Solitons and phase domains during the cooling of a one-dimensional ultra-cold gas

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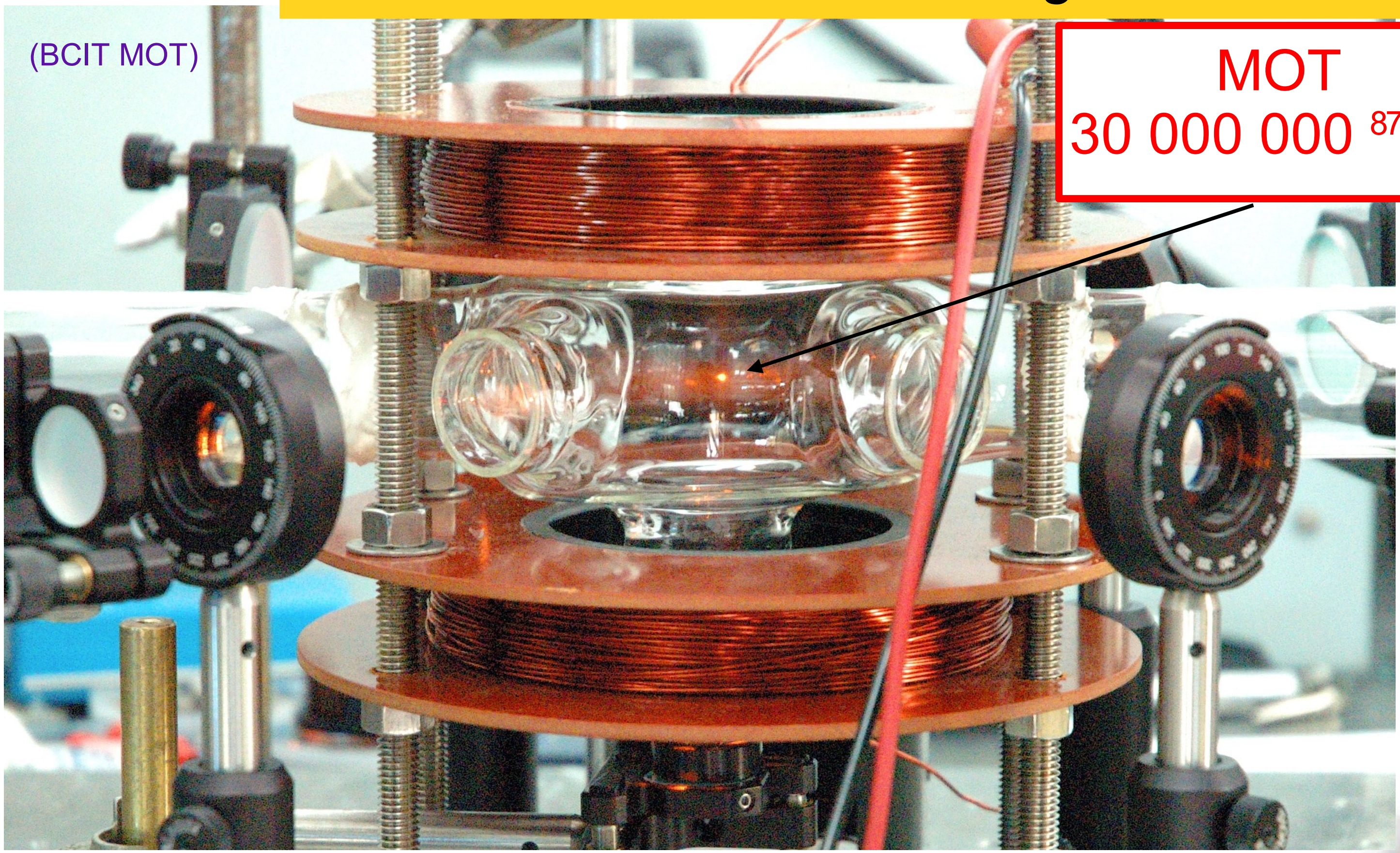
Cover of Phys. Rev. Lett., 1 Apr 2011
E. Witkowska, PD, M. Gajda, K. Rzążewski, PRL **106**, 135301 (2011)

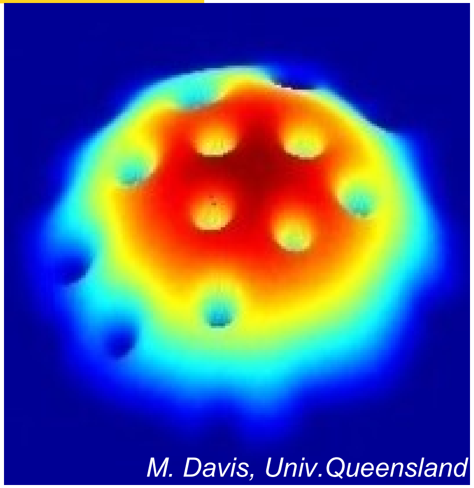
Outline

- The one-dimensional Bose gas
- Two views on what goes on during cooling
  - Defect creation (Kibble-Zurek mechanism)
  - Thermal phase fluctuations (at long times)
- Simulation method
- Simulation results
  - BEC regime
  - Quasicondensate regime
- How one scenario goes into the other
- Weak domains, strong defects

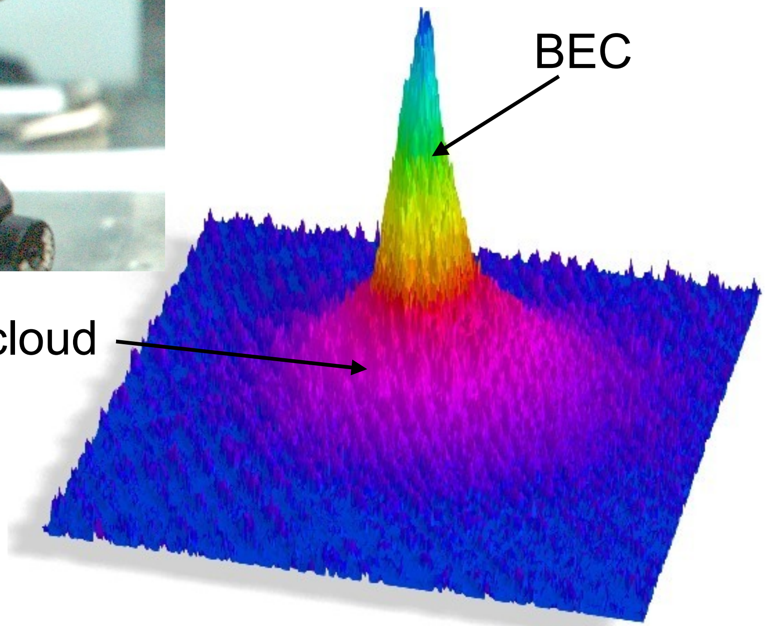
Bose-Einstein condensate

"most atoms are in a single orbitalof the trap"

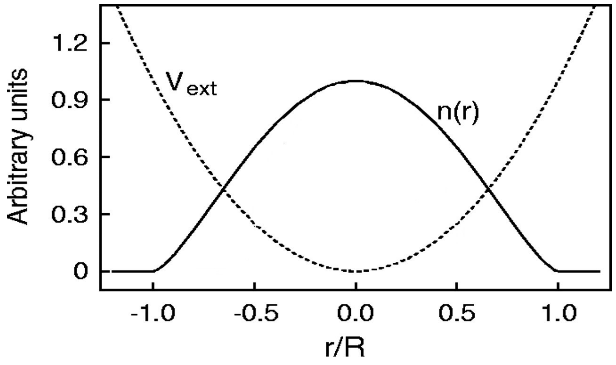




3. vortices → superfluidit



2. thermal cloud



1. non-uniform density in the trap

Cooling stage 1 - 100 μK

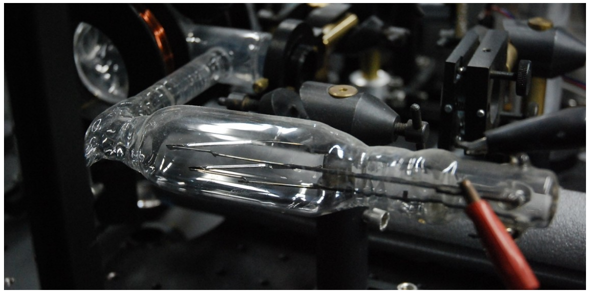
NASA/ESA

Coldest known natural object
Boomerang nebula ≈1K
(Joule-Thompson effect)

(microwave background radiation: 2.7K)

A teraz najzimniejsze znane obiekty *w ogóle* ...

