Superfluid excitations of Fermi dipolar gases

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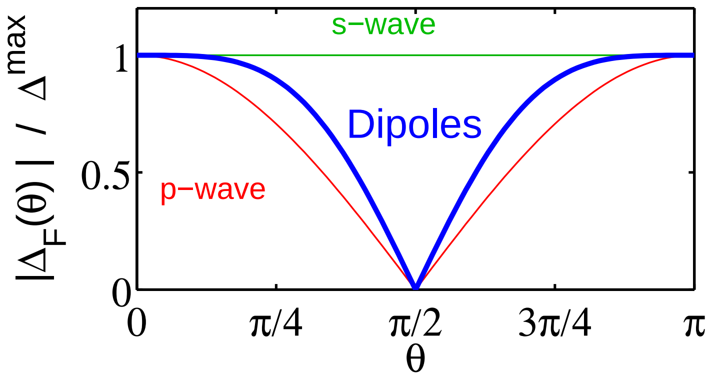
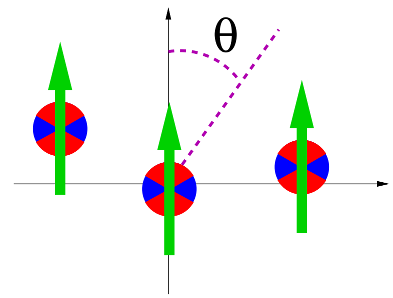
BCS-like superfluidity



BCS Pairing gap
on Fermi surface
has zeros



Baranov et al. PRA **66**, 013606 (2002)

- Node structure anlogous to:
  - Polar phase of $^3\mathrm{He}$. (never experimentally realized)
  - Heavy-fermion superconductors like $\mathrm{UPt_3}$. (messy)

- Gap zero allows quasiparticles down to $T \to 0$

- New regime when $\hbar\omega \lesssim k_B T \ll k_B T_c$ not seen in standard BCS

comparison to standard BCS

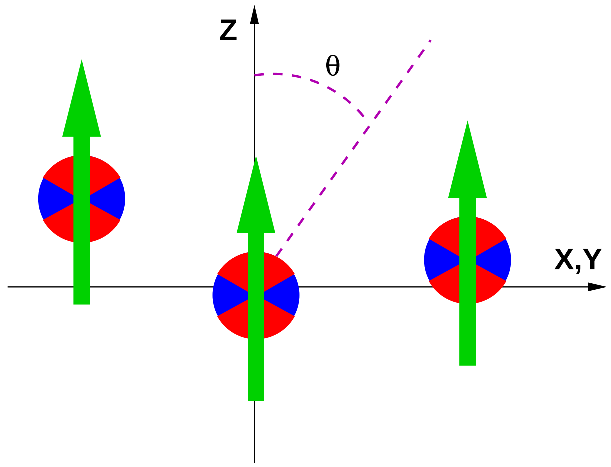
dipole–dipole potential

- long range interaction
  $\longrightarrow$ Needs 1 spin component
- always partly attractive
  BCS pairing if *polarised*
- Energy gap has nodes
- *Anisotropic*

contact s-wave $\uparrow\downarrow$ potential

- short range interaction
  $\longrightarrow$ Needs 2 spin components
  (Pauli blocking)
- arttractive or repulsive
  BCS pairing only if $a_s < 0$
- Energy gap always $\neq 0$
- *Isotropic*

uniform 3D gas



Essential features of superfluid physics seen in center of trapped system

- Cold: $T < T_c^{BCS}$
- static external field (E or B) $\Longrightarrow$ full polarisation
- single-species (spin polarised)
- dilute $\Longrightarrow$ Energy dominated by Fermi sea to leading order
- short-range interaction assumed negligible (Fermi exclusion, no p-wave resonances)

