Violation of the Cauchy-Schwartz inequality with matter waves

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Experiment

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Marie Bonneau
Guthrie Partridge
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Raphael Lopes

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Theory

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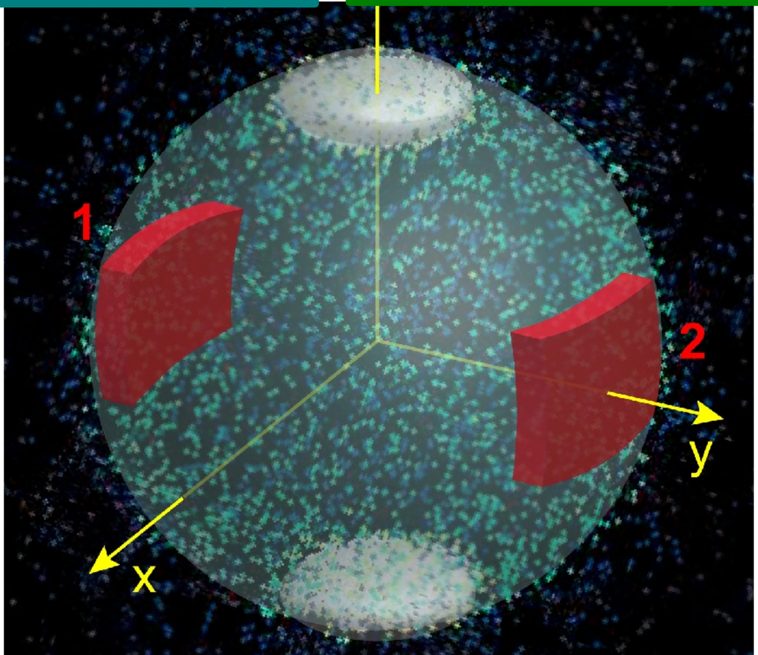


By way of motivation: quantum nonlocality

Bell's theorem:
Any hidden variable theory consistent with relativity cannot describe all the phenomena of quantum mechanics

- 1935 EPR paradox
  "quantum mechanics is incomplete"
- 1964 Bell's inequality
  "either quantum mechanics or local realism"

$$\frac{1}{\sqrt{2}}\left(|\uparrow\uparrow\rangle + \langle|\downarrow\downarrow\right)$$

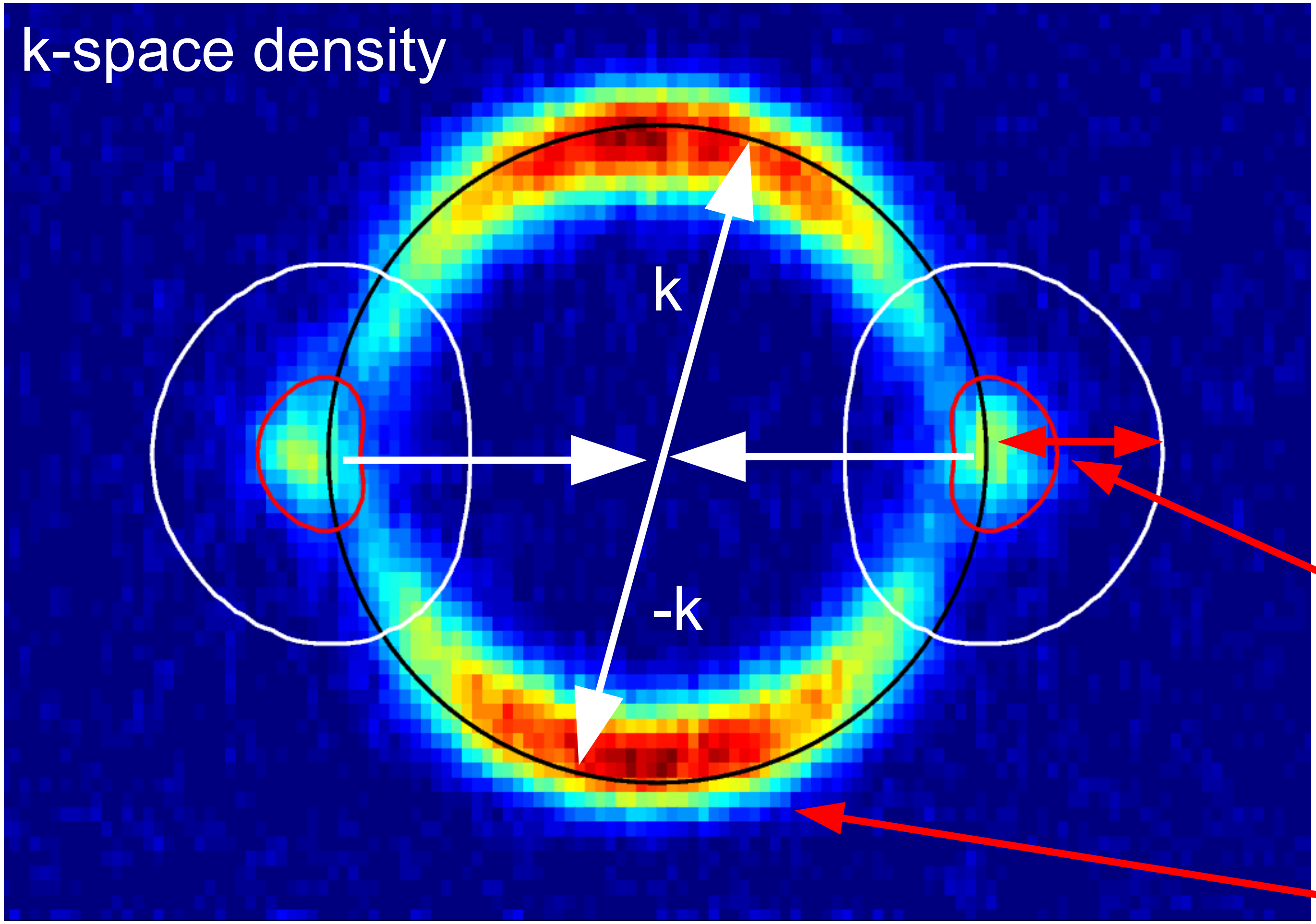
- Experiments:
  - 1972 Freedman & Clauser
  - 1981 Aspect *[photons]* ----- "looks like it's quantum mechanics"
  - 1998 Zeilinger (closed locality loophole)
  - 2001 Wineland (closed detection loophole) *[ion internal states]*
  - 2007 Zeilinger (ruled out Leggett-type non-local realism)
  - 2009 Martinis *[solid state qubits]*