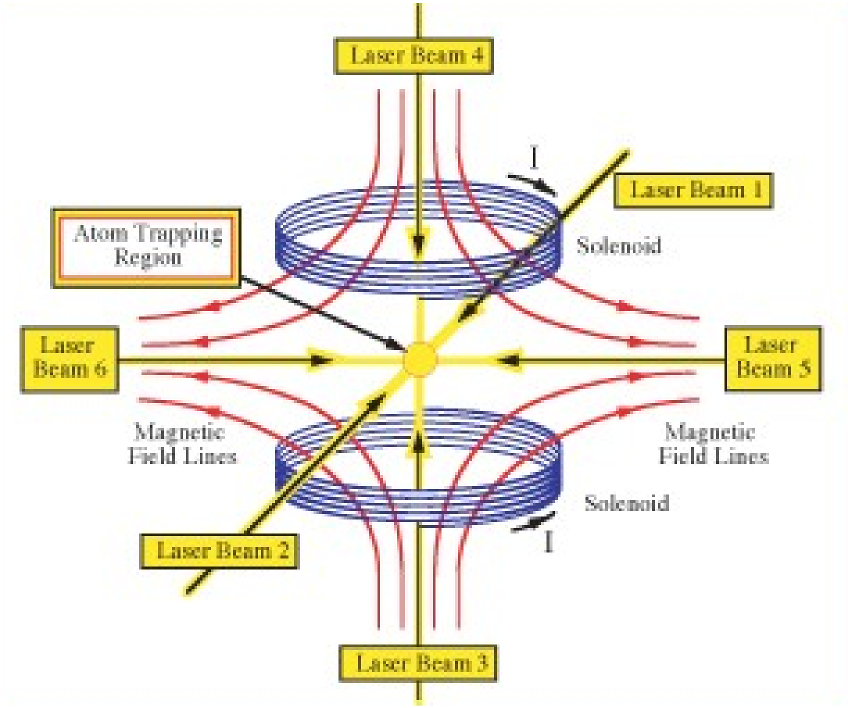
1. Dispenser (500-1000K)



M. Główdź, IF PAN

Rb
Cs
Li
K
Na

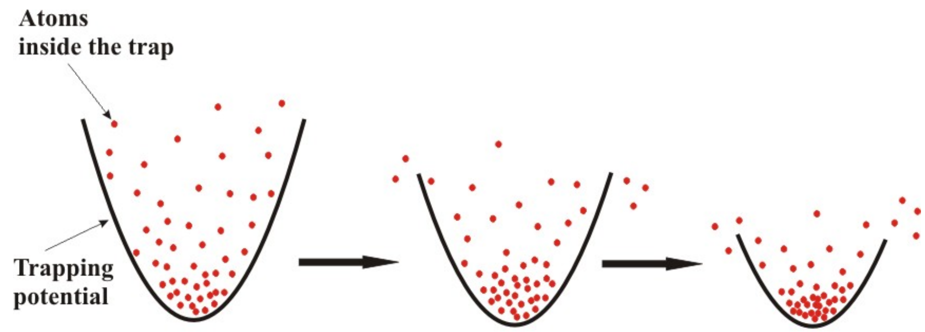
2. Laser cooling
+ MOT(*Magneto-optical trap)*
≈100μK



A. Murray, Manchester Univ.

Cooling stage 2 – 100 nK

3. Evaporative cooling
+ magneto-optical / optical dipole trap
(≈100nK)



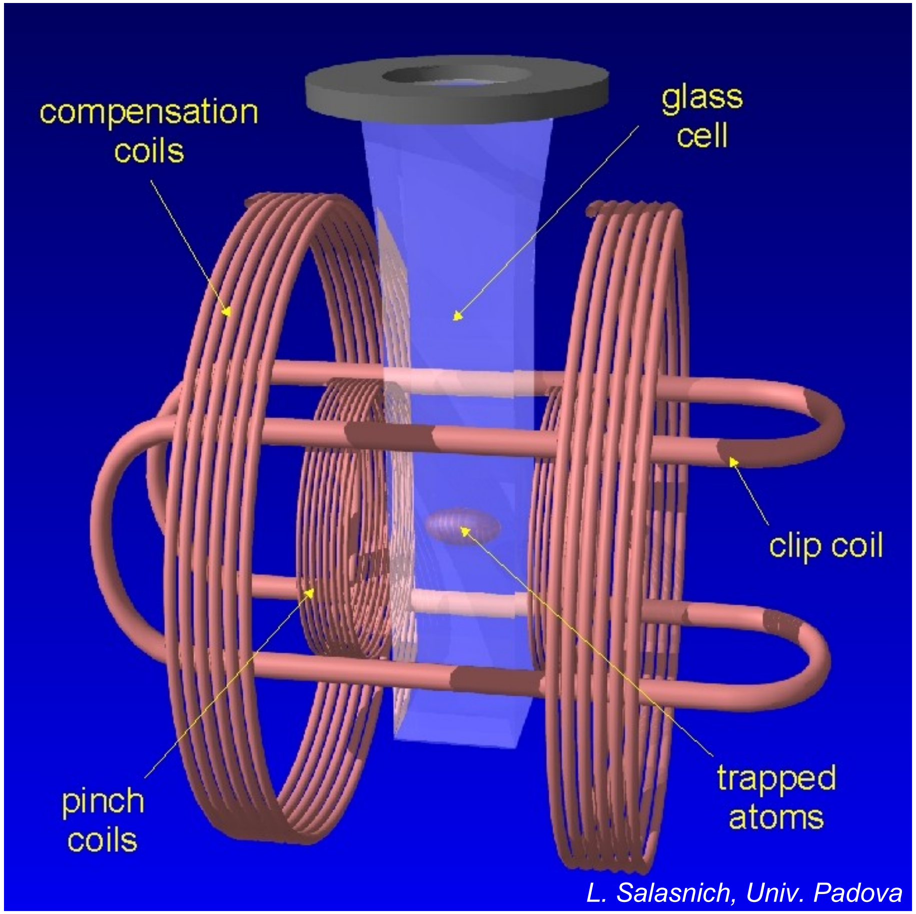
G. Raithel, Univ. Michigan

Typically, 10^5-10^7 atoms

Max density 10^{12}-10^{14} cm^{-3}

[c.f. $3X10^{19}$ cm^{-3} in the air]

