Phase and density correlations in Bose gases after a quantum quench

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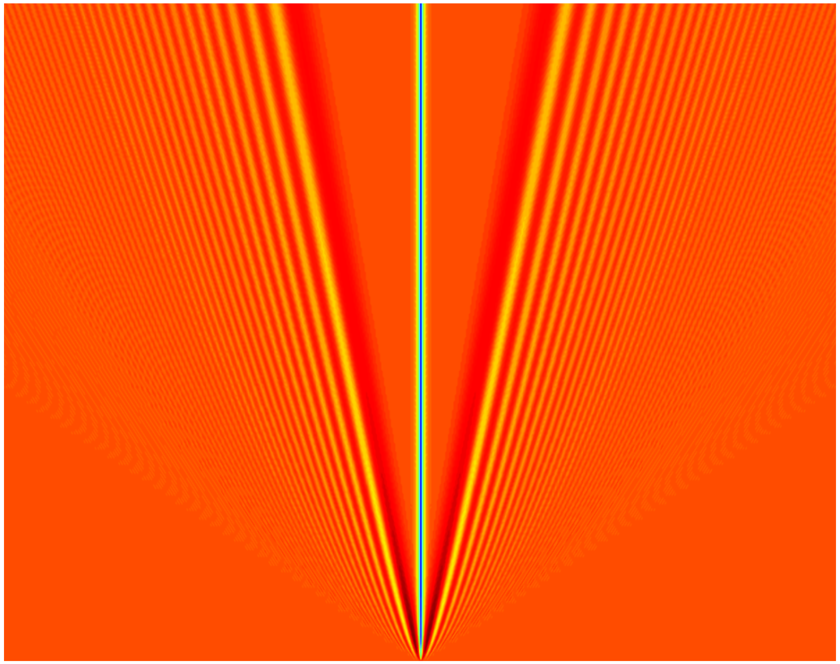
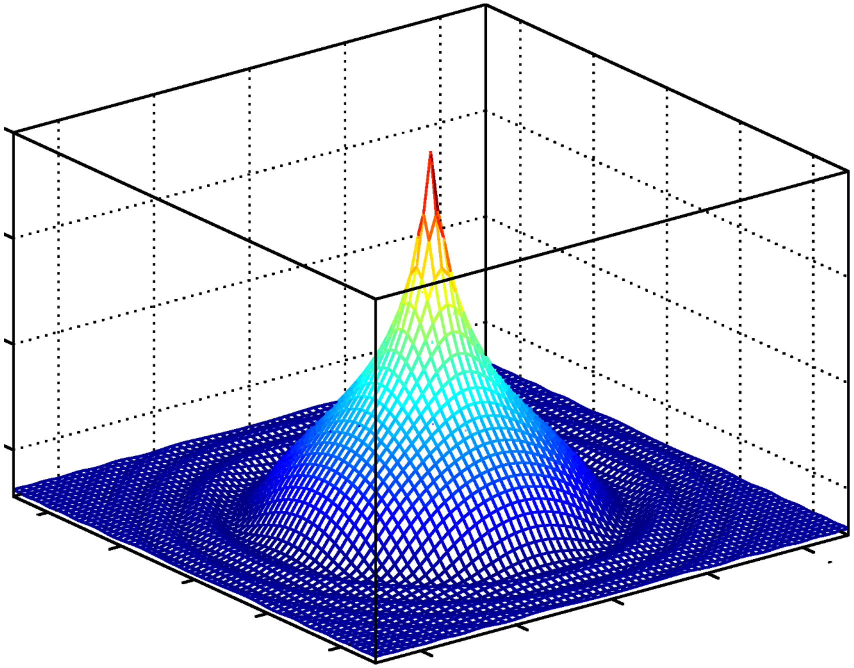


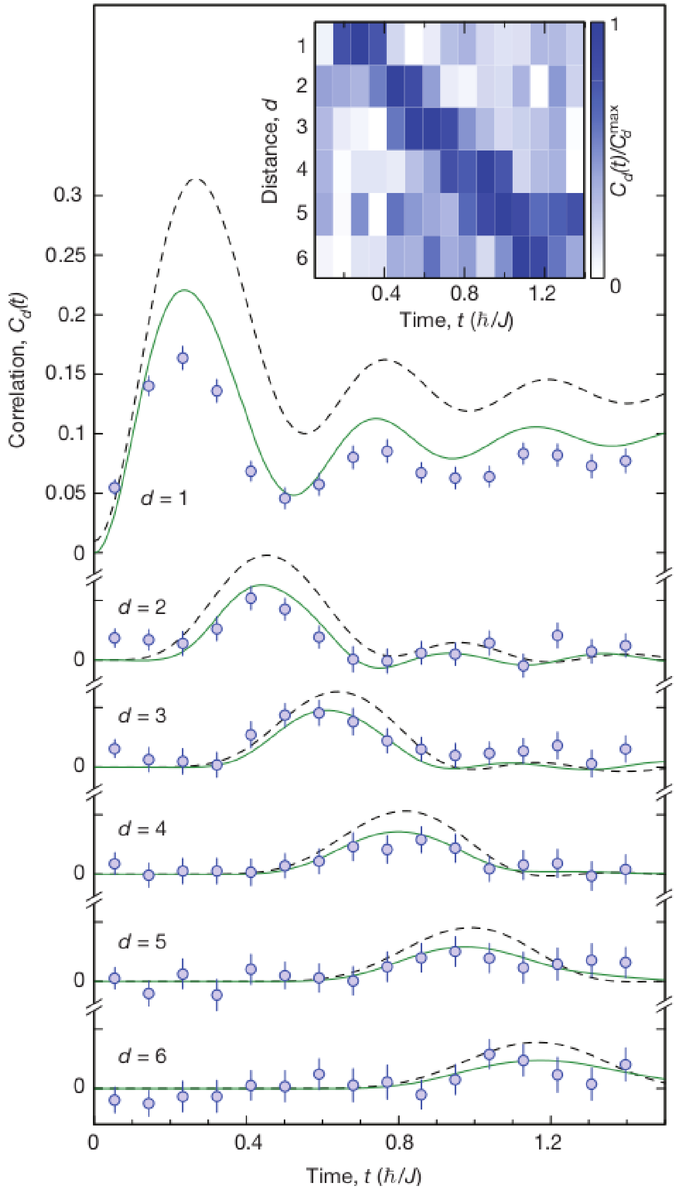


Quantum quench

- Sudden (diabatic) change of a system parameter
- e.g. In ultracold gases, usually means a change of the interaction strength *g*
    * Feshbach resonance
    * Low dimensional gases: change of the trapping potential in the collapsed dimensions

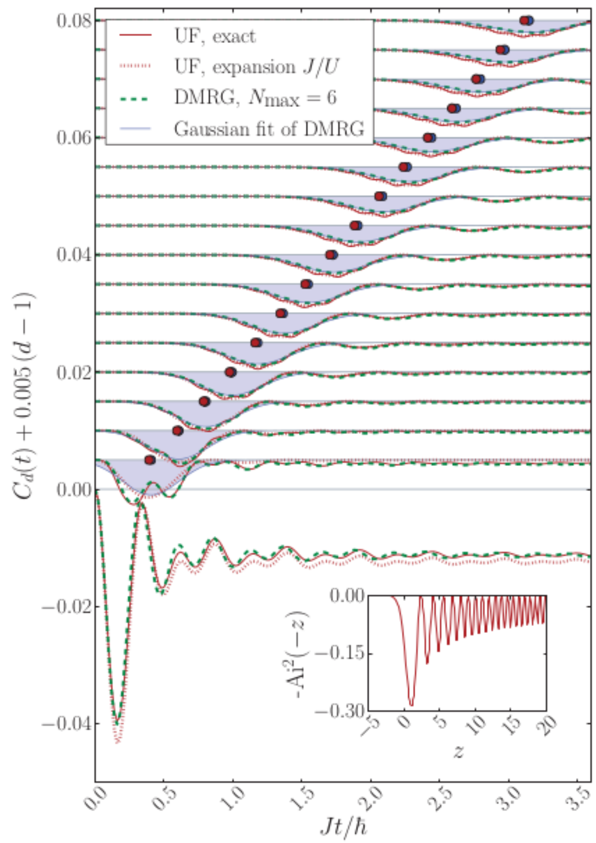
$$g_{1D} \approx \frac{g}{2\pi l_\perp^2} = 2\hbar a_s \omega_\perp \qquad l_\perp = \sqrt{\frac{\hbar}{m\omega_\perp}}$$

Correlations after a quantum quench



Parity correlations. Mott insulator U/J: **40→9**

Cheneau, Barmettler, Poletti, Endres, Schauss, Fukuhara, Gross, Bloch, Kollath, Khur, Nature **481,** 484 (2012)

