# The two kinds of noise when simulating quantum dynamics: thermal and quantum

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# Ultracold boson gas

$$\widehat{H} = \int d^3\mathbf{x} \left\{ \widehat{\Psi}^\dagger(\mathbf{x}) \left[ V(\mathbf{x}) - \frac{\hbar^2}{2m}\nabla^2 \right] \widehat{\Psi}(\mathbf{x}) + \frac{g}{2}\, \widehat{\Psi}^\dagger(\mathbf{x})^2 \widehat{\Psi}(\mathbf{x})^2 \right\}$$

Bose field $\widehat{\Psi}(\mathbf{x})$

## Phase-space treatments:

$$|\Psi\rangle \longrightarrow \{ \quad \psi^{(1)}(\mathbf{x}), \psi^{(2)}(\mathbf{x}), \dots, \psi^{(\mathcal{S})}(\mathbf{x}) \quad \}$$

N-body state

*S* samples of 1-body wavefunction, *or a related quantity*

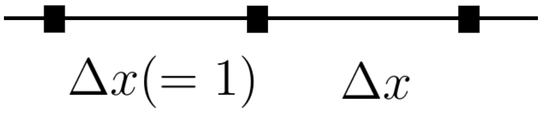
## Dynamics:

$$i\hbar \frac{\partial}{\partial t}|\Psi\rangle = \widehat{H}|\Psi\rangle \longrightarrow \frac{d\psi(\mathbf{x})}{dt} = A[\psi(\mathbf{x})] + \underbrace{B[\psi(\mathbf{x})]\xi(\mathbf{x},t)}_{\textit{possibly}}$$

## Noise:

$$\langle \xi(\mathbf{x},t)\xi(\mathbf{x}',t')\rangle = \frac{\delta_{tt'}\delta_{\mathbf{x},\mathbf{x}'}}{\Delta\mathbf{x}\Delta t} \qquad \langle \xi(\mathbf{x},t)\rangle = 0$$

# Quantum & thermal fluctuations: toy model



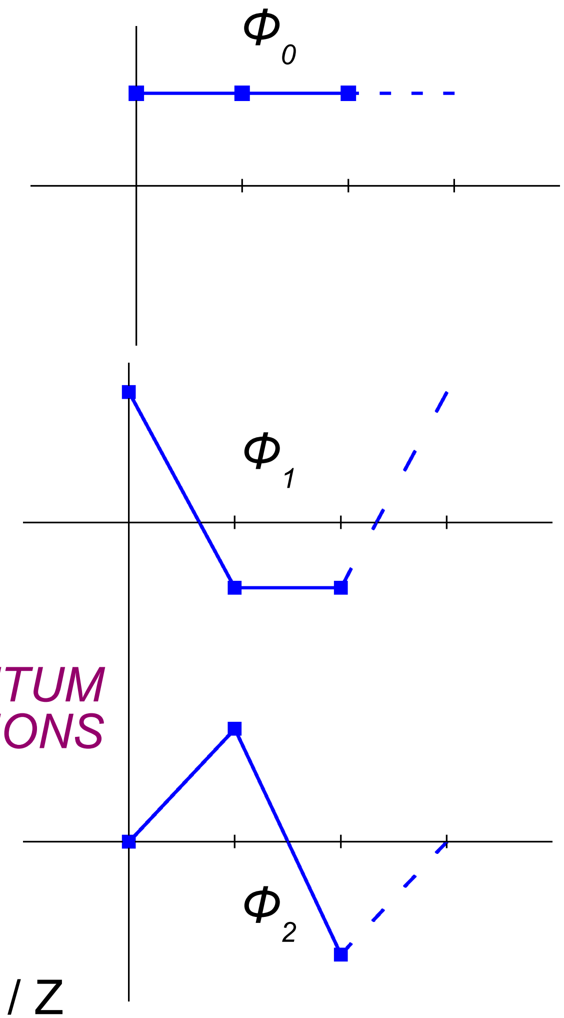

*Box, periodic boundary*

*3 PARTICLES*

$$\hat{H} = -\hat{\Psi}_j^\dagger \frac{\nabla^2}{2} \hat{\Psi}_j + \frac{g}{2\Delta x} \hat{\Psi}_j^{\dagger 2} \hat{\Psi}_j^2$$

g=0 ground state: $|\Psi_G\rangle = |300\rangle$

*3 particles in state $\Phi_0$*
*0 particles in state $\Phi_1$*
*0 particles in state $\Phi_2$*




g=0.3 ground state:

$|\Psi_G\rangle = 0.997\,|300\rangle$

$+\ 0.052\,(\,|102\rangle - |120\rangle) + 0.005\,|012\rangle + 0.003\,|030\rangle$

samples of 1-particle density: 99.630 % $\Phi_0$

*0.185 % $\Phi_1$ and $\Phi_2$* ← *T=0 QUANTUM FLUCTUATIONS*

g=0 T=0.5 thermal state

$\rho = \{\, |300\rangle\langle 300| + e^{-1.5/T}\,(\,|\lambda_1\rangle\langle\lambda_1| + |\lambda_2\rangle\langle\lambda_2| + e^{-3/T}(|\lambda_3\rangle\langle\lambda_3| + \ldots) + \ldots \,\} / Z$

samples of 1-particle density: 96.511 % $\Phi_0$

*1.745 % $\Phi_1$ and $\Phi_2$* ← *g=0 THERMAL FLUCTUATIONS*

# The usual T > 0 methods

## Bogoliubov

$$\widehat{\Psi}(\mathbf{x},t) = \phi(\mathbf{x},t) + \widehat{\delta}(\mathbf{x},t)$$

condensate

the rest (ASSUMED SMALL)

- Both thermal and quantum fluctuations
- Must have dominant condensate
- Only very low temperatures

## classical field: PGPE, SGPE, ...

$$\widehat{\Psi}(\mathbf{x}) = \sum_j \phi_j(\mathbf{x})\widehat{a}_j \to \sum_{E_j<E_{\mathrm{cut}}} \phi_j(\mathbf{x})\alpha_j$$

mode amplitude operator

complex number amplitude random phase, magnitude

- ONLY thermal fluctuations
- No need for condensate
- Almost all temperatures
- Suffer from cutoff-dependence

## truncated Wigner

Like c-fields, but quantum fluctuations are added in vacuum noise initial conditions

- Both thermal and quantum fluctuations
- Any temperature
- Suffer from cutoff-dependence
- Suffer from bogus scattering as $t > 0$
- Impractical if vacuum noise too strong (i.e. modes with occupation ~O(1) studied)

## positive-P

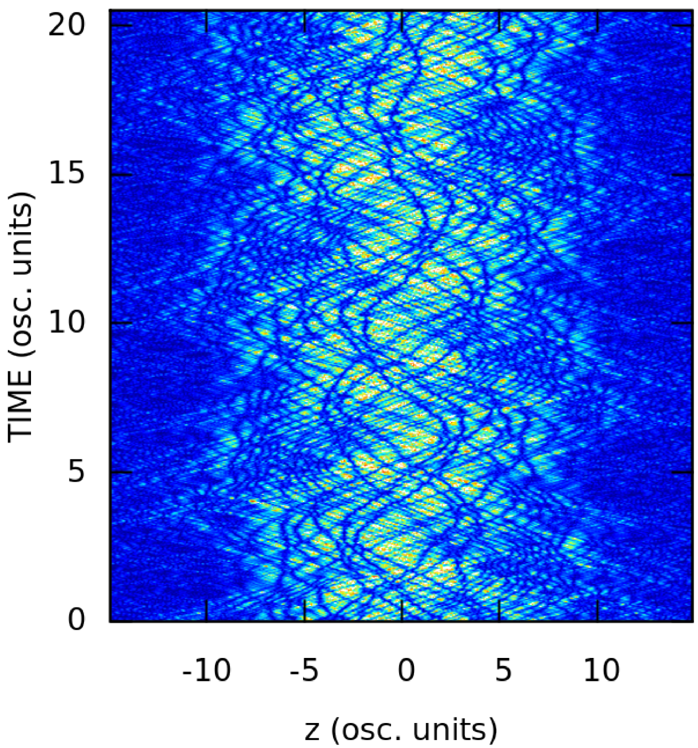
$$\widehat{\rho} = \int P\left[\psi, \widetilde{\psi}\right] \, |\psi\rangle\langle\widetilde{\psi}| \, \mathcal{D}^2\psi(\mathbf{x}) \, \mathcal{D}^2\widetilde{\psi}(\mathbf{x})$$

Probability distribution of bra & ket coherent fields $\psi(\mathbf{x}), \widetilde{\psi}(\mathbf{x})$

- Full quantum mechanics
- All fluctuations
- Any temperature
- Restricted to relatively short times by noise

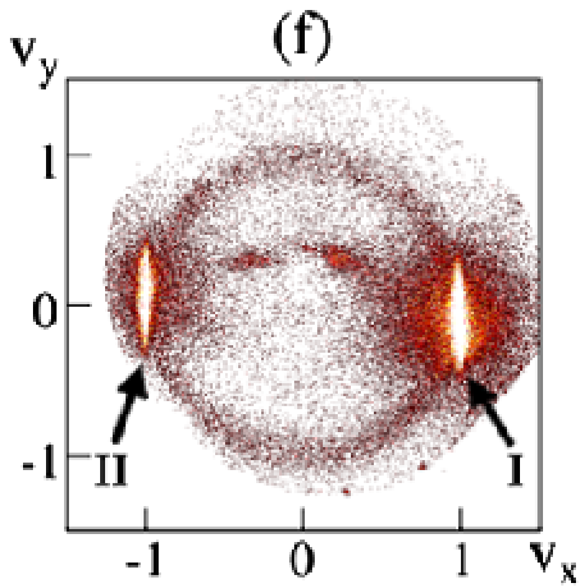
# Examples of spontaneous (quantum fluctuation) phenomena at “high” temperature

