Superfluidity and excitations in fermionic dipolar gases

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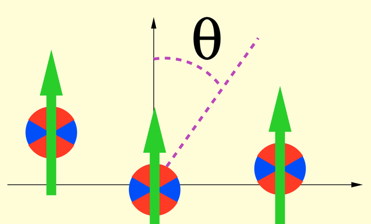
Model: uniform 3D gas



DIPOLAR INTERACTION:

$$V_D(\mathbf z) = \frac{|\mathbf d|^2}{|\mathbf z|^3}\left(1 - 3\cos^2\theta\right)$$

- FULLY POLARIZED by external field
- SINGLE-SPECIES
  –yet interacting due to long range of V_D
- SUPERFLUID : $T < T_c^{\rm BCS}$
- DILUTE (Fermi sea dominates energy)
- UNIFORM–relevant also for
  Local Density approximation in trap

Mean field Hamiltonian

$$\widehat H_{\rm eff} = \int d^3\mathbf x\,\Big\{$$
$$\widehat\Psi^\dagger(\mathbf x)\left(\frac{\hbar^2\nabla^2}{2m} - E_F\right)\widehat\Psi(\mathbf x) \qquad \text{Kinetic}$$
$$+\tfrac12\int d^3\mathbf y\,\Delta^*(\mathbf x,\mathbf y)\,\widehat\Psi(\mathbf x)\widehat\Psi(\mathbf y) + \text{h.c.} \quad \text{BCS}$$
$$+\int d^3\mathbf y\,F^{\rm ex}(\mathbf x,\mathbf y)\,\widehat\Psi^\dagger(\mathbf x)\widehat\Psi(\mathbf y) \quad \Big\}\ \text{Exchange}$$

with gap and $F^{\rm ex}$ consistency equations:

$$\Delta(\mathbf x,\mathbf y) = V_D(\mathbf x-\mathbf y)\left\langle\widehat\Psi(\mathbf x)\widehat\Psi(\mathbf y)\right\rangle$$
$$F^{\rm ex}(\mathbf x,\mathbf y) = V_D(\mathbf x-\mathbf y)\left\langle\widehat\Psi(\mathbf x)\widehat\Psi^\dagger(\mathbf y)\right\rangle$$

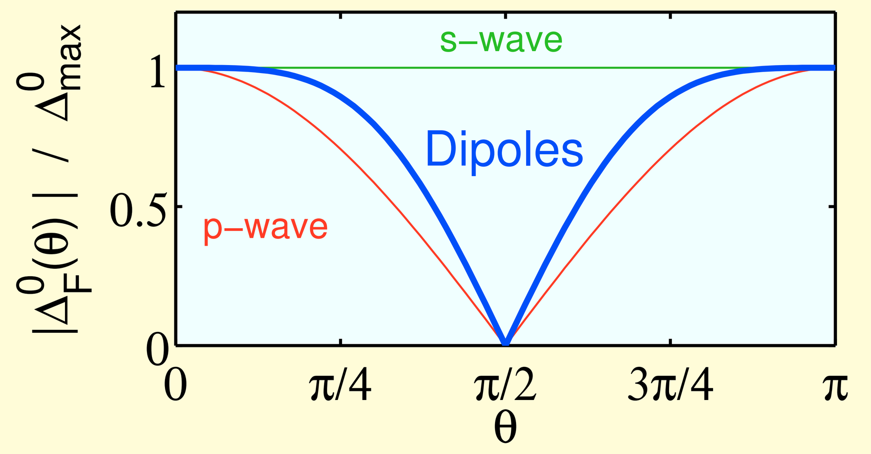
Ground state

BOGOLIUBOV QUASIPARTICLE **spectrum**

$$E^0_{\mathbf k} = \sqrt{|\Delta^0_{\mathbf k}|^2 + \left(\frac{\hbar^2|\mathbf k|^2}{2m} - E_F\right)^2} \geq |\Delta^0_{\mathbf k}|$$