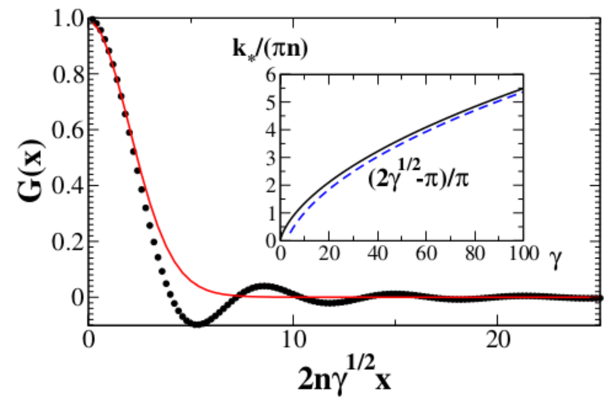


Density correlations

Mott insulator U/J: **∞→18**

Barmettler, Cheneau, Poletti, Kollath, PRL **85,** 053625 (2012)

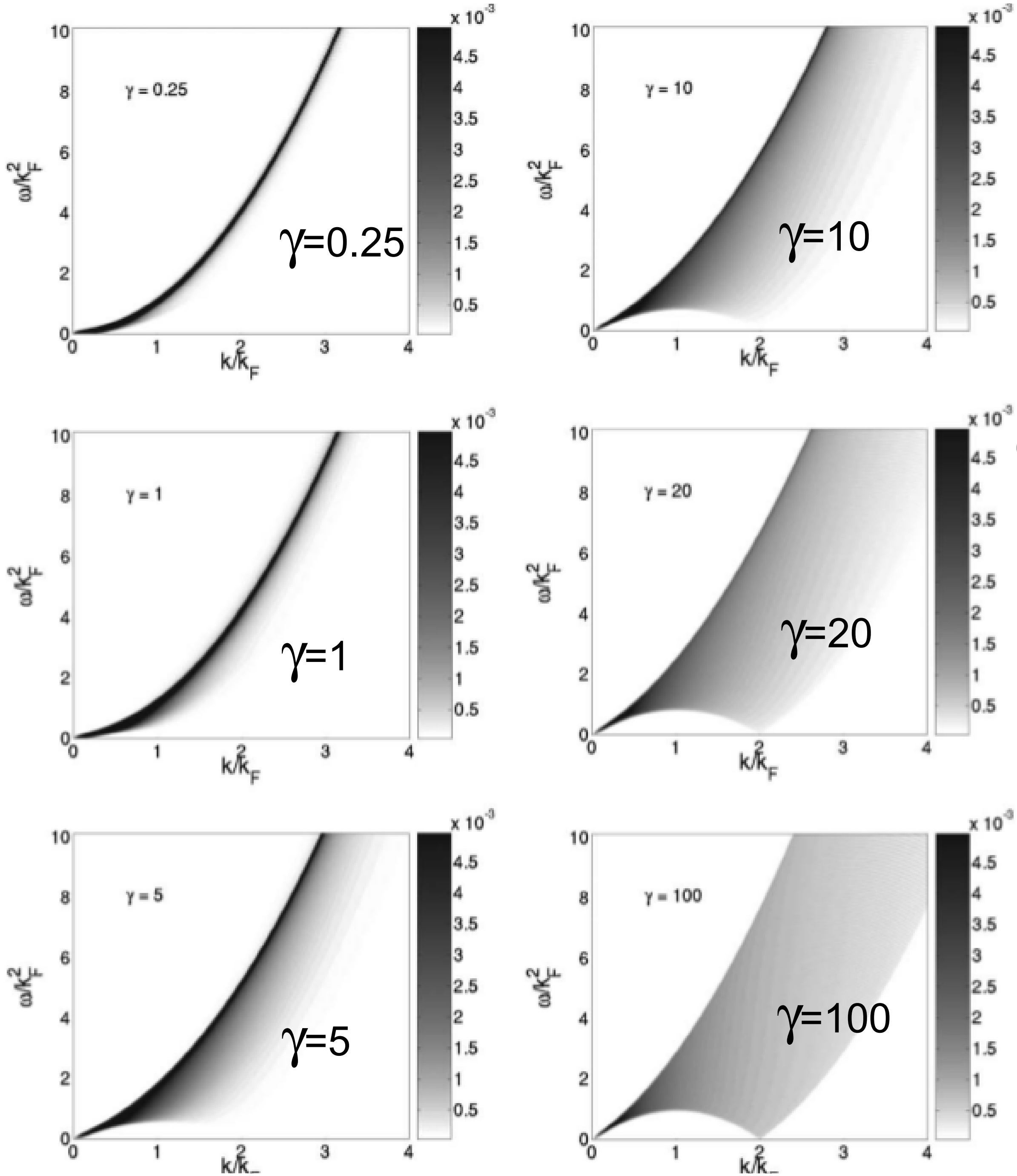
$$\gamma = \frac{g_{1D}}{\rho}$$



Greens function, Lieb Liniger gas, γ: **0 → 400**

Kormos, Shashi, Chou, Imambekov, ArXiv:1204.3889 (2012)

Correlations after a quantum quench



$$\gamma = \frac{g_{1D}}{\rho}$$

Uniform gas
Dynamical structure factor
(Fourier transform of density over x and t)

Caux, Calabrese, PRA **74,** 031605R (2006)

Caux, Calabrese, Slavnov, J Stat Mech **2007,** P01008 (2007)

Initial interest – simple test case

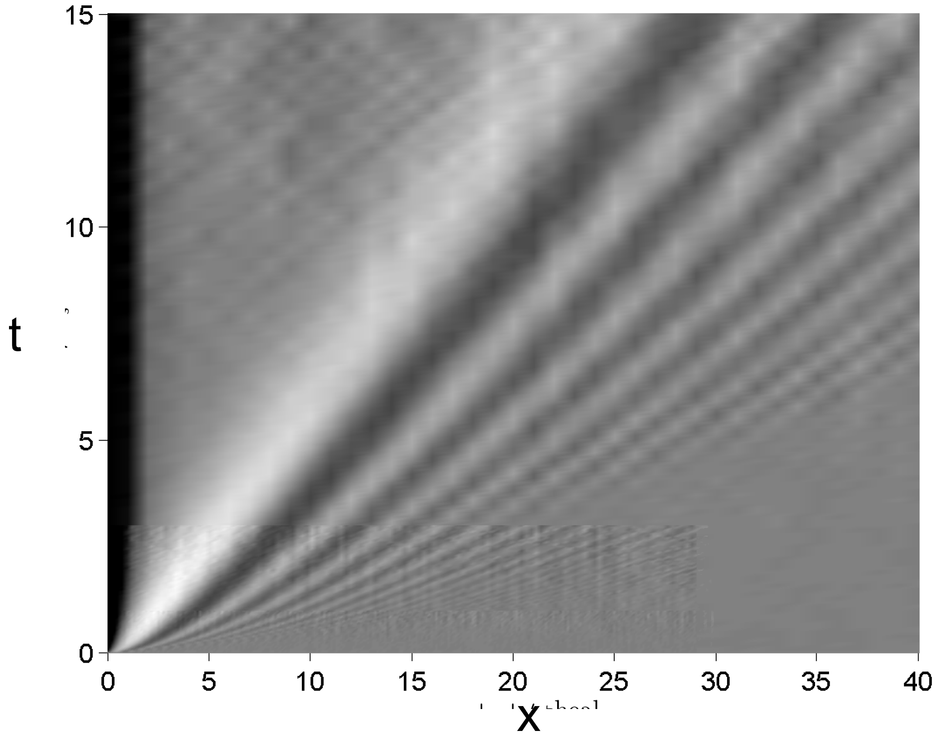
<u>*UNIFORM GAS*</u>

t<0 T=0 ideal gas (BEC)

t>0 finite interaction strength g $\gamma \lesssim 1$

Calculations with the positive-P method (complete many-body dynamics)

1D $g^{(2)}(0,x)(t)$



System description

$$\widehat{H} = \int d^d\mathbf{x} \left\{ -\frac{\hbar^2}{2m} \widehat{\Psi}^\dagger(\mathbf{x}) \nabla^2 \widehat{\Psi}(\mathbf{x}) + \frac{g}{2} \widehat{\Psi}^\dagger(\mathbf{x})^2 \widehat{\Psi}(\mathbf{x})^2 \right\}$$

Introduce physically relevant units

$$\hbar = 1, \qquad m = 1 \quad (\text{Boson mass}), \qquad \xi = \hbar / \sqrt{2m\rho g} = 1.$$

This sets $g = 1/2\rho$

Obtain time units - “healing time” $t_\xi = 1 = \hbar / 2\rho g$

$$\widehat{H} \;=\; \int dx \left\{ -\frac{1}{2} \widehat{\Psi}^\dagger(x) \nabla^2 \widehat{\Psi}(x) + \frac{1}{4\rho} \widehat{\Psi}^{\dagger 2}(x) \widehat{\Psi}^2(x) \right\}$$

Only <u>one relevant parameter</u>
(particles per healing length)

$$\rho = \frac{1}{\sqrt{2\gamma}} = \frac{\sqrt{2}}{\pi} K$$

In 1D

Bogoliubov approximation for $\gamma \ll 1$

Define $$\widehat{\Psi}(x) = \phi_0(x)\,\widehat{a}_0 + \delta\widehat{\Psi}(x)$$