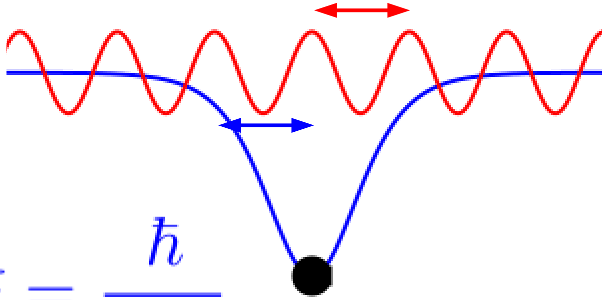
Now consider separated entangled atomic pairs:
- entangled mass
- objects with internal structure

BEC collision

- Above the speed of sound $v_s = \sqrt{\mu/m}$ a condensate no longer behaves as a superfluid

$$\lambda_v = \frac{2\pi\hbar}{mv}$$





$$\xi = \frac{\hbar}{mv_s}$$

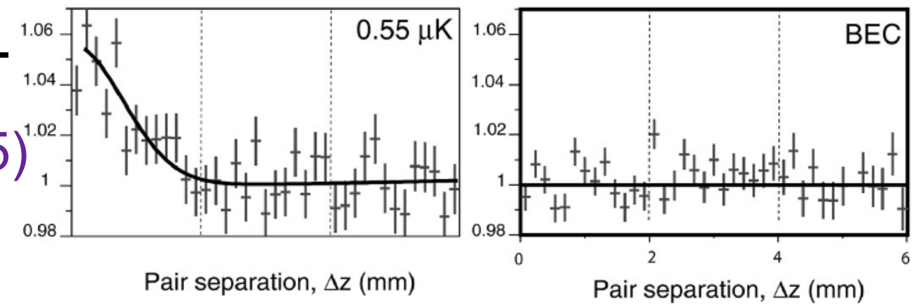
BEC width in K-space is v_s

Scattered atoms Well separated From BEC

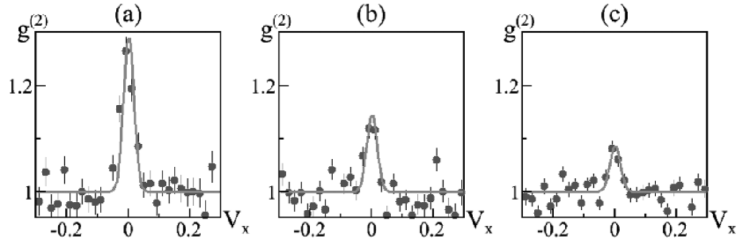
PD, Ziń, Chwedeńczuk, Trippenbach, EPJD **65,** 19 (2011)

So far in Orsay

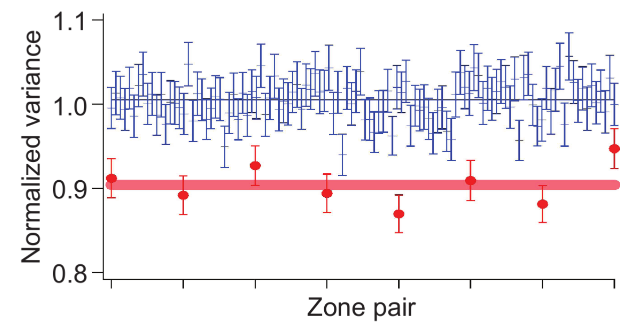
- 2005 single atom measurements, HBT
  M. Schellekens *et al,* Science **310**, 648 (2005)



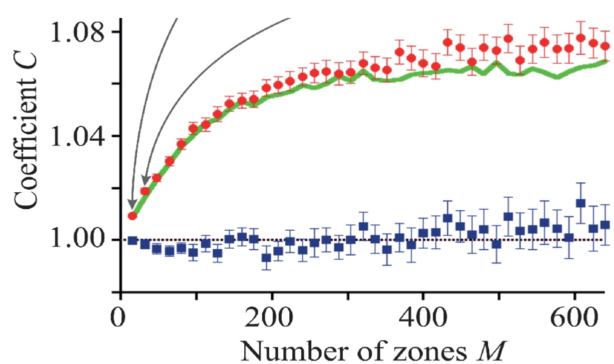
- 2007 Observation of correlations across halo
  A. Perrin *et al,* PRL **99**, 150405 (2007)