Gap: $\Delta^0_{\mathbf k} = \frac{1}{(2\pi)^3}\int d^3\mathbf r\, e^{-i\mathbf k\cdot\mathbf r}\Delta(\mathbf x,\mathbf x-\mathbf r)$

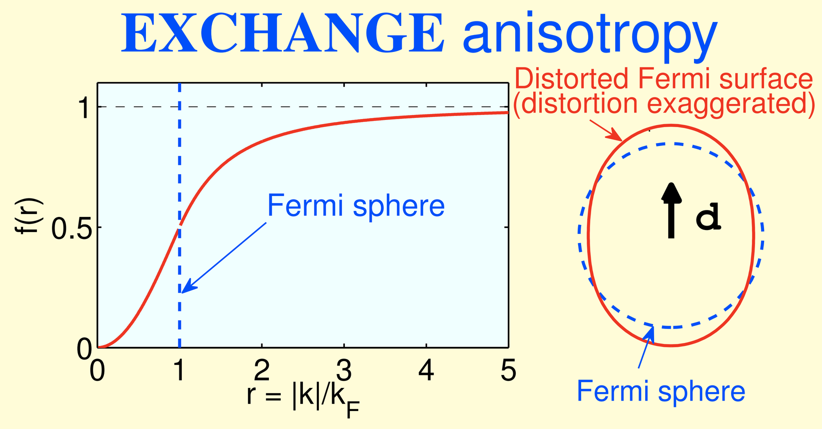
BCS GAP on Fermi surface



Baranov et al. PRA **66**, 013606 (2002)

- Node structure of gap anlogous to:
  - Polar phase of ^{3}He (hypothetical)
  - Heavy-fermion superconductors
- Gap zero allows plentiful quasiparticles even as $T \to 0$

EXCHANGE anisotropy



$$F^{\rm ex}_{\mathbf k} = \frac{4\pi}{3}n|\mathbf d|^2\left(1 - 3\cos^2\theta\right)f(r) \ll E_F$$
$$\left(F^{\rm ex}_{\mathbf k} = \frac{1}{(2\pi)^3}\int d^3\mathbf r\, e^{-i\mathbf k\cdot\mathbf r}F^{\rm ex}(\mathbf x,\mathbf x-\mathbf r)\right)$$

Experimental prospects

$$T_c^{\rm BCS} = 1.44\,E_F\,e^{-\frac{\pi}{2|a_D|k_F}};\qquad |a_D| = \frac{2m|\mathbf d|^2}{\pi^2\hbar^2}$$

Baranov et al., PRA **66**, 013606 (2002)

Comparison to **RECENT VALUES:**

K.-K. Ni et al., arXiv:0808.2963

$d = 0.566\,\text{D}$, $n \sim 10^{12}/\text{cm}^3$ $\Longrightarrow T_c^{\rm BCS} \approx 1.6\text{nK}$ small! :–(

However, with $10\times$ more density (plausible), one would have

$T_c^{\rm BCS} \approx 40\text{nK} \sim T_F$:–)

Collective Excitations

- Weak perturbations $\phi(t) \ll \Delta$ of the ground state order parameter.
- Short-range form factor
  $A(\mathbf r)\ :\ k_F|\mathbf r| \sim (E_F/\Delta^0) \ll 1.$

$$\Delta(\mathbf x,\mathbf y,t) = \Delta^0(\mathbf x-\mathbf y) + 2iA(\mathbf x-\mathbf y)\,\phi(\mathbf x,t).$$

- Diagonalize new $\widehat H_{\rm eff}$ with Bogoliubov transform ⇒ perturbed eigenfunctions.
- Keep only lowest order in ϕ.
- $F^{\rm ex}$ is negligible here ⟹ discard.
- Impose gap equation to give ⇓

