Quantum dynamics of correlated atom pairs

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Contributions

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Mariusz Gajda (Warsaw IFPAN)
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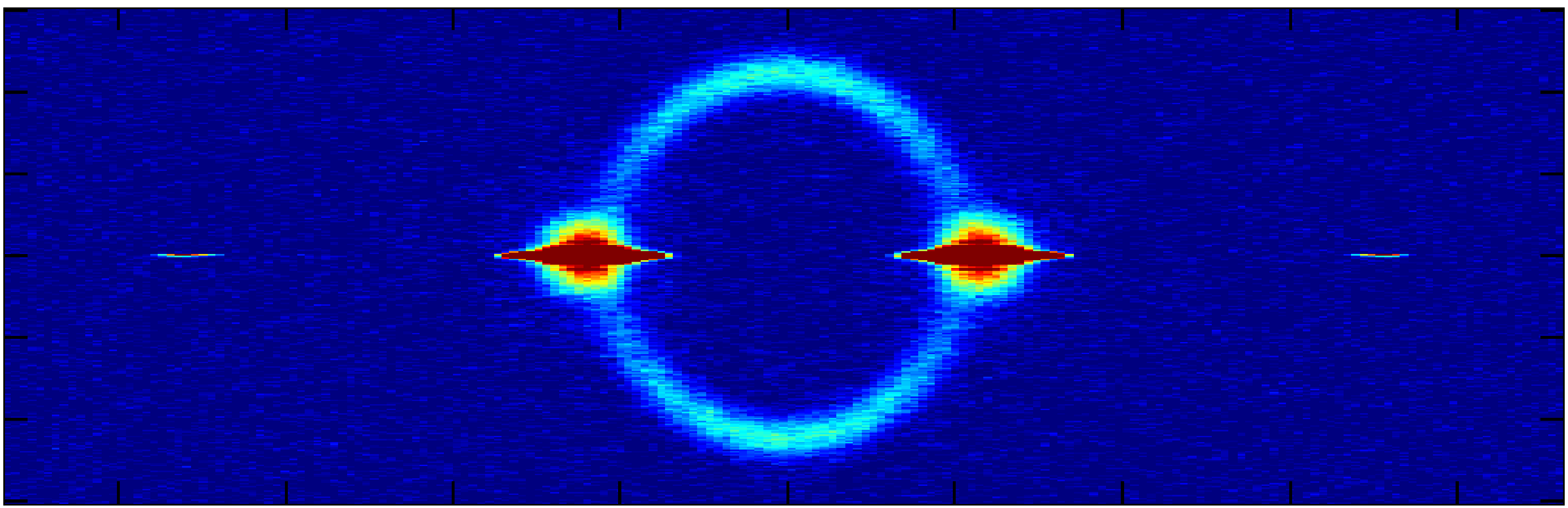
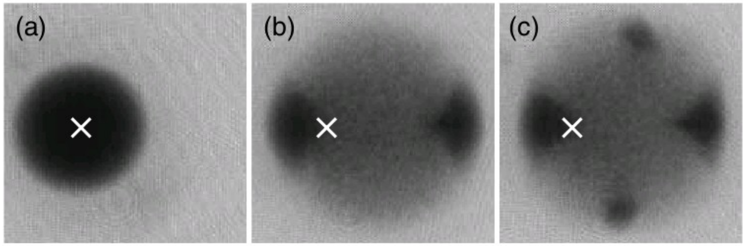
Kazimierz Rzążewski (Warsaw CFT)



Outline

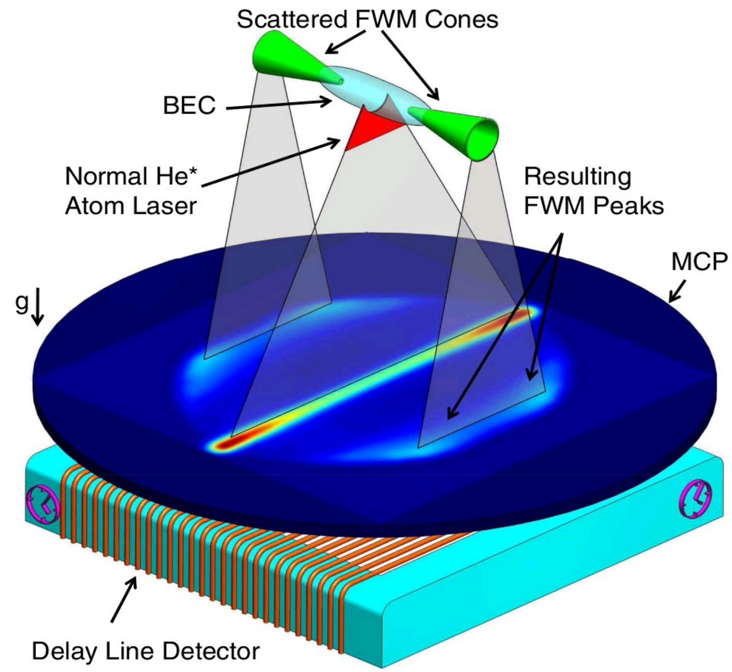
1. Supersonic pair creation
2. Positive-P/Bogoliubov method
3. He* pair scattering at T=0
4. Quasicondensate 0 < T < ~Tc

