Validity and benchmarking of c-fields descriptions of the 1D interacting Bose gas at nonzero temperatures

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Abstract: Classical or c-fields are a way to tractably describe the thermal state of a quantum Bose gas with an ensemble of complex wave functions. They are widely used as one of the only practical methods for quasicondensates and partially condensed gases but have never been checked with exact results when interactions are strong. We benchmark them to the exact Yang & Yang solutions for the interacting 1D uniform gas. This allow us to determine the range of validity and specify good values for the cuttoff parameter that is essential for accurate results.

$$\boldsymbol{H} = \int dx\ \hat{\boldsymbol{\Psi}}^{\dagger}(x)\left[\frac{p^2}{2m} + V_{pot}(x)\right]\hat{\boldsymbol{\Psi}}(x) \ + \frac{g_0}{2}\int dx\ \hat{\boldsymbol{\Psi}}^{\dagger}(x)\hat{\boldsymbol{\Psi}}^{\dagger}(x)\hat{\boldsymbol{\Psi}}(x)\hat{\boldsymbol{\Psi}}(x)$$

$$\hat{\Psi}(x) = \sum_{k}^{K_c} \phi_k(x)\ \hat{a}_k$$

C-fields approximation:

$$\hat{a}_k, \hat{a}^{\dagger}{}_k \ \rightarrow\ \alpha_k, \alpha_k^{\dagger}$$

CUTOFF

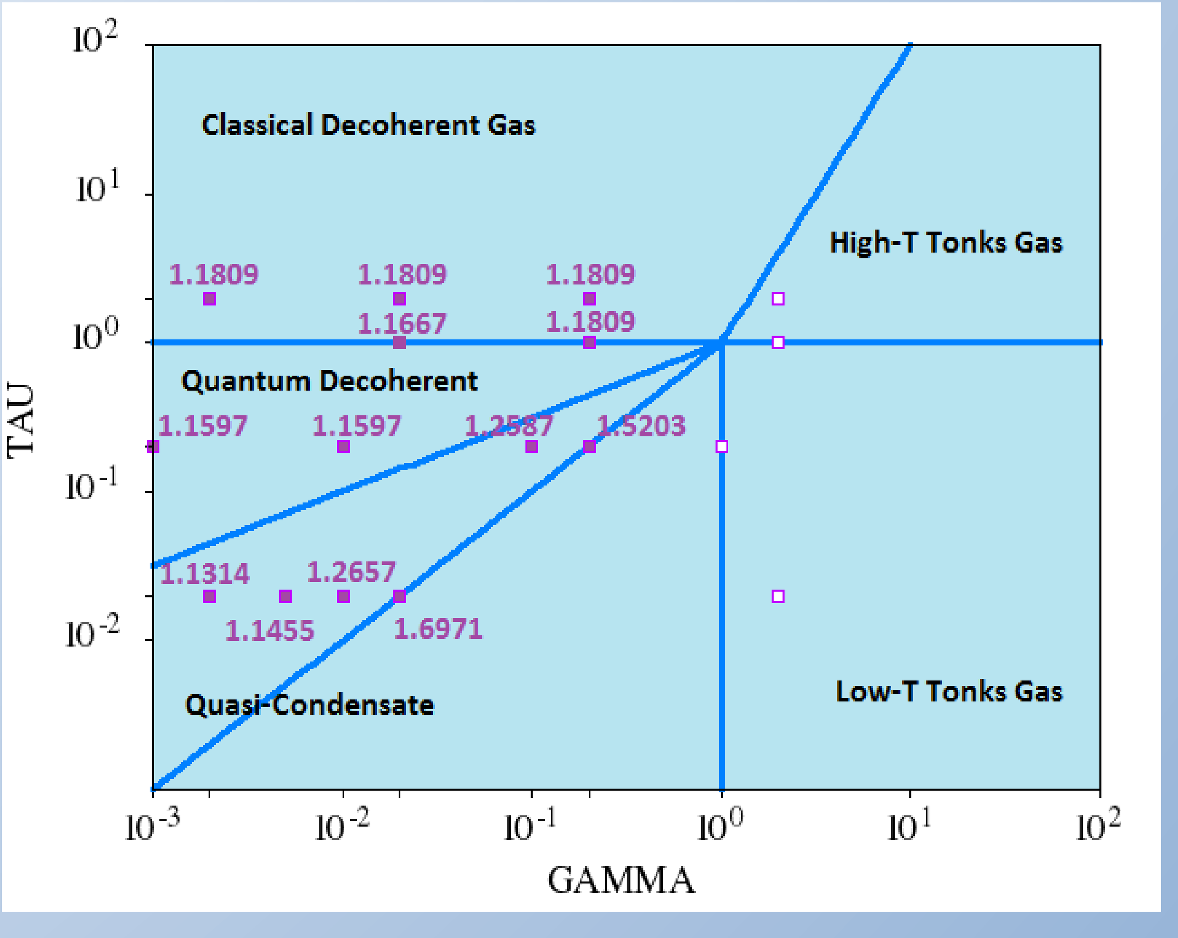
C-fields approximation requires finite numbers of degrees of freedom. In practice this is assured by defining the last state that can be occupied in phase space, labeled by the CUTOFF momentum Kc:

$$\boldsymbol{K_c = f_c \sqrt{2\,T}}$$

Probabilistic properties of the condensate were benchmarked for small interactions **[1]**, **[2]**, in trap **[4]**.

We consider the local density approximation (LDA) and use the grand canonical ensemble (GCE) which is a useful description in such a situation. We benchmark physical observables against the exact Yang & Yang **[5]** solution, precisely matching the density n and parameters γ, τ, and varying the cutoff.

OPTIMAL CUTOFF $\boldsymbol{f_c = \frac{K_c}{\sqrt{2\,T}}}$



$$\tau = \frac{m}{\hbar^2}\frac{2T}{n^2}$$

$$\gamma = \frac{m}{\hbar^2}\frac{g_0}{n}$$

RELATIVE ERROR IN OBSERVABLES

$$RMS(g^{(2)}, \varepsilon) := \sqrt{\left(\frac{g^{(2)}}{g^{(2)}_{YY}} - 1\right)^2 + \left(\frac{\varepsilon}{\varepsilon_{YY}} - 1\right)^2}$$

$g^{(2)}, g^{(2)}_{YY}$ – density-density corelation function; $\varepsilon, \varepsilon_{YY}$ – energy per particle;

$g^{(2)}_{YY}\ \epsilon\ \left\{g^{(2)}_{YY}(0), g^{(2)}_{YY}(TH)\right\}$, $\varepsilon_{YY}\ \epsilon\ \{\ \varepsilon_{kin}, \varepsilon_{tot}, \varepsilon^{TH}_{tot}\ \}$, where TH indicates removed shot noise