Hamiltonian

$$\widehat{H} \;=\; \text{K.E.} \;+\; \frac{1}{2}\int d^3x\, d^3y \left\{\, \widehat{\Psi}^{\dagger}_x \widehat{\Psi}_x\, V_D(x-y)\, \widehat{\Psi}^{\dagger}_y \widehat{\Psi}_y \,\right\}$$

Resulting effective BCS mean-field Hamiltonian

$$\widehat{H}_{\text{eff}} = \frac{1}{2}\int d^3x\, d^3y \Big\{ \quad \frac{\hbar^2}{m}\, \widehat{\Psi}^{\dagger}_x\, \nabla^2 \widehat{\Psi}_x\, \delta(x-y) \qquad \textit{Kinetic}$$

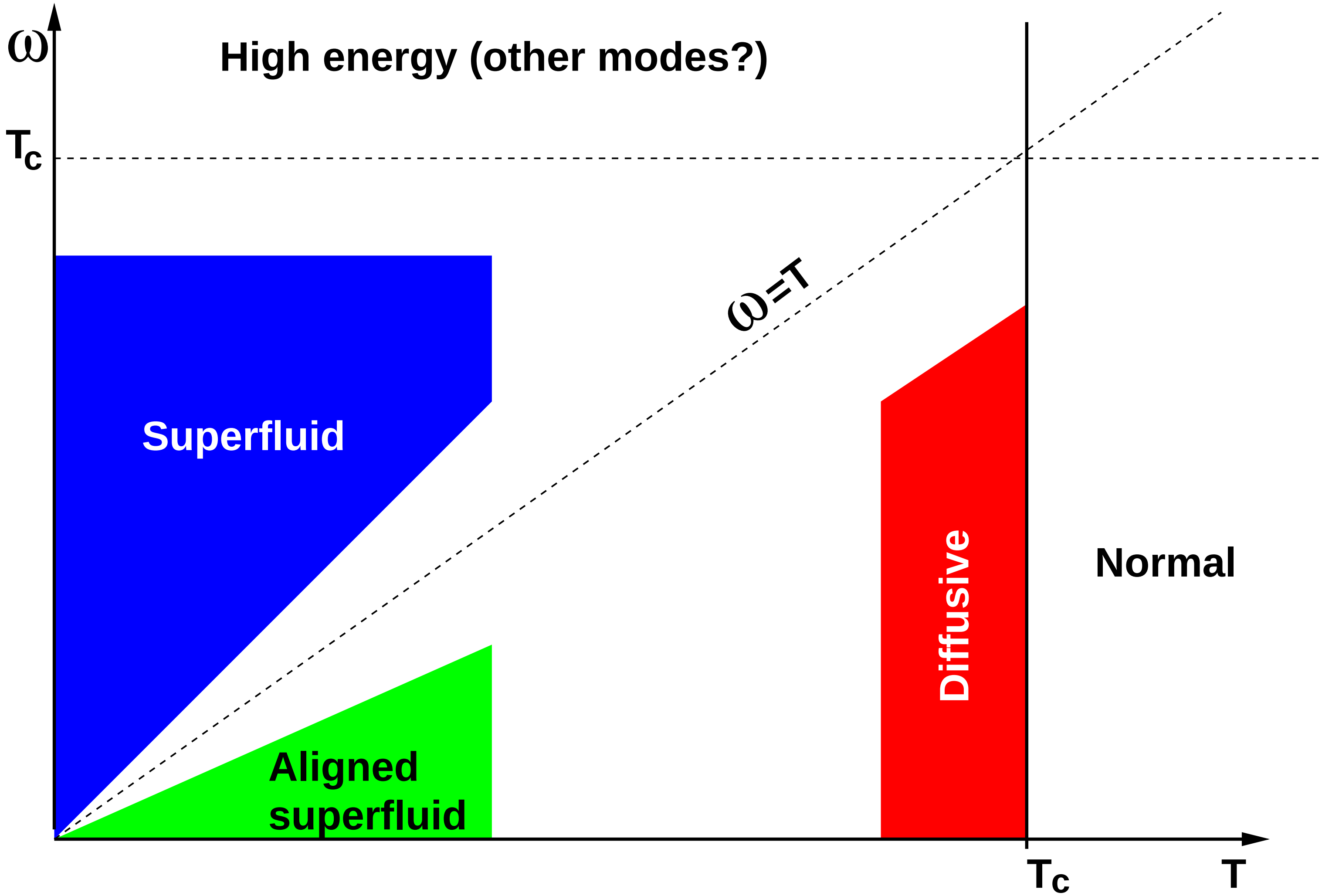
$$\Delta^*(x-y)\, \widehat{\Psi}_x \widehat{\Psi}_y - \Delta(x-y)\, \widehat{\Psi}^{\dagger}_x \widehat{\Psi}^{\dagger}_y \qquad \textit{BCS}$$