### Spontaneous solitons in trapped gas



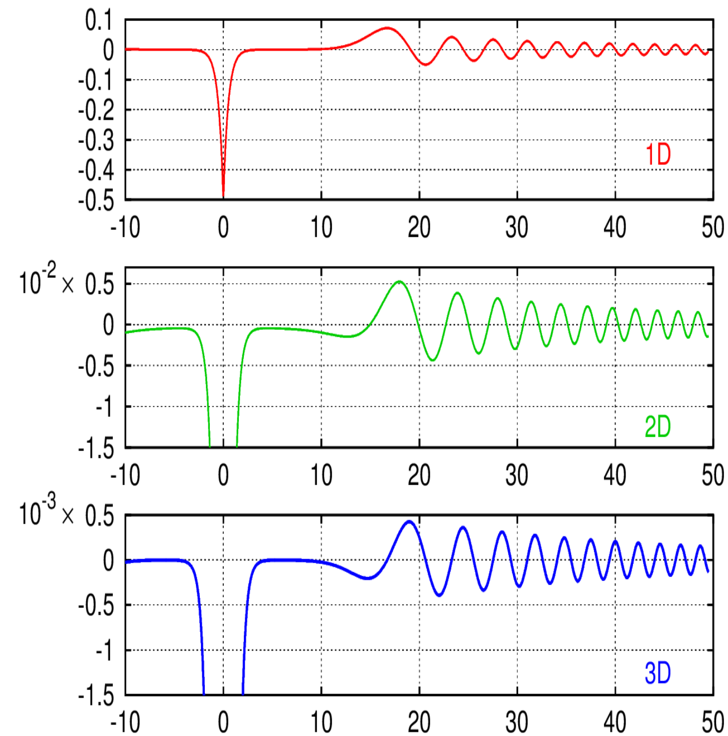

Karpiuk, PD, Bienias, Witkowska, Pawłowski, Gajda, Rzążewski, Brewczyk, PRL **109**, 205302 (2012)

### Pair scattering out of colliding quasicondensates




Perrin, Chang, Krachmalnicoff, Schellekens, Boiron, Aspect, Westbrook, PRL **99,** 150405 (2007)

### Pair creation in-situ after a quantum quench




PD, Stobińska, arXiv:1310.1301; Carusotto, Balbinot,Fabbri, Recati, EPJD **56**, 391 (2010)

<u>Q:  How to get thermal and spontaneous fluctuations together when:</u>

- Temperature is too high for a condensate (Bogoliubov excluded)
- Thermal state required (positive-P excluded)
- Low occupied modes needed (truncated Wigner excluded)

# T=0 STAB method – BEC collision

$$\widehat{\Psi}(\mathbf{x},t) = \phi(\mathbf{x},t) + \widehat{\delta}(\mathbf{x},t)$$

(Stochastic Time-Adaptive Bogoliubov)

