Can one have a consistent c-field description of ultracold Bose gases?

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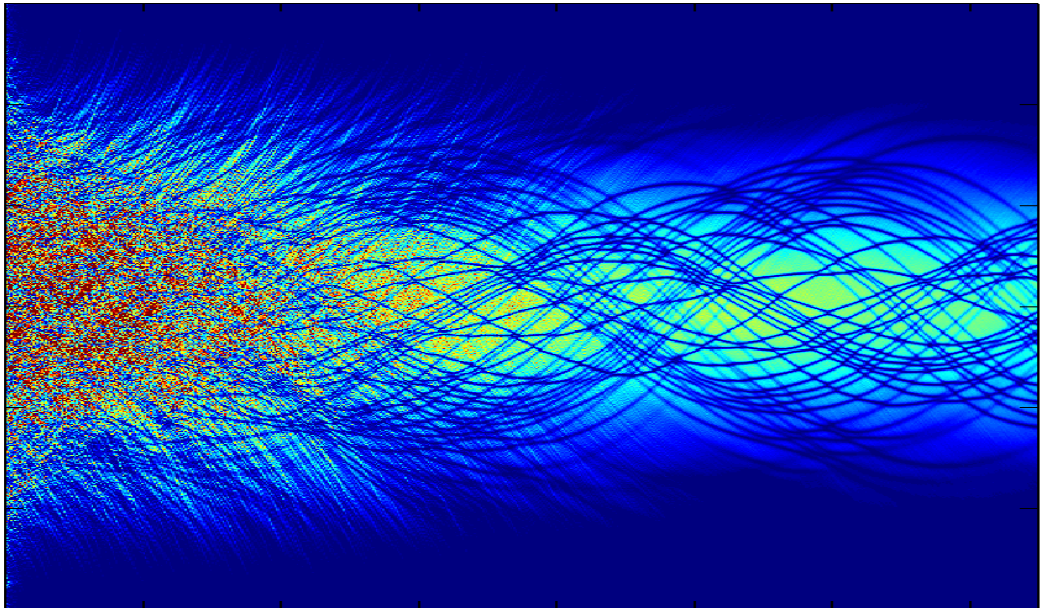
arXiv:1504.06154

Thermal states and defects

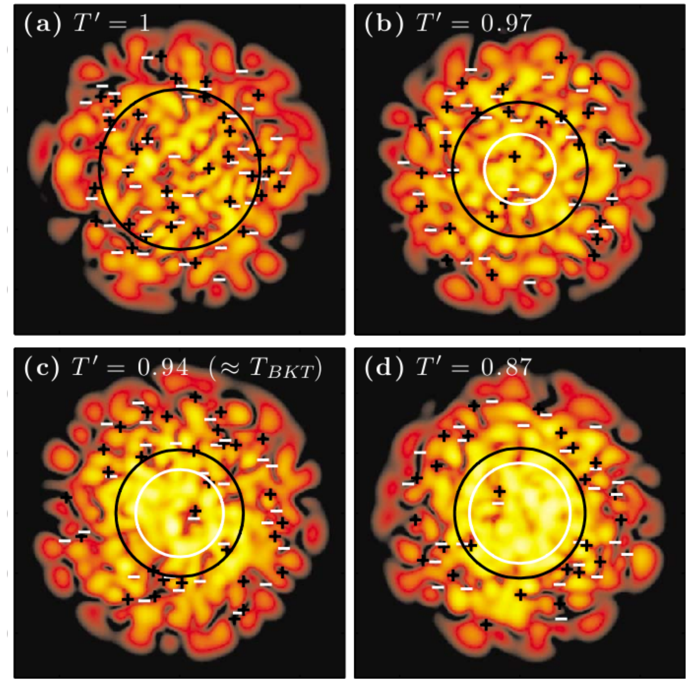
Evaporative cooling (temperature quench)

Witkowska, PD, Gajda, Rzążewski, *PRL* **106**, 135301 (2011)