$$W(x-y)\, \widehat{\Psi}^{\dagger}_x \widehat{\Psi}_y \quad \Big\} \qquad \textit{Hartree}$$

Gap consistency equation

$$\Delta(x-y) = V_D(x-y) \left\langle \widehat{\Psi}_x \widehat{\Psi}_y \right\rangle_{\text{eff}}$$

Collective excitation regimes



Weak Phase perturbations of the ground state order parameter

$$\Delta_0(x-y) \quad \rightarrow \quad \Delta(x,y) = \Delta_0(x-y)\; e^{2i\phi(x,t)}$$

Goldstone mode

Experimental prospects

$$T_c = 1.44 E_F \exp\left(-\frac{\pi}{2|a_D|k_F}\right)$$

$\Longrightarrow$ *Effective* scattering length a_D:

$$a_D = -2m\left(\frac{d}{\pi\hbar}\right)^2$$

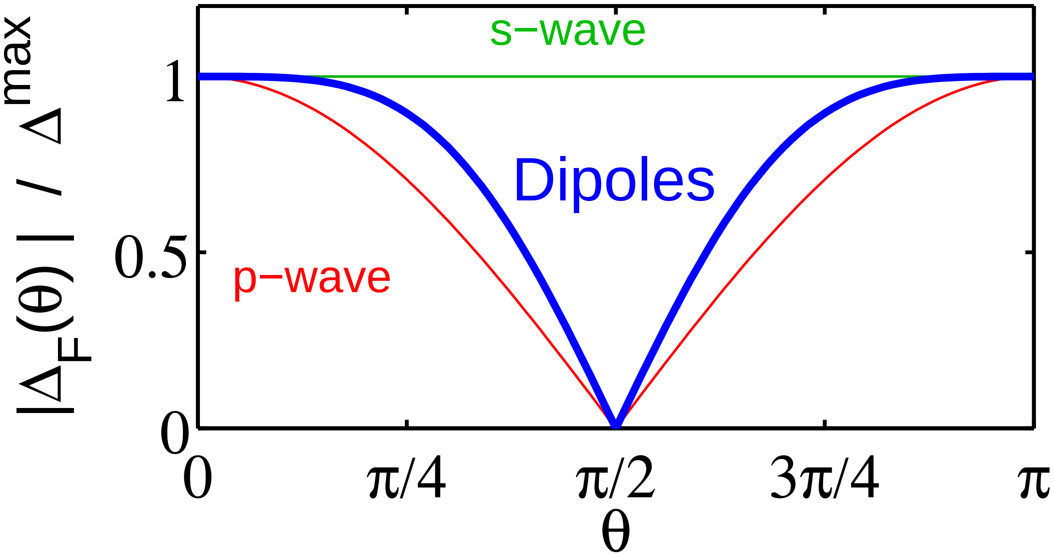
Baranov et al., PRA **66**, 013606 (2002)

