Excitations of ultracold Fermi dipolar gases

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Overview

- Motivation
  - Comparison with standard BCS gas
  - Condensed matter analogue
  - Recent experimental values with KRb
- The uniform 3D gas
- Quasiparticle spectrum
  - Gap zero
- Collective excitations
  - $T \sim T_c$
  - $T \rightarrow 0$
  - New regime: $\hbar\omega \ll T \ll T_c$
  - Deflected superfluid current

BCS superfluidity

dipole–dipole potential	contact s-wave $\uparrow\downarrow$ potential
• long range interaction $\longrightarrow$ Needs 1 spin component	• short range interaction $\longrightarrow$ Needs 2 spin components (Pauli blocking)
• *Anisotropic*	• *Isotropic*
• always partly attractive BCS pairing if *polarised*	• arttractive or repulsive BCS pairing only if $a_s < 0$
• Energy gap has nodes	• Energy gap always $\neq 0$

Condensed matter analogue

- The node structure of the order parameter is similar to that of solid state and liquid He phases, e.g.:
  - Polar phase of $^3\mathrm{He}$.
    (Never experimentally realized) Aoyama & Ikeda PRB 73, 060504 (2006), Elbs etal. arXiv:0707.3544
  - Heavy-fermion superconductors like UPt_3.
    (Difficult to get pure system, many potential phases)

- Dipole gas is a much "cleaner" system.
  - $\widehat{H}$ well known
  - spin degrees of freedom can be removed.

- Potentially well controllable $[: -)]$

Critical Temperature for BCS

s-wave ↑↓ gas:

$$T_c = 0.28\, E_F \exp\left(-\frac{\pi}{2|a_s|k_F}\right)$$

Dipole gas:

Baranov, Mar'enko, Rychkov, Shlyapnikov, PRA **66**, 013606 (2002)

$$T_c = 1.44\, E_F \exp\left(-\frac{\pi}{2|a_D|k_F}\right)$$

$\Longrightarrow$ *Effective* scattering length a_D:

$$a_D = -2m\left(\frac{d}{\pi\hbar}\right)^2$$

T_c rises strongly with $a_D \propto md^2$

Experimental values with $^{40}\mathrm{K}^{87}\mathrm{Rb}$

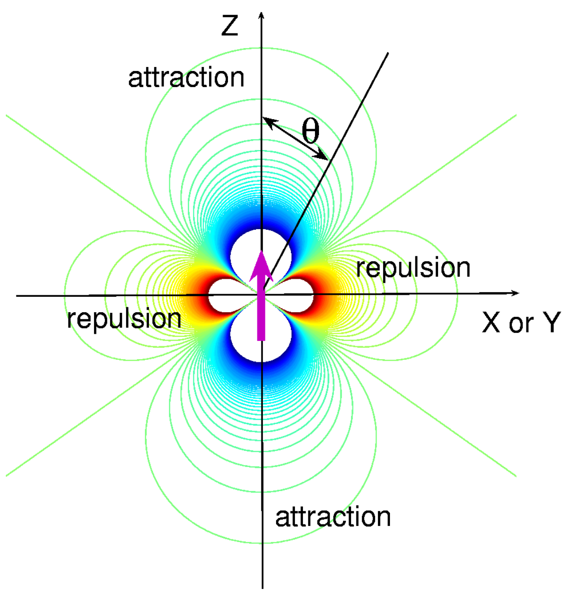
D. Jin group, JILA, based on talk by K.-K. Ni at DAMOP

- Molecules formed via STIRAP (65% efficiency) in deeply bound (7 THz) triplet vibrational state.
- Density $\sim 10^{12}/\mathrm{cm}^3$
- $T = 3T_F$ (300 nK) $: -)$
- Dipole moment $d \approx 0.1D$. (Hence $a_D \approx -500\mathrm{nm}$)
- Lifetime: several 100μs. $: -($
- They expect to be able to go deeper to $d \approx 1D$
  Then, one would have $T_c \sim T_F$

