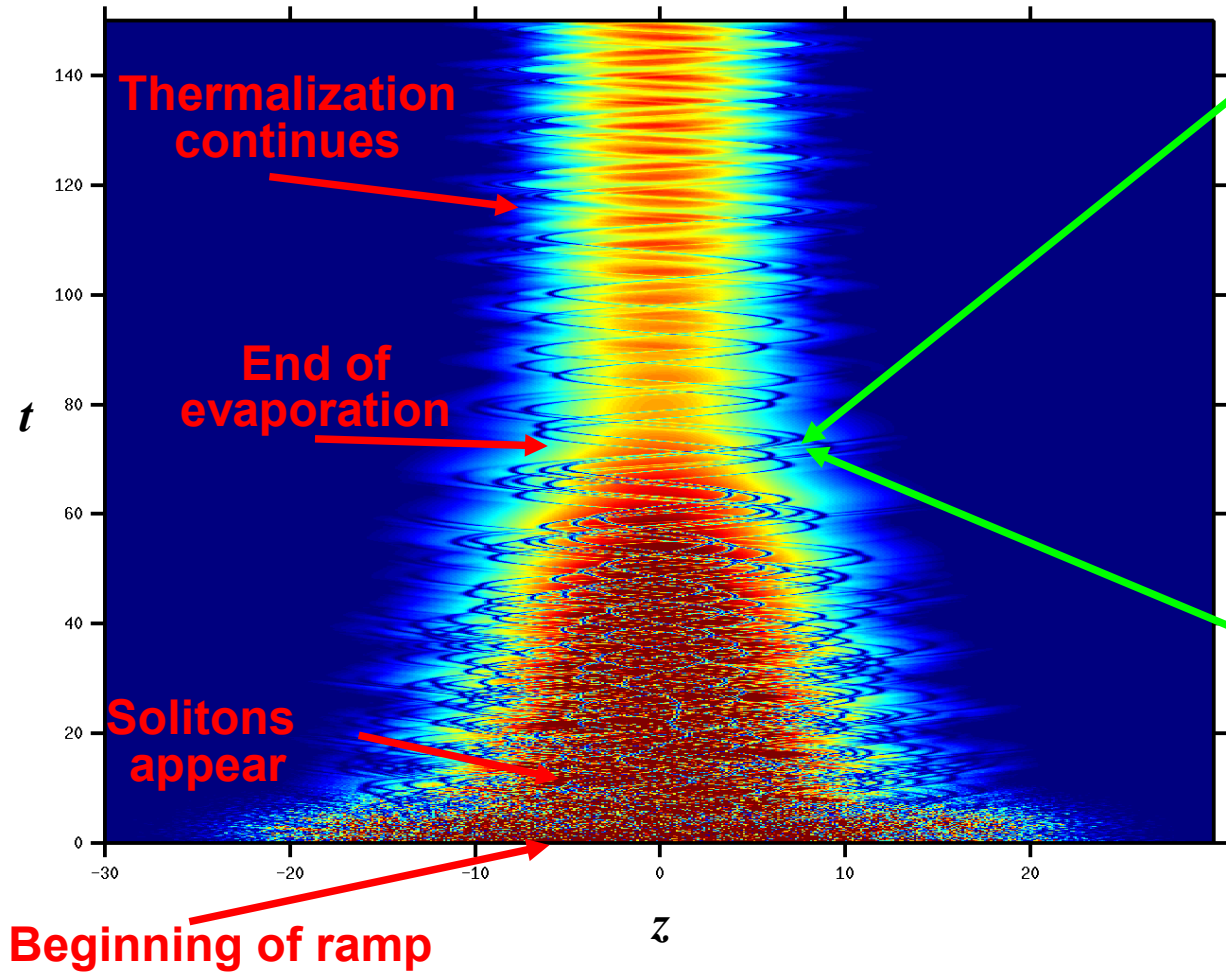
$$N \approx \frac{2\hat{X}}{f\hat{\xi}} = \frac{2\Delta^2}{f\lambda_{\text{dB}}^2} \left(\frac{\tau_0}{\tau_Q} \right)^{(1+2\nu)/(1+\nu z)}$$

Quench time
= evaporation time t ?