- 2010 sub-Poissonian fluctuations of number difference across halo
  J.C. Jaskula *et al,* PRL **105**, 190402 (2010)



- 2012 Cauchy-Schwartz inequality violation
  K. V. Kheruntsyan *et al,* PRL **108**, 260401 (2012)



Cauchy-Schwartz inequality

- For vectors:

$$|\vec{x} \cdot \vec{y}| \leq |\vec{x}| \ |\vec{y}|$$

- Limit on allowed fluctuations of random variables:

$$\mathrm{Cov}\left[X, Y\right] \leq \sqrt{\mathrm{Var}\left[X\right] \mathrm{Var}\left[Y\right]}$$

- Obeyed by observables in classical physics, and <u>any ensemble from which single values are drawn</u>
- Simplest test of stronger-than-classical correlation
- Precursor, necessary condition of Bell tests etc.

Cauchy Schwartz – quantum formulation

- Consider 2 boson modes $\widehat{a}_1 \quad \widehat{a}_2$
- and simultaneous detection of two particles, e.g.
  - expectation value of the number of particles detected in mode 1: $\langle \widehat{a}_1^\dagger \widehat{a}_1^\dagger \widehat{a}_1 \widehat{a}_1 \rangle$
  - expectation value of the number of particles detected taking one from each mode: $\langle \widehat{a}_1^\dagger \widehat{a}_2^\dagger \widehat{a}_2 \widehat{a}_1 \rangle$
- Cauchy-Schwartz inequality for the results of these measurements:

$$\langle \widehat{a}_1^\dagger \widehat{a}_2^\dagger \widehat{a}_2 \widehat{a}_1 \rangle^2 \leq \langle \widehat{a}_1^{\dagger 2} \widehat{a}_1^2 \rangle \langle \widehat{a}_2^{\dagger 2} \widehat{a}_2^2 \rangle$$

$$\boxed{g_{12}^{(2)} \leq \sqrt{g_{11}^{(2)} g_{22}^{(2)}}}$$

$$g_{ij}^{(2)} = \frac{\langle \widehat{a}_i^\dagger \widehat{a}_j^\dagger \widehat{a}_j \widehat{a}_i \rangle}{\langle \widehat{a}_j^\dagger \widehat{a}_j \rangle \langle \widehat{a}_i^\dagger \widehat{a}_i \rangle}$$

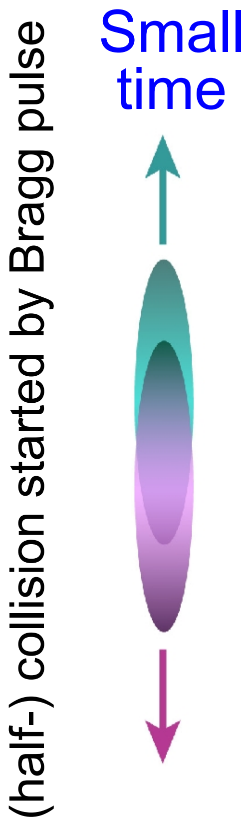
Cauchy-Schwartz inequality – violation

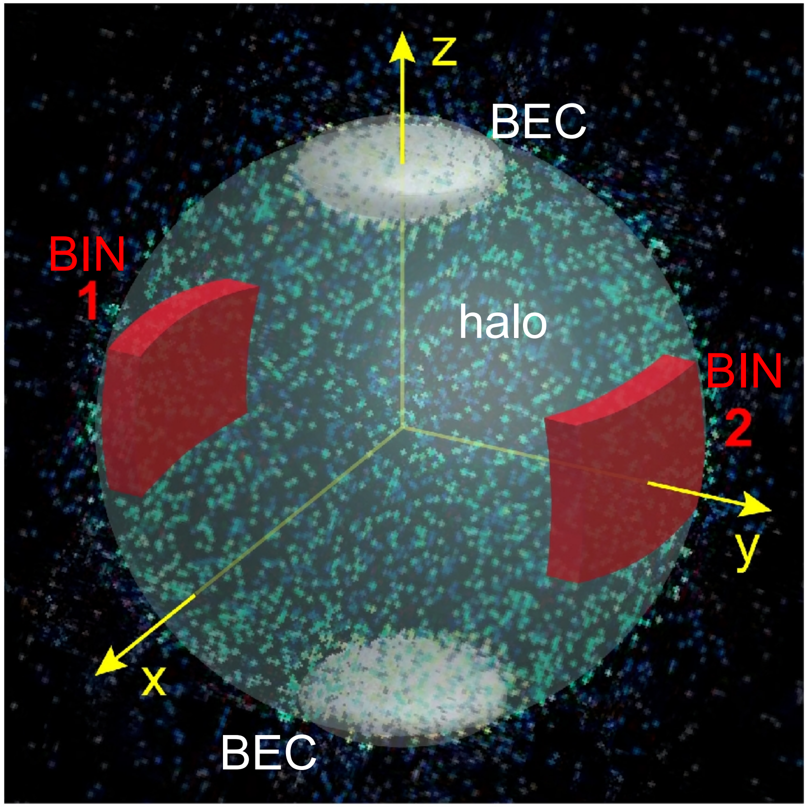
$$\boxed{g_{12}^{(2)} > \sqrt{g_{11}^{(2)} g_{22}^{(2)}}}$$