Supersonic pair creation

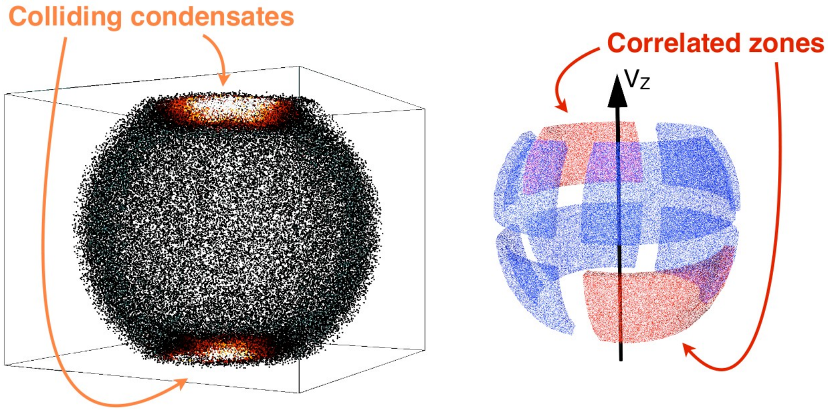


Vogels, Xu, Ketterle, PRL **89,** 020401 (2002)

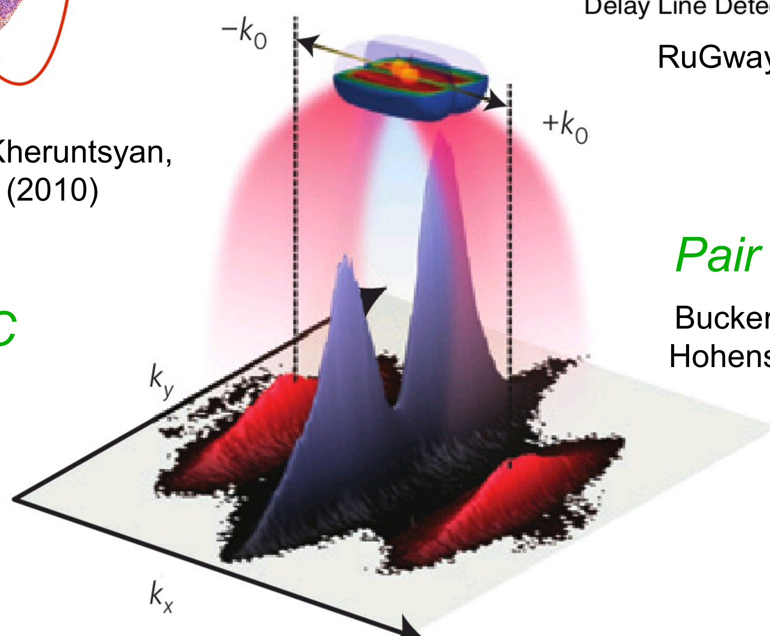
BEC Collisions



RuGway, Hodgman, Dall, Johnsson, Truscott, PRL **107,** 075301 (2011)



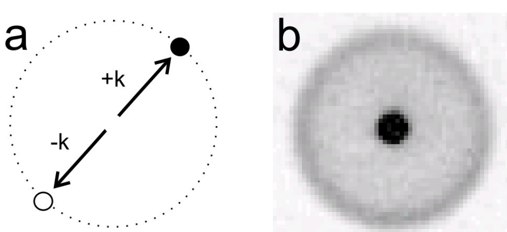
Jaskula, Bonneau, Partridge, Krachmalnicoff, PD, Kheruntsyan, Aspect, Boiron,Westbrook, PRL **105,** 190402 (2010)



Pair emission from a 1D gas

Bucker, Grond, Manz, Berrada, Betz, Koller, Hohenster, Schumm, Perrin, Schmiedmayer, Nature Phys. **7,** 608 (2011)

Dissociation of molecular BEC

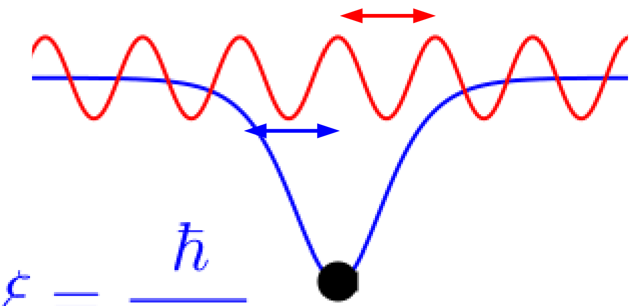


Greiner, Regal, Stewart, Jin, PRL **94,** 110401 (2005)