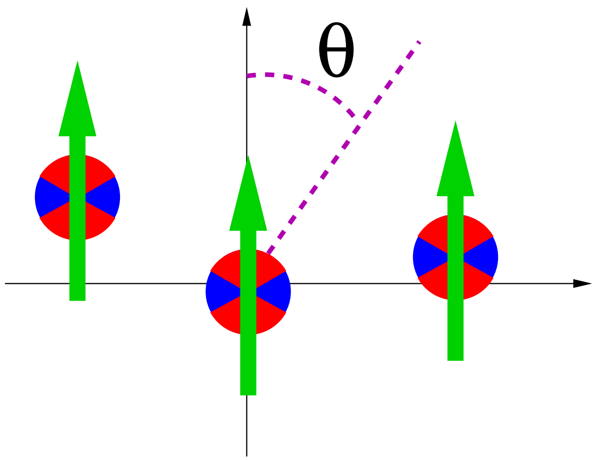
RECENT VALUES (D. Jin group, JILA, based on talk by K.-K. Ni at DAMOP)

- Density $\sim 10^{12}/\mathrm{cm}^3$
- $T = 3T_F$ (300 nK) $:-)$
- Dipole moment $d \approx 0.1D$. (Hence $a_D \approx -500\mathrm{nm}$)
- Expect to go to $d \approx 1D$. Then,

one would have $T_c \sim T_F$

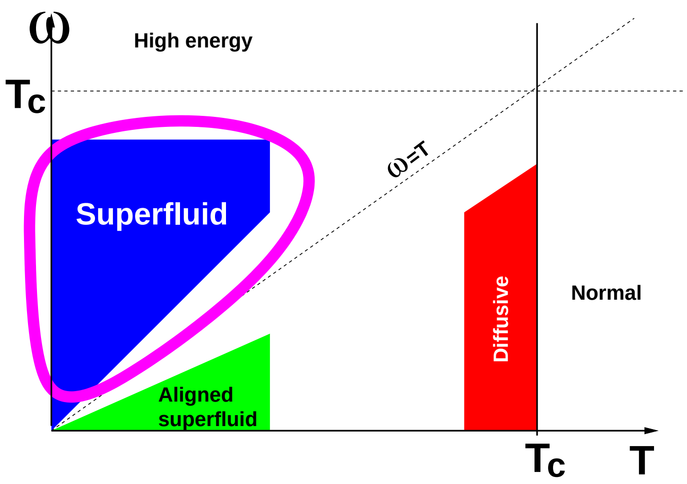
Quasiparticle energy gap





- NODE in plane $\perp$ to polarisation
- Breaking a pair costs $\geq 2|\Delta(\theta)|$.
- Dipoles: Easy to excite a pair in plane $\perp$ to polarisation because energy cost is small. For all T
- $\uparrow\downarrow$gas: Appreciable energy cost of excitations always.