Assume $$\frac{dN}{N} = \Delta = \int dx \,\left\langle \delta\widehat{\Psi}^\dagger(x)\delta\widehat{\Psi}(x)\right\rangle \ll 1$$

Basis $$\delta\widehat{\Psi}(x) = \sum_{k\neq 0}\left[\widehat{b}_k(t)u_k(x) + \widehat{b}_k^\dagger(t)v_k^*(x)\right]$$

Keep only terms *O(≤ 2)* in $\delta\widehat{\Psi}(x)$: $$\widehat{H} = \text{const.} + \sum_{k\neq 0}\omega_k\widehat{b}_k^\dagger\widehat{b}_k$$

This can be solved for eigenvalues and eigenstates

$$\omega_k = \sqrt{\frac{k^2}{2}\left(\frac{k^2}{2}+1\right)}$$

Bogoliubov operators $\widehat{b}_k$ can be written

in terms of momentum space modes (plane waves) $\widehat{a}_k(0)$

Initial state $|I\rangle$ had vacuum in all excited modes

$$\widehat{a}_k(0)|I\rangle = \langle I|\widehat{a}_k^\dagger(0) = 0 \qquad \forall k \neq 0.$$

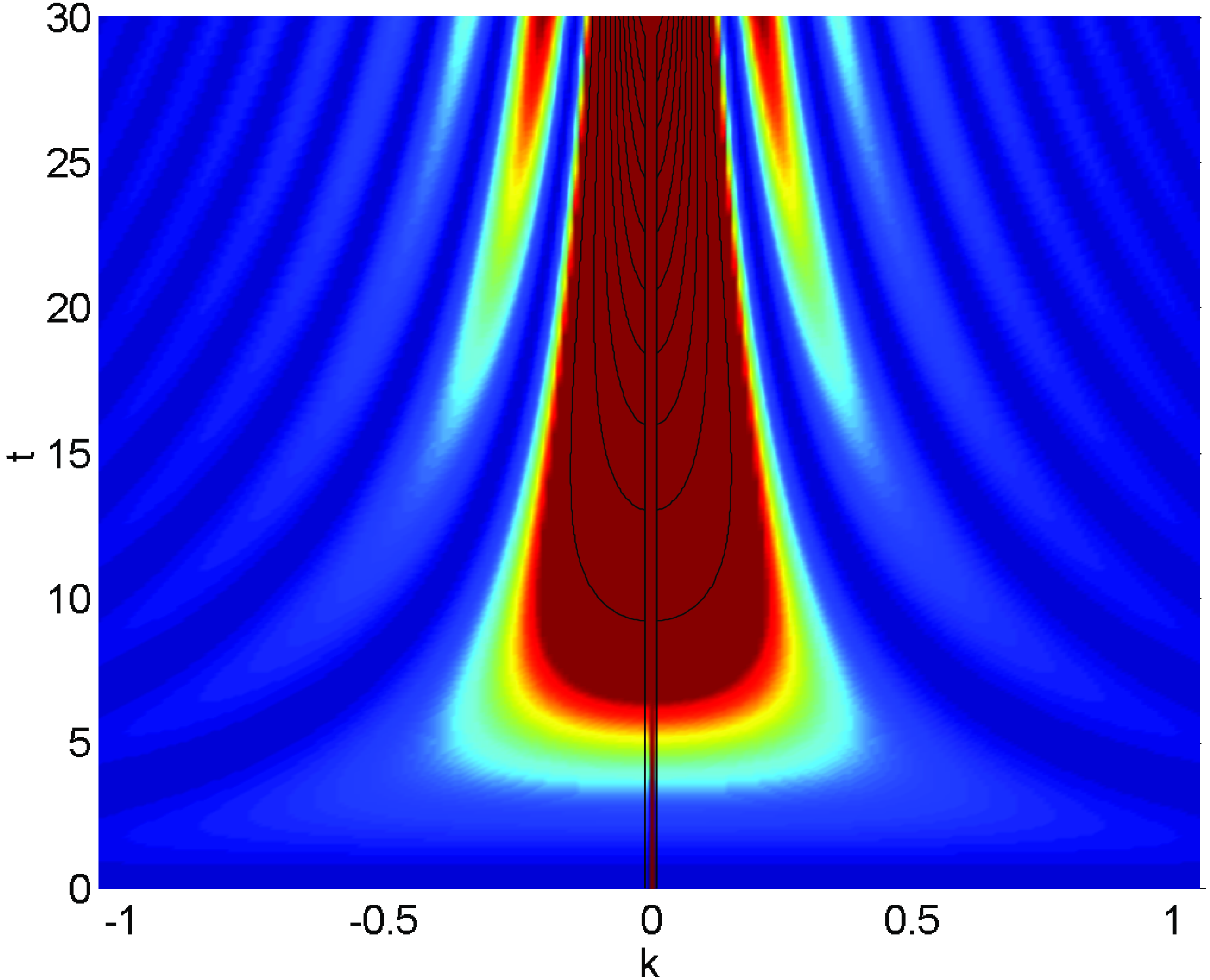
====> <u>All observables can be found</u>

Momentum density

$$\rho_k = \langle \widehat{\Psi}_k^\dagger \widehat{\Psi}_k \rangle = \frac{1}{4\Delta k} \left(\frac{\sin \omega_k t}{\omega_k} \right)^2 \quad \forall k \neq 0$$

$$\Delta k = 2\pi / L$$

ρ(k) colors truncate at ρ(k)=1000, then contours at 2000, 4000, ...



Momentum correlations

$$g^{(1)}(k,k') = \delta_{kk'}$$

$$g^{(2)}(k,k') = \begin{cases} 2 & \text{if } k' = k \\ 2 + 1/\rho_k \Delta k & \text{if } k' = -k \\ 1 & \text{if } |k'| \neq |k| \end{cases}$$

$$\Delta k = 2\pi/L$$

Spatial correlations

$$
\begin{aligned}
g^{(1)}(x, x+y) &= \frac{\langle \widehat{\Psi}^\dagger(x)\widehat{\Psi}(x+y)\rangle}{N} \\
&= 1 - \frac{1}{8N}\sum_{k\neq 0}\frac{1}{\omega_k^2}\left[1 - \cos 2\omega_k t - \cos ky + \cos(ky + 2\omega_k t)\right]
\end{aligned}
$$

$$
\begin{aligned}
g^{(2)}(x, x+y) &= \frac{1}{N^2}\langle \widehat{\Psi}^\dagger(x)\widehat{\Psi}^\dagger(x+y)\widehat{\Psi}(x)\widehat{\Psi}(x+y)\rangle \\
&= 1 - \frac{1}{4N}\sum_{k\neq 0}\frac{k^2}{\omega_k^2}\left[\cos ky - \cos(ky + 2\omega_k t)\right]
\end{aligned}
$$

$$
\omega_k = \sqrt{\frac{k^2}{2}\left(\frac{k^2}{2} + 1\right)}
$$

Continuum limit

$$g^{(1)}(0,y)(t) = 1 - \frac{1}{\rho}\int_0^{k_{\max}} dk \left(\frac{1-\cos 2\omega_k t}{2+k^2}\right) \begin{cases} \frac{1}{2\pi k^2}\ (1-\cos ky) & \text{1D} \\ \frac{1}{4\pi k}\ (1 - J_0\left[k|y|\right]) & \text{2D} \\ \frac{1}{4\pi^2}\ \left(1 - \frac{\sin ky}{ky}\right) & \text{3D} \end{cases}$$

$$g^{(2)}(0,y)(t) = 1 - \frac{1}{\rho}\int_0^{k_{\max}} dk \left(\frac{1-\cos 2\omega_k t}{2+k^2}\right) \begin{cases} \frac{1}{\pi}\ \cos ky & \text{1D} \\ \frac{k}{2\pi}\ J_0\left[k|y|\right] & \text{2D} \\ \frac{k^2}{2\pi^2}\ \frac{\sin ky}{ky} & \text{3D} \end{cases}$$