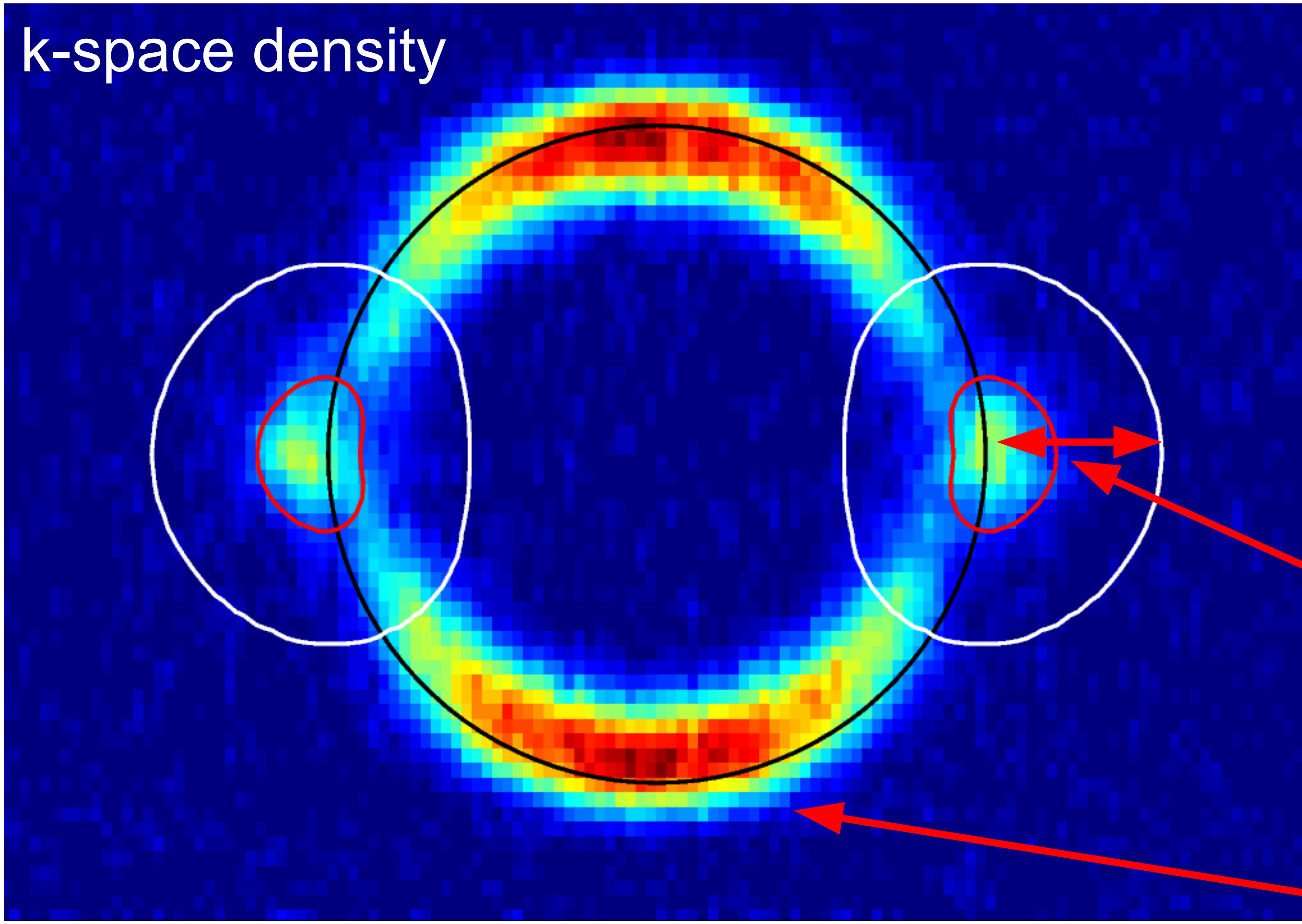
Supersonic scattering

- Above the speed of sound $v_s = \sqrt{\mu/m}$ a condensate no longer behaves as a superfluid

$$\lambda_v = \frac{2\pi\hbar}{mv}$$



$$\xi = \frac{\hbar}{mv_s}$$



BEC width in K-space is v_s

Scattered atoms Well separated From BEC

PD, Ziń, Chwedeńczuk, Trippenbach, EPJD **65,** 19 (2011)

Bogoliubov pair creation

$$\hat{H} = \int dx \left\{ \hat{\Psi}^\dagger(x) \left[V(x) - \frac{\hbar^2}{2m} \nabla^2 \right] \hat{\Psi}(x) + \frac{g}{2} \hat{\Psi}^\dagger(x)^2 \hat{\Psi}(x)^2 \right\}$$

$$\boxed{\widehat{\Psi}(\mathbf{x},t) = \underbrace{\phi(\mathbf{x},t)}_{\text{BEC}} + \underbrace{\widehat{\delta}(\mathbf{x},t)}_{\text{incoherent part, "scattered" atoms}}}$$

Assume small $\widehat{\delta}(\mathbf{x},t)$

$$\hat{H}_{\text{eff}} = \int d^3\mathbf{x}\, \hat{\delta}^\dagger(\mathbf{x}) H_0(\mathbf{x}) \hat{\delta}(\mathbf{x})$$

K.E. + trap

$$+2g \int d^3\mathbf{x}\, |\phi(\mathbf{x})|^2 \hat{\delta}^\dagger(\mathbf{x}) \hat{\delta}(\mathbf{x})$$

Potential from BEC
For scattered atoms

$$+\frac{g}{2} \int d^3\mathbf{x}\, \phi(\mathbf{x})^2 \hat{\delta}^\dagger(\mathbf{x}) \hat{\delta}^\dagger(\mathbf{x}) + \text{ h.c.}$$

Pair creation

Bogoliubov hurdles

- Looks like a linear problem, so why not just diagonalize $\hat{H}_{\mathrm{eff}}$ and have everything, but.......

1. The numerical lattice might be too large ($10^6 - 10^7$ points in a 3D calculation)

   (note also the ***"human time"*** bottleneck!)
2. BEC evolves "under" the Bogoliubov field
   $\rightarrow$ would have to re-diagonalize at each time step
3. Assumption of small $\widehat{\delta}(\mathbf{x}, t)$ may fail

- Diagonalization can be avoided by using the positive-P representation

Positive-P representation

Drummond, Gardiner J. Phys. A **13,** 2353 (1980)

Hilbert space dimension

n^M

$$\widehat{\rho} = \int P\left[\psi, \widetilde{\psi}\right] \left|\psi\right\rangle \left\langle\widetilde{\psi}\right| \mathcal{D}^2\psi(\vec{x})\mathcal{D}^2\widetilde{\psi}(\vec{x})$$

Probability distribution of bra & ket coherent fields $\psi(x),\ \widetilde{\psi}(x)$