Our physical system

uniform 3D gas

$$V_D(R,\theta) = \frac{d^2}{R^3}\left(1 - 3\cos^2\theta\right)$$



- Cold: $T < T_c^{BCS}$
- static external field (E or B)
  $\Longrightarrow$ full polarisation
- single-species (spin polarised)
- dilute $\Longrightarrow$ Energy dominated by Fermi sea to leading order
- short-range interaction assumed negligible (Fermi exclusion, no p-wave resonances)

Uniform gas: Motivation

- Global shape of trapped cloud dominated by Hartree energy:

$$E_d \approx \int d^3x\, d^3y\, V_d(x-y)\, \langle n(x)\rangle\, \langle n(y)\rangle$$

- Not very sensitive to temperature

- Statics and dynamics of the shape of a trapped cloud

  Theory: Góral, Englert, Rzążewski PRA **63**, 033606 (2001)

  Góral, Brewczyk, Rzążewski PRA **67**, 025601 (2003)

  Baranov, Dobrek, Lewenstein PRL **92**, 250403 (2004)

  Experiment: TBA?

- Essential features of superfluid physics seen best in uniform system (local density approximation).

Hamiltonian

$$\widehat{H} = \text{K.E.} + \frac{1}{2}\int d^3x\, d^3y \left\{ \widehat{\Psi}_x^\dagger \widehat{\Psi}_x \, V_D(x-y) \, \widehat{\Psi}_y^\dagger \widehat{\Psi}_y \right\}$$

- $\widehat{\Psi}_x$ is the anihilating Fermi field operator at point x.

BCS Mean field theory: Postulate the quadratic effective Hamiltonian:

$$\widehat{H}_{\text{eff}} = \frac{1}{2}\int d^3x\, d^3y \Big\{ \quad \frac{\hbar^2}{m} \widehat{\Psi}_x^\dagger \nabla^2 \widehat{\Psi}_x \delta(x-y) \qquad \textit{Kinetic}$$

$$\Delta^*(x-y)\,\widehat{\Psi}_x \widehat{\Psi}_y - \Delta(x-y)\,\widehat{\Psi}_x^\dagger \widehat{\Psi}_y^\dagger \qquad \textit{BCS}$$

$$+W(x-y)\,\widehat{\Psi}_x^\dagger \widehat{\Psi}_y \quad \Big\} \qquad \textit{Hartree}$$

- With some “appropriate” $\Delta(x-y)$ and $W(x-y)$

Gap equation

Choose $\Delta(x-y)$ and $W(x-y)$ to minimise the full Free energy

$$F = \langle \widehat{H} \rangle_{\text{eff}} - \mu N - TS$$

when calculated with eigenstates of $\widehat{H}_{\text{eff}}$.

Obtain:

$$\Delta(x-y) = V_D(x-y) \left\langle \widehat{\Psi}_x \widehat{\Psi}_y \right\rangle_{\text{eff}}$$ *GAP*

$$W(x-y) = -V_D(x-y) \left\langle \widehat{\Psi}^\dagger_x \widehat{\Psi}_y \right\rangle_{\text{eff}}$$ *"Hartree" field*

Δ, W and Ψ must be self-consistent.