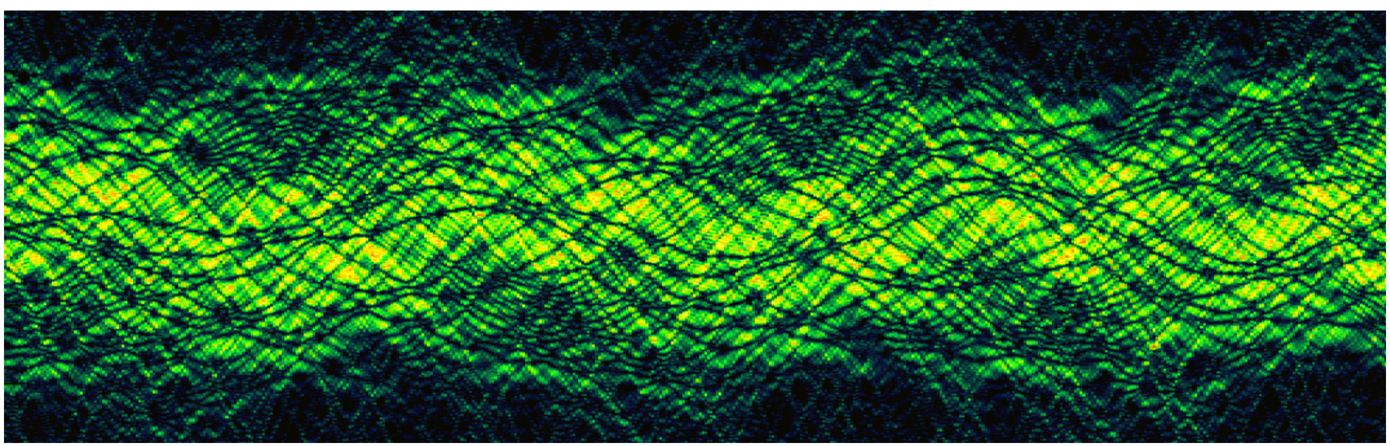
Vortex pairs in 2D gas

Bisset, Davis, Simula, Blakie, *PRA* **79**, 033626 (2009)



Solitons in thermal equilibrium state

Karpiuk, PD, Bienias, Witkowska, Pawłowski, Gajda, Rzążewski, Brewczyk, *PRL* **109**, 205302 (2012)



Classical fields approximation

Full quantum field → Ensemble of complex-fields

$$\hat{\Psi}(\mathbf{x}) = \sum_k \hat{a}_k \psi_k(\mathbf{x}) \rightarrow \left\{ \sum_{k\in\mathcal{C}} \xi_k \psi_k(\mathbf{x}) \right\}$$

The dreaded cutoff k_c

Assume highly occupied modes

Replace mode amplitude operators $\hat{a}_k$

with complex number amplitudes ξ_k

"Quantum field theory, without discretized particles"

Evolution: nonlinear Schrodinger equation

$$i\hbar \frac{d\phi(x)}{dt} = \left[H_0(x) + g|\phi(x)|^2 \right] \phi(x)$$

Developed by many authors:

A. Sinatra, M. Brewczyk, M. Gajda, M. Davis, K. Rzazewski, K. Burnett, E. Witkowska, … *(no particular order)*

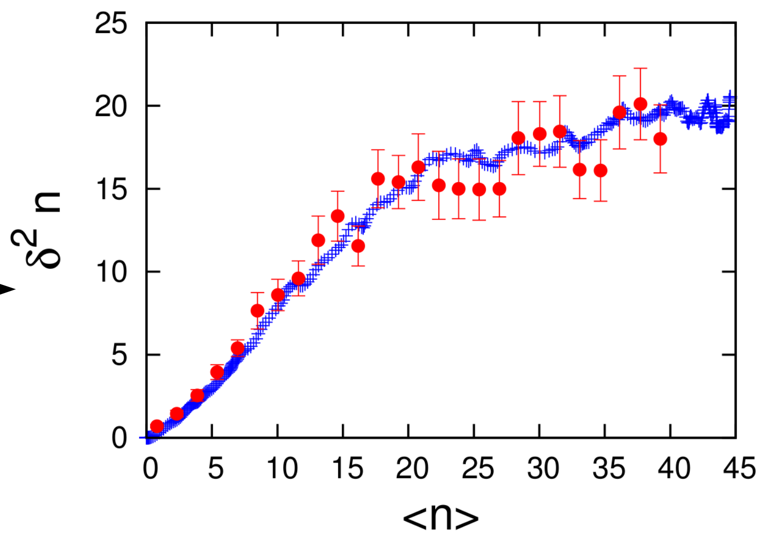
Useful Reviews: M. Brewczyk *et al*, J. Phys B **40,** R1 (2007);

P. Blakie *et al.* Adv. Phys. **57**, 363 (2008)

Qualitative or quantitative?