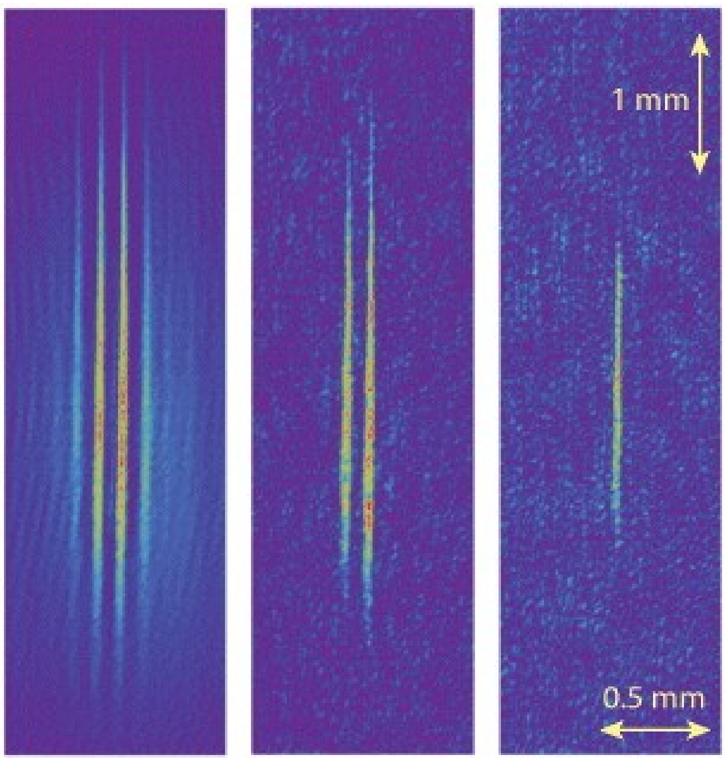
→ BEC



One-dimensional Bose gas

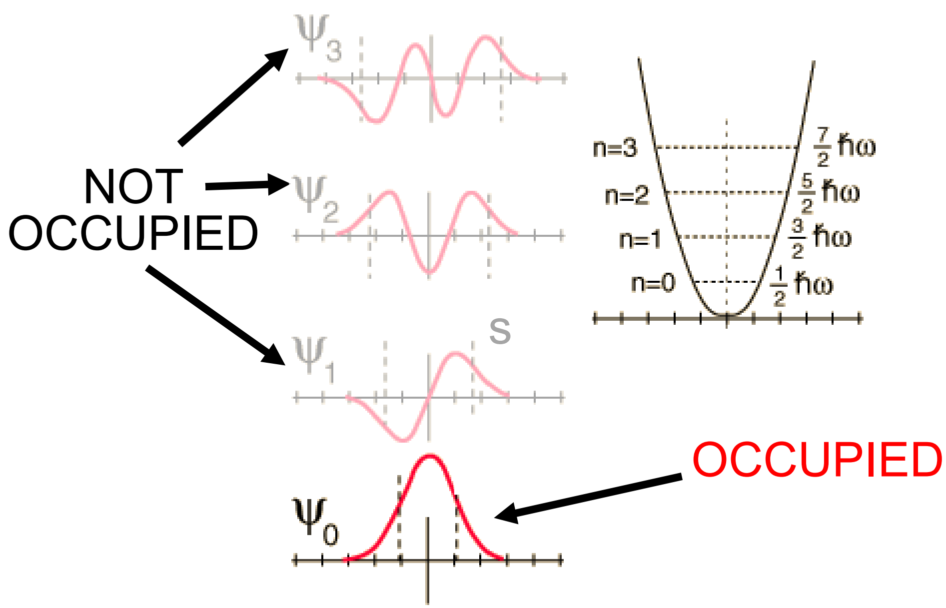
$$\hat{H} = \int dx \left\{ \hat{\Psi}^\dagger(x) \left[V(x) - \frac{\hbar^2}{2m} \nabla^2 \right] \hat{\Psi}(x) + \frac{g}{2} \hat{\Psi}^\dagger(x)^2 \hat{\Psi}(x)^2 \right\}$$

- $N \sim 10^3$-10^4
- $g > 0$ → repulsive contact interactions



I. Llorente Garcia *et al.* New J. Phys. **12**, 093017 (2010)

IN THE NARROW DIRECTIONS:



Need $h\omega_{long} \ll k_B T \ll h\omega_{narrow}$

Quasicondensate

- In the uniform 1D gas, there is no true condensate for T>0
- *However:* finite coherence length l_ϕ

$$g^{(1)}(x,x') \sim \exp\left[-\frac{|x-x'|}{l_\phi}\right] \quad ; \quad l_\phi \sim \frac{N^{2/3}}{T}$$

$$g^{(1)}(0,x) = \frac{\langle \widehat{\Psi}^\dagger(0)\widehat{\Psi}(x)\rangle}{[\rho(0)\rho(x)]^{1/2}}$$

- $\rightarrow$ In the trap, BEC occurs when $L < l_\phi$

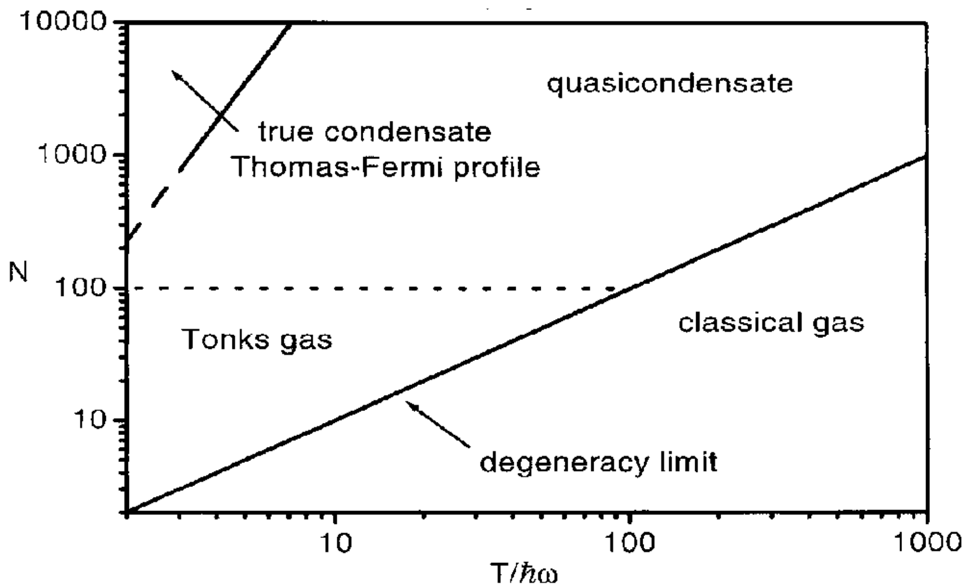
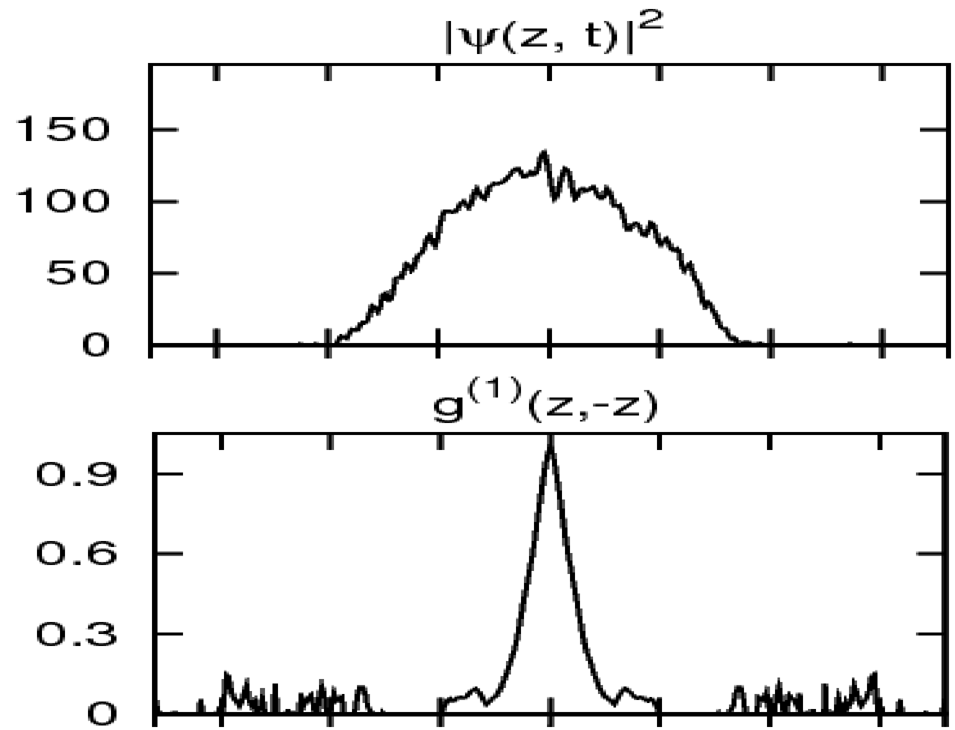


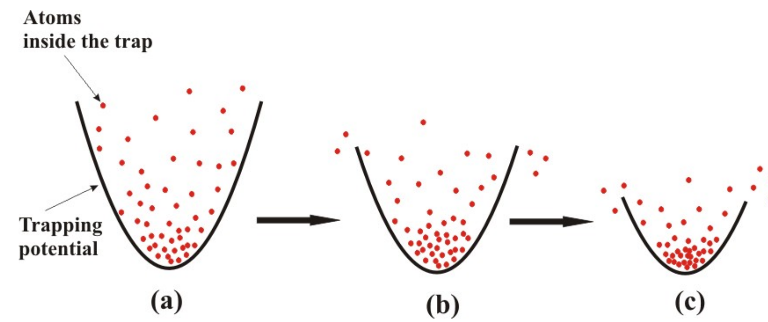
FIG. 1. Diagram of states for a trapped 1D gas.