min $RMS(g^{(2)}, \varepsilon)$ for $g^{(2)}_{YY}(0)$ & ε_{kin}

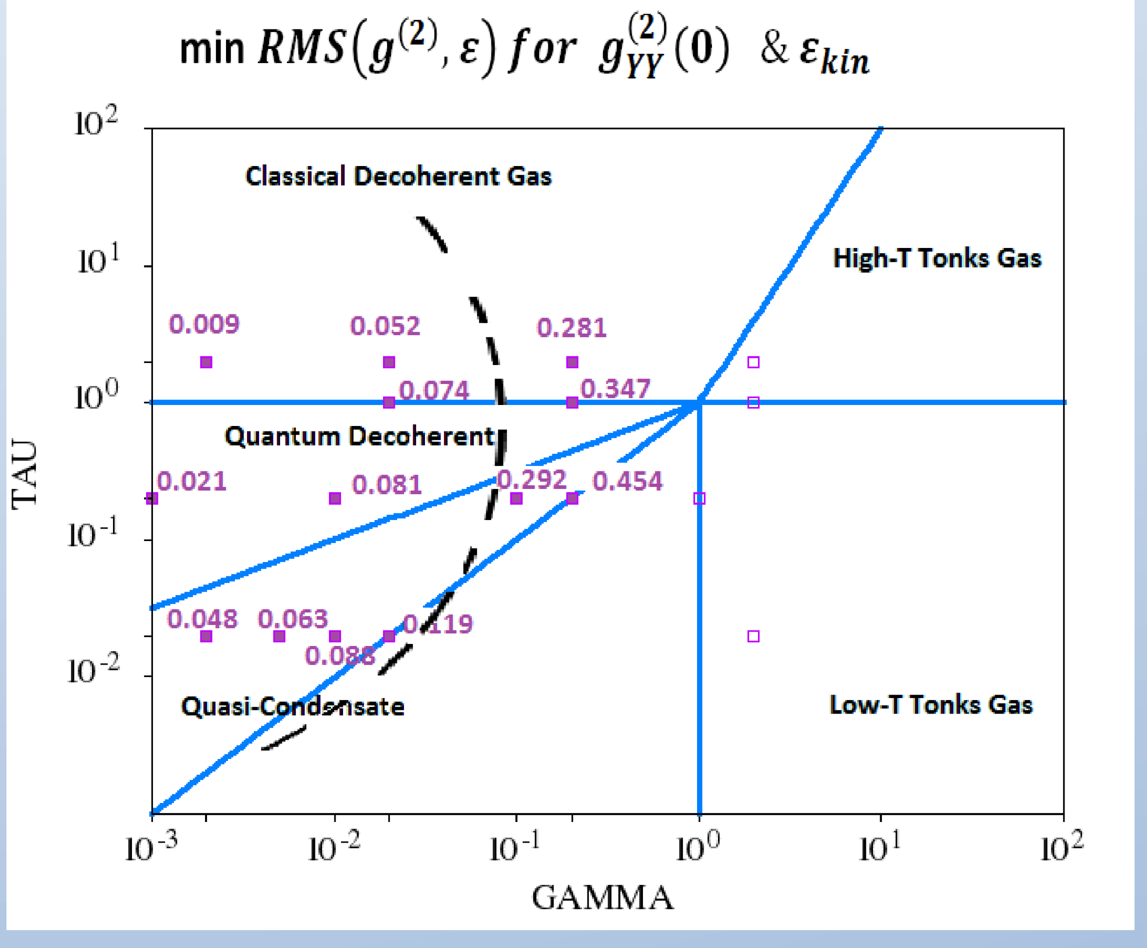


Table of $RMS(g^{(2)}, \varepsilon)$ *for* $\tau = 0.2$

GAMMA=0.001

	(TH,TH)	(TH,0)	(0,TH)	(0,0)	(TH,Ekin)	(0,Ekin)
X0	164.198	164.198	164.243	163.891	164.198	164.328
Y_RMS	0.011263	0.011263	0.0205788	0.0212211	0.0122033	0.0207189

GAMMA=0.01

	(TH,TH)	(TH,0)	(0,TH)	(0,0)	(TH,Ekin)	(0,Ekin)
X0	159.255	157.412	159.453	155.414	163.489	163.008
Y_RMS	0.0359435	0.0328583	0.0792083	0.0770908	0.0387142	0.0810479