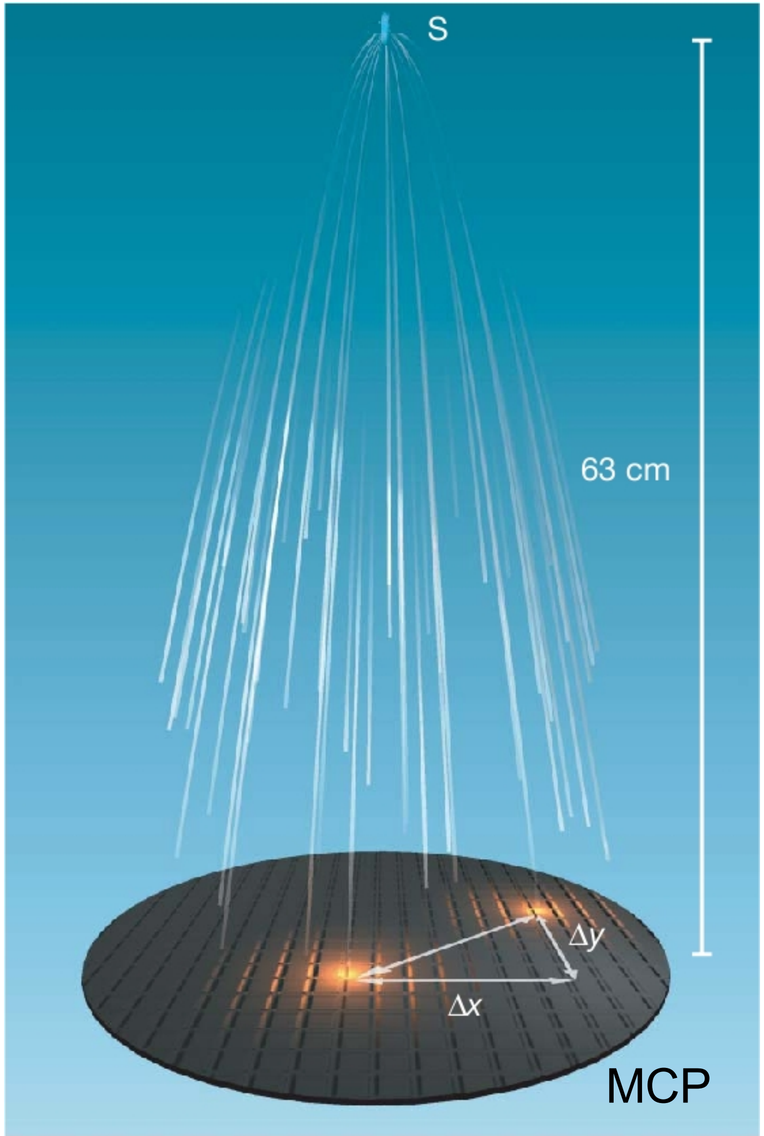
- Stronger than any possible classical correlations
- Cross-correlations between particles in two different modes are larger than correlations between particles in the same mode
- No underlying probability distribution of particle counts in individual modes
- Particle counts not described by random variables.
- Violated in the past in quantum optics:

- Clauser, Phys. Rev. D **9**, 853 (1974)
- Kimble Dagenais, Mandel, PRL **39**, 691 (1997)
- Zou, Wang, Mandel, Opt. Commun. **84**, 351 (1991)
- Marino, Boyer, Lett, PRL **100**, 233601 (2008)

Experiment – setup, bins







short time momentum distribution
=
long time position distribution

Simulation - t-dependent Bogoliubov approx.

PD, Chwedeńczuk, Ziń, Trippenbach, PRA **83,** 063625 (2011)

Useful to see what's going on (only have access to the final distribution in experiment)

$$\widehat{\Psi}(\mathbf{x},t) = \phi(\mathbf{x},t) + \widehat{\delta}(\mathbf{x},t)$$

condensate

Bogoliubov fluctuation field – *MUST BE "small"*

Bogoliubov is "easily solvable". However,
3D simulation: 10^7 spatial grid points. H = 10 000 000 x 10 000 000 matrix?
<u>Did not try to diagonalize</u>

Treat only $\widehat{\delta}(\mathbf{x},t)$ using positive-P representation

See also Wigner treatment:
Sinatra, Castin & Lobo J. Mod Opt **47,** 2629 2000

$$i\hbar\frac{d\phi(x)}{dt} = \left[H_0(x) + g|\phi(x)|^2\right]\phi(x)$$ GP mean field