“Zero” temperature regime: superfluid

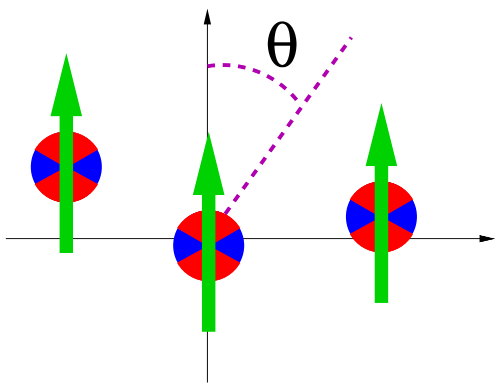


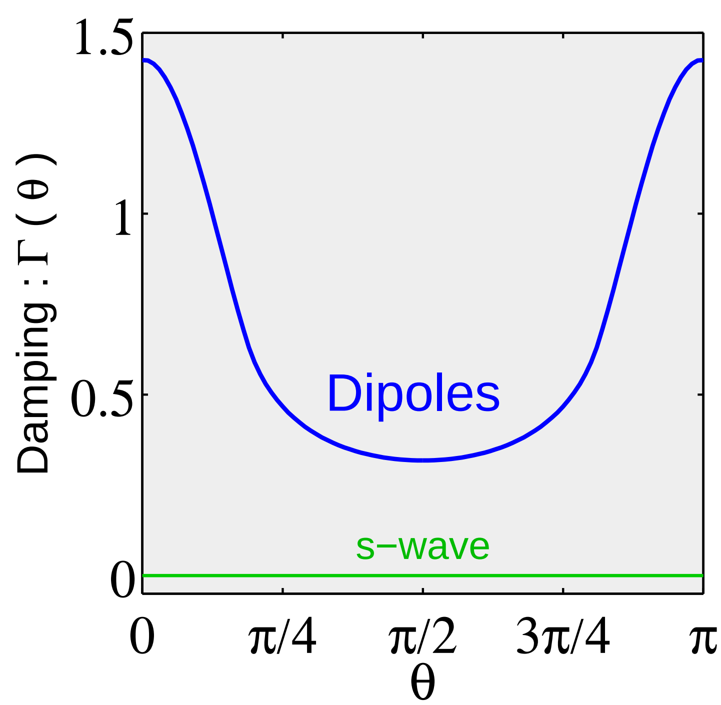
Anisotropic, nonzero damping

(damping absent for ↑↓ gas)

Beliaev process:

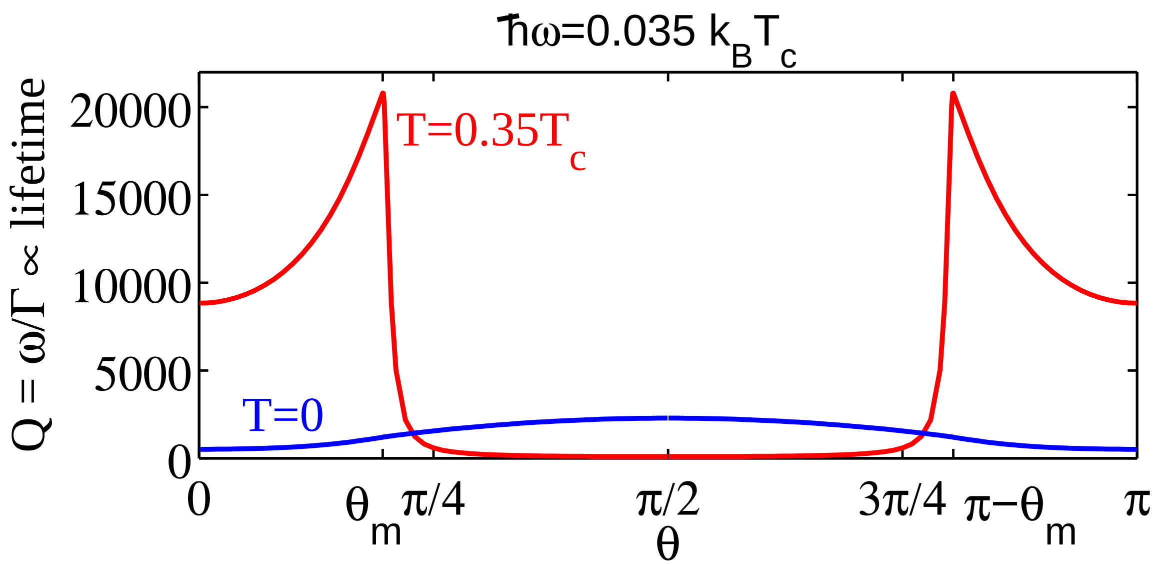
collective $\Longrightarrow$ $2\times$quasipart.