Uniform gas

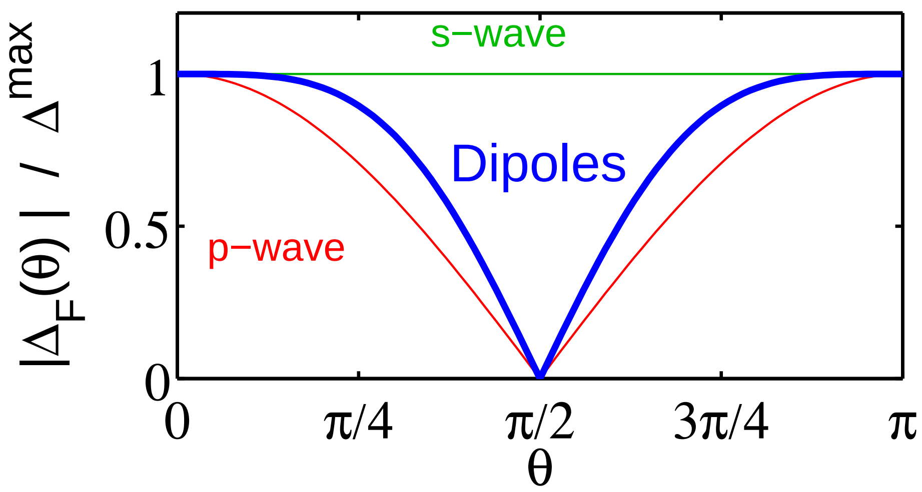
In k–space

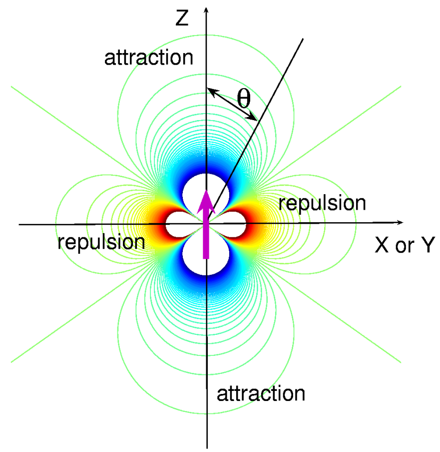
$$\widehat{H}_{\text{eff}} = \frac{1}{2}\int d^3k \left\{ \left(\frac{\hbar^2 k^2}{m} - 2\mu - W(k) \right) \widehat{\Psi}^\dagger_k \widehat{\Psi}_k + \Delta^*(k) \widehat{\Psi}_k \widehat{\Psi}_{-k} - \Delta(k) \widehat{\Psi}^\dagger_k \widehat{\Psi}^\dagger_{-k} \right\}$$

- Order parameter $\Delta(k) \neq 0$ corresponds to BCS pairing of k and $-k$ atoms.
- $\Delta(k)$ is anisotropic and has nodes on the Fermi surface (unlike s-wave $\uparrow\downarrow$gas)
- $W(k)$ is a minor energy shift of Fermi surface

  $\Longrightarrow$ ignore it in leading order

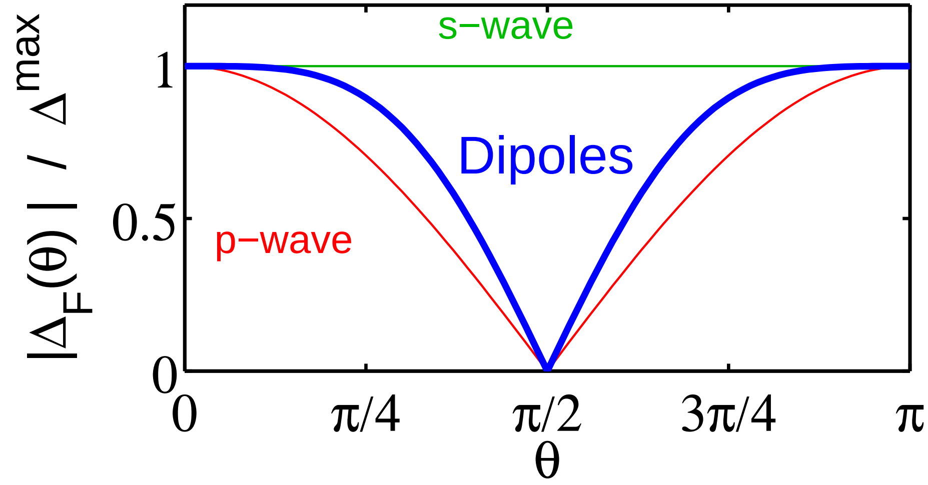
BCS gap $\Delta_F(\theta)$ on Fermi surface





- NODE in plane $\perp$ to polarisation
- Breaking a pair costs $\geq 2|\Delta(\theta)|$.
- Dipoles: Easy to excite a pair in plane $\perp$ to polarisation because energy cost is small.
- $\uparrow\downarrow$gas: Appreciable energy cost of excitations always.