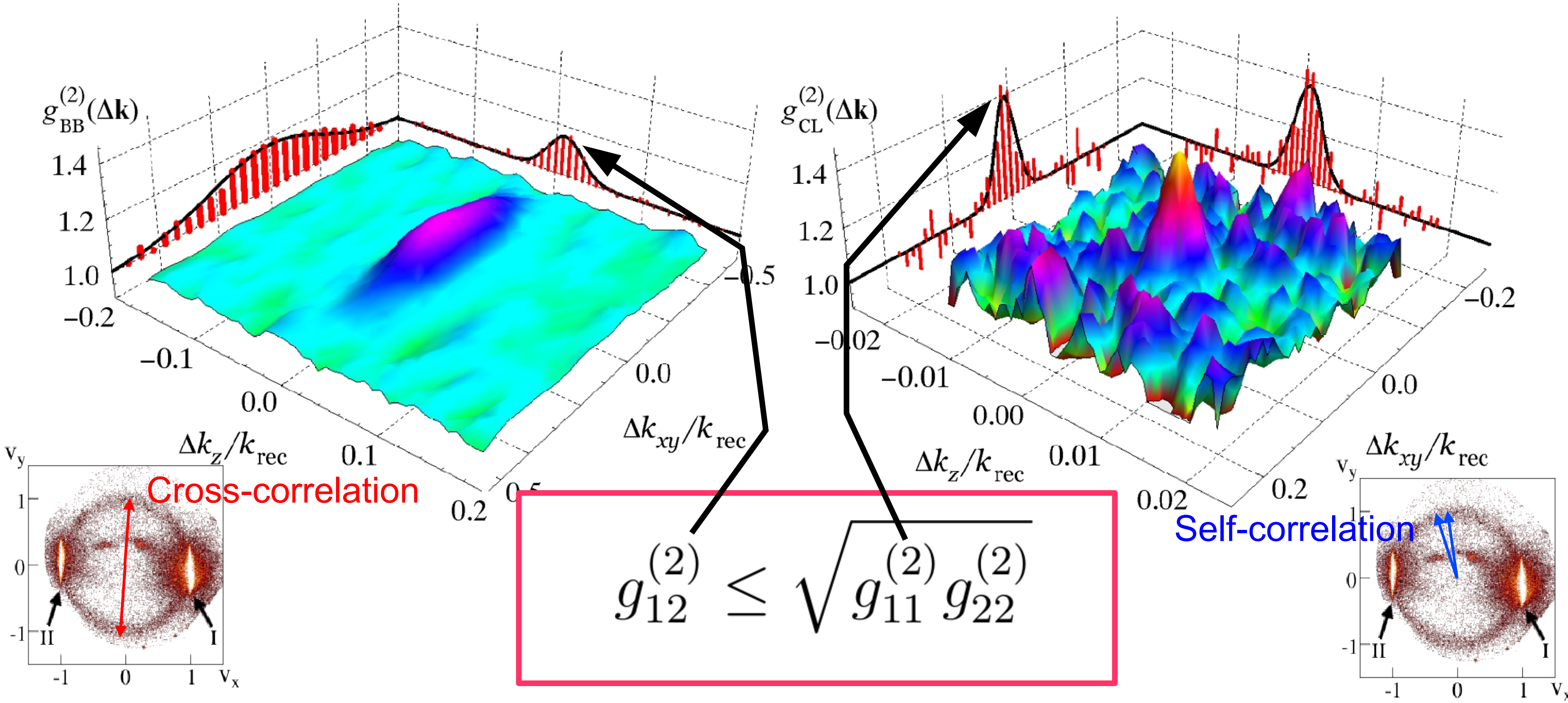
$$i\hbar\frac{d\psi(x)}{dt} = \left\{H_0(x) + 2g|\phi(x)|^2\right\}\psi(x) + g\phi(x)^2\widetilde{\psi}(x)^* + \sqrt{i\hbar g}\,\phi(x)\xi(x,t)$$

$$i\hbar\frac{d\widetilde{\psi}(x)}{dt} = \left\{H_0(x) + 2g|\phi(x)|^2\right\}\widetilde{\psi}(x) + g\phi(x)^2\psi(x)^* + \sqrt{i\hbar g}\,\phi(x)\widetilde{\xi}(x,t)$$

Can use plane wave basis ---> no diagonalizing of 10^7 X 10^7 matrices :)

Experiment - correlations

Kheruntsyan, Jaskula, PD, Bonneau, Partridge, Ruaudel, Boiron, Lopes, Westbrook, PRL **108**, 260401 (2012)



$$g^{(2)}_{12} \leq \sqrt{g^{(2)}_{11}\, g^{(2)}_{22}}$$

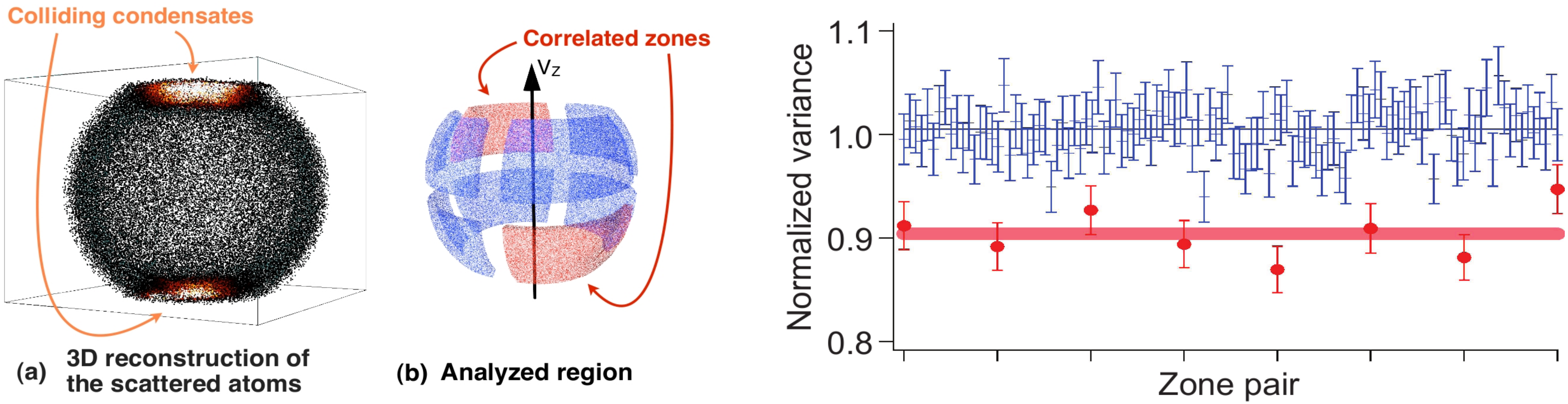
- Two local modes (measurements)
  → use peak values
- Peak values seem to imply no CS violation