- The distribution P is positive & real
- Density matrix $\widehat{\rho} \leftrightarrow$ distribution P for the fields $\psi(x), \widetilde{\psi}(x)$
  $\leftrightarrow$ random samples of the fields
- From n^M variables we get samples x M

n = Hilbert space dimension at one point

M = numerical lattice size

Schrodinger → Langevin equations

Evolution of $\widehat{\rho}$ → diffusive evolution of P
(Fokker-Planck equation)

→ random walk of samples of $\psi(x), \widetilde{\psi}(x)$

Observables

$$\langle \hat{O} \rangle = \mathrm{Tr}\left[\hat{\rho}\,\hat{O}\right]$$

Expectation values of observables → moments of P

→ stochastic averages of samples $\psi(x), \widetilde{\psi}(x)$

<u>As samples → ∞ we get better precision</u>

$$\boxed{\rho_1(\mathbf{x},\mathbf{x}') = \left\langle \hat{\Psi}^\dagger(\mathbf{x})\hat{\Psi}(\mathbf{x}') \right\rangle = \mathrm{Re}\langle \widetilde{\psi}(\mathbf{x})^* \psi(\mathbf{x}')\rangle_{st}}$$

Bare positive-P equations

PD, Drummond PRL **98,** 120402 (2007)

$$i\hbar\frac{d\psi(x)}{dt} = \left\{H_0(x) + g\,\widetilde{\psi}^*(x)\psi(x) + \sqrt{i\hbar g}\,\xi(x,t)\right\}\psi(x)$$

$$i\hbar\frac{d\widetilde{\psi}(x)}{dt} = \left\{H_0(x) + g\,\psi^*(x)\widetilde{\psi}(x) + \sqrt{i\hbar g}\,\widetilde{\xi}(x,t)\right\}\widetilde{\psi}(x)$$

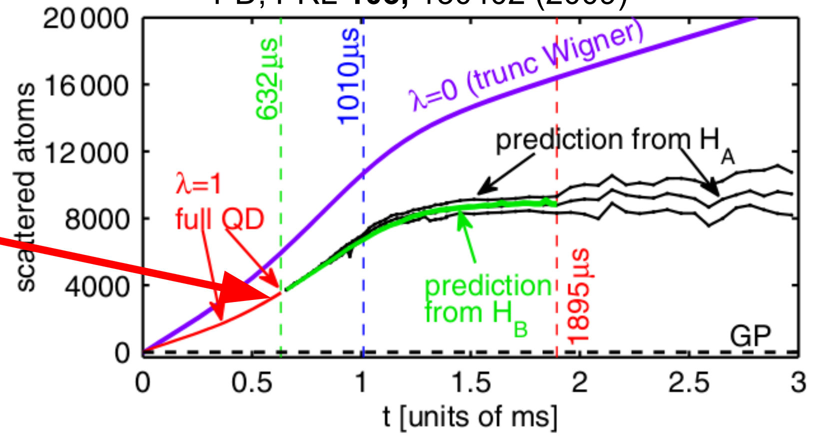
Mean field GP equation

Rest of quantum mechanics

Gaussian real white noise $\xi(x,t),\ \widetilde{\xi}(x,t)$

trouble: noise amplification

PD, PRL **103,** 130402 (2009)



Bogoliubov positive-P equations

PD, Chwedeńczuk, Ziń, Trippenbach, PRA **83,** 063625 (2011)

Krachmalnicoff *et al*, PRL **104,** 150402 (2010)

$$\boxed{\widehat{\Psi}(\mathbf{x},t) = \phi(\mathbf{x},t) + \widehat{\delta}(\mathbf{x},t)}$$

condensate

Bogoliubov fluctuation field – *MUST BE "small"*

Treat only $\widehat{\delta}(\mathbf{x},t)$ using positive-P representation

$$i\hbar\frac{d\phi(x)}{dt} = \left[H_0(x) + g|\phi(x)|^2\right]\phi(x)$$

Mean field

$$i\hbar\frac{d\psi(x)}{dt} = \left\{H_0(x) + 2g|\phi(x)|^2\right\}\psi(x) + g\phi(x)^2\widetilde{\psi}(x)^* + \sqrt{i\hbar g}\,\phi(x)\xi(x,t)$$

$$i\hbar\frac{d\widetilde{\psi}(x)}{dt} = \left\{H_0(x) + 2g|\phi(x)|^2\right\}\widetilde{\psi}(x) + g\phi(x)^2\psi(x)^* + \sqrt{i\hbar g}\,\phi(x)\widetilde{\xi}(x,t)$$

Now equations are linear ------> no blow-up of noise :)

Can use plane wave basis ---> no diagonalizing of 10^6 X 10^6 matrices :)
---> less human time used! :)

He* BEC collisions (T=0)

Frequency doubling

Coherent

Energy stolen from mf(t)