- For many problems, classical fields (c-fields) are the only viable method.
  - Especially when single realizations are needed
- Perennial questions:
  - Fine, but, are the effects real?
  - is it quantitative or only qualitative?
  - what was the cutoff used?
- Perennial answers:
  - It's okay if there are many particles
  - Can work very well →

Local density fluctuations in a trapped 1D bose gas



Karpiuk, PD, Bienias, Witkowska, Pawłowski, Gajda, Rzążewski, Brewczyk, *PRL* **109**, 205302 (2012)

The cutoff..

- The cutoff k_c is a very important parameter. Recommendations differ, though:

Study	Cutoff energy suggestions
Ideal gases Canonical ensemble Consideration of number of excited atoms Witkowska, Gajda, Rzazewski, *PRA* **79**, 033631 (2009)	Uniform: 0.30 k_BT in 1D Trapped: 1.0 k_BT in 1D Other values in 2D, 3D
SGPE calculations of interacting gas Cockburn, Negretti, Proukakis, Henkel, *PRA* **83**, 043619 (2011)	Match particle number in truncated Wigner description to ideal gas
Brewczyk etal Brewczyk, Gajda, Rzążewski, *J. Phys. B* **40**, R1 (2007)	Match energy in high E modes to k_BT (equipartition) ~ 1 particle in high E modes
Consideration of damping rates Sinatra, Lobo, Castin, *J. Phys. B* **35**, 3599 (2002)	~< k_BT ~ 10 particles in mode below cutoff
Widely used rule of thumb	k_BT + chemical potential

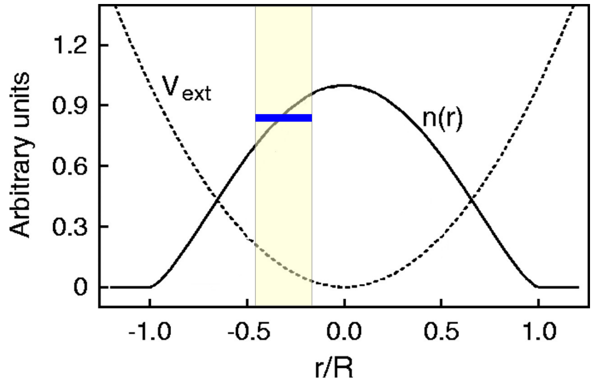
<u>Initial plan</u>: benchmark 1D quasicondensate with exact solution

Yang, Yang, *J. Math. Phys.* **10**, 1115 (1969)

<u>Realization:</u> even ideal gas is not well understood

Generic case: uniform section of gas

- Local Density approximation (LDA)
  - → Grand Canonical ensemble
  - (rest of gas acts as a reservoir)



Units:

Dimensionless temperature

$$\tau = \frac{T}{T_d}$$

Ideal gas degeneracy temperature

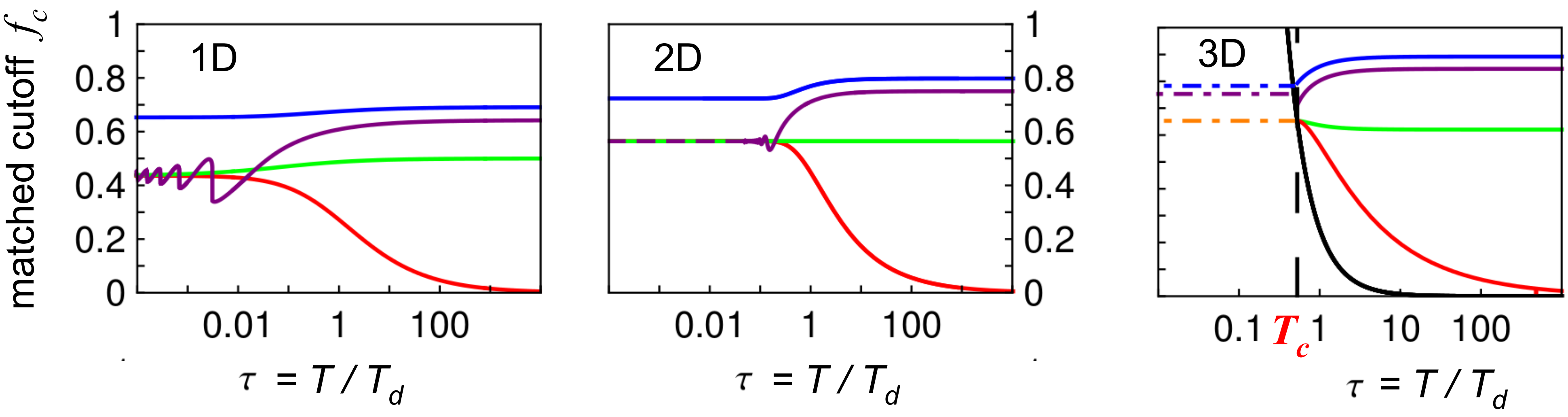
Dimensionless cutoff $f_c = \frac{k_c}{k_T}$