D. Petrov, G. Shlyapnikov, J. Walraven, PRL **85,** 3745 (2000)

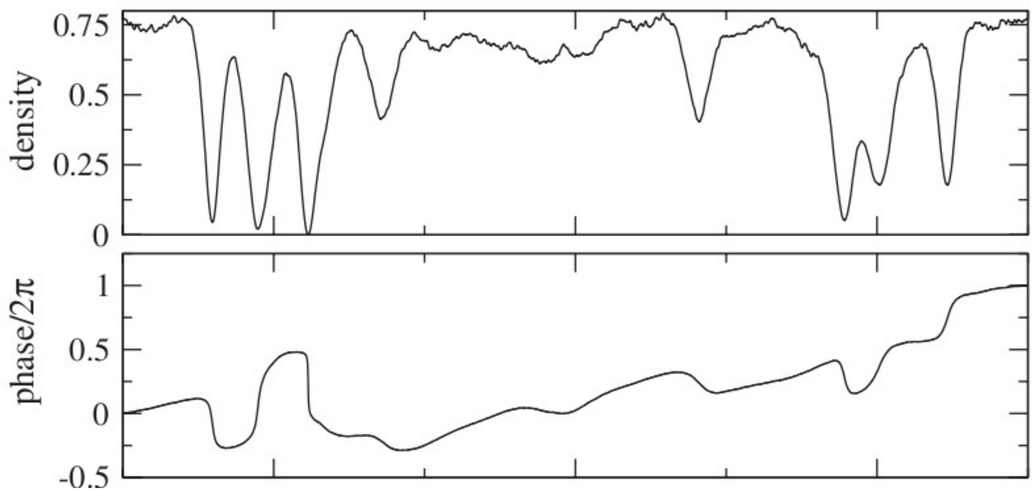


But – what actually goes on during the cooling?

Aim:
to reconcile two aspects of the cooled state



(1): Solitons formed in a quench via Kibble-Zurek mechanism

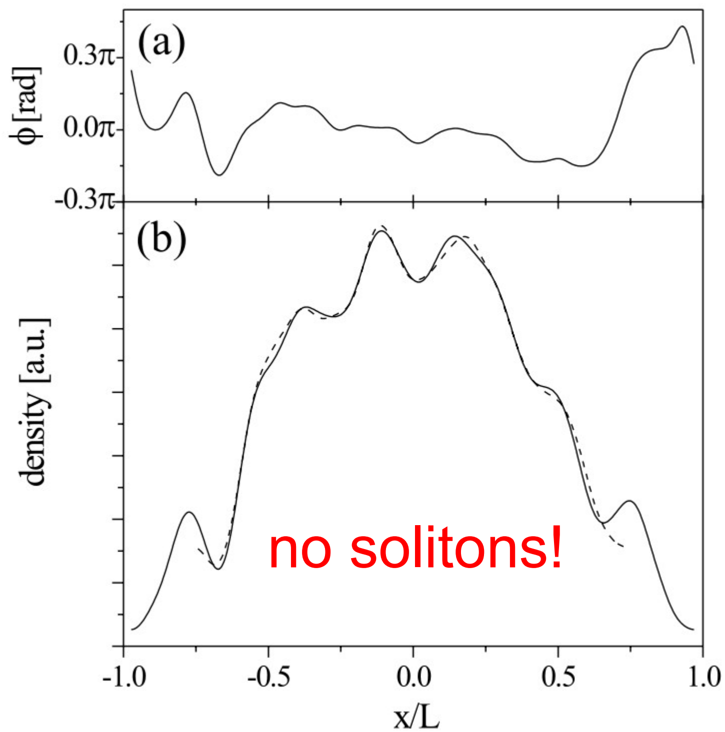


B. Damski, W. Żurek, PRL **104,** 160404 (2010)

Quench of μ in thermal bath

What about a realistic quench of temperature?

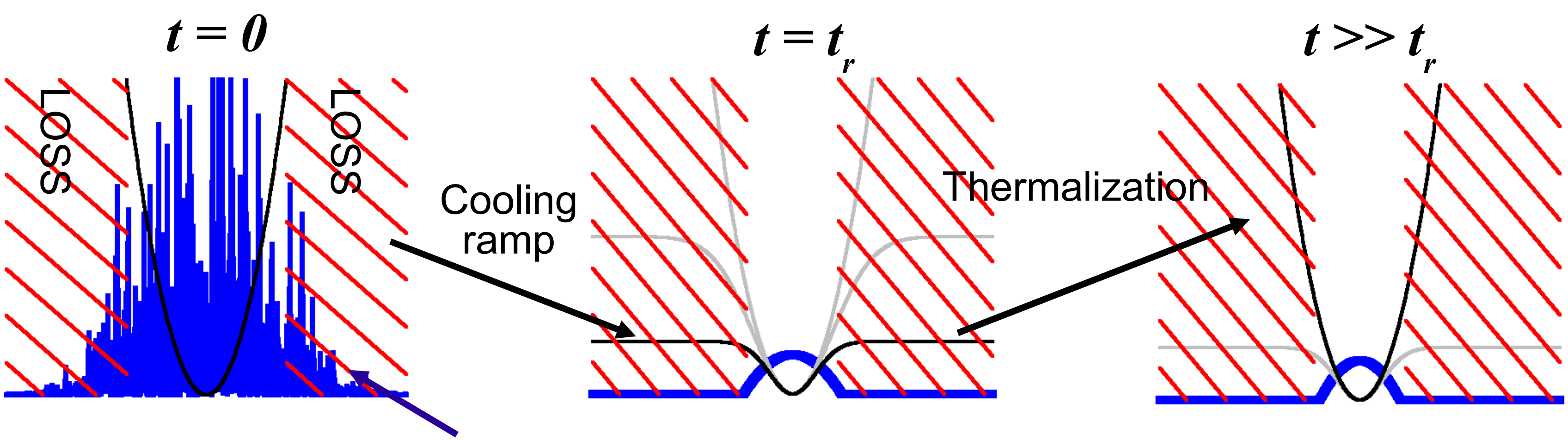
(2): Smooth quasicondensate phase in thermal equilibrium



S. Dettmer *et al*, PRL **87,** 160406 (2001)g

Evaporative cooling of 1D Bose gas

THE MODEL



- Initial condition: gas at thermal equilibrium, above T_c

E. Witkowska, M. Gajda, K. Rzążewski Opt. Commun. **283**, 671 (2010)

Simulation: *c-field method*

"Quantum field theory, without discretized particles"

$$\hat{\Psi}(x) \rightarrow \psi(x)$$

$$i\partial_t \psi(z,t) = [H(z,t) - i\Gamma(z,t)]\psi(z,t),$$

$$H(z,t) = -\frac{1}{2}\frac{\partial^2}{\partial z^2} + V(z,t) + g_{1D}|\psi(z,t)|^2$$

Developed by many authors:

A. Sinatra, M. Brewczyk, M. Gajda, M. Davis, K. Rzazewski, K. Burnett, E. Witkowska, … *(no particular order)*

Useful papers: M. Brewczyk *et al,* J. Phys B **40,** R1 (2007);

P. Blakie *et al.* Adv. Phys. **57**, 363 (2008)

In more detail

e.g. free space : plane wave basis

Full quantum field → c-fields

$$\Psi(\mathbf{r}) = \sum_{\mathbf{k}} a_{\mathbf{k}} \frac{1}{\sqrt{V}} e^{i\mathbf{k}\mathbf{r}} \longrightarrow \Phi(\mathbf{r}) = \sum_{|\mathbf{k}|\leqslant \mathbf{K}_{\max}} \alpha_{\mathbf{k}} \frac{1}{\sqrt{V}} e^{i\mathbf{k}\mathbf{r}}$$

Replace mode amplitude operators $a_{\mathbf{k}}$
with complex number amplitudes $\alpha_{\mathbf{k}}$

Thermal initial state:

- $|\alpha_{\mathbf{k}}|^2$ Distributed according to Bose-Einstein distribution
- Phase of $\alpha_{\mathbf{k}}$ is random
- Use many realizations to get thermal ensemble