Main halo

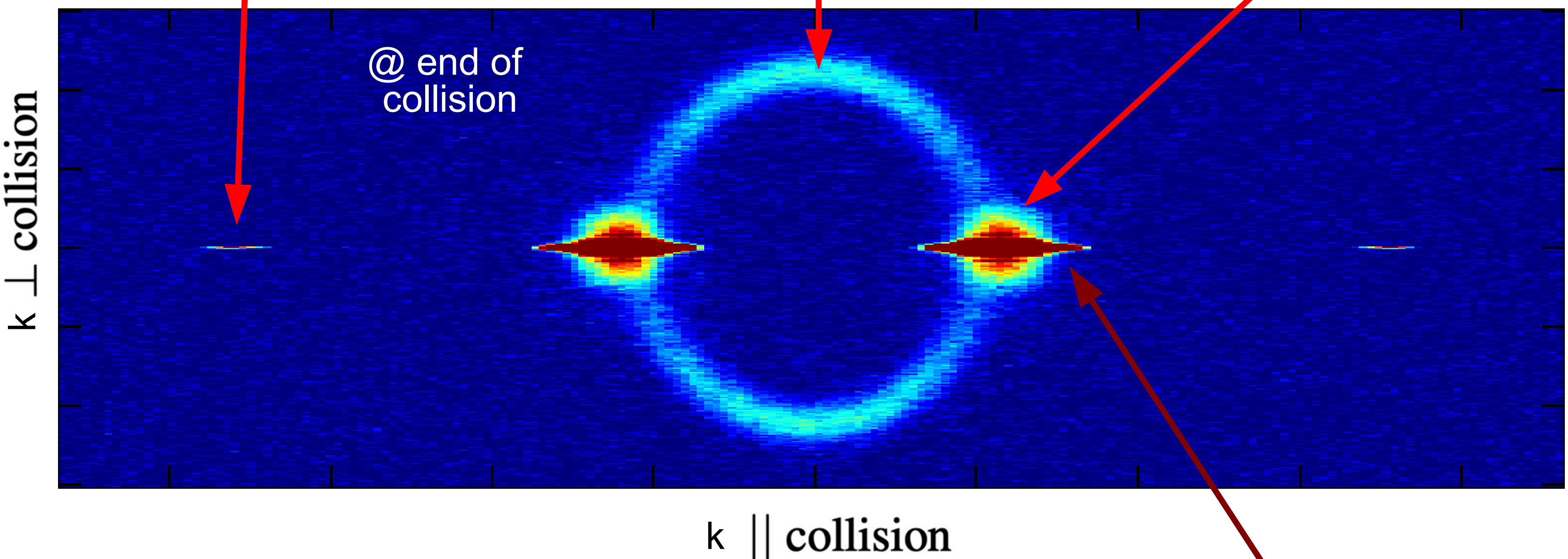
k, -k pairs

Energy resonant

Mean-field Induced halo

$k_o \pm \delta k$ pairs

Energy stolen from mf(t)

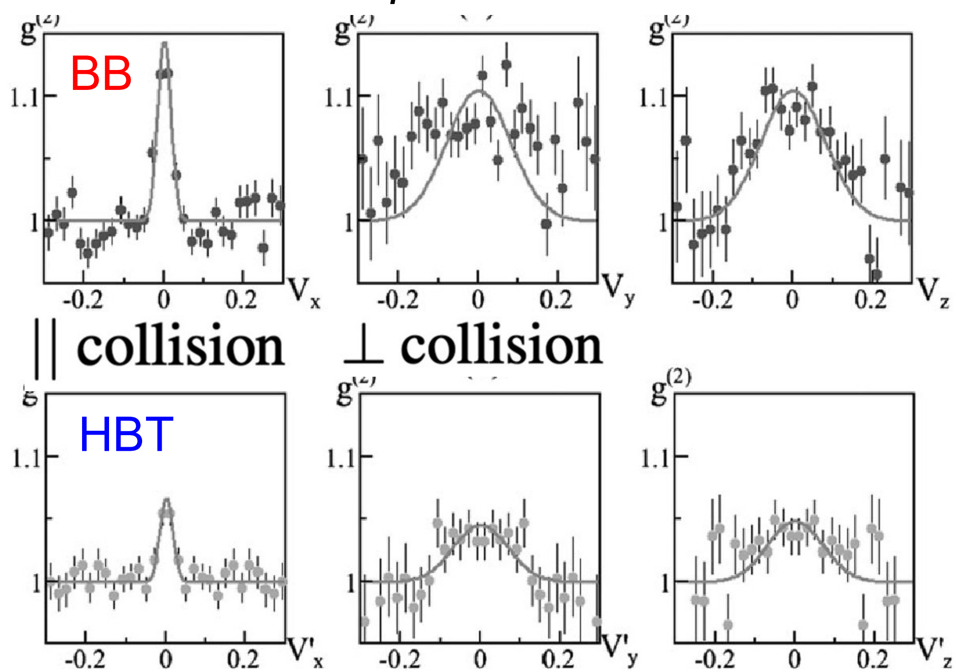


BEC

As per experiment:
Krachmalnicoff, Jaskula, Bonneau, Leung, Partridge, Boiron, Westbrook, PD, Zin, Trippenbach, Kheruntsyan, PRL **104,** 150402 (2010)