Experiment – number squeezing

Jaskula, Bonneau, Partridge, Krachmalnicoff, PD, Kheruntsyan, Aspect, Boiron, Westbrook, *PRL* ***105***, *190402 (2010)*

- However, experiment showed number difference squeezing between opposite regions of the halo.(sub-Poissonian fluctuations)

$$\mathrm{var}\,[n_1 - n_2] < \langle n_1 + n_2 \rangle$$



- Which is equivalent to Cauchy-Schwartz violation in a symmetric two-mode situation.

$$\eta^2 = \frac{\Delta(n_1 - n_2)^2}{n_1 + n_2} \geq 1 \qquad \text{classically}$$

- So what gives? <u>Two-mode assumption!</u>

Multimode Cauchy-Schwartz violation

Kheruntsyan, Jaskula, PD, Bonneau, Partridge, Ruaudel, Boiron, Lopes, Westbrook, PRL **108**, 260401 (2012)

Consider 2 BINS not 2 MODES

Bin averaged correlations

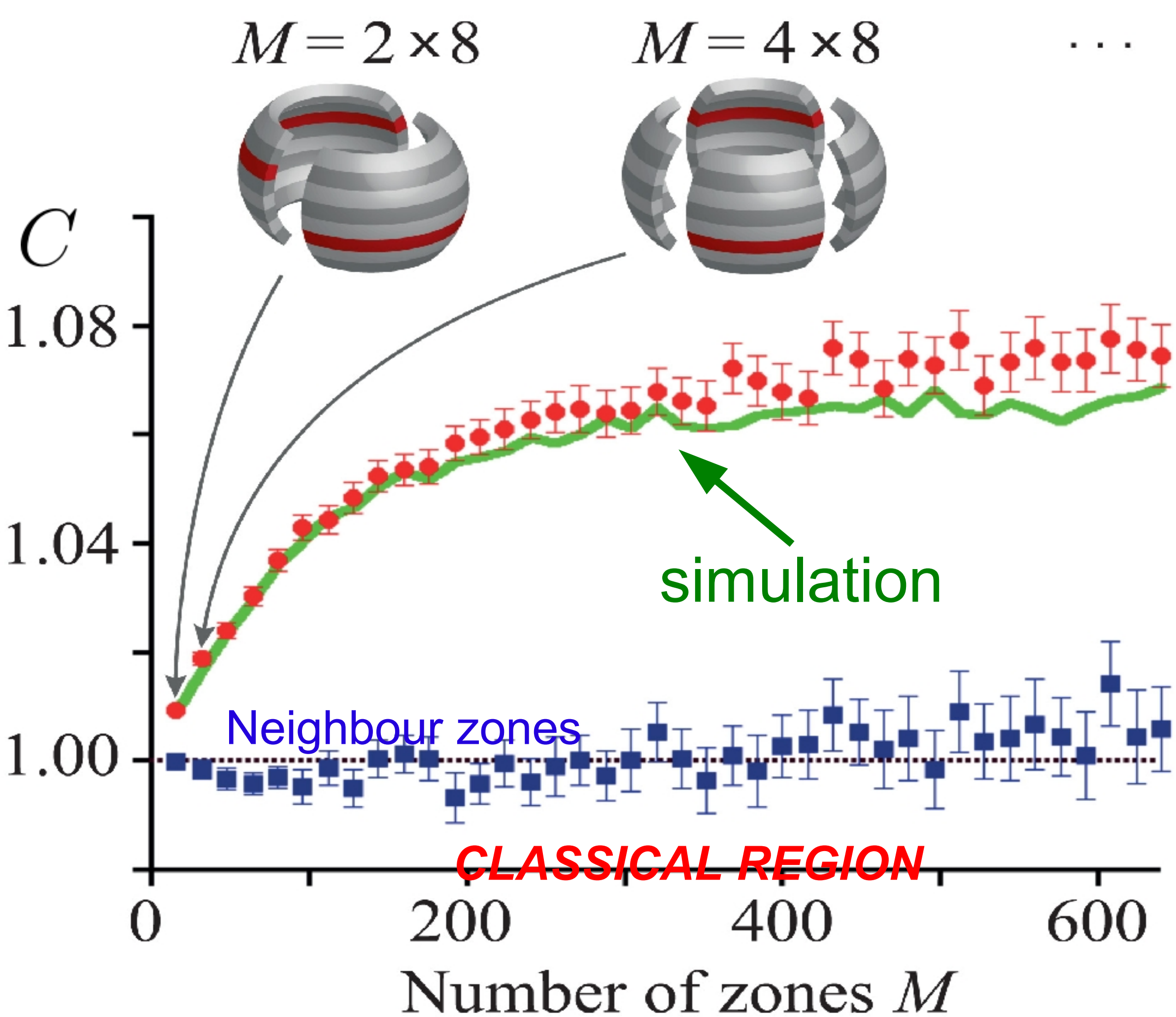
$$\widehat{N}_i = \int_{\text{bin } j} \widehat{\Psi}^\dagger(k)\widehat{\Psi}(k)\, dk$$