Consequences of pole in Δ



	$\uparrow\downarrow$ gas	dipoles
damping of sound at $T = 0$	0	nonzero
Specific heat at low T	$\sim \exp(-\Delta/T)$	$\sim T^2$
normal component at low T	$\sim \exp(-\Delta/T)$	polynomial in T

Low energy collective modes

Phase perturbations of the ground state order parameter (Goldstone mode)

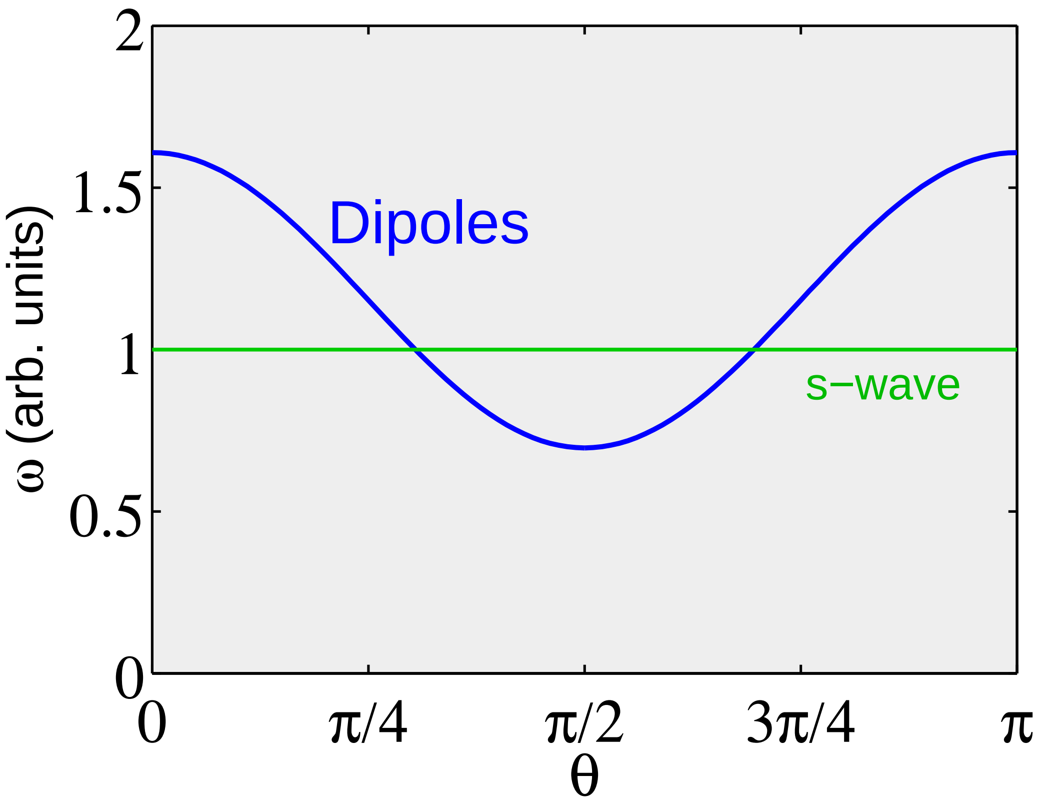
$$\boxed{\Delta_0(x-y) \quad \rightarrow \quad \Delta(x,y) = \Delta_0(x-y)\, e^{2i\phi(x,t)}}$$

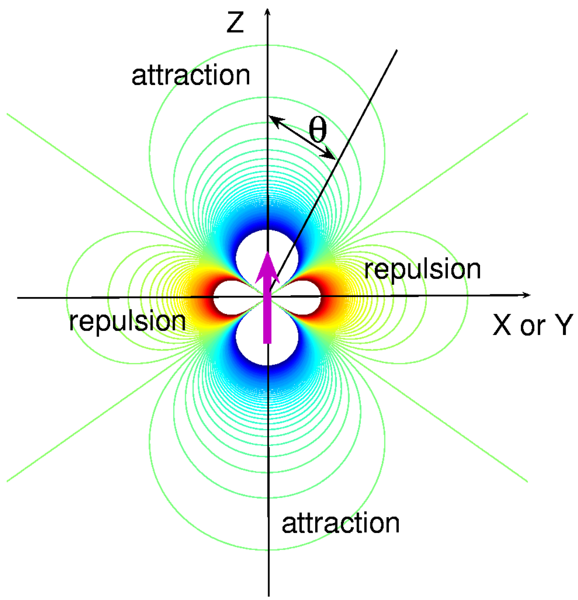
Assumptions:

- Low energy ($\hbar\omega \ll \Delta_0^{\max}$)
- Phase perturbations only (amplitude perturbations are gapped)
- Low ω $\Longrightarrow$ long wavelength ($k \ll k_F$)
  $\Longrightarrow$ insensitive to small-scale of $|x-y|$ $\Longrightarrow$ $\phi \approx \phi(x \text{ only })$
- Weak perturbation $\Longrightarrow$ lowest order in ϕ

$T \approx T_c$

$$\omega = -i \left(\frac{7\zeta(3)}{6\pi^3}\right)\left(\frac{\hbar v_F^2}{T_c}\right) k^2 \left(1 + \frac{3}{2\pi^2}(1 + 3\cos 2\theta)\right)$$





- Purely diffusive (as for standard short-range ↑↓gas)
- Anisotropic (differently to ↑↓gas)

$T \to 0$ Anisotropic damping of sound

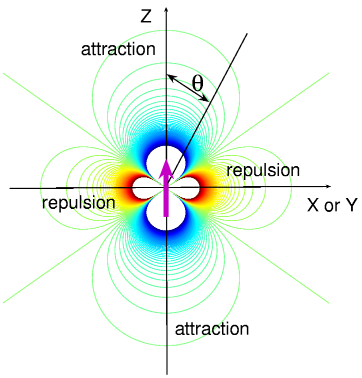
$$\omega = \left(\frac{v_F}{\sqrt{3}}\right) k \left\{1 - i \left(\frac{\hbar\omega_{\mathrm{Bog}}}{\Delta_{\max}}\right) \Gamma(\theta)\right\}$$