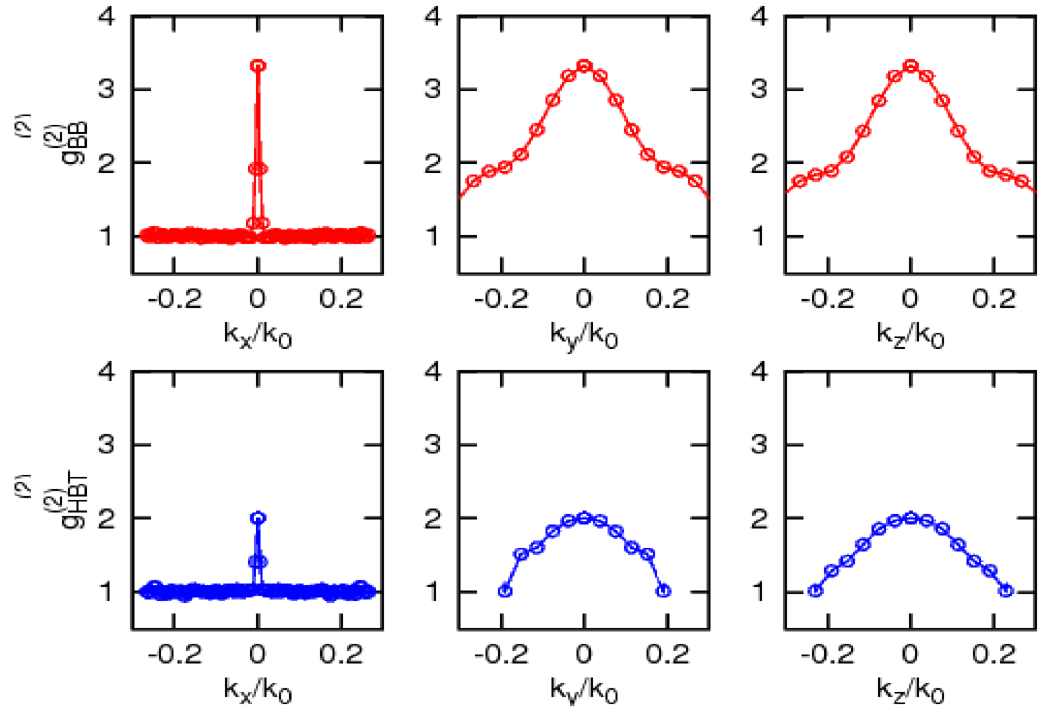
Halo correlations

experiment



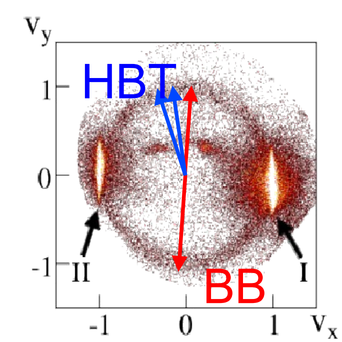
Perrin, Chang, Krachmalnicoff, Schellekens, Boiron, Aspect, Westbrook, PRL **99**, 150405 (2007)

simulation



Pair correlations along collision

experiment			numerics		
BB width	$\frac{\text{BB width}}{\text{HBT width}}$	$\frac{\text{BB height}}{\text{HBT height}}$	BB width	$\frac{\text{BB width}}{\text{HBT width}}$	$\frac{\text{BB height}}{\text{HBT height}}$
$0.017\ k_0$	1.1	2.1	$0.004\ k_0$	1	2.2



Krachmalnicoff, Jaskula, Bonneau, Leung, Partridge, Boiron, Westbrook, PD, Zin, Trippenbach, Kheruntsyan, PRL **104,** 150402 (2010)

Cauchy-Schwartz inequality violation

Kheruntsyan, Jaskula, PD, Bonneau, Partridge, Ruaudel, Boiron, Lopes, Westbrook, arXiv:1204.0058

Cross-correlation ≤ Auto correlation

e.g. currents

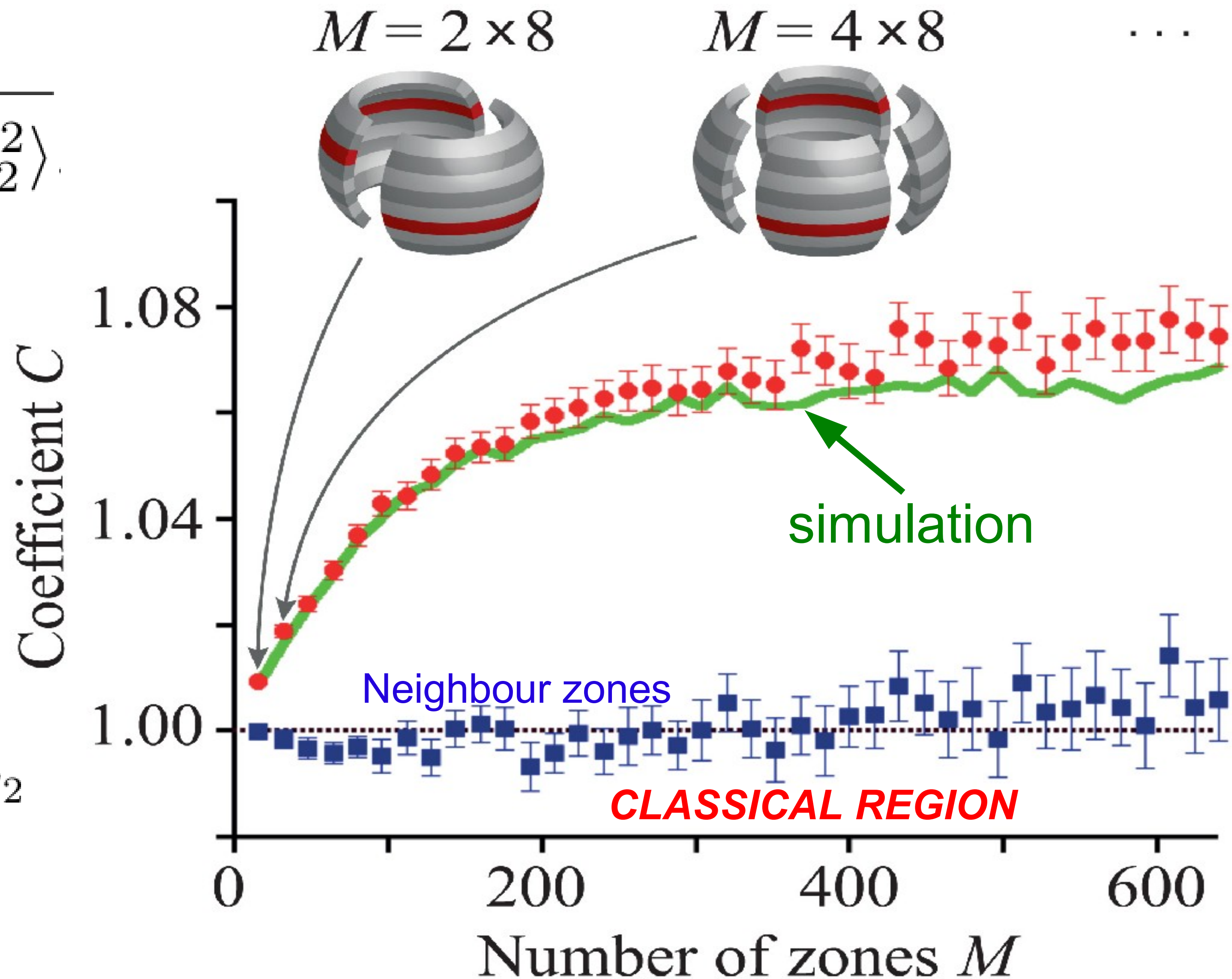
$$|\langle I_1 I_2\rangle| \leq \sqrt{\langle I_1^2\rangle\langle I_2^2\rangle}$$