Convenient single-dimensional integrals

Bose-Hubbard correspondence

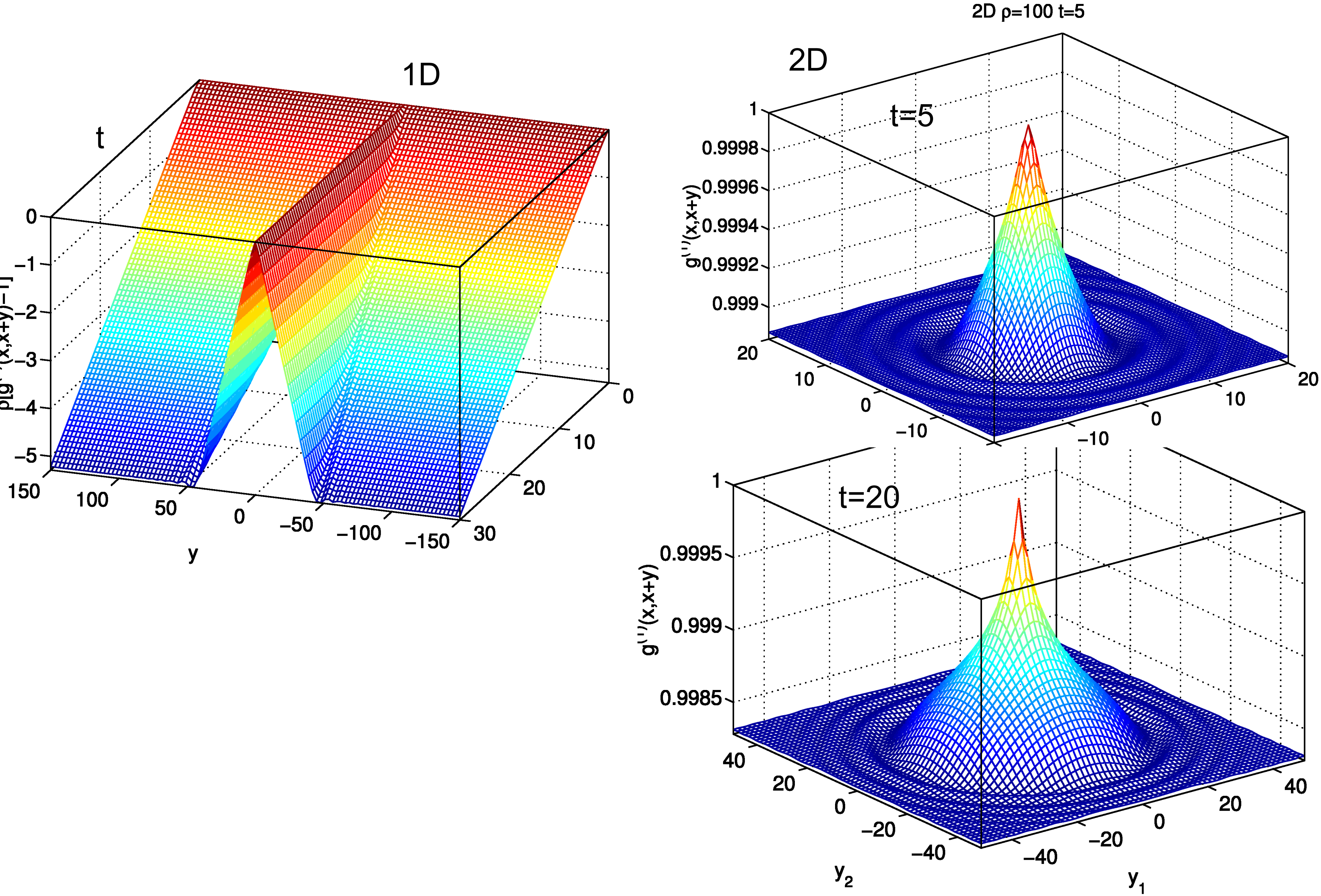
Momentum cutoff and
Numerical lattice spacing are related

$$k_{\mathrm{max}} = \frac{\pi}{\Delta x}$$

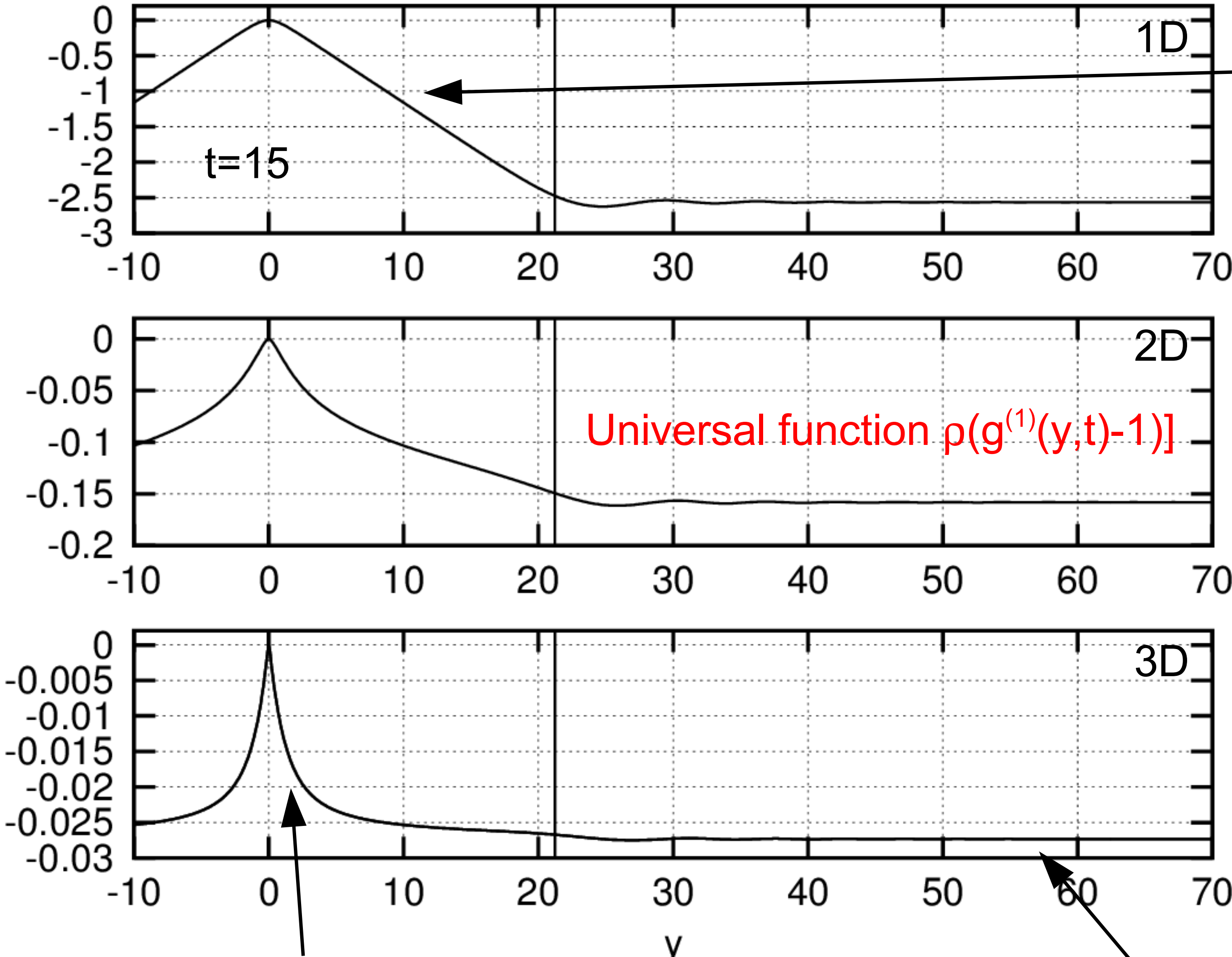
Equivalent Bose-Hubbard Hamiltonian has:

$$\frac{J}{U} = \frac{2\rho}{\pi}\, k_{\mathrm{max}}$$

$g^{(1)}(y,t)$ phase correlations



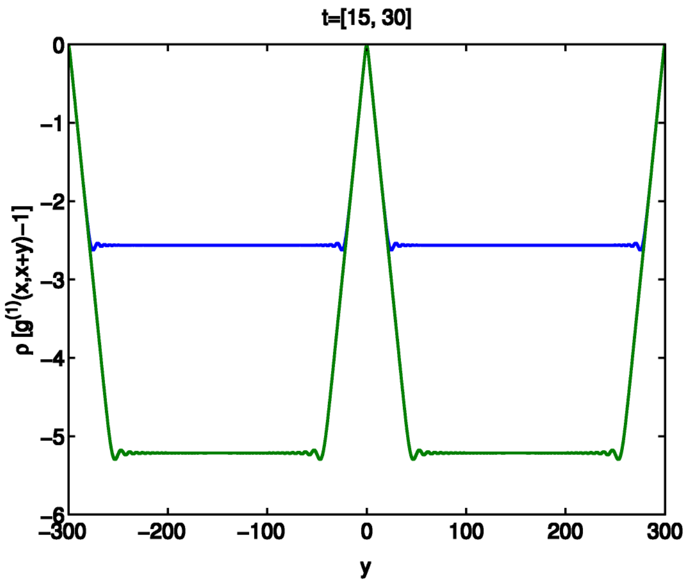
Spatial $g^{(1)}(y,t)$



$$-\frac{y\sqrt{2}-1}{8\sqrt{2}\rho} \qquad d=1$$

$$-\frac{1}{8\pi\rho}\left\{\gamma+\log\frac{y}{\sqrt{2}}\right\} \qquad d=2$$

$$-\frac{1}{8\pi\rho\sqrt{2}} \qquad d=3$$



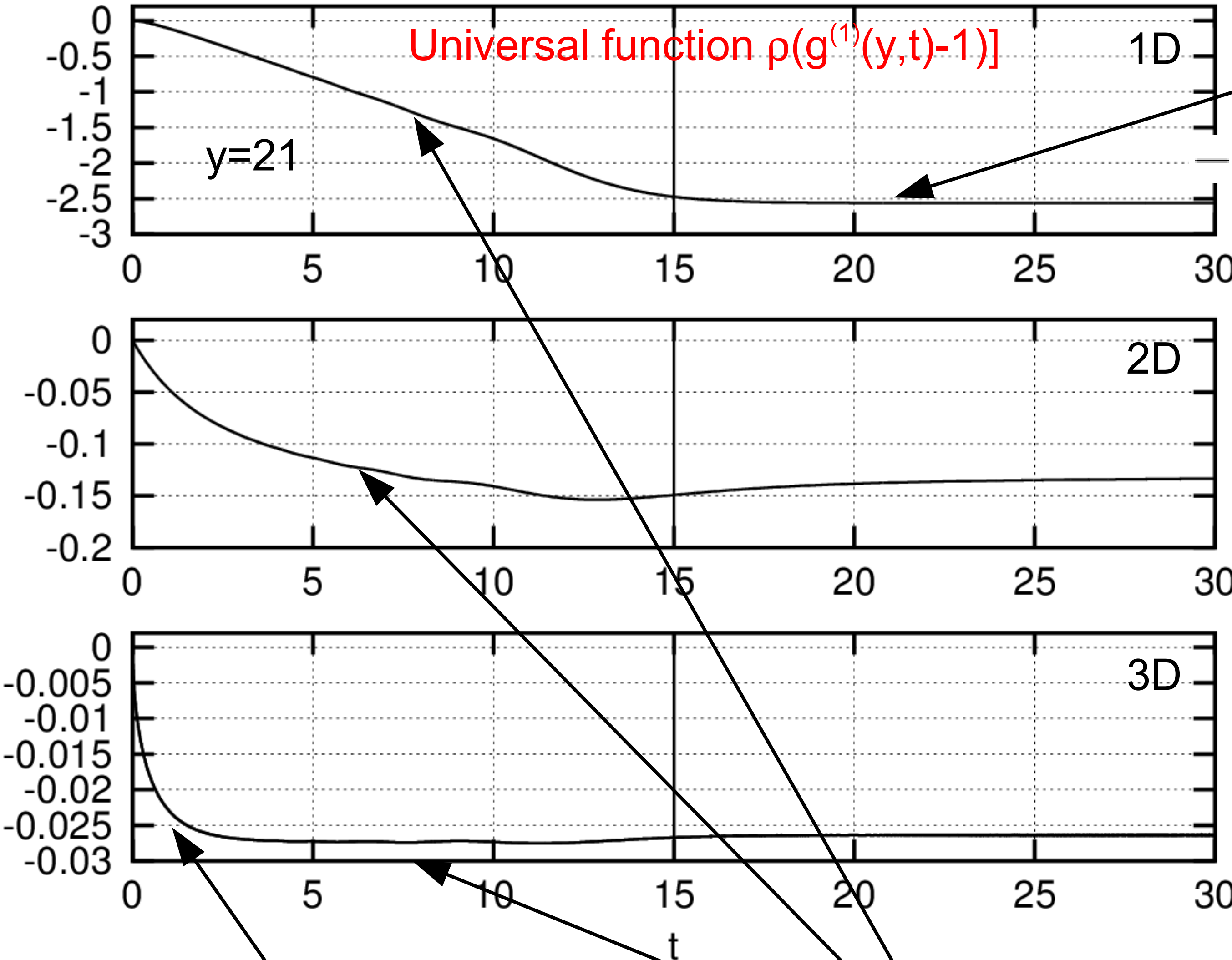
$$-\frac{y^2}{8\sqrt{2}\rho} \qquad d=1$$

$$-\frac{y^2}{16\pi\rho}\left\{1-\gamma-\log\frac{y}{\sqrt{2}}\right\} \qquad d=2$$

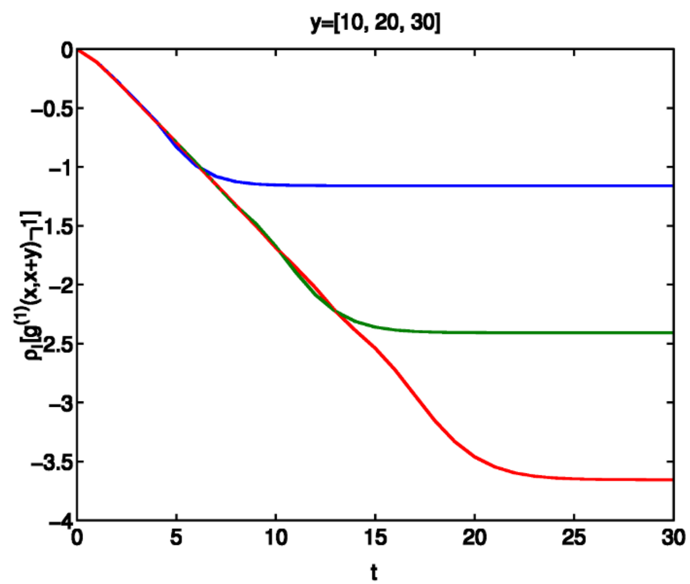
$$-\frac{y}{16\pi\rho} \qquad d=3$$

$$g^{(1)}(\infty,t)=1-\Delta(t)$$

Temporal $g^{(1)}(y,t)$



$$
\begin{aligned}
&-\frac{y^2}{8\sqrt{2}\rho} && d=1\\
&-\frac{y^2}{16\pi\rho}\left\{1-\gamma-\log\frac{y}{\sqrt{2}}\right\} && d=2\\
&-\frac{y}{16\pi\rho} && d=3
\end{aligned}
$$

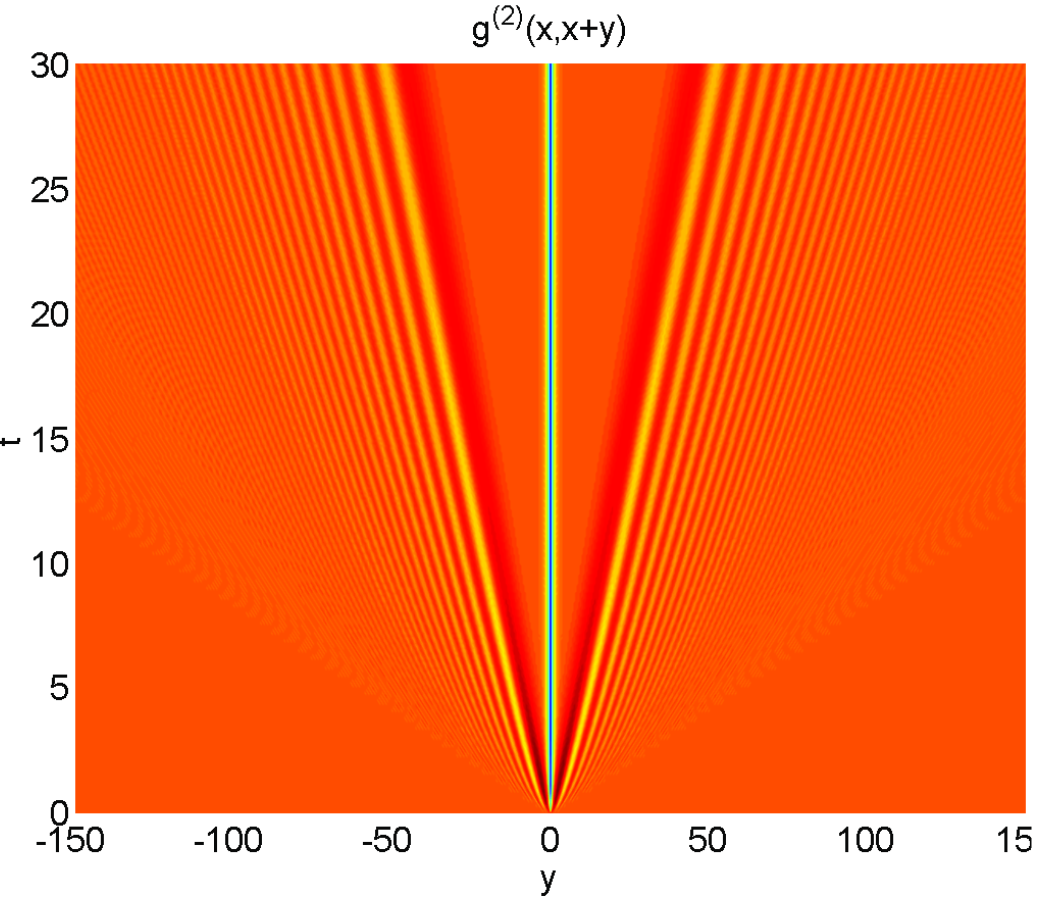


$$
\begin{aligned}
&-\frac{t^{3/2}}{3\rho\sqrt{2\pi}} && d=1\\
&-\frac{t}{16\rho} && d=2\\
&-\frac{\sqrt{t}}{4\rho\sqrt{2\pi^3}} && d=3
\end{aligned}
$$