GAMMA=0.1

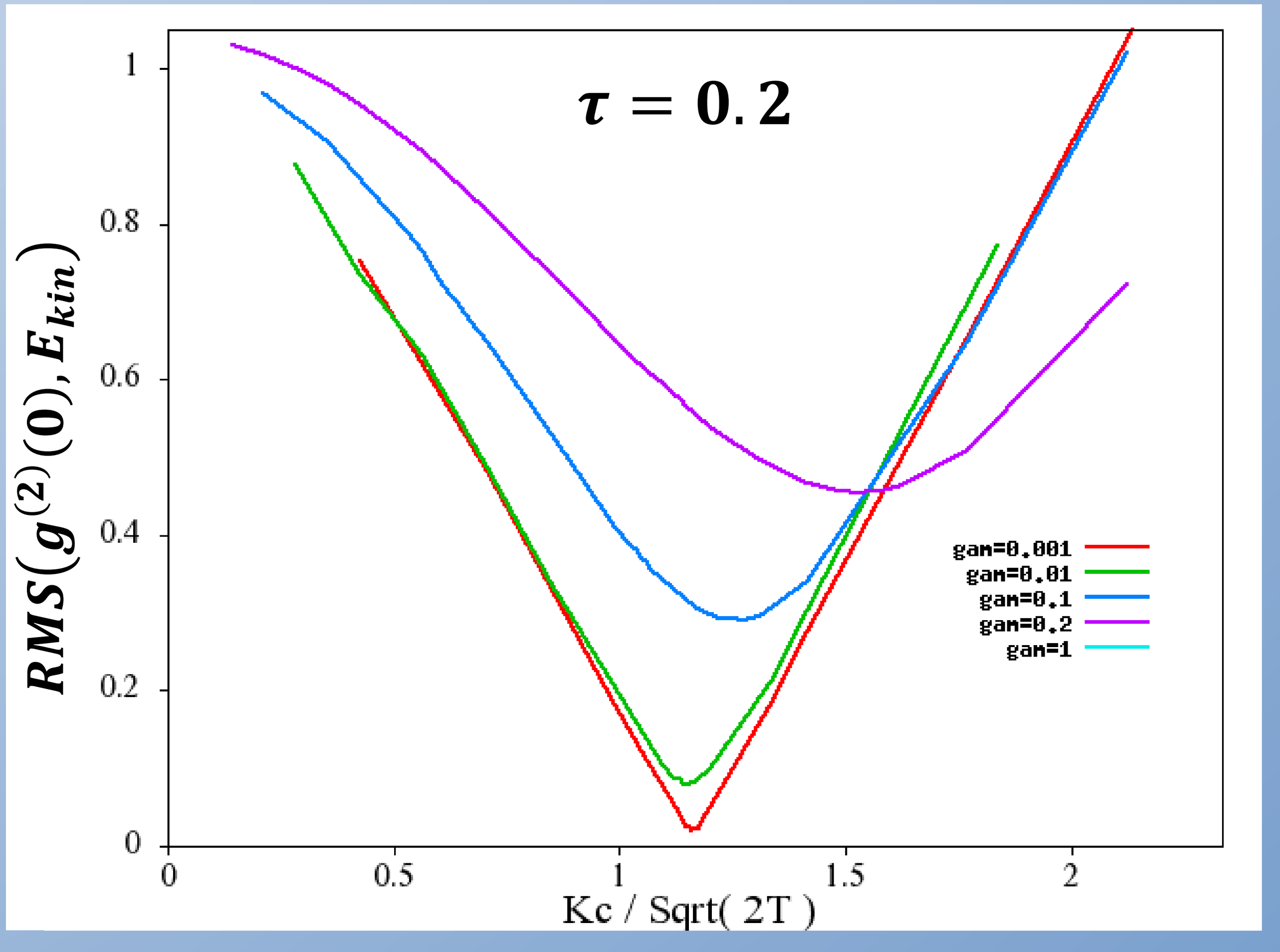
	(TH,TH)	(TH,0)	(0,TH)	(0,0)	(TH,Ekin)	(0,Ekin)
X0	148.808	85.0086	83.8075	-	180.976	177.777
Y_RMS	0.0488432	0.00720279	0.249572	-	0.0676026	0.291863

GAMMA=0.2

	(TH,TH)	(TH,0)	(0,TH)	(0,0)	(TH,Ekin)	(0,Ekin)
X0	161.488	-	-	-	222.015	215.702
Y_RMS	0.0512825	-	-	-	0.07674	0.454345