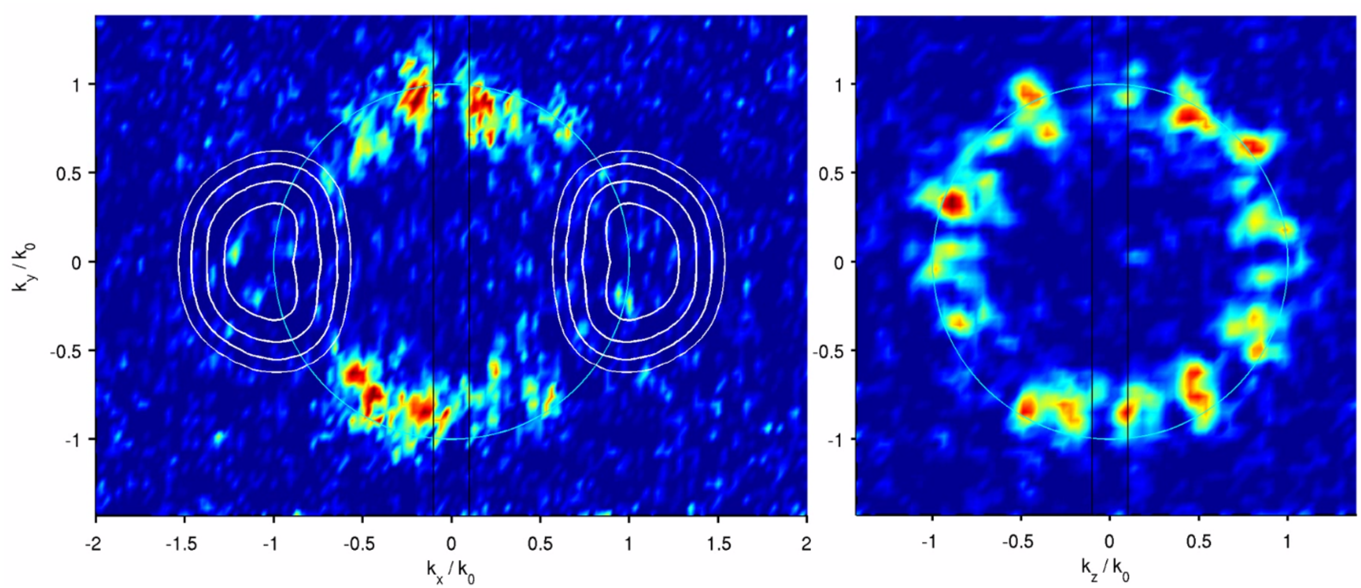
Treat condensate using GP equation

Treat $\widehat{\delta}(\mathbf{x},t)$ using positive-P representation

$$i\hbar\frac{d\phi(x)}{dt} = \left[H_0(x) + g|\phi(x)|^2\right]\phi(x)$$

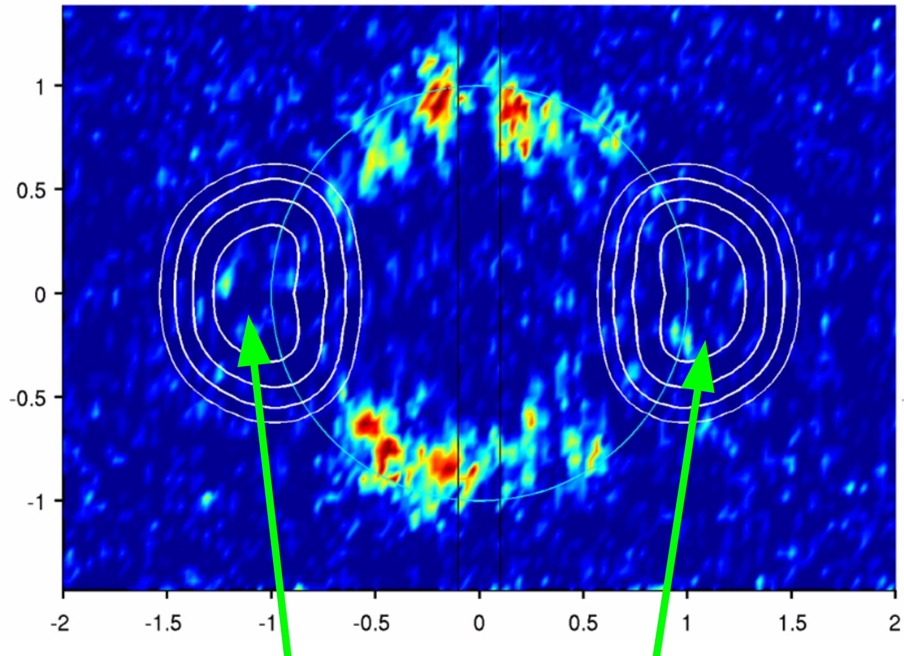
$$i\hbar\frac{d\psi(x)}{dt} = \left\{H_0(x) + 2g|\phi(x)|^2\right\}\psi(x) + g\phi(x)^2\widetilde{\psi}(x)^* + \sqrt{i\hbar g}\,\phi(x)\xi(x,t)$$

$$i\hbar\frac{d\widetilde{\psi}(x)}{dt} = \left\{H_0(x) + 2g|\phi(x)|^2\right\}\widetilde{\psi}(x) + g\phi(x)^2\psi(x)^* + \sqrt{i\hbar g}\,\phi(x)\widetilde{\xi}(x,t)$$