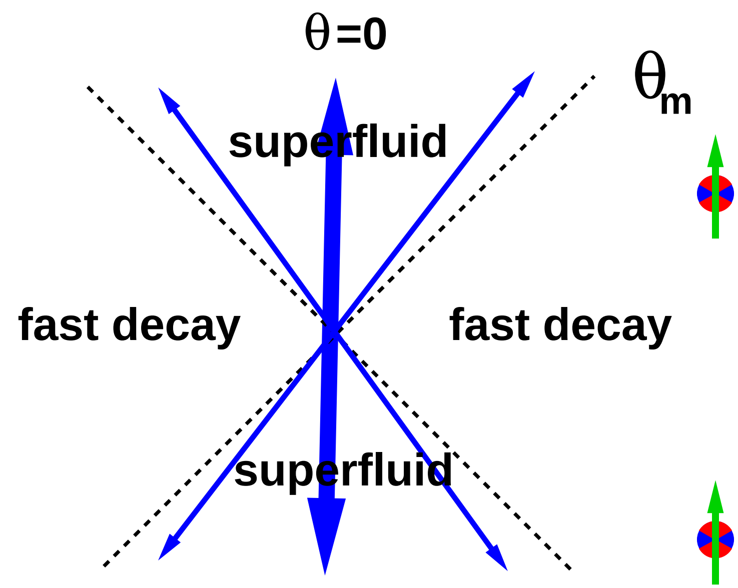




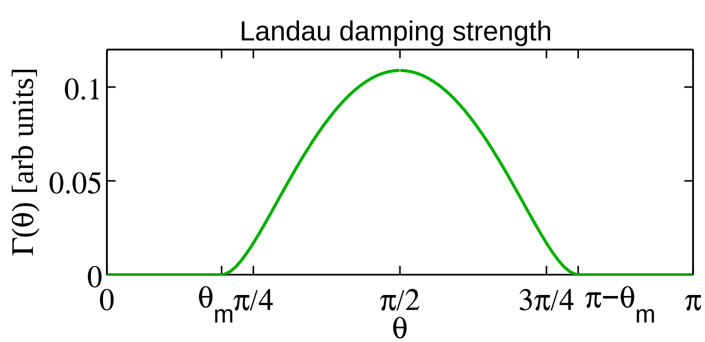
Aligned superfluid

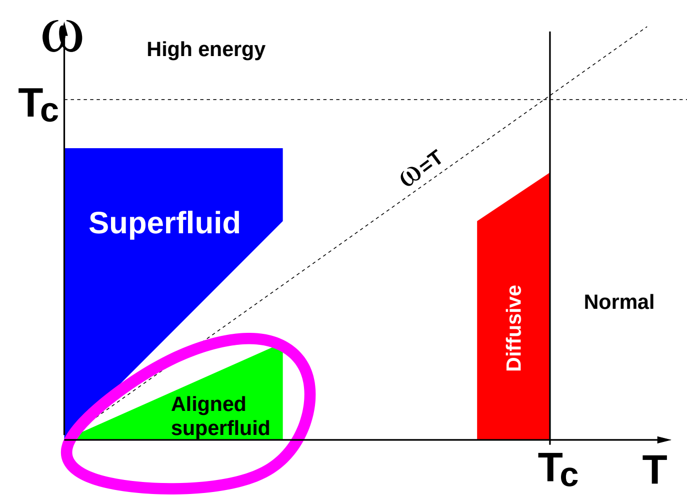
(No s-wave ↑↓gas analogue)