$$\overline{\mathcal{G}}^{(2)}_{ij} = \langle : \widehat{N}_i \widehat{N}_j : \rangle$$

$$C = \overline{\mathcal{G}}^{(2)}_{12} / \sqrt{\overline{\mathcal{G}}^{(2)}_{11}\overline{\mathcal{G}}^{(2)}_{22}}$$

$$C \leq 1 \qquad \text{classically}$$

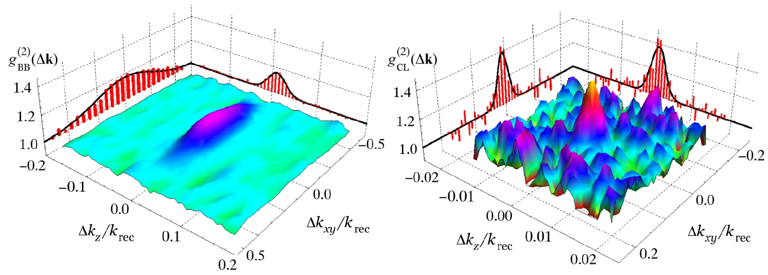
compare $g^{(2)}_{12} \leq \sqrt{g^{(2)}_{11}\, g^{(2)}_{22}}$



Multi-mode : simple model

- Gaussian ansatz for correlations

$$g^{(2)}_{\mathrm{bb;cl}}(\vec{\Delta x}) = 1 + h_{\mathrm{bb;cl}} e^{-|\vec{\Delta x}|^2/2\sigma^2_{\mathrm{bb;cl}}}$$



- Multimode relationships:

$$\eta^2 = 1 + (2\pi)^{3/2}\overline{n}\left[h_{\mathrm{cl}}\sigma^3_{\mathrm{cl}} - h_{\mathrm{bb}}\sigma^3_{\mathrm{bb}}\right]$$

LARGE BIN $\gg \sigma$

$$\eta^2 = 1 + \overline{n}(\Delta x)^3\left[h_{\mathrm{cl}} - h_{\mathrm{bb}}\right]$$