Evolution: nonlinear Schrodinger equation

$$i\partial_t \psi(z,t) = [H(z,t) - i\Gamma(z,t)]\psi(z,t),$$

$$H(z,t) = -\frac{1}{2}\frac{\partial^2}{\partial z^2} + V(z,t) + g_{1D}|\psi(z,t)|^2$$

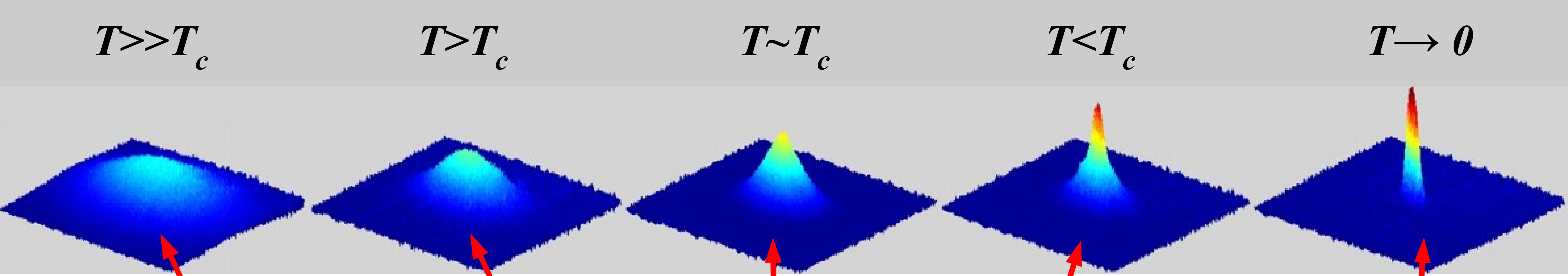
Validity

$$\left[\hat{\Psi}(x), \hat{\Psi}^\dagger(x')\right] = \delta(x - x') \qquad \rightarrow \qquad [\psi^*(x), \psi(x')] = 0$$