Strong Landau damping $\Gamma \propto \frac{1}{\omega}$





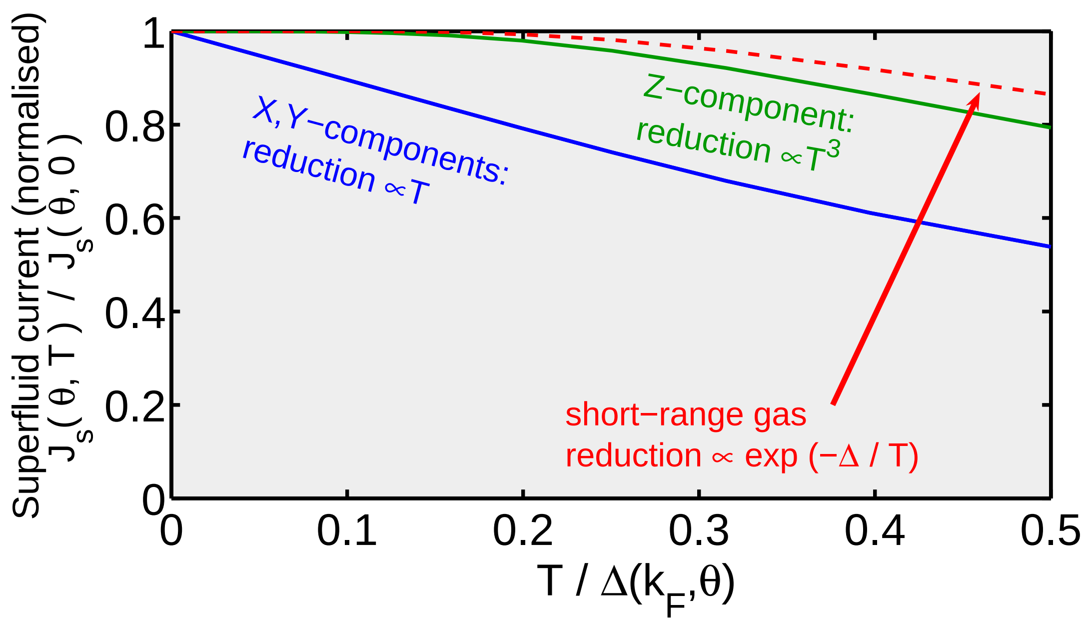
Deflected current response

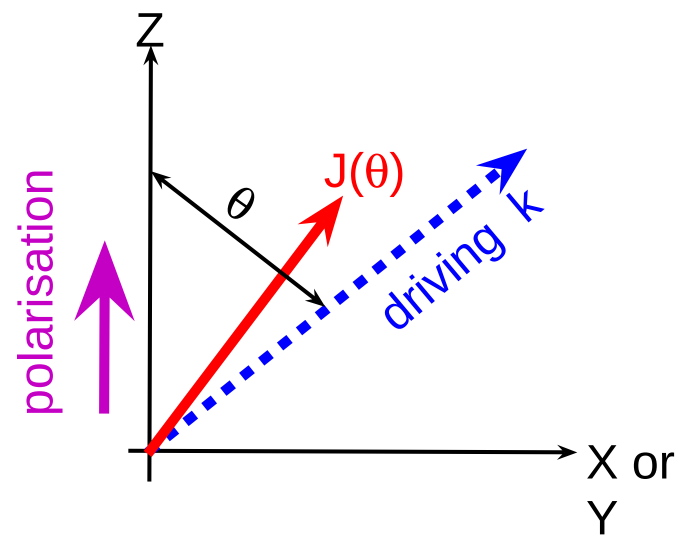
- Current response J_s to an EXTERNAL phase perturbation of the gap

$$\Delta(x,y,t) = \Delta_0(x-y)e^{2i\phi(x,t)}$$

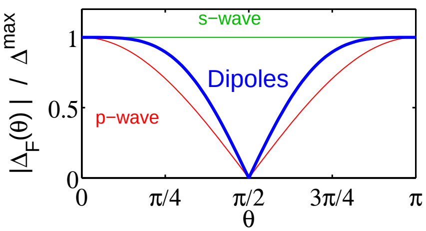
- Strable driving frequency ω, wave-vector k, in direction θ.