SMALL BIN $\ll \sigma$

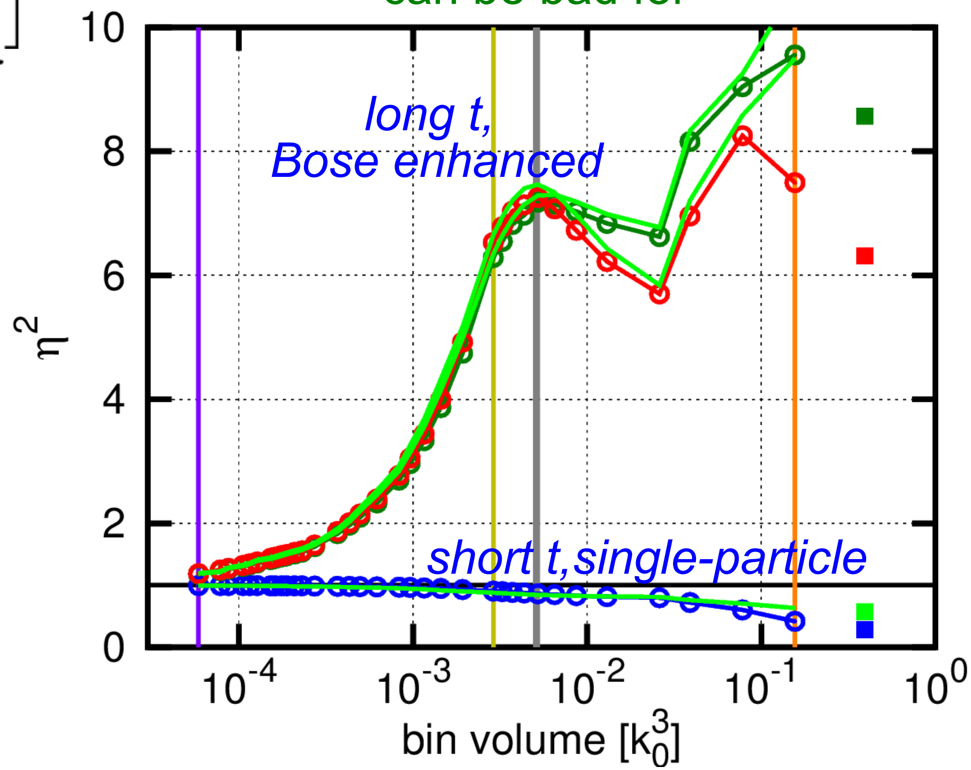
<u>DEPENDENCES:</u>

correlation peak

coherence volume

bin size

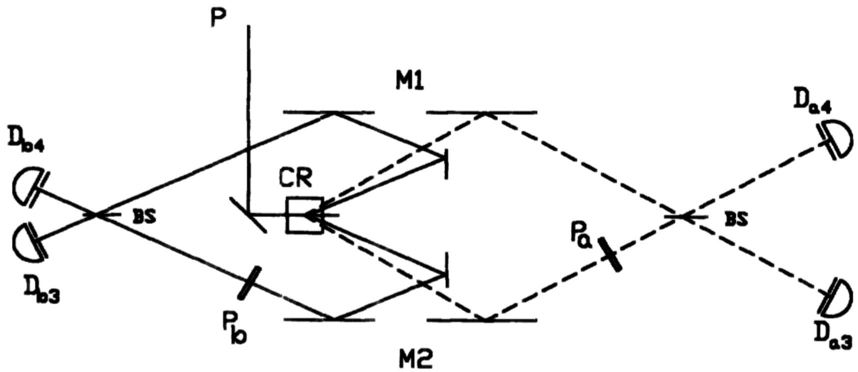
Dense halo can be bad for η^2



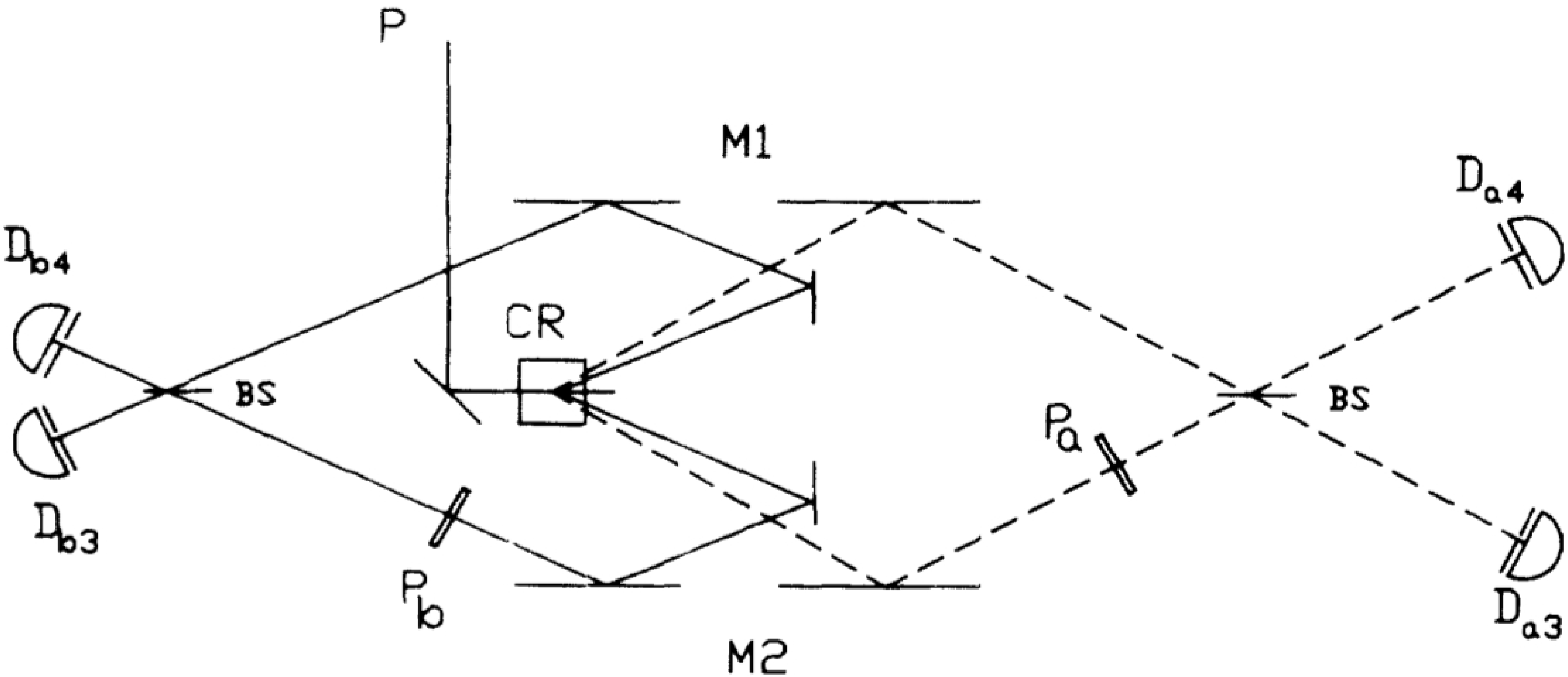
PD, T. Wasak, J. Chwedeńczuk, M. Trippenbach, unpublished

Conclusions

- Cauchy-Schwartz violation with massive, separated particles
  - *Simplest nonclassicality test*
  - *Precursor of Bell tests on massive particles*
- Multimode nature of correlations can be crucial
  - *Coherence volume and bin volume important*
- Bogoliubov theory with a very large lattice tractable
  - *With stochastic approaches: positive-P or Wigner*
- Outlook: Bell tests?
  - *Rarity-Tapster scheme*



Rarity & Tapster, PRL **64**, 2495 (1990)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M1
D_{a4}
D_{b4}
CR
BS
BS
P_a
D_{b3}
P_b
D_{a3}
M2