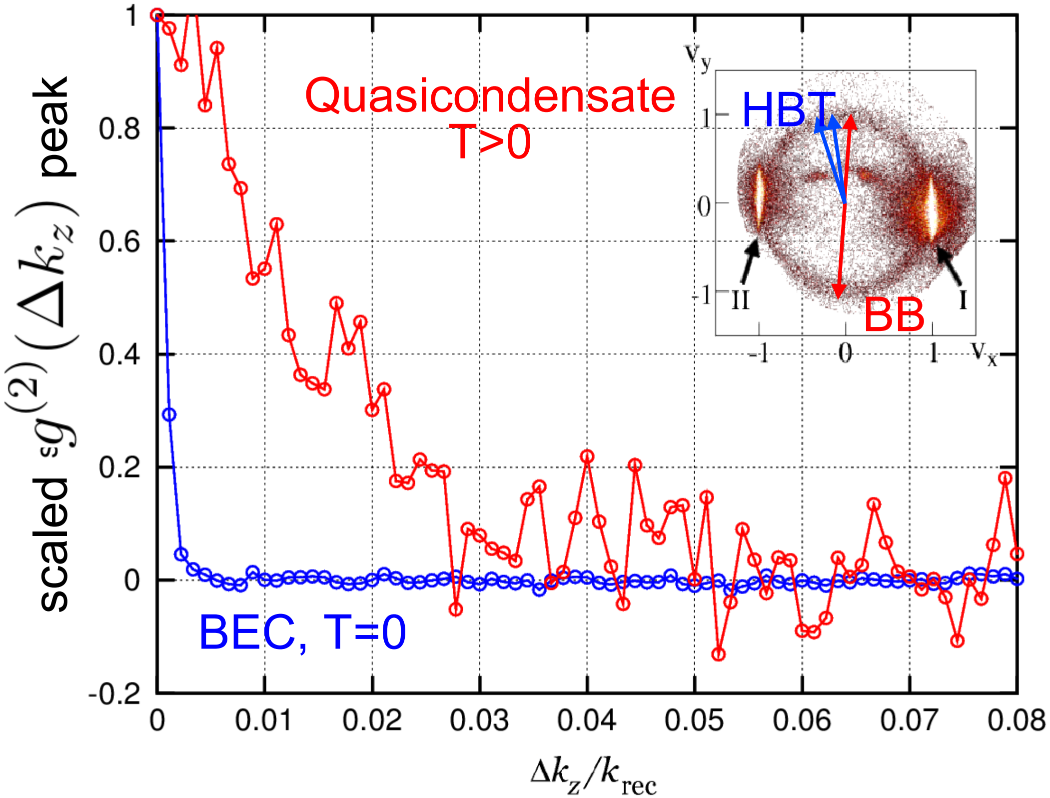



PD, Wasak, Zin, Chwedenczuk, Trippenbach, PRA **88**, 013617 (2013)

# T>0 Adaptation



Use c-field ensemble for $\phi(\mathbf{x}, t)$




Westbrook group experiment,
e.g. Kheruntsyan, Jaskula, PD, Bonneau, Partridge, Ruaudel, Lopes, Boiron, Westbrook, PRL **108, 260401** (2012)

| method | coherence length $l_\phi$ | Pair correlation $g^{(2)}(k,-k)$ width |
|---|---|---|
| EXPERIMENT | $\sim 45\mu$m | $0.017\, k_{\mathrm{rec}}$ |
| $T = 0$ STAB | $\infty$ | $0.0004\, k_{\mathrm{rec}}$ |
| $T > 0$ adaptation | $45\mu$m | $0.013\, k_{\mathrm{rec}}$ |

Caveat:
Scattered field is only accurate
When well separated from source

# In situ situations - SGPE

$$i\hbar\frac{d\psi(\mathbf{x})}{dt} = (1 - i\gamma)\left\{V(\mathbf{x}) - \frac{\hbar^2\nabla^2}{2m} - \mu + g|\psi(\mathbf{x})|^2\right\}\psi(\mathbf{x}) + \sqrt{\gamma T}\,\eta(\mathbf{x},t)$$

Gross-Pitaevskii equation (GPE)

reservoir coupling
(weak $\gamma \ll 1$ )

Complex white noise

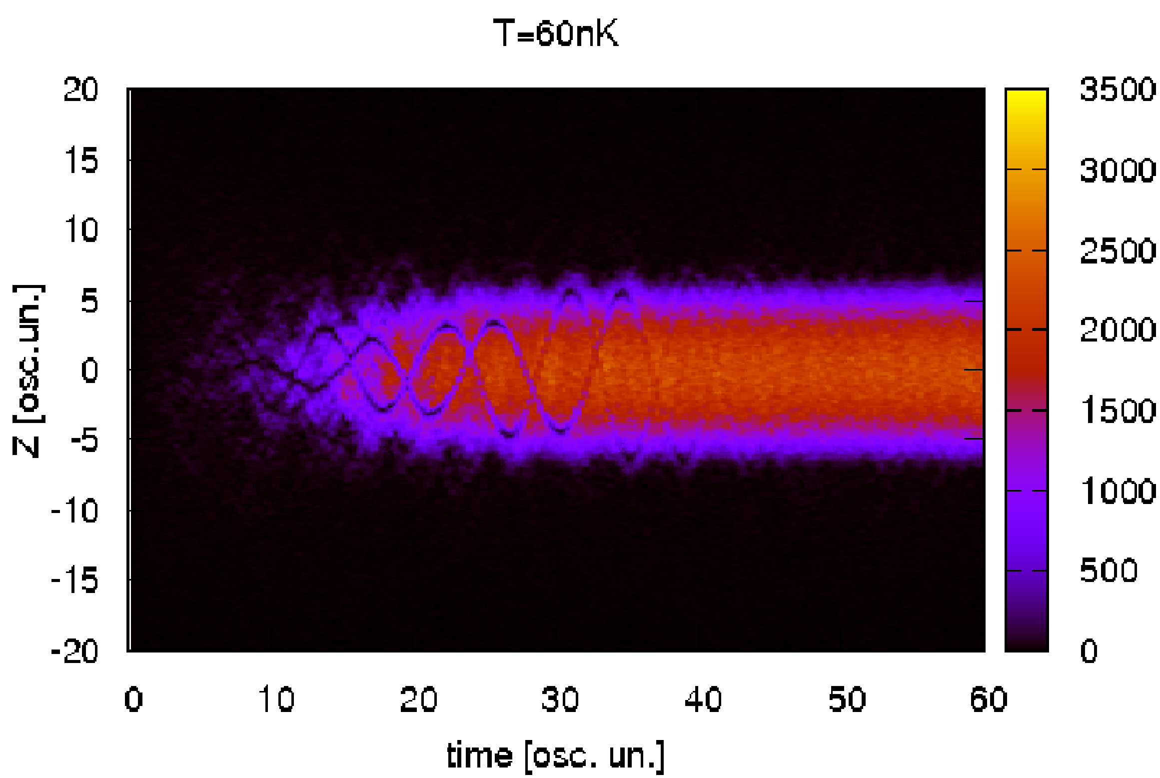
<u>General assumptions:</u>

*Low energy modes* (high occupation):