$$k_T = \frac{2\pi}{\Lambda_T}$$

Thermal de Broglie wavelength $\Lambda_T = \sqrt{\frac{2\pi\hbar^2}{mk_B T}}$

Cutoff optimum for different observables

Pietraszewicz, PD, arXiv:1504.06154



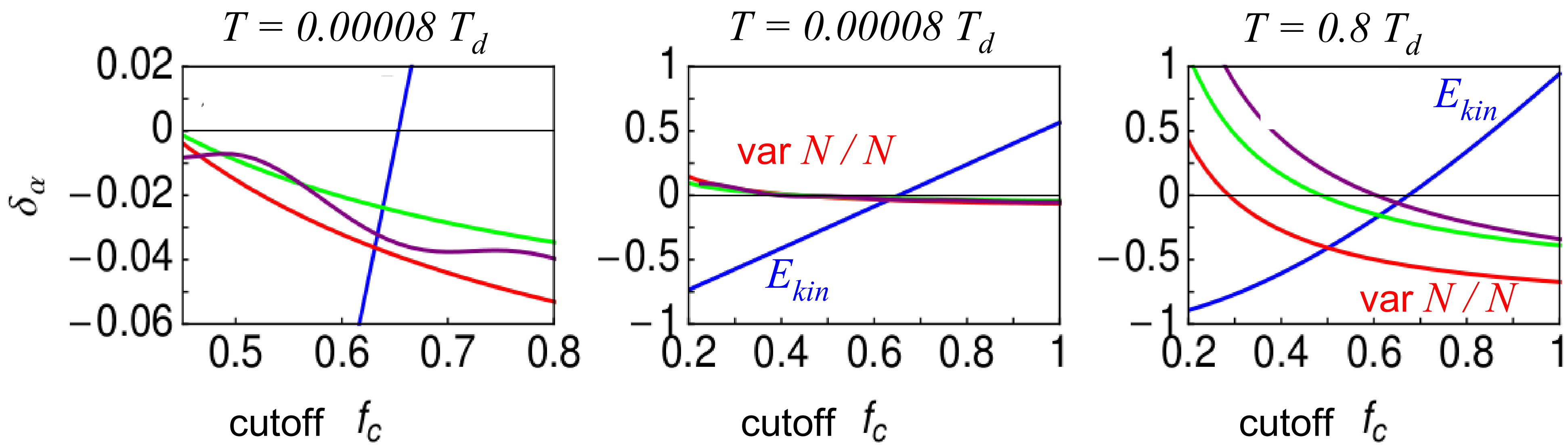
- E_{kin} Kinetic energy per particle
- varN / N Coarse-grained fluctuations
- l_{pg} phase grain volume (~ coherence length l_{Φ})
- Half-width of $g^{(1)}(x)$
- ρ_0 condensate fraction

Most extreme behaviour

Accuracy

Pietraszewicz, PD, arXiv:1504.06154

Single observable error $\delta_\alpha(\tau, f_c) := \frac{\Delta\alpha}{\alpha} = \left(\frac{\alpha^{(\mathrm{cf})}(\tau, f_c)}{\alpha^{(\mathrm{id})}(\tau)} - 1\right)$