$$0 = \frac{\phi_{\mathbf k}A_{\mathbf M}}{4E^0_{\mathbf m}E^0_{\mathbf n}}\left\{\left(\frac{T^0_{\mathbf n}-T^0_{\mathbf m}}{2}\right)\left[\frac{(E^0_{\mathbf n}+\varepsilon_{\mathbf n})(E^0_{\mathbf m}-\varepsilon_{\mathbf m})\pm\Delta^0_{\mathbf n}\Delta^0_{\mathbf m}}{\hbar\omega-E^0_{\mathbf n}+E^0_{\mathbf m}+i0} - \frac{(E^0_{\mathbf n}-\varepsilon_{\mathbf n})(E^0_{\mathbf m}+\varepsilon_{\mathbf m})\pm\Delta^0_{\mathbf n}\Delta^0_{\mathbf m}}{\hbar\omega+E^0_{\mathbf n}-E^0_{\mathbf m}+i0}\right] + T^0_{\mathbf n}\left[\frac{(E^0_{\mathbf n}+\varepsilon_{\mathbf n})(E^0_{\mathbf m}+\varepsilon_{\mathbf m})\mp\Delta^0_{\mathbf n}\Delta^0_{\mathbf m}}{\hbar\omega-E^0_{\mathbf n}-E^0_{\mathbf m}+i0}\right] - T^0_{\mathbf m}\left[\frac{(E^0_{\mathbf n}-\varepsilon_{\mathbf n})(E^0_{\mathbf m}-\varepsilon_{\mathbf m})\mp\Delta^0_{\mathbf n}\Delta^0_{\mathbf m}}{\hbar\omega+E^0_{\mathbf n}+E^0_{\mathbf m}+i0}\right]\right\} - \phi_{\mathbf k}\int d^3\mathbf z\, e^{-i\mathbf M\cdot\mathbf z}\frac{A(\mathbf z)}{V_d(\mathbf z)}$$

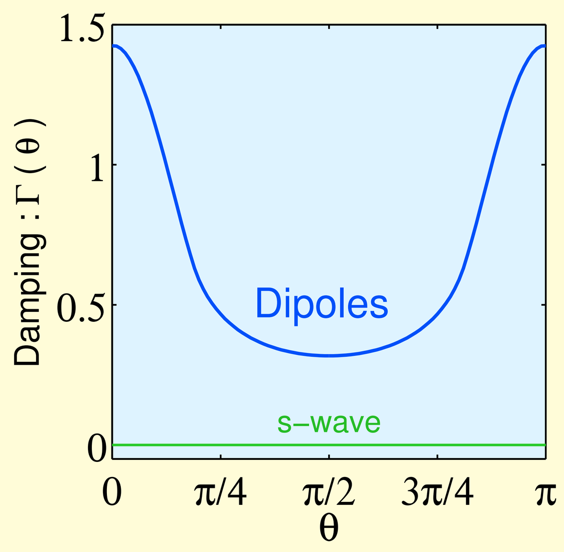
where $\mathbf n = \mathbf M + \mathbf k/2$, $\mathbf m = -\mathbf M + \mathbf k/2$, $\varepsilon_{\mathbf k} = \hbar^2k^2/2m - E_F$, $T^0_{\mathbf p} = \tanh(E^0_{\mathbf p}/2T)$, and upper/lower sign corresp. to real/imag $\phi(\mathbf x)$.

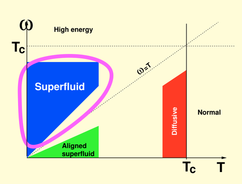
At low energies one has conformal modes $A(\mathbf z) = \Delta^0(\mathbf z)$, and observable quantities are obtained from an effective Lagrangian for $\phi(\mathbf x)$.

"Zero temperature" regime – superfluid

Nonzero anisotropic damping

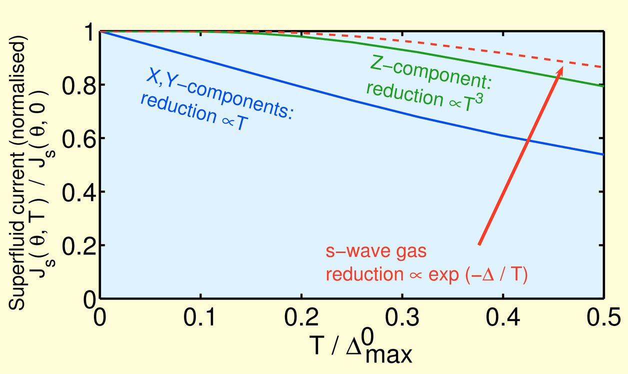
$$\omega - i\Gamma = \pm\frac{1}{\sqrt3}|\mathbf k|v_F\left[1 - i\left(\frac{\hbar\omega}{\Delta^0_F(0)}\right)\Gamma(\theta)\right]$$