C-field

*High energy modes* (low occupation):

Thermal reservoir



# Observation

*__Observation:__ the positive-P equations (full quantum mechanics) for a Bose field coupled to a naïve Markovian thermal reservoir*

$$i\hbar\frac{d\psi(\mathbf{x})}{dt} = \left\{V(\mathbf{x}) - \frac{\hbar^2\nabla^2}{2m} + g\widetilde{\psi}^*\psi - i\overline{\gamma} + \sqrt{i\hbar g}\,\xi(\mathbf{x},t)\right\}\psi + \sqrt{\overline{\gamma}T}\,\eta(\mathbf{x},t)$$

$$i\hbar\frac{d\widetilde{\psi}(\mathbf{x})}{dt} = \left\{V(\mathbf{x}) - \frac{\hbar^2\nabla^2}{2m} + g\psi^*\widetilde{\psi} - i\overline{\gamma} + \sqrt{i\hbar g}\,\widetilde{\xi}(\mathbf{x},t)\right\}\widetilde{\psi} + \sqrt{\overline{\gamma}T}\,\eta(\mathbf{x},t)$$

Real white noises (quantum fluctuations)

Complex white noise (thermal fluctuations)

*Are quite similar to the SGPE equation (with a less naïve reservoir):*

$$i\hbar\frac{d\psi(\mathbf{x})}{dt} = (1-i\gamma)\left\{V(\mathbf{x}) - \frac{\hbar^2\nabla^2}{2m} - \mu + g|\psi|^2\right\}\psi + \sqrt{\gamma T}\,\eta(\mathbf{x},t)$$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# Hybrid: thermal Initial conditions, full evolution

*This suggests a hybrid equation that would include:*

- *both thermal and quantum fluctuations:*
- *equilibrated thermal cloud from SGPE*
- *quantum fluctuations, atom pair production, etc. from positive-P*
- *probably the positive-P instability?*

$$i\hbar\frac{d\psi(\mathbf{x})}{dt} = (1-i\gamma)\left\{V(\mathbf{x}) - \frac{\hbar^2\nabla^2}{2m} - \mu + g\widetilde{\psi}^*\psi + \sqrt{i\hbar g}\,\xi(\mathbf{x},t)\right\}\psi + \sqrt{\gamma T}\,\eta(\mathbf{x},t)$$

$$i\hbar\frac{d\widetilde{\psi}(\mathbf{x})}{dt} = (1-i\gamma)\left\{V(\mathbf{x}) - \frac{\hbar^2\nabla^2}{2m} - \mu + g\psi^*\widetilde{\psi} + \sqrt{i\hbar g}\,\widetilde{\xi}(\mathbf{x},t)\right\}\widetilde{\psi} + \sqrt{\gamma T}\,\eta(\mathbf{x},t)$$

Quantum fluctuations    Thermal fluctuations

_Observables_ are calculated as by replacing: $\widehat{\Psi}(\mathbf{x}) \to \psi(\mathbf{x}),\qquad \widehat{\Psi}^\dagger(\mathbf{x}) \to \widetilde{\psi})(\mathbf{x})^*$
In normally ordered moments.

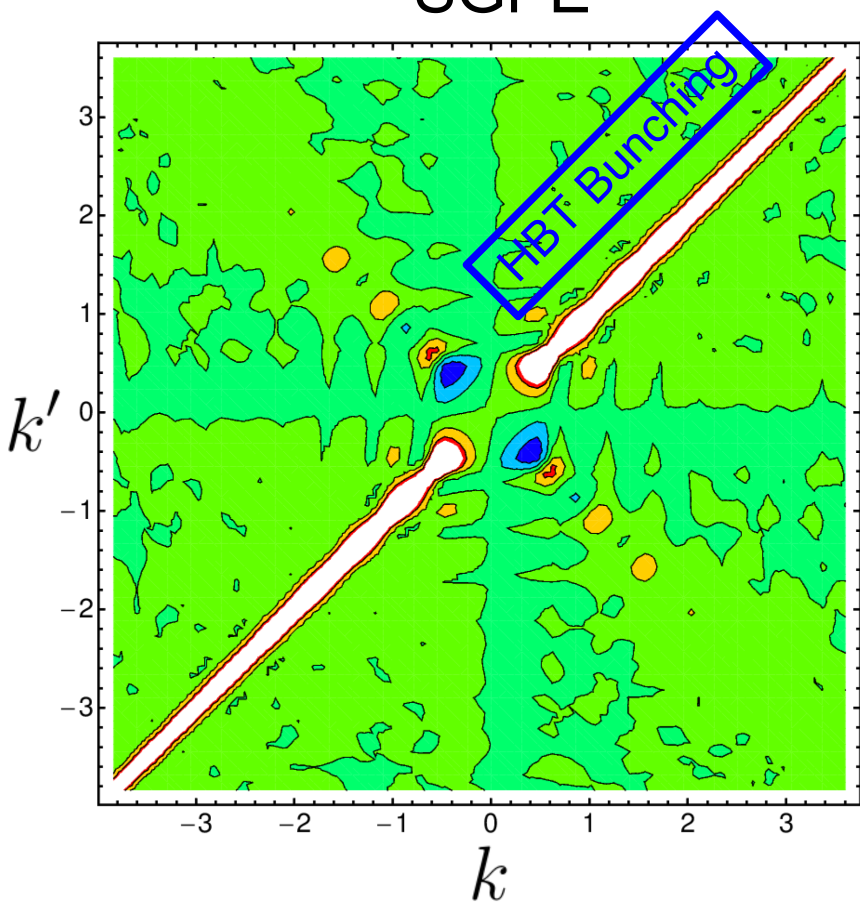
e.g. $$\rho_1(\mathbf{x},\mathbf{x}') = \left\langle \hat{\Psi}^\dagger(\mathbf{x})\hat{\Psi}(\mathbf{x}')\right\rangle = \mathrm{Re}\langle\widetilde{\psi}(\mathbf{x})^*\psi(\mathbf{x}')\rangle_{\text{samples}}$$