Deflected current

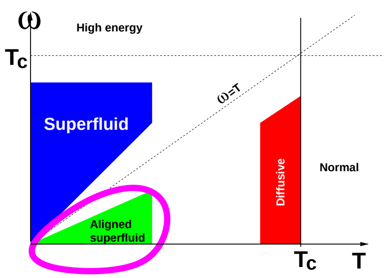


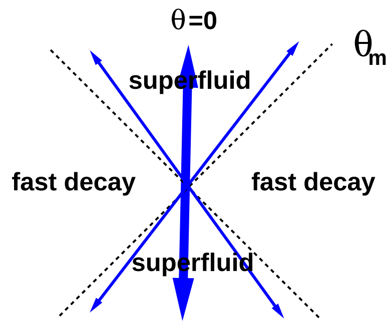


Summary

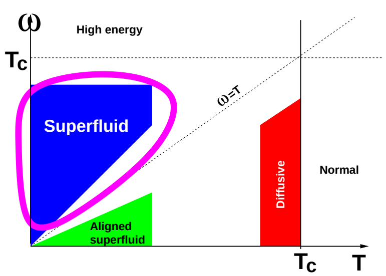


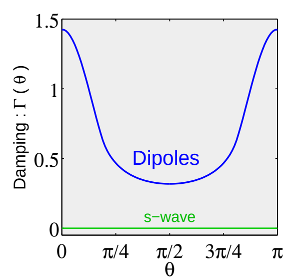
Quasiparticles down to $T = 0$



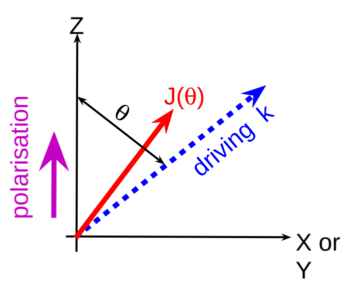


Aligned superfluid:
(Novel regime)





Damping at $T = 0$



Deflected currents

Fun:

Full consistency equation in k-space

$$-\frac{\phi_{\mathbf{k}}\Delta^0_{\mathbf{M}}\tau^0_{\mathbf{M}}}{2E^0_{\mathbf{M}}} = \frac{\phi_{\mathbf{k}}\Delta^0_{\mathbf{M}}}{4E^0_{\mathbf{m}}E^0_{\mathbf{n}}}\left\{\left(\frac{\tau^0_{\mathbf{n}}-\tau^0_{\mathbf{m}}}{2}\right)\left[\frac{(E^0_{\mathbf{n}}+\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}-\varepsilon_{\mathbf{m}})+\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega-E^0_{\mathbf{n}}+E^0_{\mathbf{m}}+i0}-\frac{(E^0_{\mathbf{n}}-\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}+\varepsilon_{\mathbf{m}})+\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega+E^0_{\mathbf{n}}-E^0_{\mathbf{m}}+i0}\right]\right.$$
$$\left.+\tau^0_{\mathbf{n}}\left[\frac{(E^0_{\mathbf{n}}+\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}+\varepsilon_{\mathbf{m}})-\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega-E^0_{\mathbf{n}}-E^0_{\mathbf{m}}+i0}\right]-\tau^0_{\mathbf{m}}\left[\frac{(E^0_{\mathbf{n}}-\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}-\varepsilon_{\mathbf{m}})-\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega+E^0_{\mathbf{n}}+E^0_{\mathbf{m}}+i0}\right]\right\}.$$

where $\mathbf{n}=\mathbf{M}+\mathbf{k}/2$, $\mathbf{m}=-\mathbf{M}+\mathbf{k}/2$, $\varepsilon_{\mathbf{k}}=\hbar^2k^2/2m-E_F$, $E^0_{\mathbf{k}}=\sqrt{\varepsilon^2_{\mathbf{k}}+(\Delta^0_{\mathbf{k}})^2}$, and $\tau^0_{\mathbf{k}}=\tanh(E^0_{\mathbf{k}}/2T)$.

- LONG wavelength $\mathbf{k}$, SHORT wavelength $\mathbf{M}$.