→ *it will be fine, ...*

.... as long as there are always many atoms involved

in whatever it is we are studying



http://www-matterwave.physics.ox.ac.uk/node/1

Mode occupation getting small

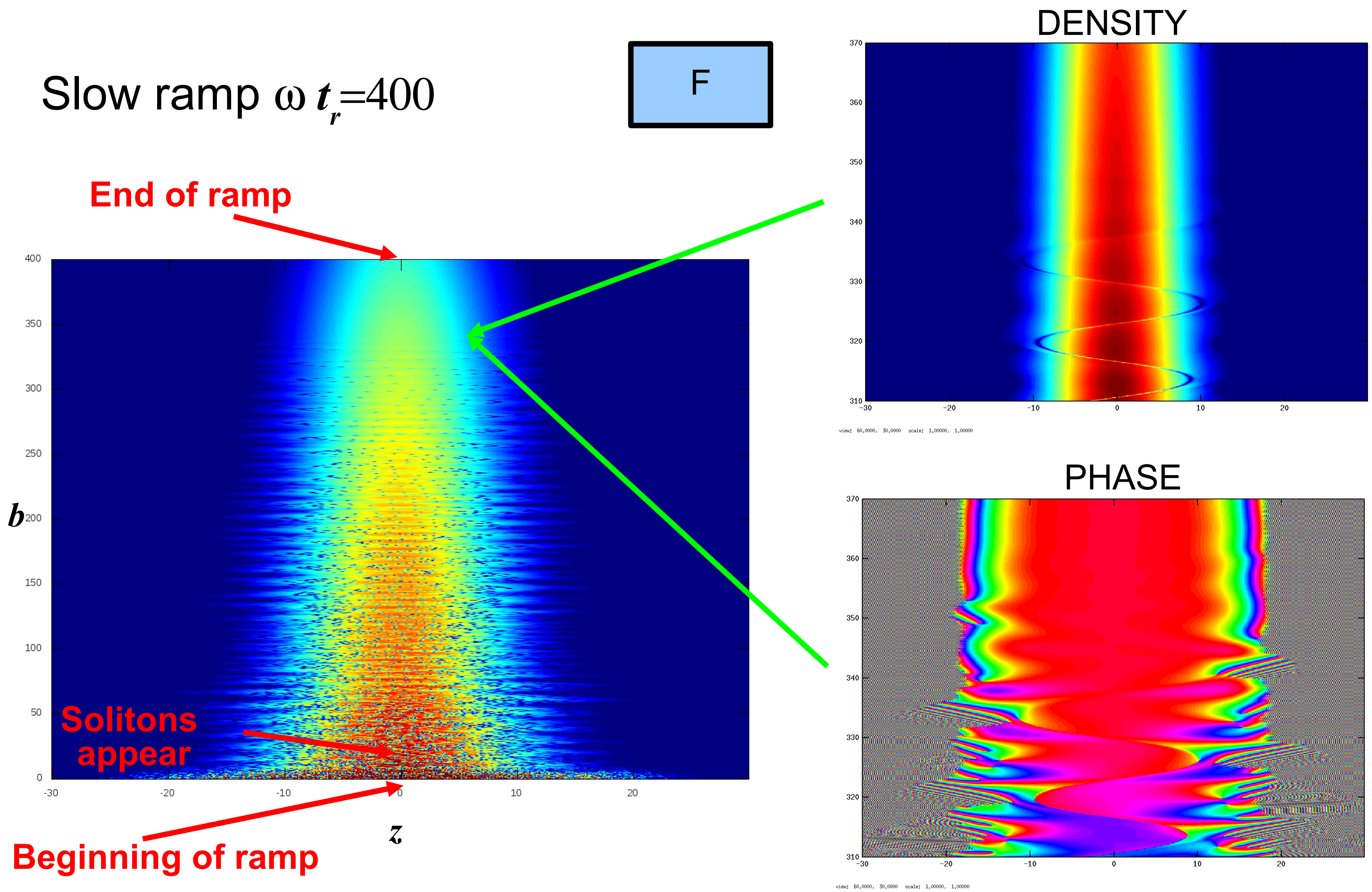
OK:
All relevant modes highly occupied

Occupation in excited modes getting small

Simulation - slow ramp → BEC

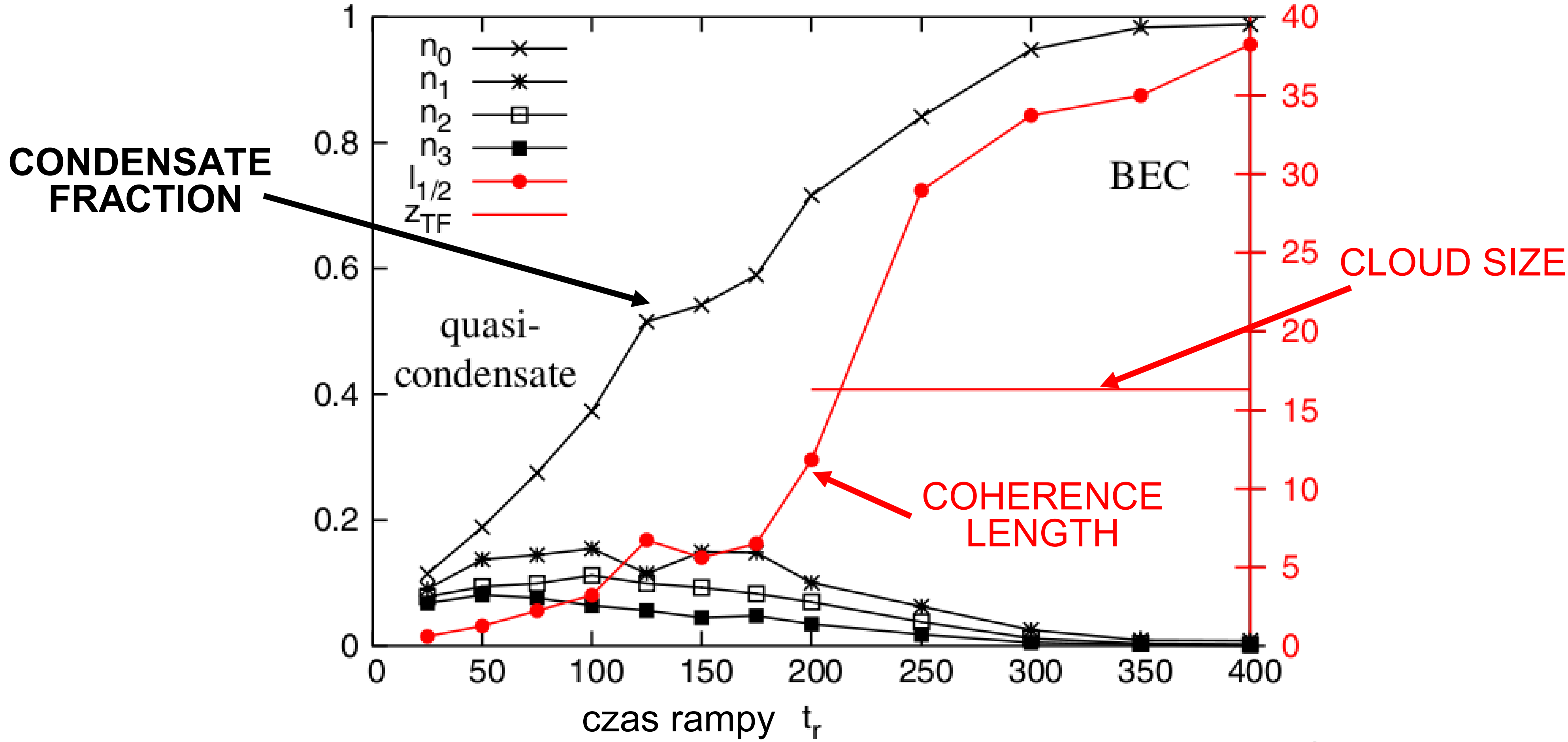
Slow ramp $\omega\, t_r = 400$

F



Ramp time

- Slow ramp → BEC
- Fast ramp → quasicondensate
- Very fast ramp → thermal gas



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Fast ramp → quasicondensate precursor

Fast ramp ωt_r=75

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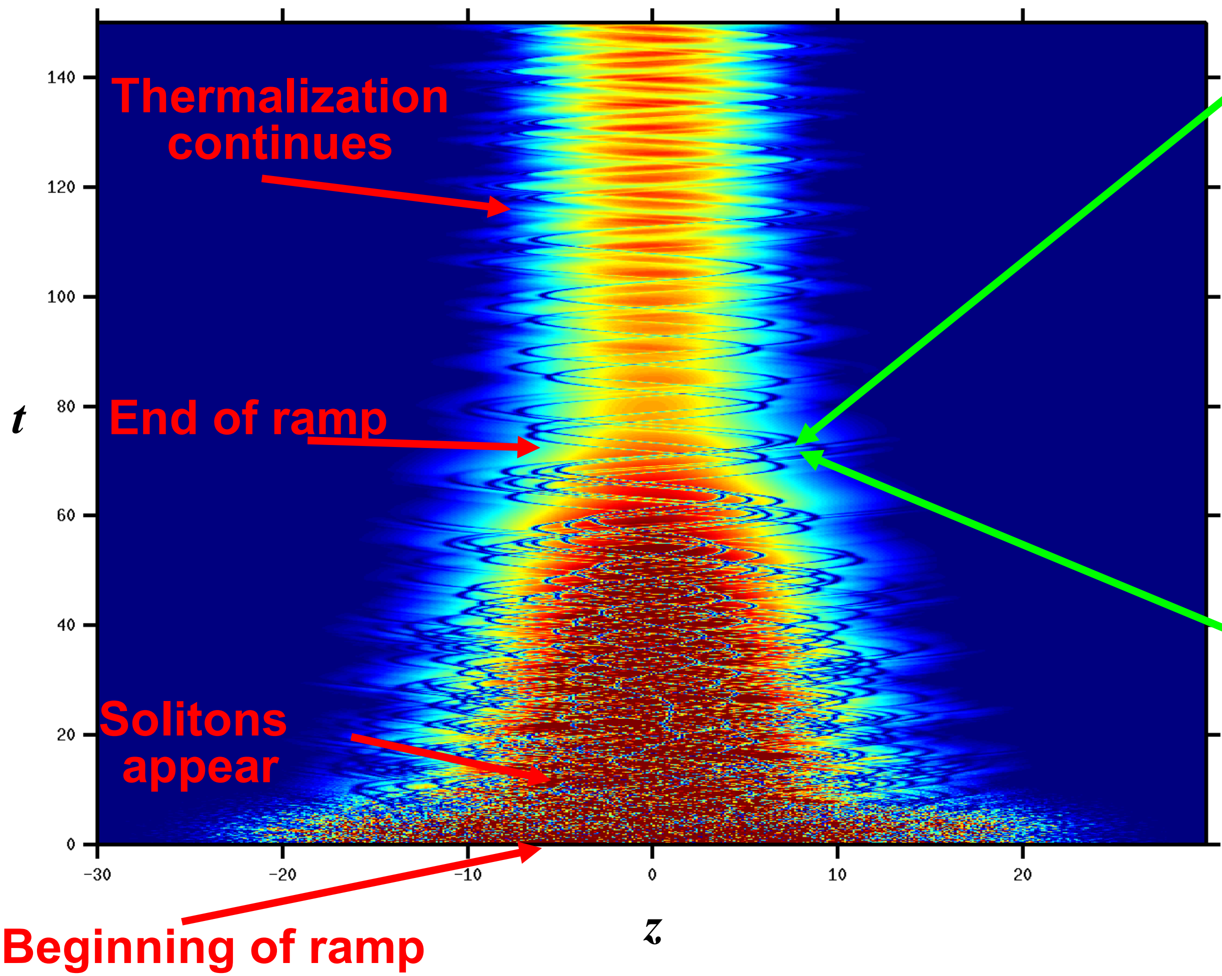
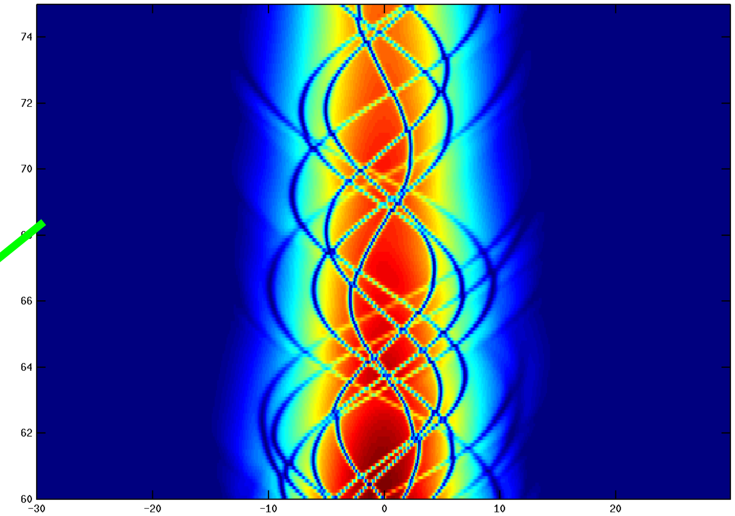