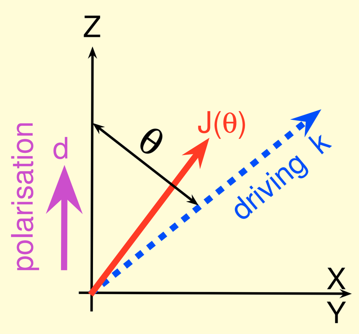


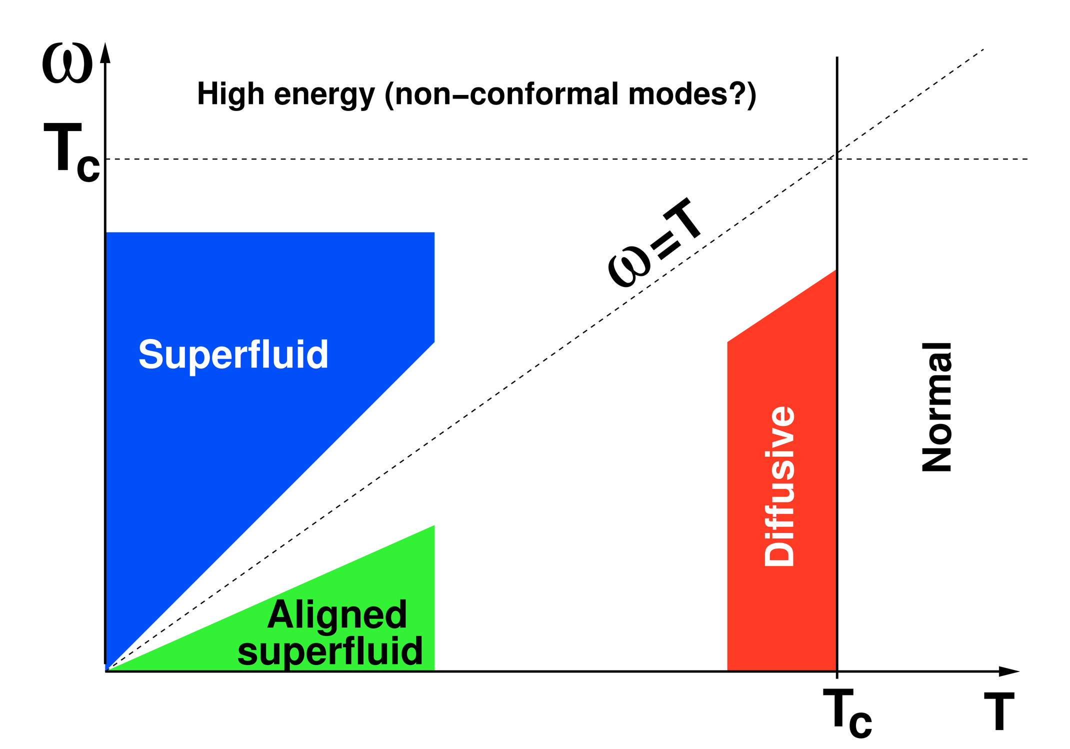


DEFLECTED SF CURRENT RESPONSE

Under an EXTERNAL phase perturbation of the gap $\Delta(x,y,t) = \Delta_0(x-y)e^{2i\phi(x,t)}$