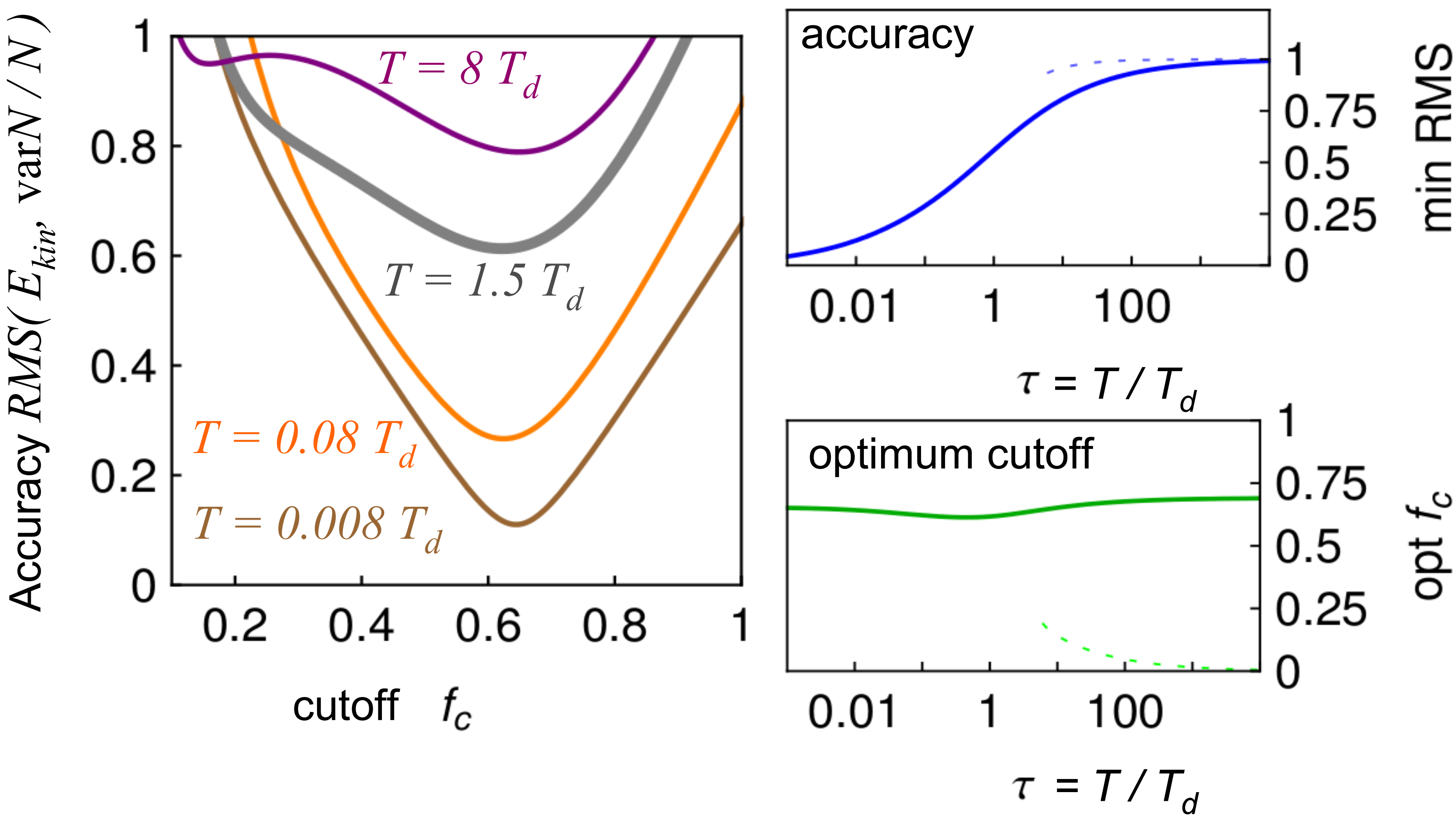
Global error $RMS_{\alpha,\beta,\ldots}(\tau, f_c) = \sqrt{\left(\delta_\alpha\right)^2 + \left(\delta_\beta\right)^2 + \ldots}$