DENSITY



PHASE

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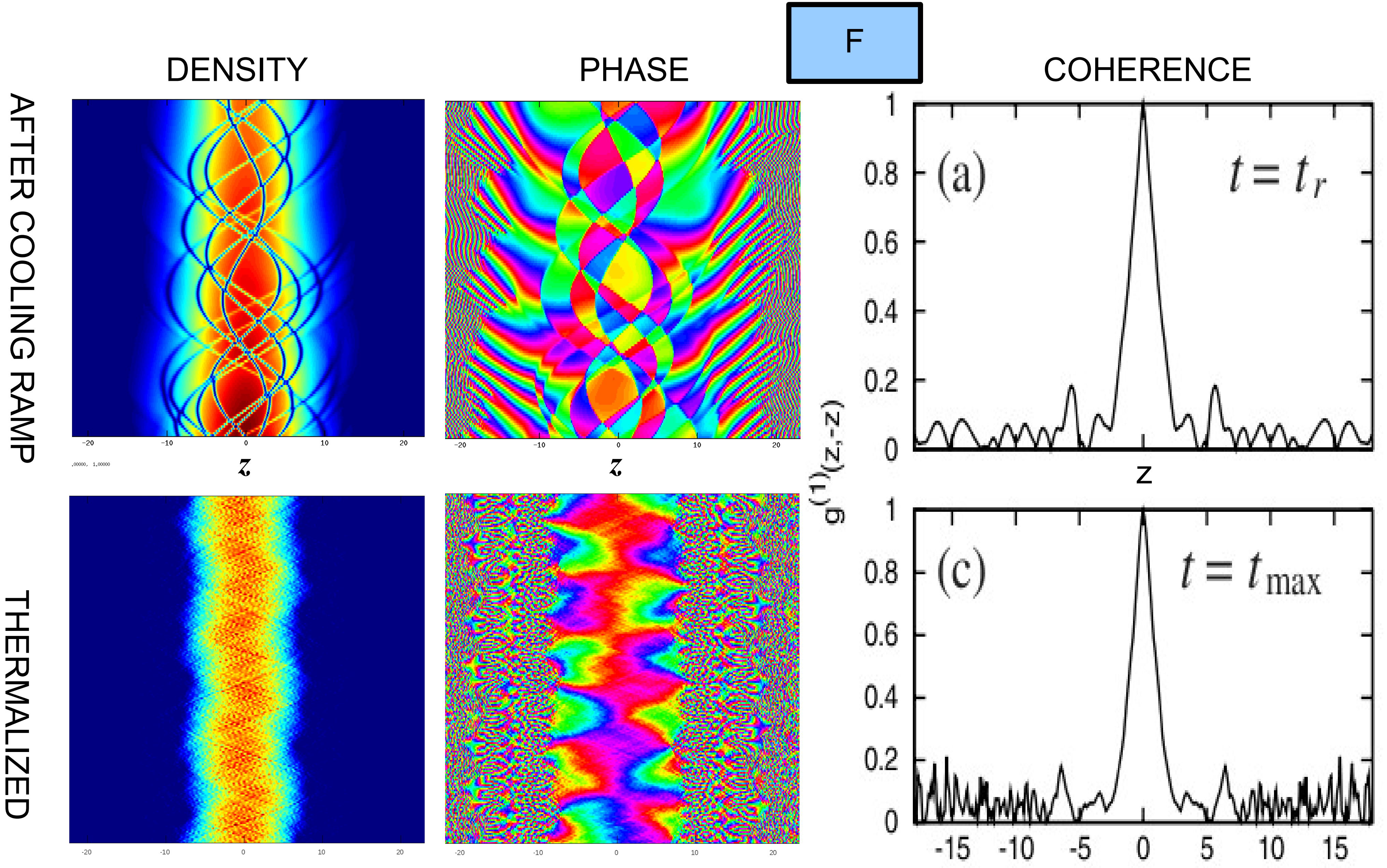
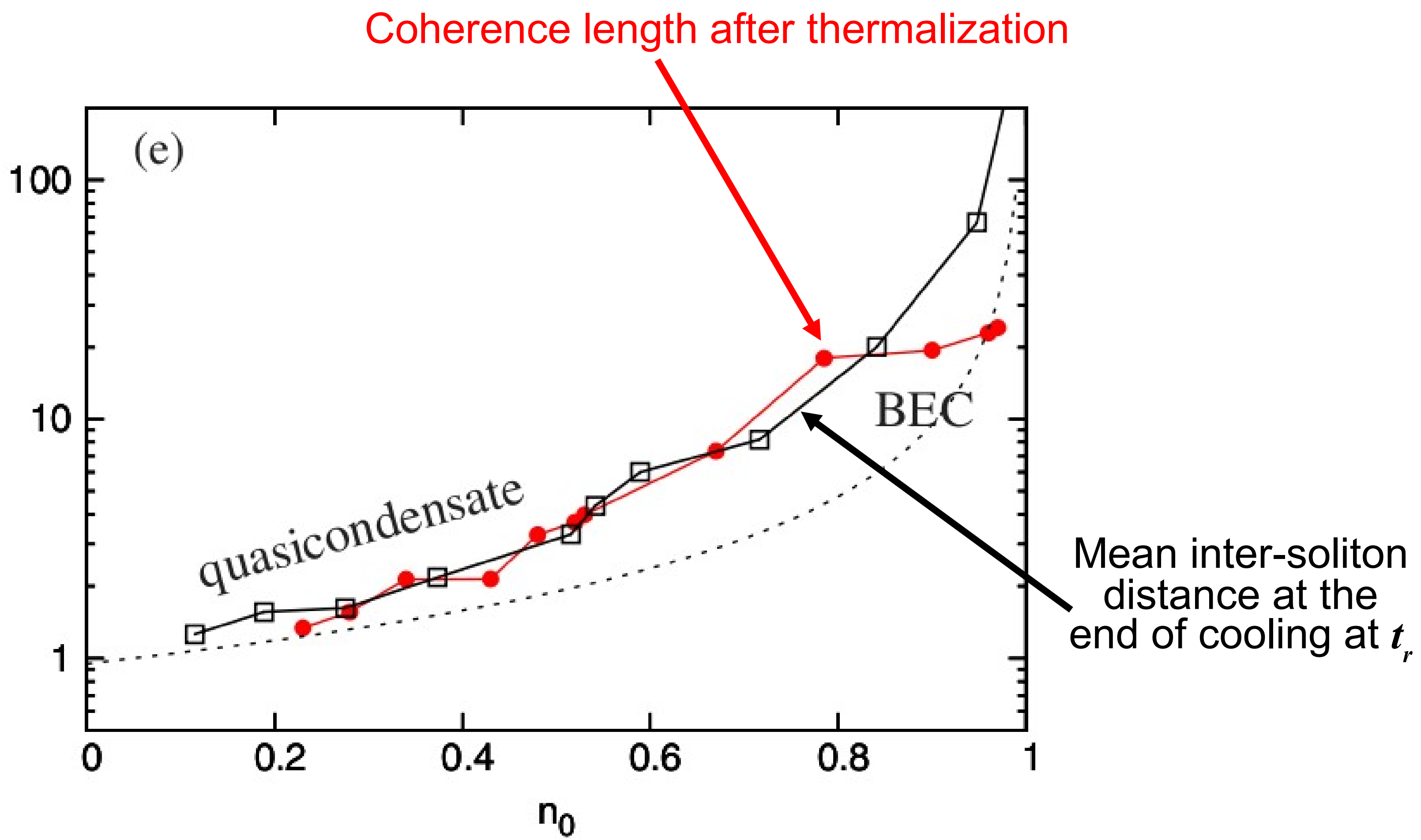
Thermalization to a quasicondensate



Solitons as the "larval stage" of equilibrium fluctuations

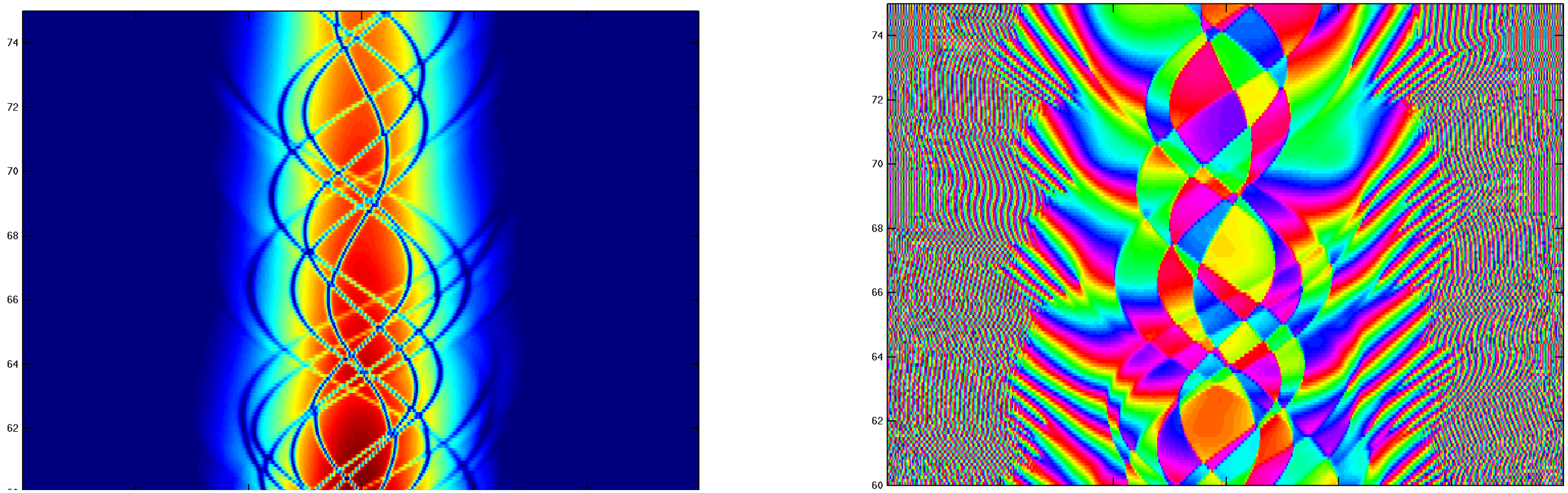


Domain formation – not like the standard story

- We did NOT see the usual scenario, where:

  "domain seeds grow with time and defects form where they meet"

DENSITY PHASE



- Instead:
  - domains are fleeting
  - solitons are the stable entities
  - coherence length conserved

- Not yet fully understood

  → we're "building a new trap"

F

Summary

- 1D evaporative cooling is quite different to the usual scenario
- Coherence length conserved during thermalization of solitons
- Solitons NOT phase domains are the long-lived objects
- *Details:*

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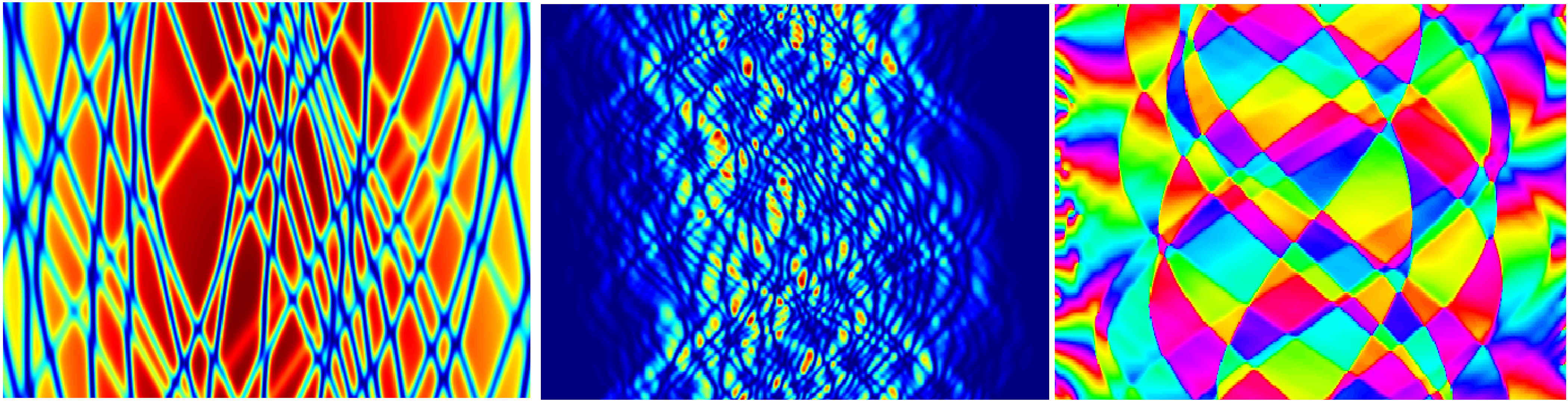
- *Movies:* www.ifpan.edu.pl/~deuar/



Solitony – Mechanizm Kibble-Żurka

- Temperatura się obniża w czasie
- Blisko Tc, czas reakcji jest za długi aby nadążyć za stanem termicznym
- Faza zostaje lokalnie “zamrożona” bez komunikacji między odległymi obszarami
- → domeny fazy
- Pomiędzy domenami tworzą się defekty (solitony w 1D)
- Im szybsza rampa, tym mniejsze domeny
- Skalowanie ilośći solitonów z prędkośćią przekraczania Tc było przewidziane.

$N_{\text{soliton}}/L \sim (1/[\text{czas rampy}])^{[\text{stała } O(1)]}$

W. Zurek, PRL **102**, 105702 (2009)

Mechanizm Kibble-Żurka

Skalowanie liczby solitonów obecnych po końcu rampy

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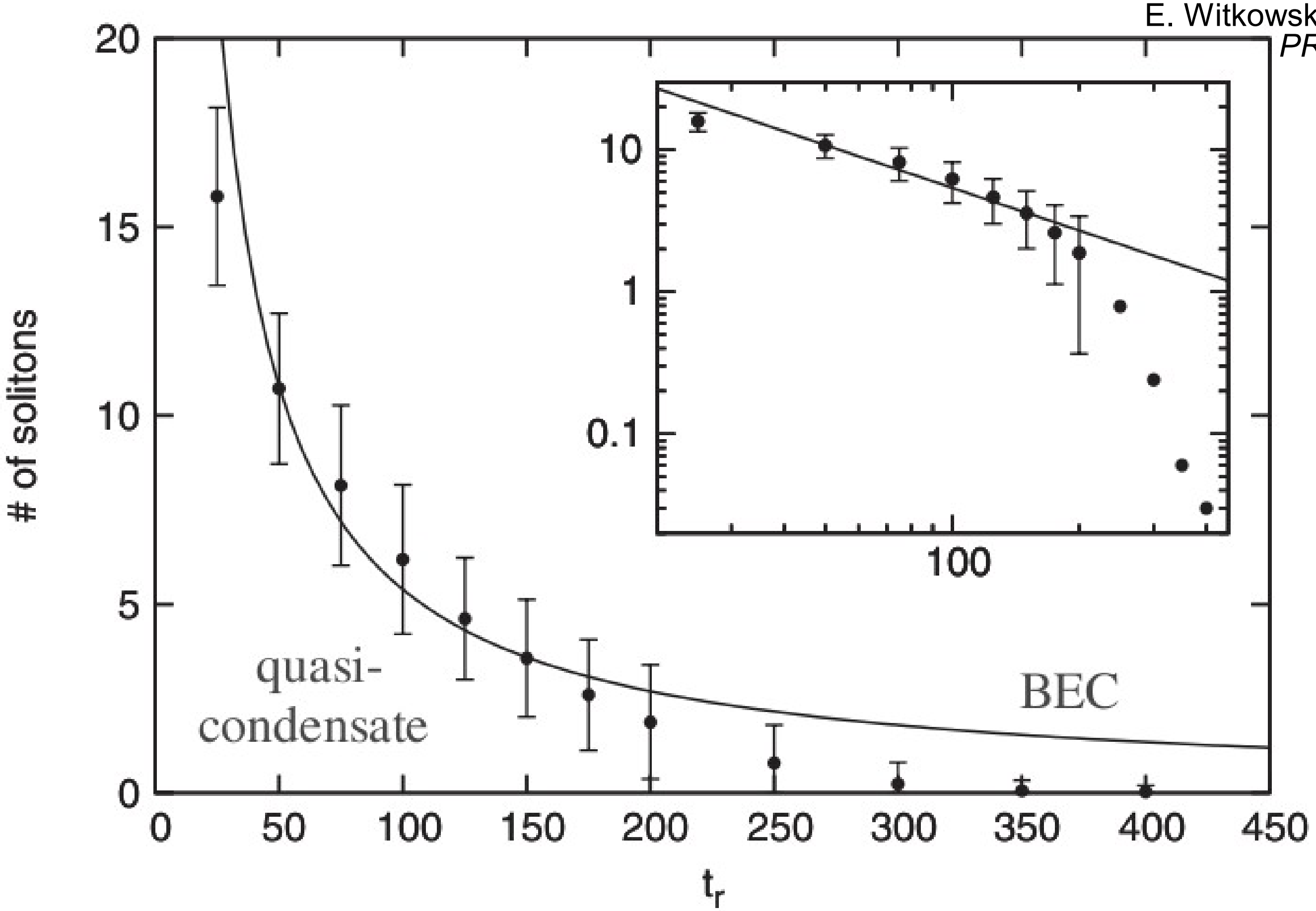
PRL **106**, 135301 (2011)



przewidziane:

$$N \approx \frac{2\hat{X}}{f\hat{\xi}} = \frac{2\Delta^2}{f\lambda_{dB}^2}\left(\frac{\tau_0}{\tau_Q}\right)^{(1+2\nu)/(1+\nu z)}$$

Quench time
~ czas rampy t_r?

~1

W. Zurek, PRL **102**, 105702 (2009)