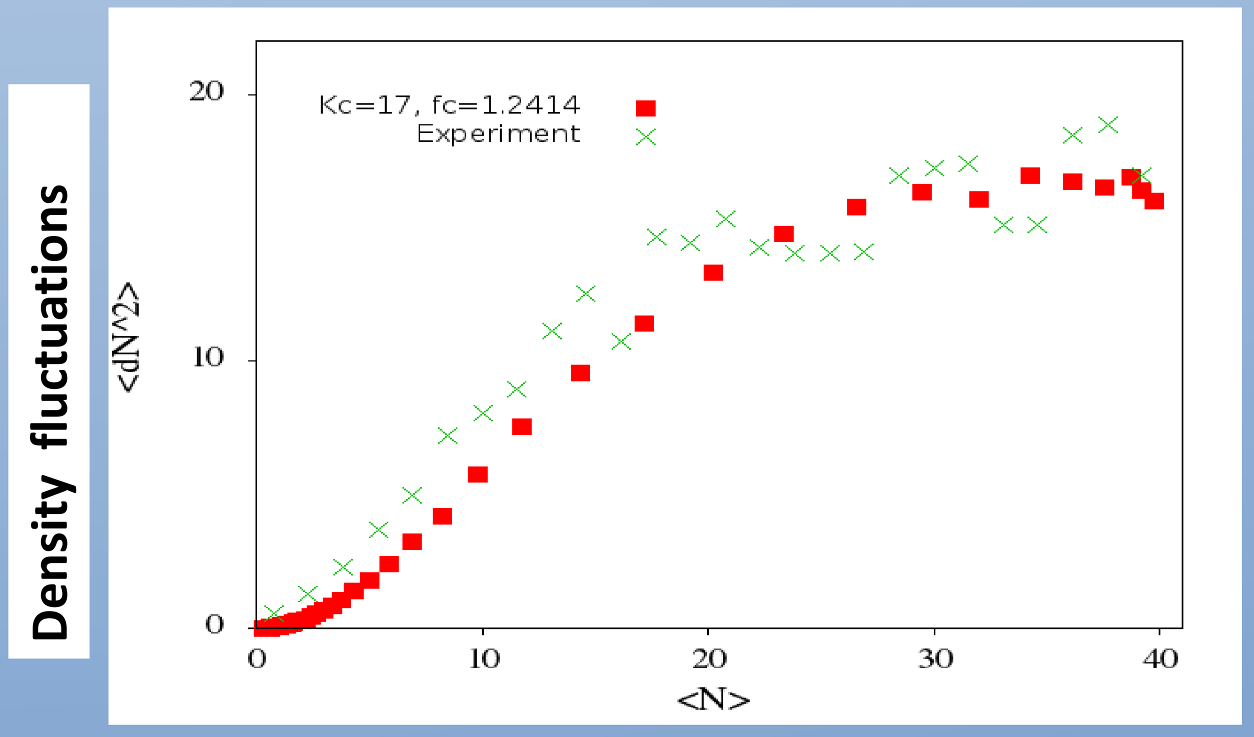
GAMMA=1

	(TH,TH)	(TH,0)	(0,TH)	(0,0)	(TH,Ekin)	(0,Ekin)
X0	-	-	-	-	-	-
Y_RMS	-	-	-	-	-	-



TEST

LDA cutoff applied to a trapped gas as in the experiment [3]: 1D quasicondensate gas in a harmonic trap at T=18nK.



Conclusions:

1. We rigorously benchamark the classical field description of **the interacting 1D Bose gas** by comparing with exact results (Yang & Yang).
2. We are striving toward determining the regime of validity of the prescription ($RMS\ \leq 0.1$) as well as the optimum values of the cutoff.
3. Choosing relevant obserbables is essential, because the optimum cutoffs and accuracy of the method do differ between observables.
4. Usefulness in practice… we have endeavoured to present an easy prescription that does not depend on trap geometries.

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[2] E. Witkowska *et. al.*, Phys. Rev. A **79**, 033631 (2009)
[3] J. Armijo *et. al.*, Phys. Rev. A **83**, 021605(R) (2011)
[4] S. P. Cockburn *et. al.*, Phys. Rev. A **83**, 043619 (2011)
[5] C. N. Yang and C. P. Yang, Journal Of Mathematical Physics **10**, 1115 (1969)

Support: the National Science Centre grant No. 2012/07/E/ST2/01389