Error in any observable will be < RMS

Kinetic energy and coarse-grained fluctuations capture most extreme behaviour

→ ***use these only***

1D



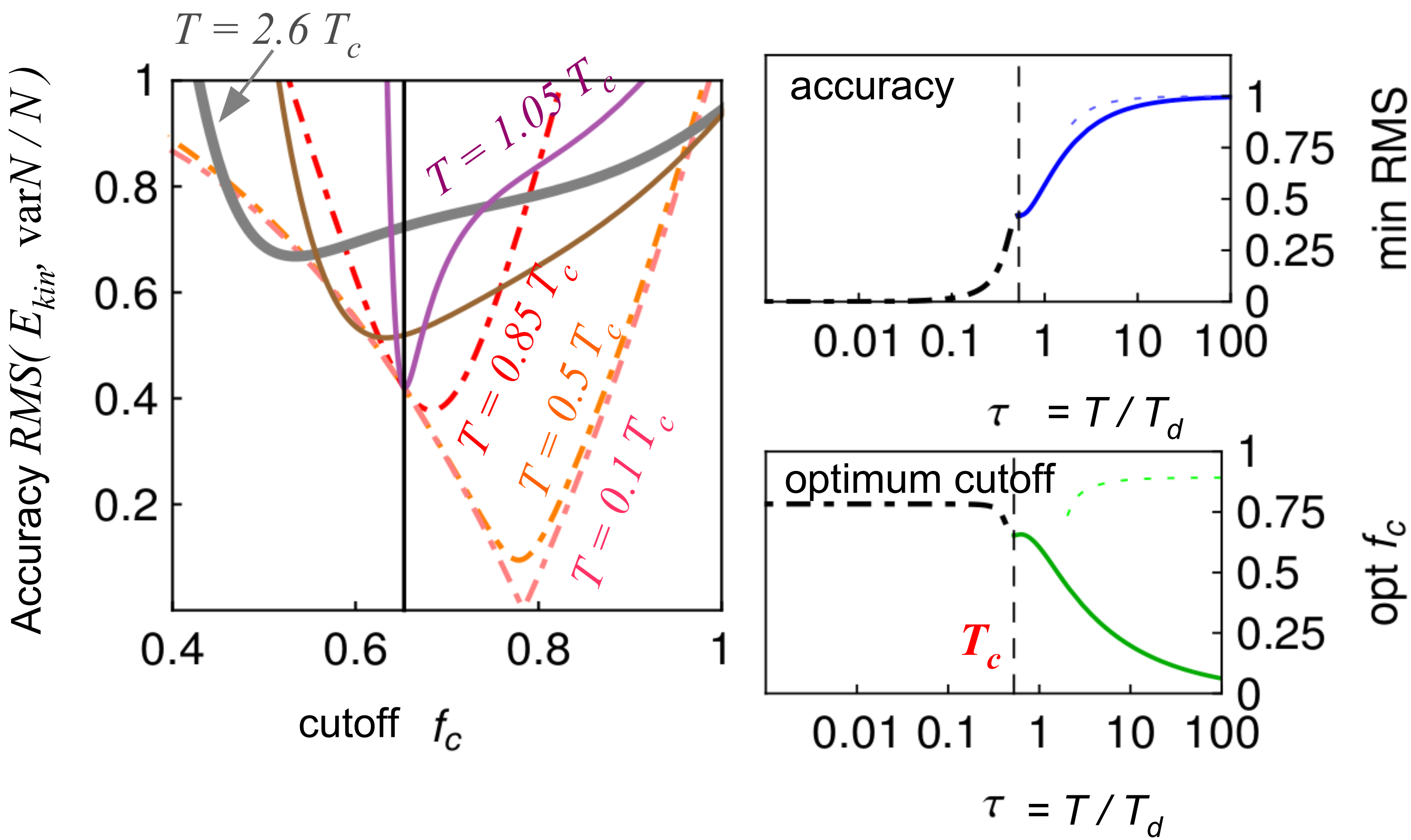
<u>Recommendation:</u>

Accuracy better than 10% for $T < 0.007\ T_d$

Use $f_c = 0.65$ (Energy cutoff = $1.3\ k_B T$)