local correlations

$$G_{12}^{(2)} \leq [G_{11}^{(2)} G_{22}^{(2)}]^{1/2}$$

Bin averaged correlations

$$C = \overline{\mathcal{G}}_{12}^{(2)} / [\overline{\mathcal{G}}_{11}^{(2)} \overline{\mathcal{G}}_{22}^{(2)}]^{1/2}$$



Classical field model for T>0

e.g. free space : plane wave basis

Full quantum field

c-fields

$$\Psi(\mathbf{r}) = \sum_{\mathbf{k}} a_{\mathbf{k}} \frac{1}{\sqrt{V}} e^{i\mathbf{kr}} \longrightarrow \Phi(\mathbf{r}) = \sum_{|\mathbf{k}|\leqslant \mathbf{K}_{\max}} \alpha_{\mathbf{k}} \frac{1}{\sqrt{V}} e^{i\mathbf{kr}}$$

Replace mode amplitude operators $a_{\mathbf{k}}$
with complex number amplitudes $\alpha_{\mathbf{k}}$

Thermal initial state:

- $|\alpha_{\mathbf{k}}|^2$ Distributed according to Bose-Einstein distribution
- Phase of $\alpha_{\mathbf{k}}$ is random
- Use many realizations to get thermal ensemble

Useful papers:

Brewczyk, Gajda, Rzazewski, J. Phys B **40,** R1 (2007)

Blakie, Bradley, Davis, Ballagh, Gardiner, Adv. Phys. **57**, 363 (2008)

Proukakis, Jackson, J. Phys A **41,** 203002 (2008)

Brewczyk, Borowski, Gajda, Rzazewski, J Phys B **37**, 2725 (2004)

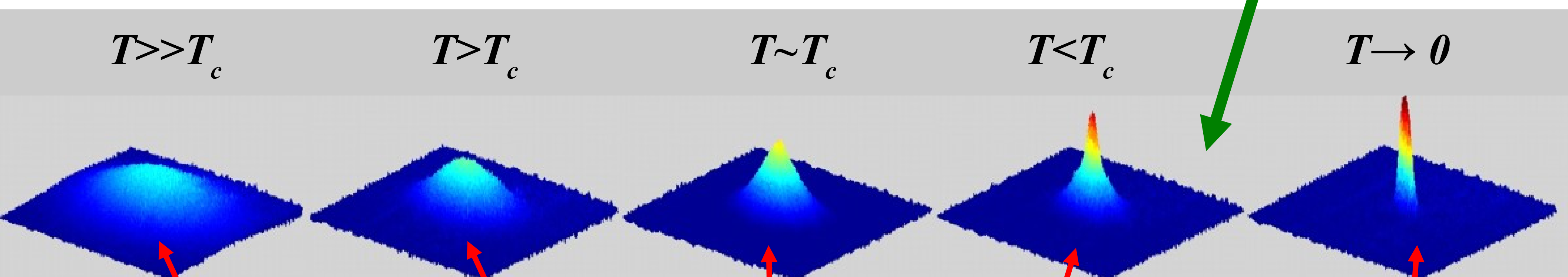
Validity of classical field

$$\left[\hat{\Psi}(x), \hat{\Psi}^\dagger(x')\right] = \delta(x - x') \qquad \longrightarrow \qquad [\psi^*(x), \psi(x')] = 0$$

→ it will be fine, ...

.... as long as there are always many atoms involved in whatever it is we are studying