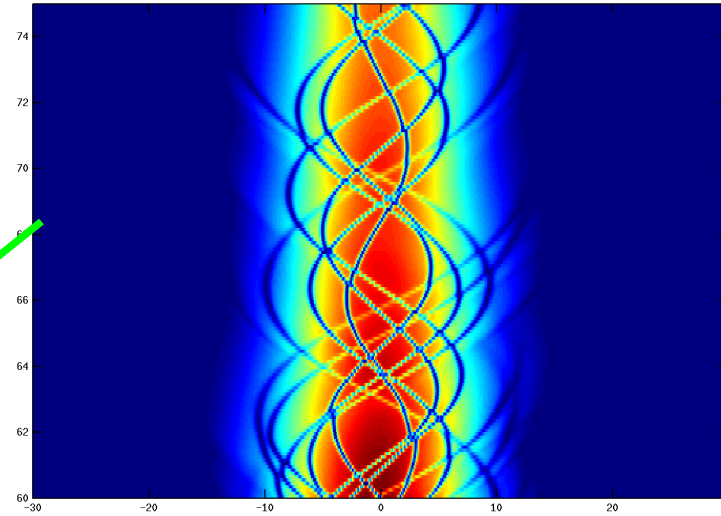
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Fast ramp \rightarrow quasicondensate precursor

Fast ramp $\omega t_r = 75$



DENSITY

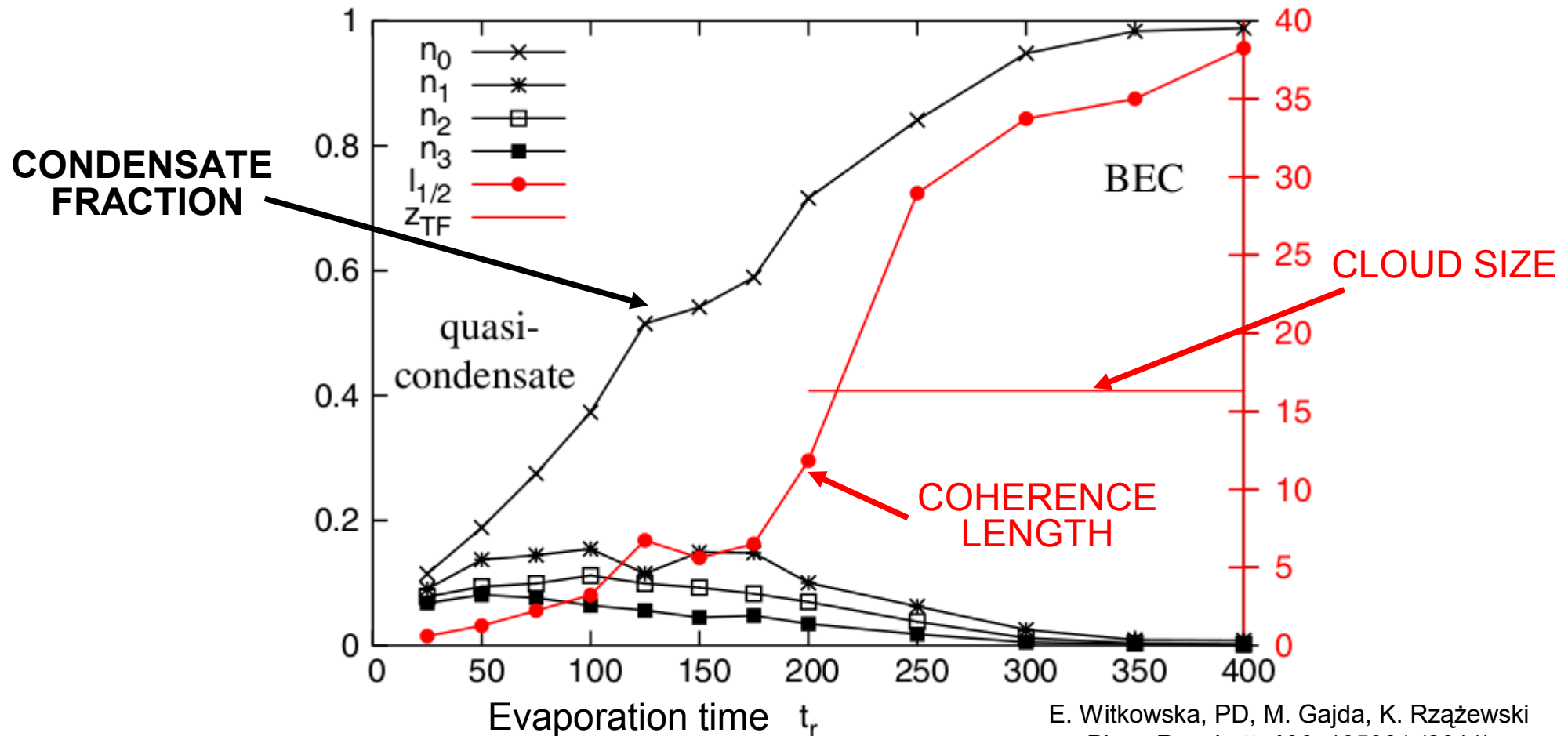


PHASE



Ramp time

- Slow ramp \rightarrow BEC
- Fast ramp \rightarrow quasicondensate
- Very fast ramp \rightarrow thermal gas



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