Pietraszewicz, PD, arXiv:1504.06154

3D



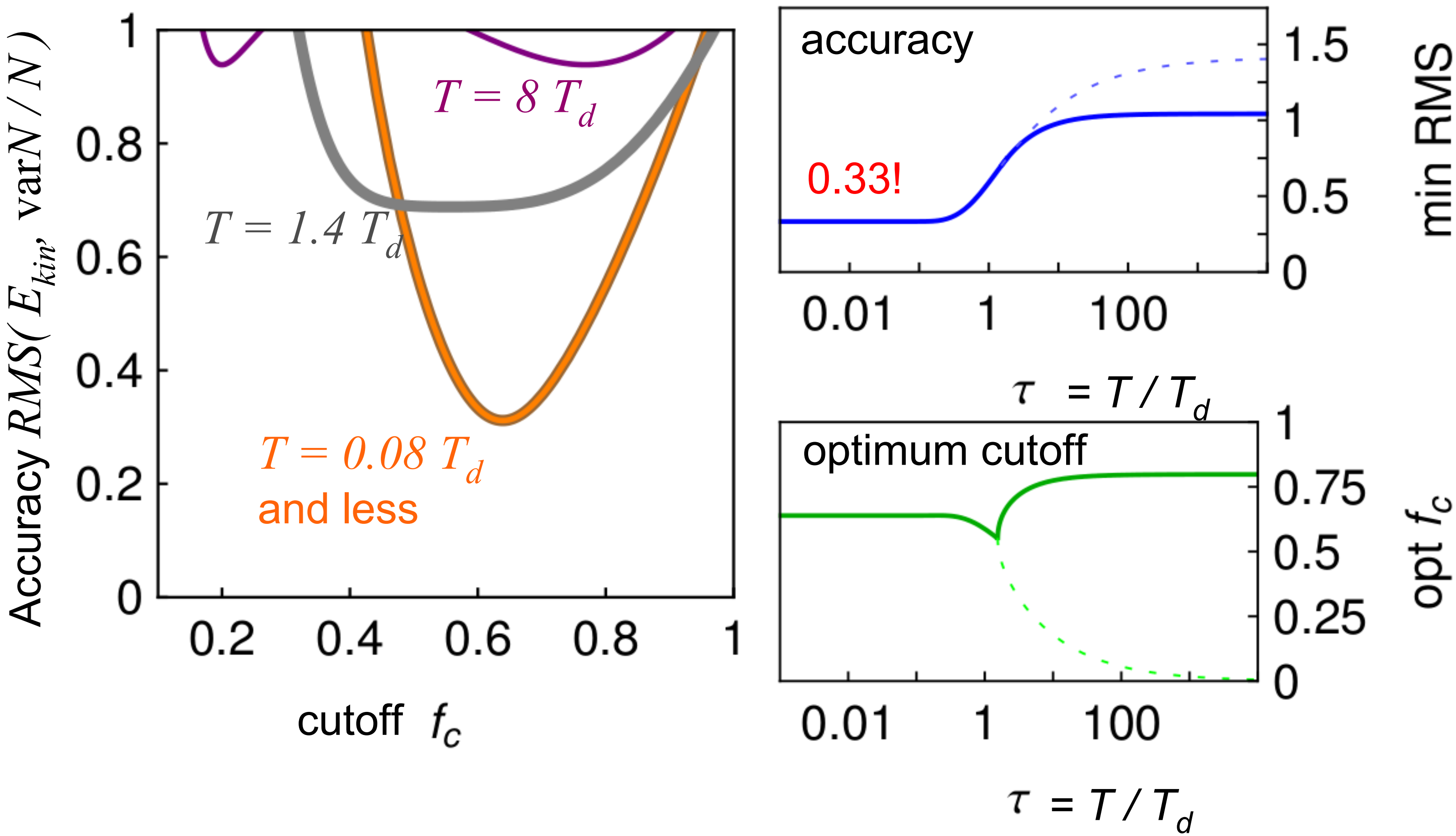
<u>Recommendation:</u>

Accuracy better than 10% for $\mathrm{T} < 0.49\, T_c$

Use $f_c = 0.78$ (Energy cutoff = $1.9\, k_B T$)

Pietraszewicz, PD, arXiv:1504.06154

2D



<u>Recommendation:</u>
Don't use classical fields,
at the least not near the ideal gas regime