Quasicondensate



http://www-matterwave.physics.ox.ac.uk/node/1

Mode occupation getting small

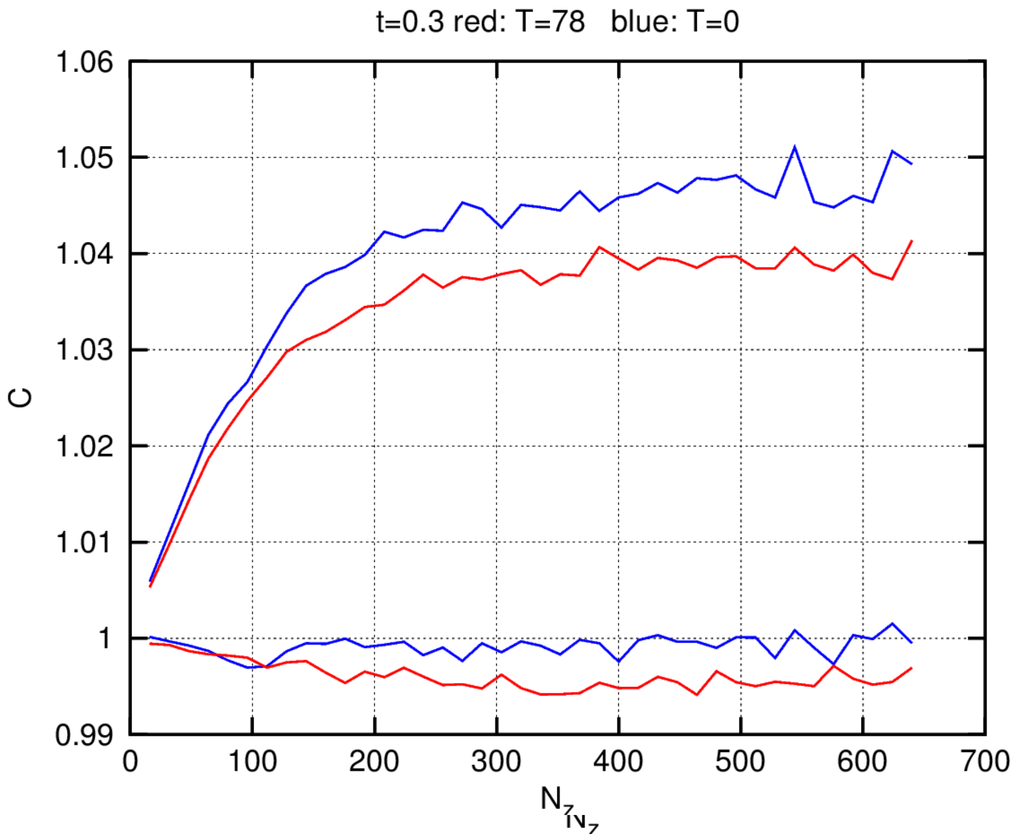
OK for Classical-field method:
All relevant modes highly occupied

Occupation in excited modes getting small

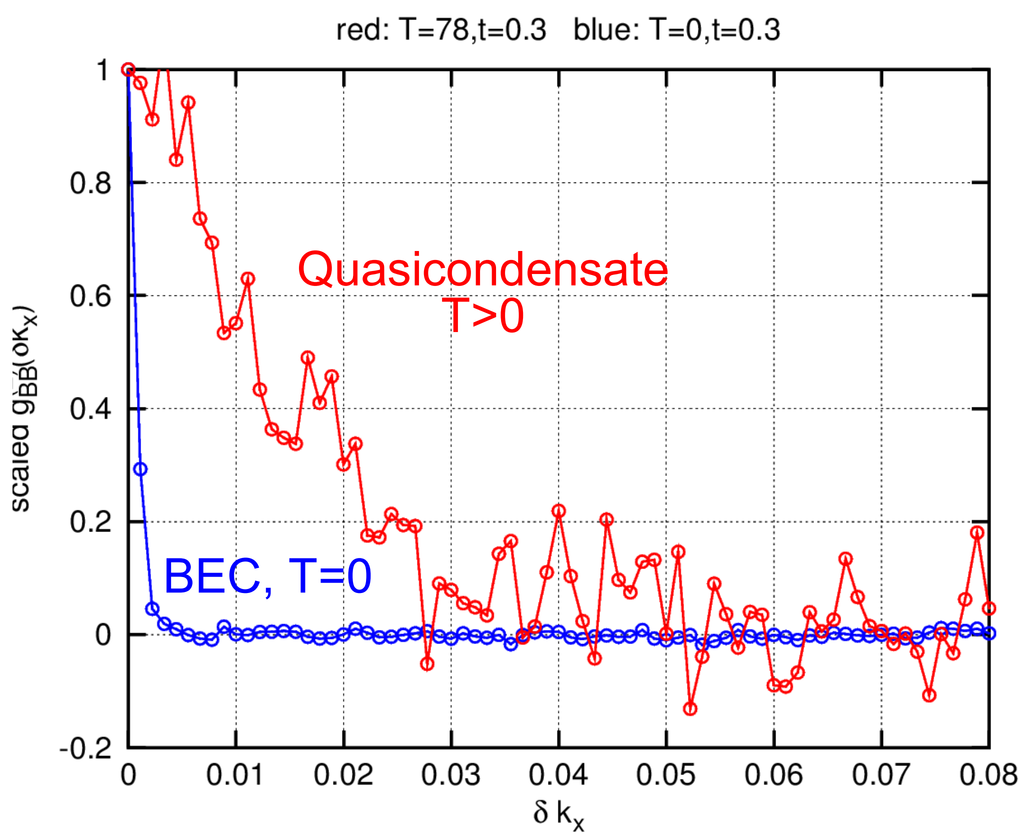
Quasicondensate effects on correlations

Initial classical field described by model of
Phase-fluctuating 3D condensates in elongated traps
Petrov, Shlyapnikov, Walraven, PRL **87**, 050404 (2001)

CS-violation not affected much



k,-k correlations strongly affected



Conclusions / Outlook

- “Straightforward” simulation of supersonic quantum dynamics

  With positive-P treatment of Bogoliubov field

- Bogoliubov dynamics made tractable for large systems

  By use of plane wave modes + stochastic noise

- Treat classical field realizations as condensates for T>0

  Quasicondensate (and near Tc?)