# Preliminary results: atom pairs in situ

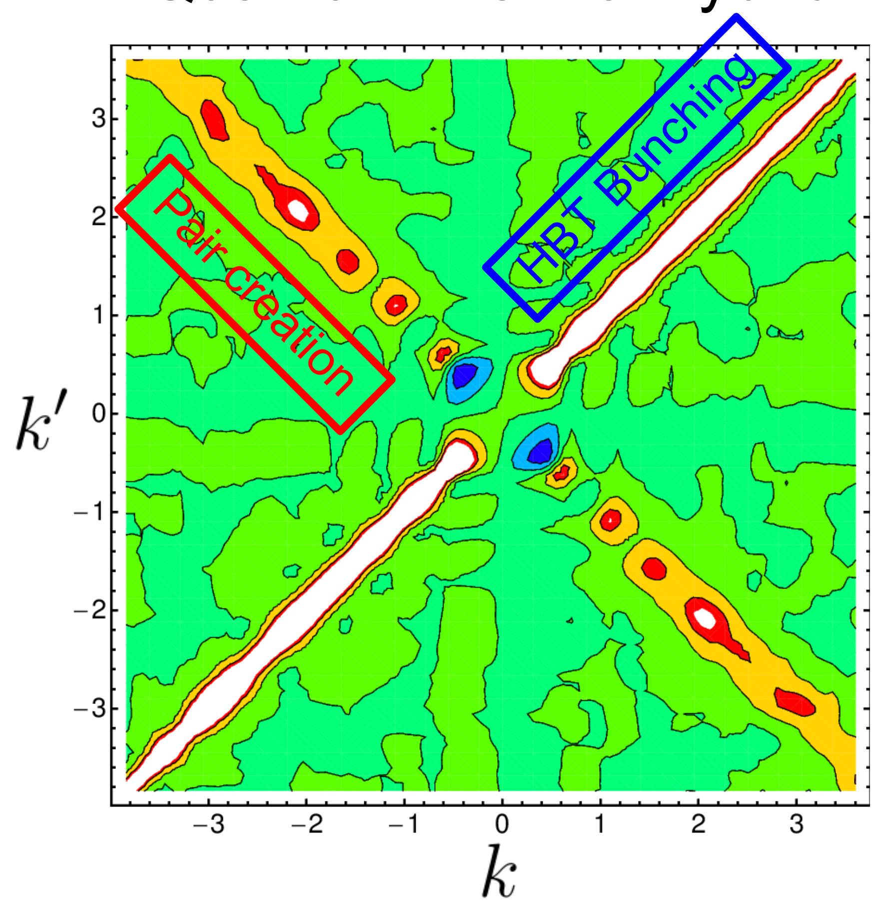
$N$=1000 $\mu$ = 22.41 $\hbar\omega$ $k_B T$ = 140 $\hbar\omega$ $T_\Phi \approx$ 900 $\hbar\omega$ $T_c \approx$ 1900 $\hbar\omega$