Pietraszewicz, PD, arXiv:1504.06154

Interacting gas benchmarking

Comparison to Yang & Yang exact solution

Yang, Yang, *J. Math. Phys.* **10**, 1115 (1969)

temperature

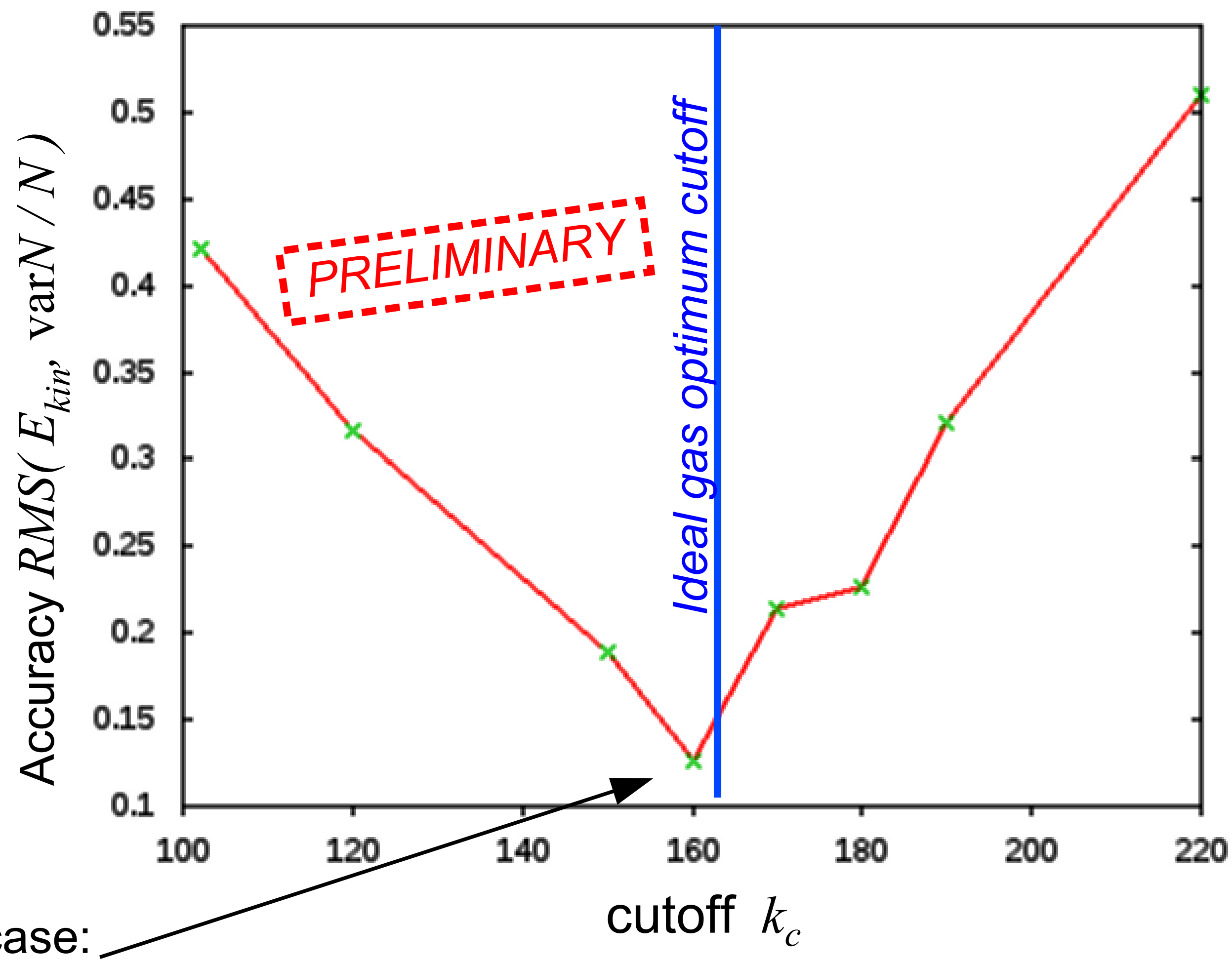
$\tau = T / T_d = 0.0016$

interaction strength

$\gamma = g / n = 0.005$

Quasicondensate:

$g^{(2)}(0) = 1.06$



Quite similar to ideal gas case:

10% accuracy

Same cutoff $1.3\ k_B T$

Summary

arXiv:1504.06154

- Cutoffs and accuracy depend strongly on the observable

  Kinetic energy and density fluctuations are most incompatible

- We found the temperatures and best cutoff for which a consistent and accurate c-field description exists in 1D and 3D.

  However, the 2D ideal gas is never well described

- Preliminary results in the interacting quasicondensate:

  Same cutoff as ideal gas, 10% accuracy also possible.