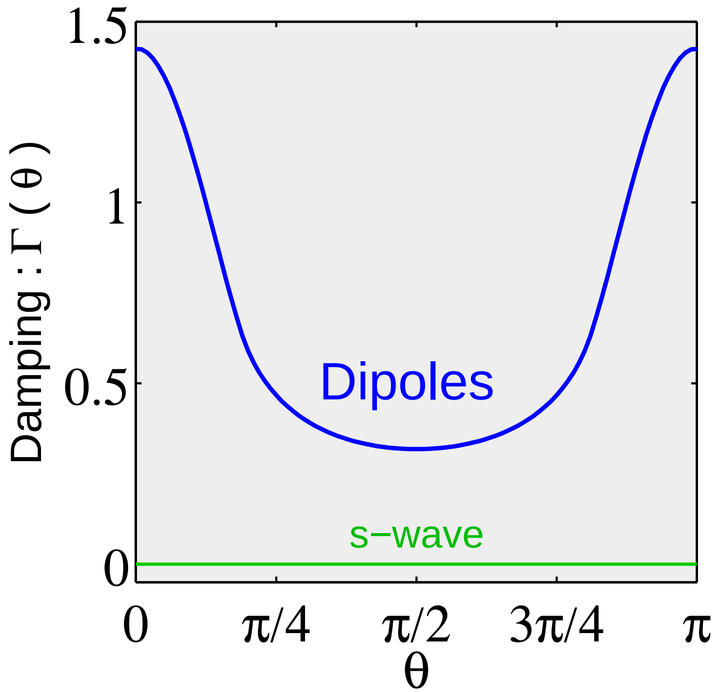
Bogoliubov sound

damping absent for ↑↓ gas

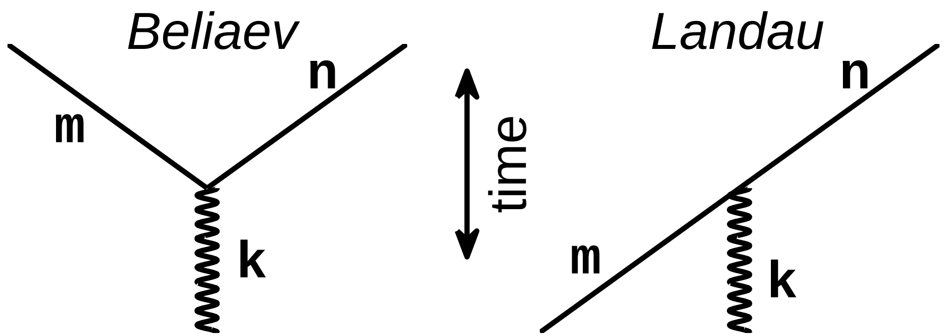
Beliaev process:

collective $\Longrightarrow$ $2\times$quasipart.

