

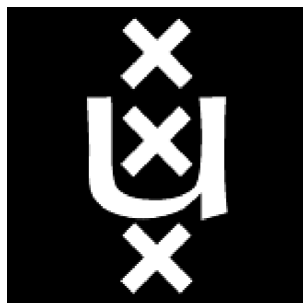
# The superfluidity of dipolar Fermi gases

Piotr Deuar<sup>(1)</sup>, Mikhail Baranov<sup>(2)</sup>, Georgy Shlyapnikov<sup>(1,3)</sup>



UNIV.  
PARIS-SUD

(1) LPTMS  
Université Paris-Sud / CNRS  
Orsay, France



(2) Van der Waals-Zeeman Instituut  
Universiteit van Amsterdam, Netherlands

(3) Institut für Theoretische Physik  
Universität Innsbruck, Austria



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# Outline

## superfluidity of dipolar fermions

1. Motivation
2. The simplest model
3. “Phase diagram” of low energy collective excitations
4. The new “aligned” superfluid

# Motivation:

- Superfluidity predicted for **single-species** Fermi gas of dipoles  
*Baranov et al. PRA 66, 013606 (2002)*  
(So far, superfluidity of fermion gases is of the two-species simple BCS sort)
- Recent **progress in cooling** of heteronuclear molecules with large dipole moment  
*K.-K. Ni et al., arXiv:0808.2963*  
⇒ Experimental realisations of the new superfluid are possible
- **Symmetry** of BCS-like gap is unlike in the standard s-wave BCS gas  
⇒ what effect does this have?
- Node structure like e.g. in **polar phase of  $^3\text{He}$** . (Never experimentally realized)  
*Aoyama & Ikeda, PRB 73, 060504 (2006), Elbs et al., arXiv:0707.3544*  
⇒ New effects not seen in condensed matter?

# comparison to standard BCS

## dipole–dipole potential

$$V(r, \theta) = \frac{d^2}{r^3} (1 - 3 \cos^2 \theta)$$

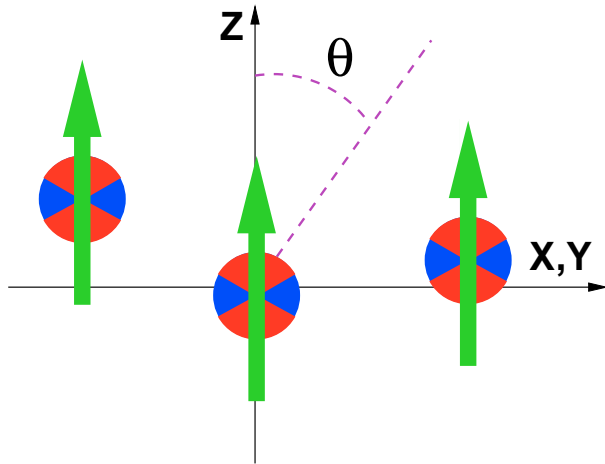
- long range interaction  
→ Needs **1 spin component**
- always partly attractive  
BCS pairing *if polarised*
- Energy gap **has nodes**
- *Anisotropic*

## contact s-wave $\uparrow\downarrow$ potential

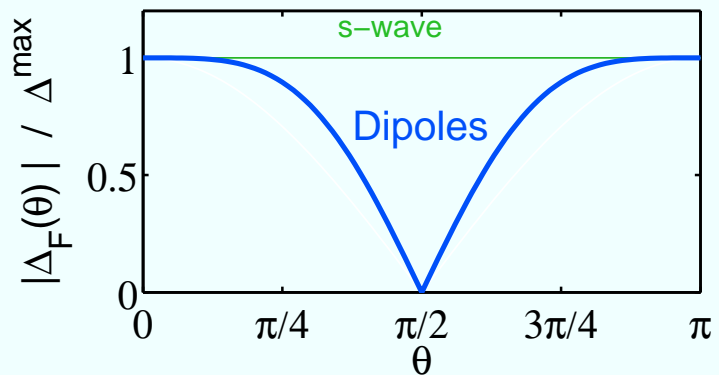
$$V(r) = g \delta(r)$$

- short range interaction  
→ Needs **2 spin components**  
(Pauli blocking)
- attractive or repulsive  
BCS pairing only *if  $a_s < 0$*
- Energy gap **always  $> 0$**
- *Isotropic*

# Uniform 3D dipolar Fermi gas



BCS pairing gap on Fermi surface has zeros



Baranov et al. PRA **66**, 013606 (2002)

- Uniform & 3D
- Cold:  $T < T_c^{BCS}$
- **static** external field (E or B)  
 $\implies$  **full polarisation**
- **single-species** (spin polarised)
- **dilute**  
 $\implies$  Energy dominated by Fermi sea  
 $\implies$  BCS-like model
- **Cooper pairs are cheap** near gap zero.