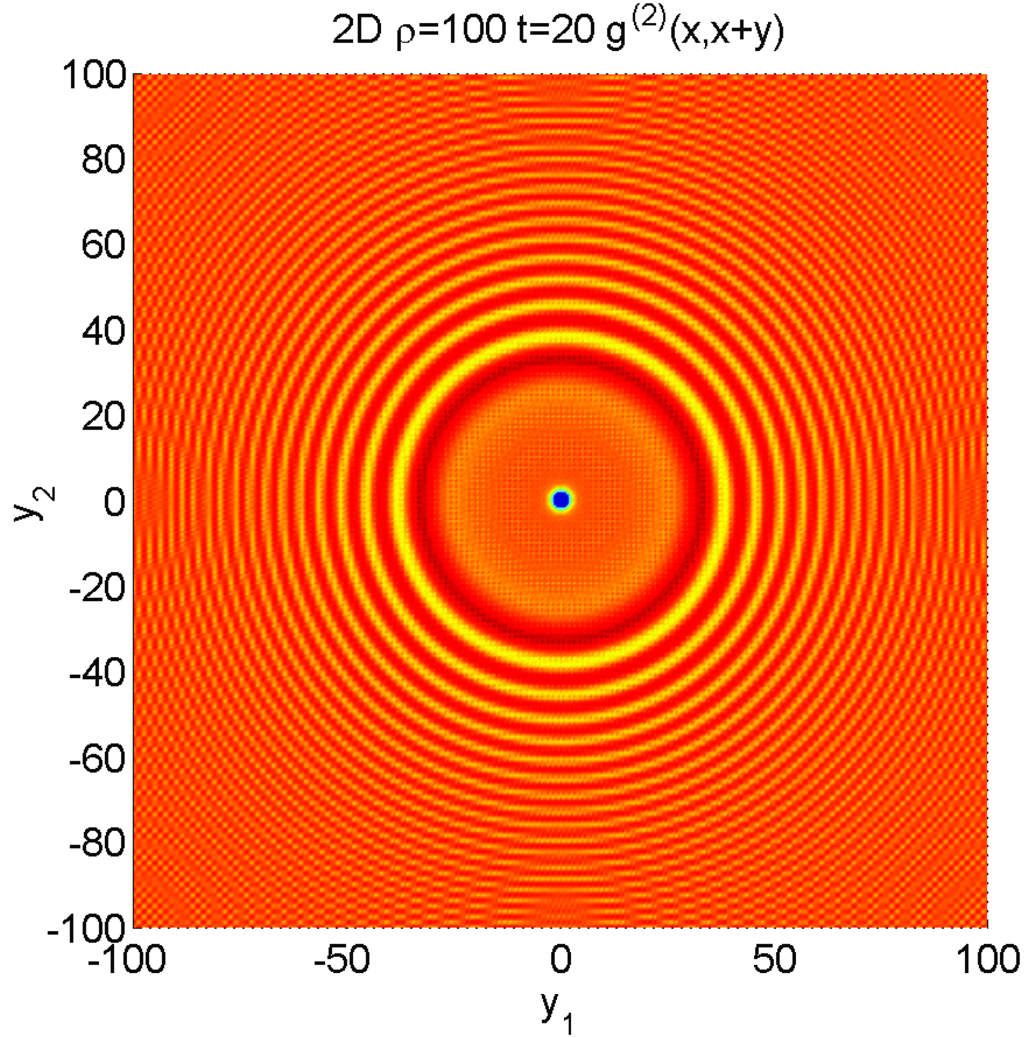
$$
g^{(1)}(\infty,t)=1-\Delta(t)\approx 1-\frac{1}{8\rho}\begin{cases}[2t-1]/\sqrt{2} & \text{1D}\\ [c_1+\log t]/\pi & \text{2D}\\ 1/\pi\sqrt{2} & \text{3D}\end{cases}
$$

$g^{(2)}(y,t)$ density correlations

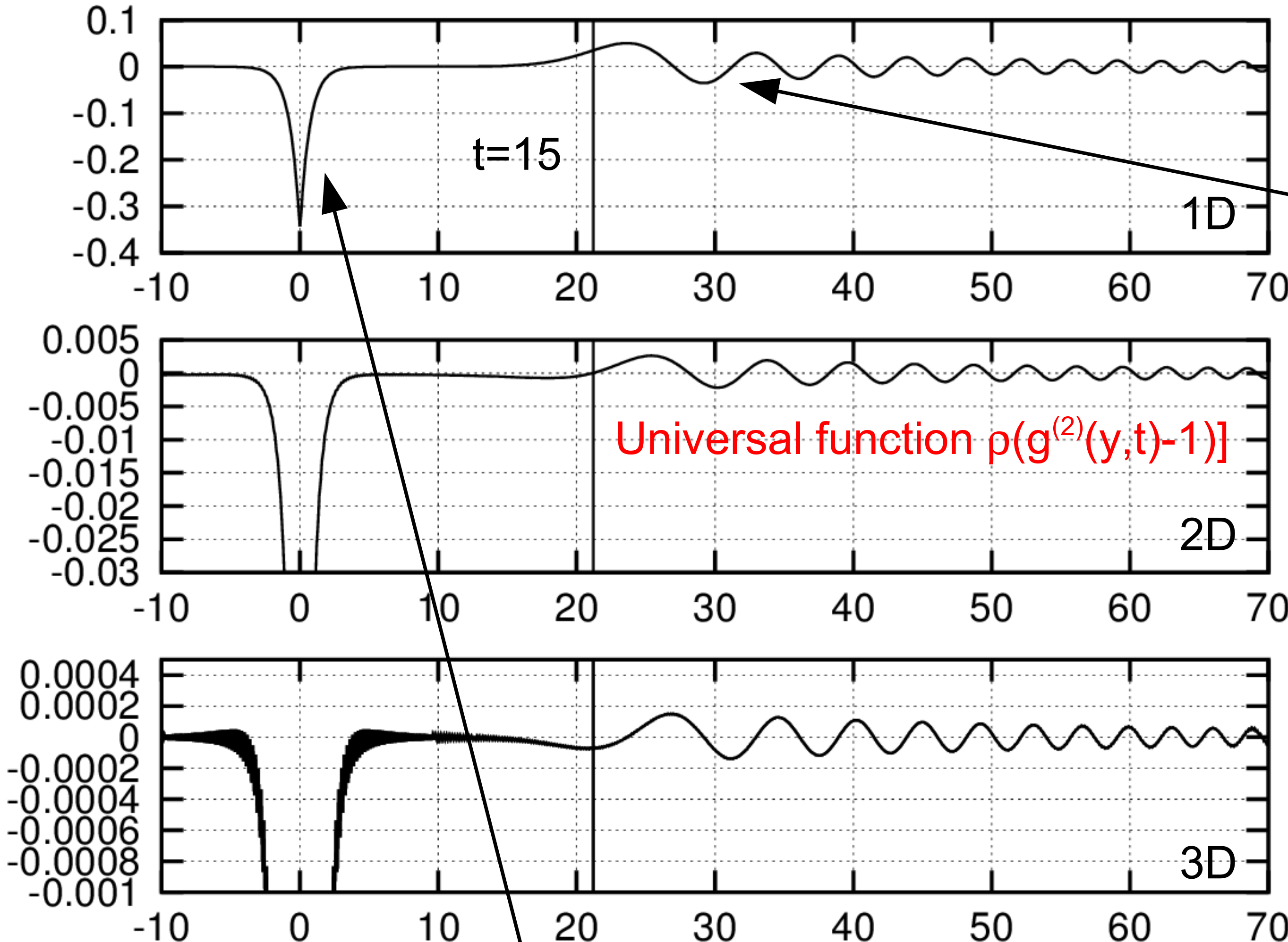
1D



2D

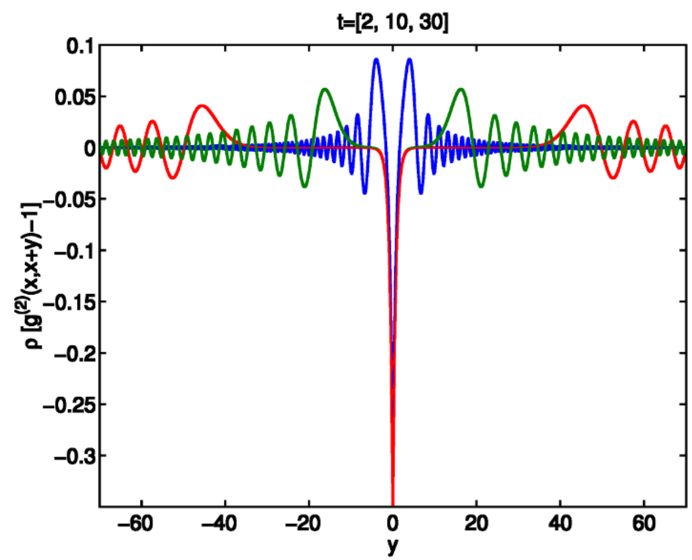


Spatial $g^{(2)}(y,t)$



$$-\begin{cases}\dfrac{1}{\rho\sqrt{8}(3t)^{1/3}}\mathrm{Ai}[x] & d=1\\ \dfrac{2^{1/4}}{4\sqrt{\pi y}(3t)^{1/2}}F_2[x] & d=2\\ \dfrac{1}{4\pi y(3t)^{2/3}}F_3[x] & d=3\end{cases}$$