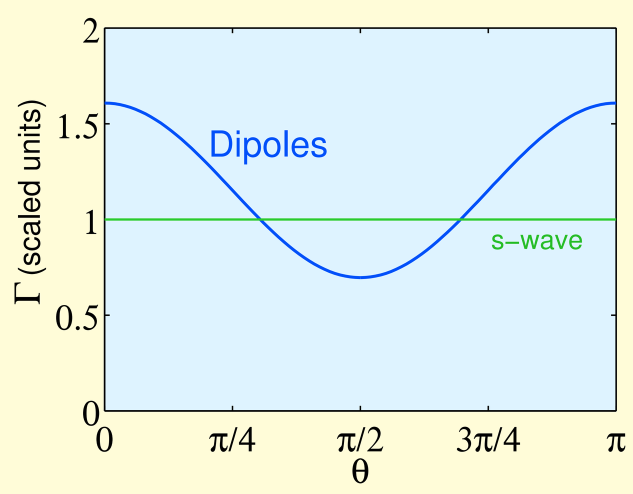


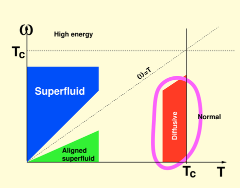




$T \approx T_c$: Diffusive regime

$$\omega = -i\Gamma_{(s\text{–wave})}\left[1 + \frac{3}{2\pi^2}(1 + 3\cos2\theta)\right]$$

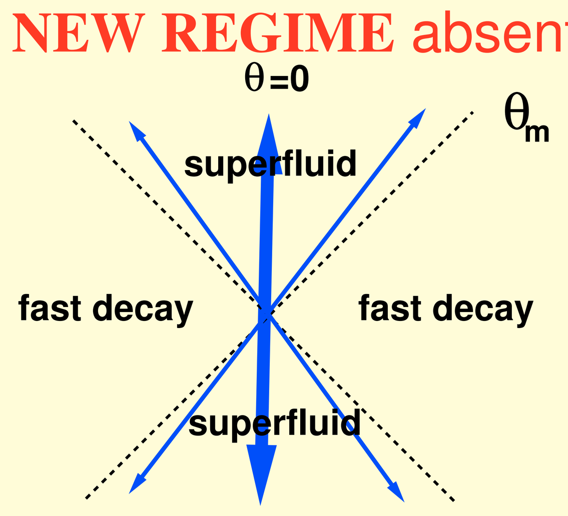


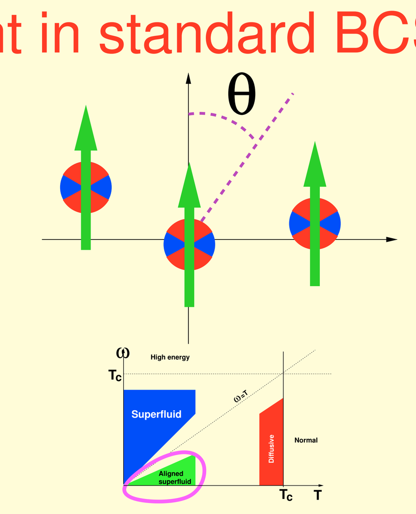


PD was supported by the European Community under the contract MEIF-CT-2006-041390.

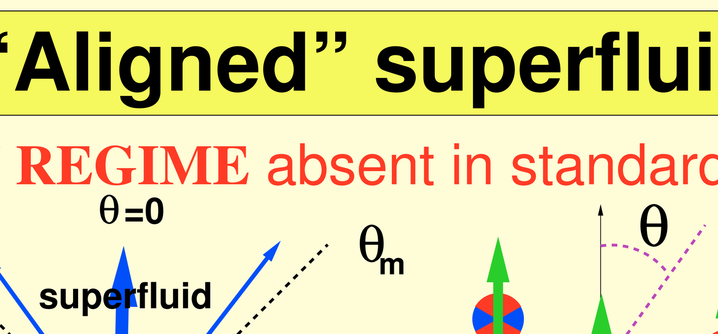
"Aligned" superfluid

NEW REGIME absent in standard BCS





- Very weak Beliaev damping
- Strong Landau damping ⊥ to dipoles
- ⟹ Good superfluid only along dipoles



EXCITATION LIFETIME