Momentum correlations $g^{(2)}(k, k')$

SGPE



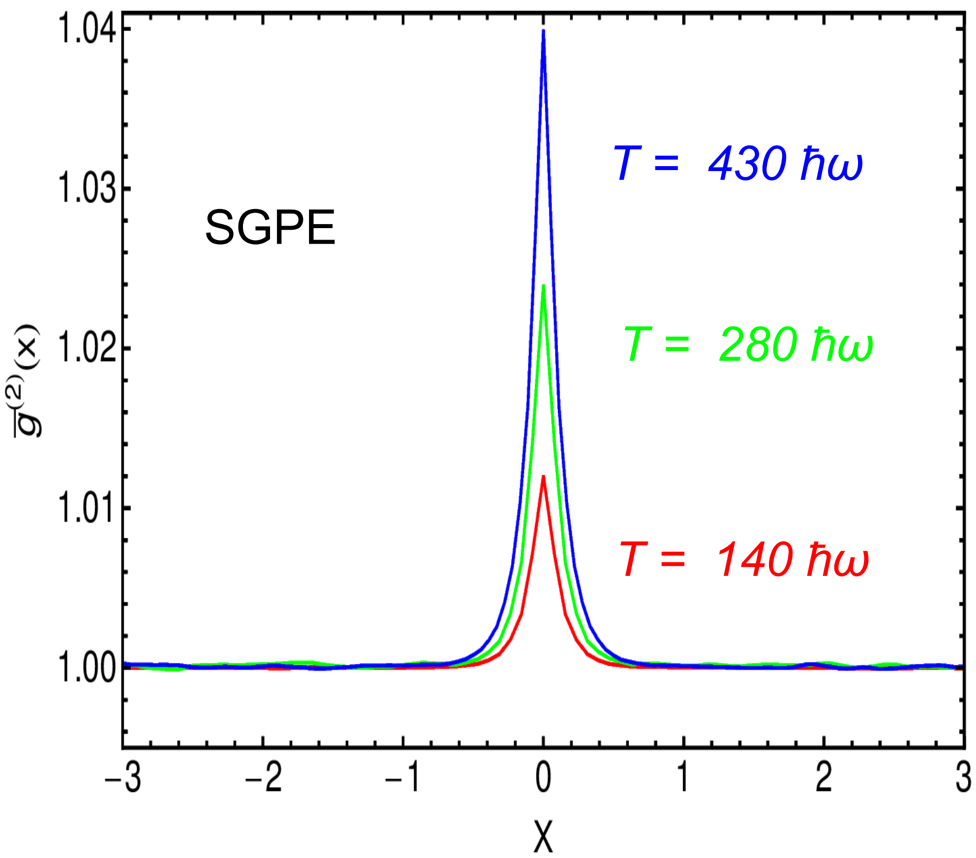

Quantum-thermal hybrid



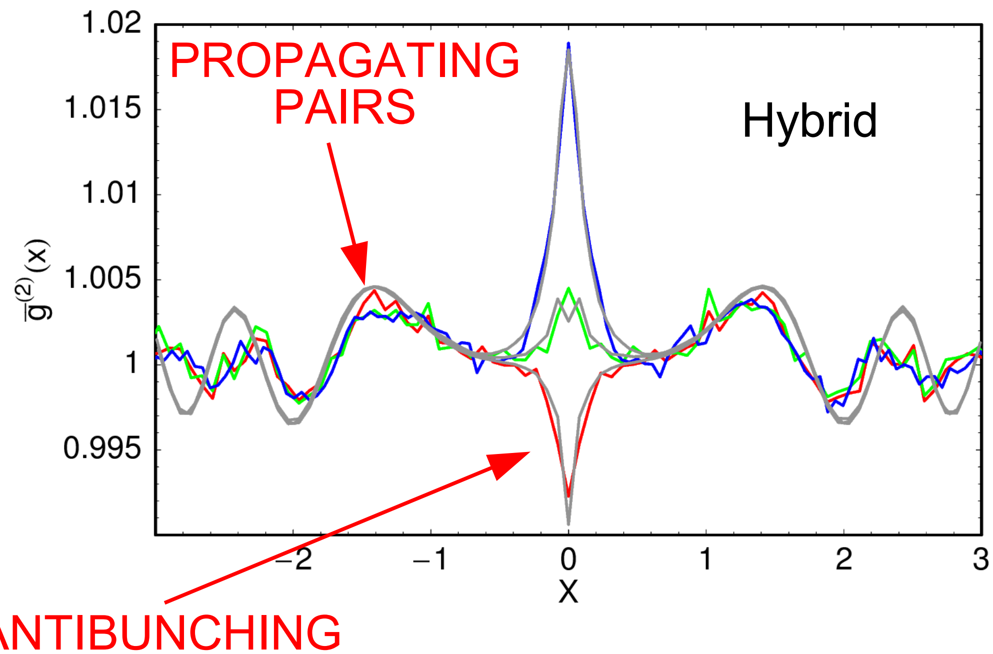
# Preliminary results: atom pairs in situ

## Spatial density correlations $g^{(2)}(0,x)$

$N$=1000 $\mu$ = 22.41 $\hbar\omega$ $T$ = 140 $\hbar\omega$ - 430 $\hbar\omega$ $T_{\Phi} \approx$ 900 $\hbar\omega$ $T_c \approx$ 1900 $\hbar\omega$




Effects of the ``quench" caused by turning on quantum fluctuations at t=0



# Scaling of spontaneous effects

Spatial density correlations $g^{(2)}(0,x)$

$\mu = 22.41\ \hbar\omega \quad T = 140\ \hbar\omega$

→ Ng = constant