$$x=\frac{-(y-t\sqrt{2})\sqrt{2}}{(3t)^{1/3}}$$



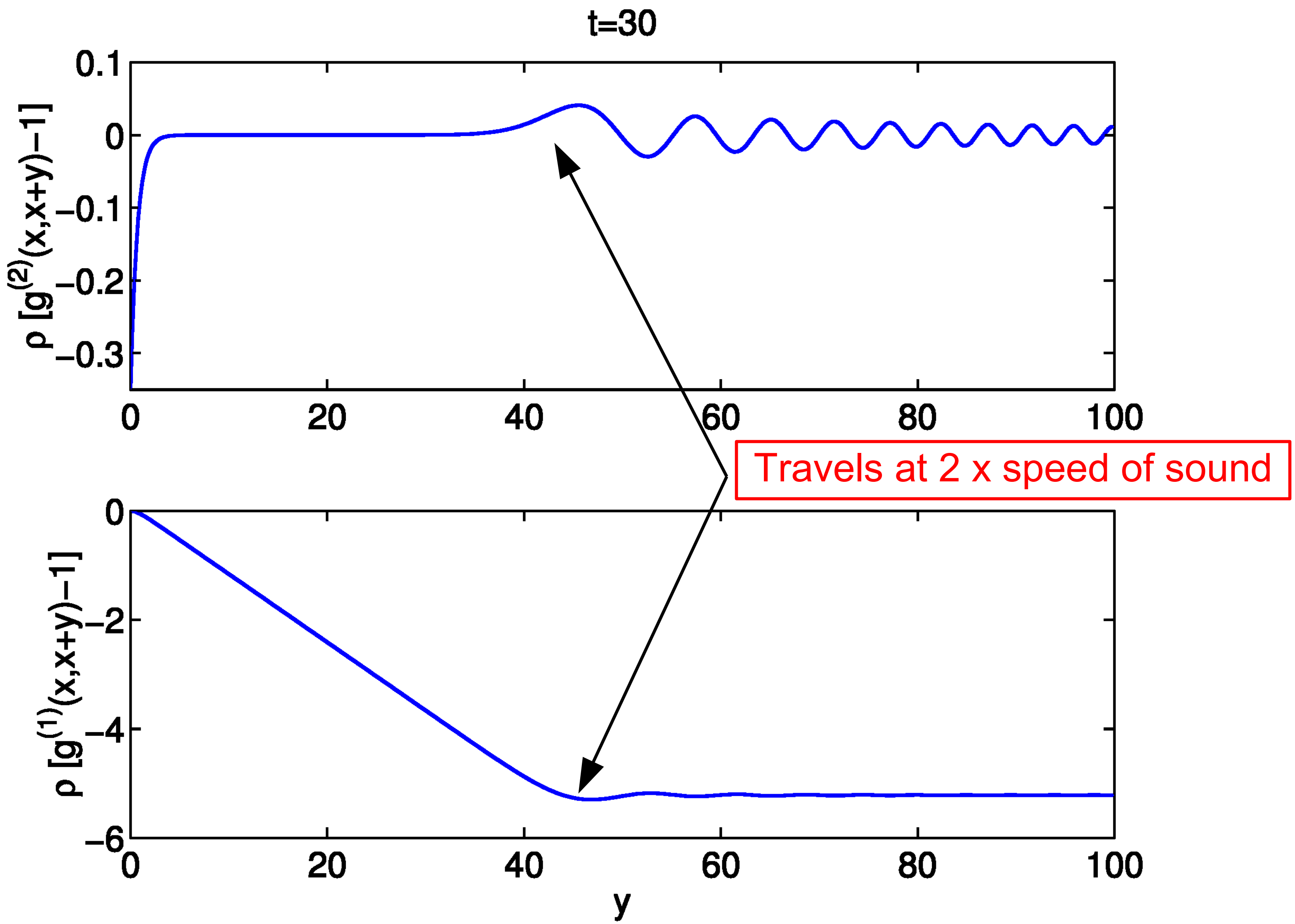
Local antibunching

$$-\begin{cases}\dfrac{e^{-y\sqrt{2}}}{2\sqrt{2}\rho} & d=1\\ \dfrac{K_0\left[y\sqrt{2}\right]}{2\pi\rho} & d=2\\ \dfrac{e^{-y\sqrt{2}}}{4\pi\rho y} & d=3\end{cases}$$

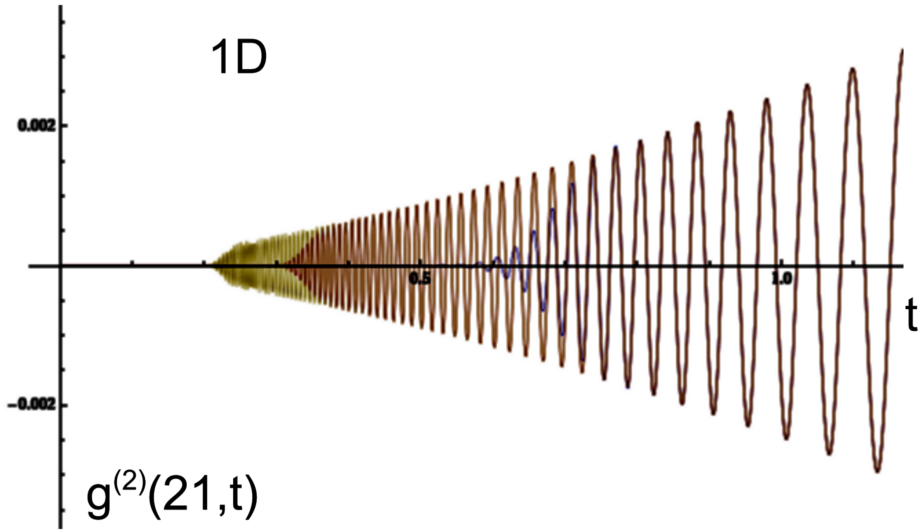
Match between features in density and phase

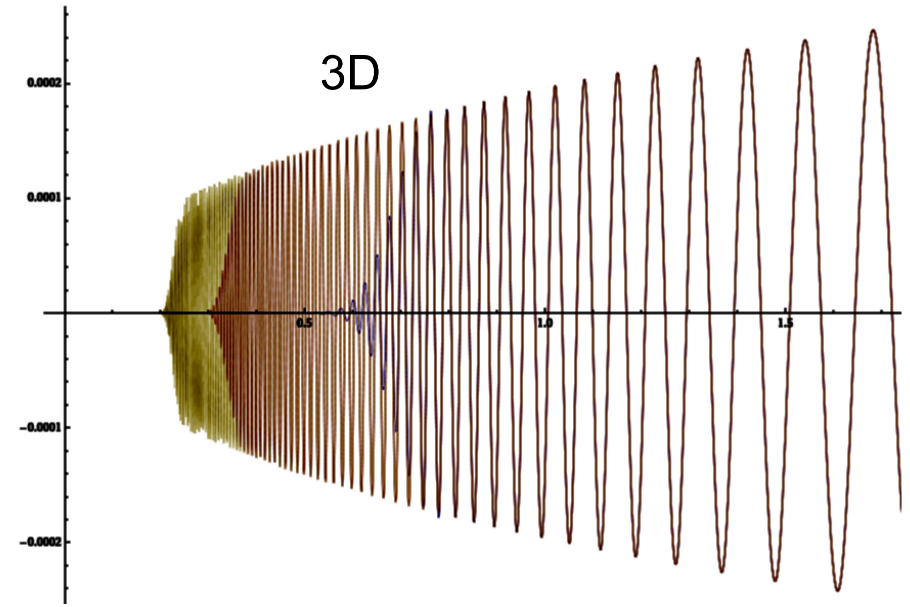
$g^{(2)}(y,t=30)$

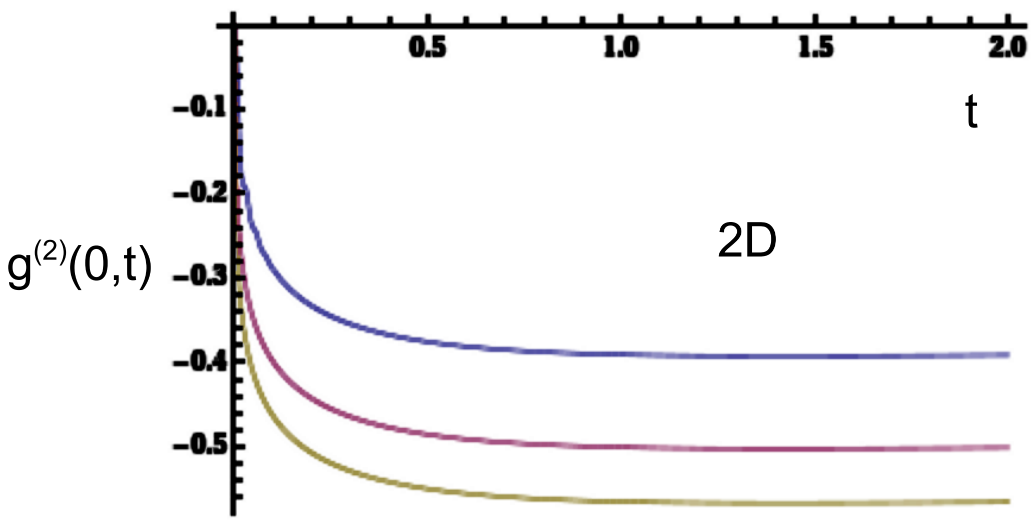
$g^{(1)}(y,t=30)$



Influence of cutoff k_{max} (B-H analogies)