# Experimental prospects for superfluidity

$$T_c^{\text{BCS}} = 1.44 E_F \exp\left(-\frac{\pi}{2|a_D|k_F}\right); \quad |a_D| = \frac{2m|\mathbf{d}|^2}{\pi^2\hbar^2}$$

Baranov et al., PRA **66**, 013606 (2002)

## Comparison to RECENT VALUES

K.-K. Ni et al., arXiv:0808.2963

$$\begin{array}{l} |\mathbf{d}| = 0.566 D \\ n \sim 10^{12}/\text{cm}^3 \end{array} \implies \boxed{T_c^{\text{BCS}} \approx 1.6\text{nK}} \quad \begin{array}{l} \text{small!} \\ :- ( \end{array}$$

However, with  $10\times$  more density (plausible?), one would have

$$\boxed{T_c^{\text{BCS}} \approx 40\text{nK} \sim T_F \quad :- )}$$

# Hamiltonian

$$\hat{H} = \text{K.E.} + \frac{1}{2} \int d^3x d^3y \left\{ \hat{\Psi}_x^\dagger \hat{\Psi}_x V_D(x-y) \hat{\Psi}_y^\dagger \hat{\Psi}_y \right\}$$

Resulting effective BCS mean-field Hamiltonian

$$\hat{H}_{\text{eff}} = \frac{1}{2} \int d^3x d^3y \left\{ \begin{array}{ll} \frac{\hbar^2}{m} \hat{\Psi}_x^\dagger \nabla^2 \hat{\Psi}_x \delta(x-y) & \text{Kinetic} \\ \Delta^*(x-y) \hat{\Psi}_x \hat{\Psi}_y - \Delta(x-y) \hat{\Psi}_x^\dagger \hat{\Psi}_y^\dagger & \text{BCS} \\ W(x-y) \hat{\Psi}_x^\dagger \hat{\Psi}_y & \text{Hartree} \end{array} \right\}$$

Gap consistency equation

$$\Delta(x-y) = V_D(x-y) \left\langle \hat{\Psi}_x \hat{\Psi}_y \right\rangle_{\text{eff}}$$

# Low energy superfluidity

Phase perturbations of the ground state order parameter

$$\Delta_0(x-y) \rightarrow \Delta(x,y) = \Delta_0(x-y) e^{2i\phi(x,t)}$$

Goldstone mode

Assumptions:

- Low energy (  $\hbar\omega \ll \Delta_0^{\max} \sim T_c$  )
- Phase perturbations only (amplitude perturbations are gapped)
- Low  $\omega \implies$  long wavelength ( $k \ll k_F$ )  
 $\implies$  insensitive to small-scale of  $|x-y| \implies \phi \approx \phi(x \text{ only} )$
- Weak perturbation  $\implies$  lowest order in  $\phi$



# Bogoliubov–de Gennes Equations

Single-particle wavefunctions  $U_v(\mathbf{r}, t)$  and  $V_v(\mathbf{r}, t)$  from a Bogoliubov diagonalization

$$\hat{\Psi}(\mathbf{r}) = \sum_v \left[ U_v(\mathbf{r}) \hat{b}_v + V_v(\mathbf{r})^* \hat{b}_v^\dagger \right]$$

obey BDG equations

$$i\hbar \frac{\partial}{\partial t} \begin{bmatrix} U_v(\mathbf{r}) \\ V_v(\mathbf{r}) \end{bmatrix} = H_0(\mathbf{r}) \begin{bmatrix} U_v(\mathbf{r}) \\ -V_v(\mathbf{r}) \end{bmatrix} - \int d^3\mathbf{r}' \begin{bmatrix} \Delta(\mathbf{r}, \mathbf{r}') V_v(\mathbf{r}') \\ \Delta^*(\mathbf{r}, \mathbf{r}') U_v(\mathbf{r}') \end{bmatrix}$$