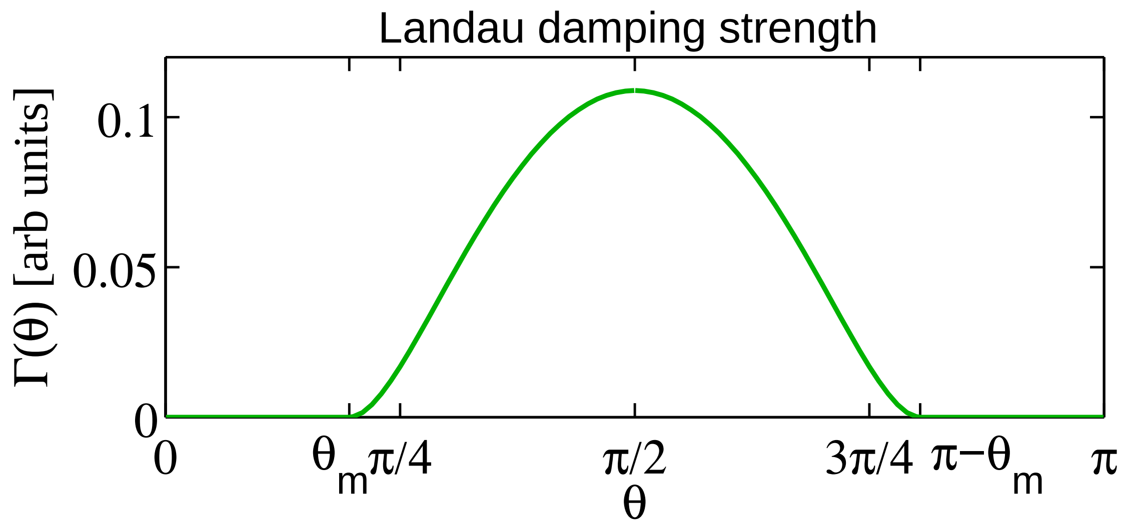




Low energy regime $h\omega \ll T \ll T_c$

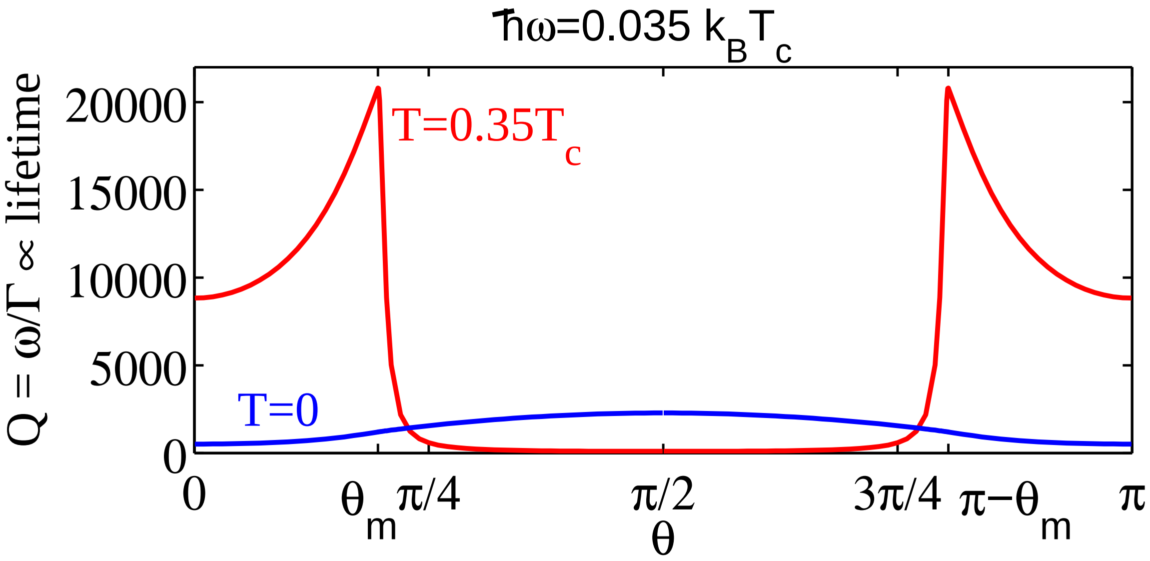


- Beliaev damping $\Gamma \propto \omega$
- Landau damping $\Gamma \propto \frac{1}{\omega}$ when $\sin^2\theta > \frac{1}{3}$



Aligned superfluid $\hbar\omega \ll T \ll T_c$

(No s-wave ↑↓gas analogue)



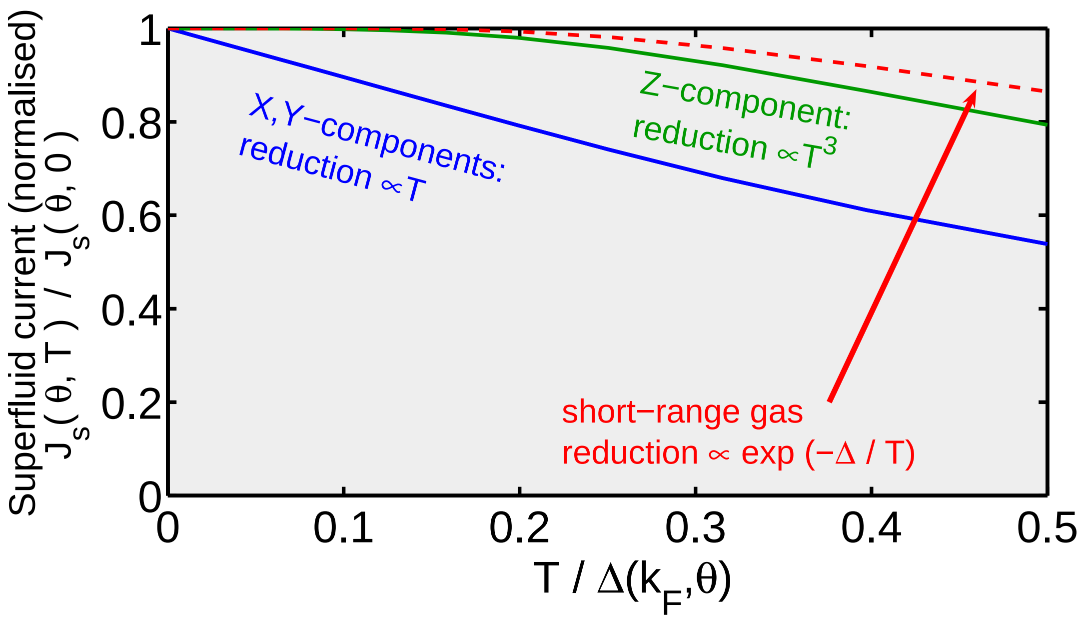
- Directions close enough to polarisation: good quality superfluid
- Directions perpendicular: Landau damping kills superfluidity

Veering superfluid current $0 < T < T_c$

- Current response J_s to an external phase perturbation of the gap

$$\Delta(x,y,t) = \Delta_0(x-y)e^{2i\phi(x,t)}$$

- Strable driving frequency ω, wave-vector k, in direction θ.



Veering current