$$\frac{J}{U} = \frac{2\rho}{\pi} k_{\max}$$

$k_{max} = 5\pi$

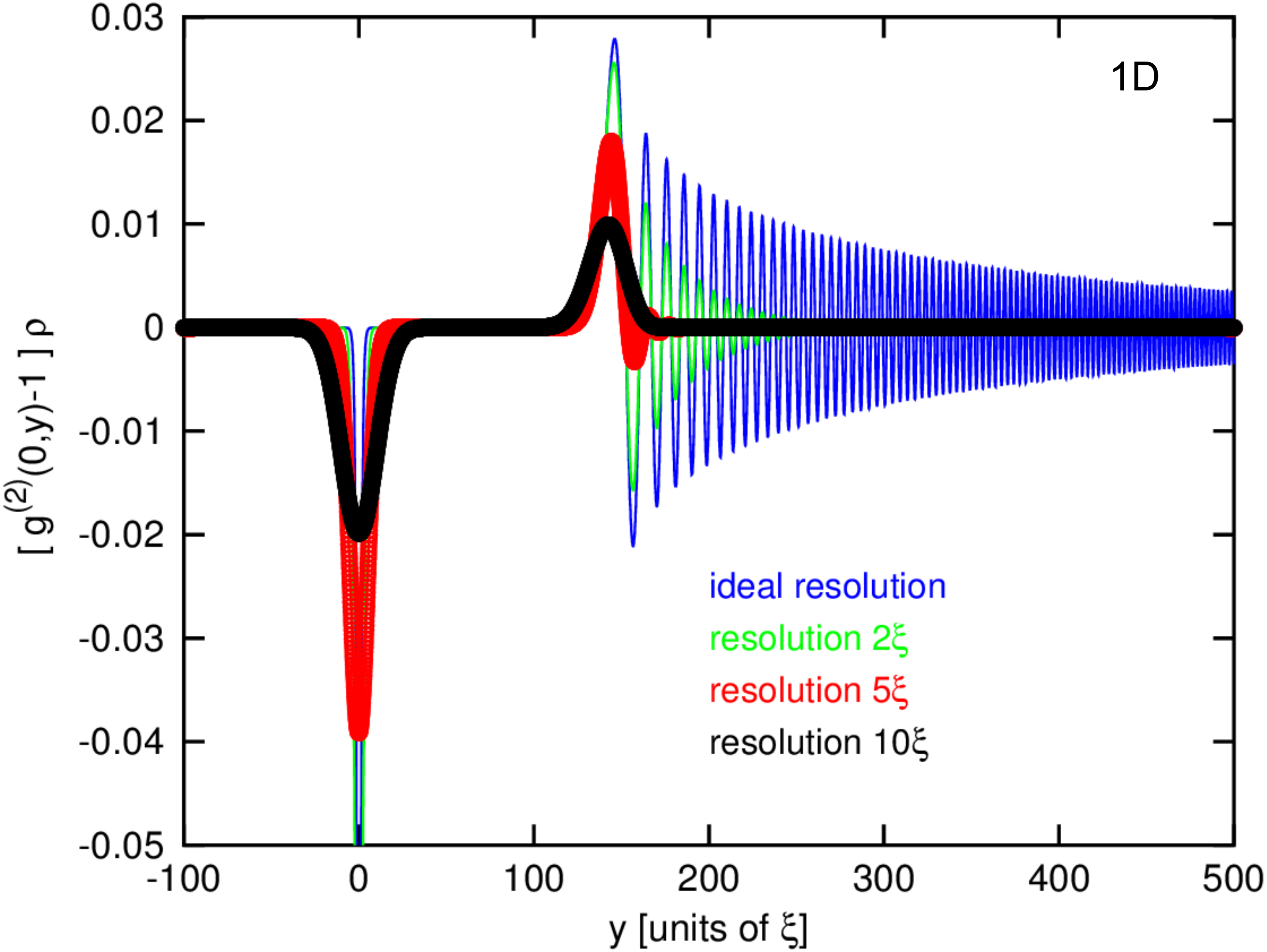
$k_{max} = 10\pi$

$k_{max} = 15\pi$

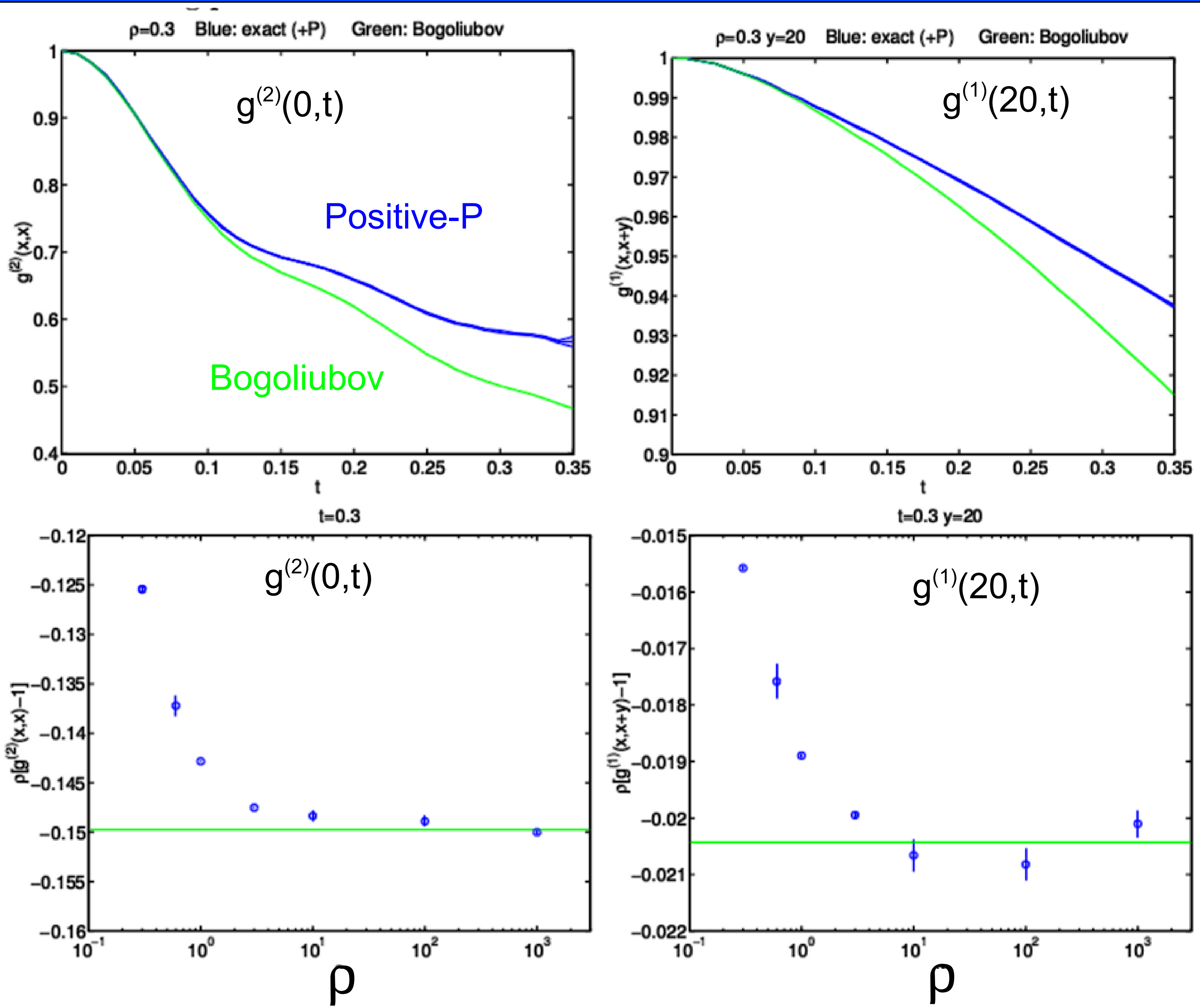
Resolution and detectability of wavefront

The issue: most structures are of healing length ξ size, but experiments do not resolve this

HOWEVER,



Small rho, large γ – deviations from Bogoliubov



Thank you

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