Expand them in terms of the uniform-gas wavefunctions  $U^0(\mathbf{r})$  and  $V^0(\mathbf{r})$  and coefficients  $C^{(\eta)} \sim O(\phi)$

$$\begin{bmatrix} U_v(\mathbf{r}) \\ V_v(\mathbf{r}) \end{bmatrix} = \sum_j \left\{ (\delta_{jv} + C_{jv}^{(1)}) \begin{bmatrix} U_v^0(\mathbf{r}) \\ V_v^0(\mathbf{r}) \end{bmatrix} + C_{jv}^{(2)} \begin{bmatrix} V_v^0(\mathbf{r})^* \\ -U_v^0(\mathbf{r})^* \end{bmatrix} \right\},$$

find  $C^{(\mu)}$  from BDG equation, and substitute it all into Gap equation, which must be satisfied up to  $O(\phi)$ .

$$\Delta(\mathbf{r}, \mathbf{r}') = \frac{V_D(\mathbf{r} - \mathbf{r}')}{2} \sum_v \tanh \left( \frac{E_v}{2k_B T} \right) [U_v(\mathbf{r}) V_v^*(\mathbf{r}') - U_v(\mathbf{r}') V_v^*(\mathbf{r})]$$

# Fun #1:

## Full consistency equation in $k$ -space

$$-\frac{\Phi_{\mathbf{k}}\Delta_{\mathbf{M}}^0\tau_{\mathbf{M}}^0}{2E_{\mathbf{M}}^0} = \frac{\Phi_{\mathbf{k}}\Delta_{\mathbf{M}}^0}{4E_{\mathbf{m}}^0E_{\mathbf{n}}^0} \left\{ \left( \frac{\tau_{\mathbf{n}}^0 - \tau_{\mathbf{m}}^0}{2} \right) \left[ \frac{(E_{\mathbf{n}}^0 + \varepsilon_{\mathbf{n}})(E_{\mathbf{m}}^0 - \varepsilon_{\mathbf{m}}) + \Delta_{\mathbf{n}}^0\Delta_{\mathbf{m}}^0}{\hbar\omega - E_{\mathbf{n}}^0 + E_{\mathbf{m}}^0 + i0} - \frac{(E_{\mathbf{n}}^0 - \varepsilon_{\mathbf{n}})(E_{\mathbf{m}}^0 + \varepsilon_{\mathbf{m}}) + \Delta_{\mathbf{n}}^0\Delta_{\mathbf{m}}^0}{\hbar\omega + E_{\mathbf{n}}^0 - E_{\mathbf{m}}^0 + i0} \right] \right. \\ \left. + \tau_{\mathbf{n}}^0 \left[ \frac{(E_{\mathbf{n}}^0 + \varepsilon_{\mathbf{n}})(E_{\mathbf{m}}^0 + \varepsilon_{\mathbf{m}}) - \Delta_{\mathbf{n}}^0\Delta_{\mathbf{m}}^0}{\hbar\omega - E_{\mathbf{n}}^0 - E_{\mathbf{m}}^0 + i0} \right] - \tau_{\mathbf{m}}^0 \left[ \frac{(E_{\mathbf{n}}^0 - \varepsilon_{\mathbf{n}})(E_{\mathbf{m}}^0 - \varepsilon_{\mathbf{m}}) - \Delta_{\mathbf{n}}^0\Delta_{\mathbf{m}}^0}{\hbar\omega + E_{\mathbf{n}}^0 + E_{\mathbf{m}}^0 + i0} \right] \right\}.$$

where  $\mathbf{n} = \mathbf{M} + \mathbf{k}/2$ ,  $\mathbf{m} = -\mathbf{M} + \mathbf{k}/2$ ,  
 $\varepsilon_{\mathbf{k}} = \hbar^2 k^2 / 2m - E_F$ ,  $E_{\mathbf{k}}^0 = \sqrt{\varepsilon_{\mathbf{k}}^2 + (\Delta_{\mathbf{k}}^0)^2}$ , and  $\tau_{\mathbf{k}}^0 = \tanh(E_{\mathbf{k}}^0 / 2T)$

$$\Delta_{\mathbf{n}}^0 = \sin\left(\frac{\pi}{2} \cos\theta_{\mathbf{n}}\right) \text{ on Fermi surface.}$$

- LONG wavelength  $\mathbf{k}$ , SHORT wavelength  $\mathbf{M} \sim 1/k_F$ .

$\implies$   $\omega(\mathbf{k}, \mathbf{M})$ . What can you do with THAT?

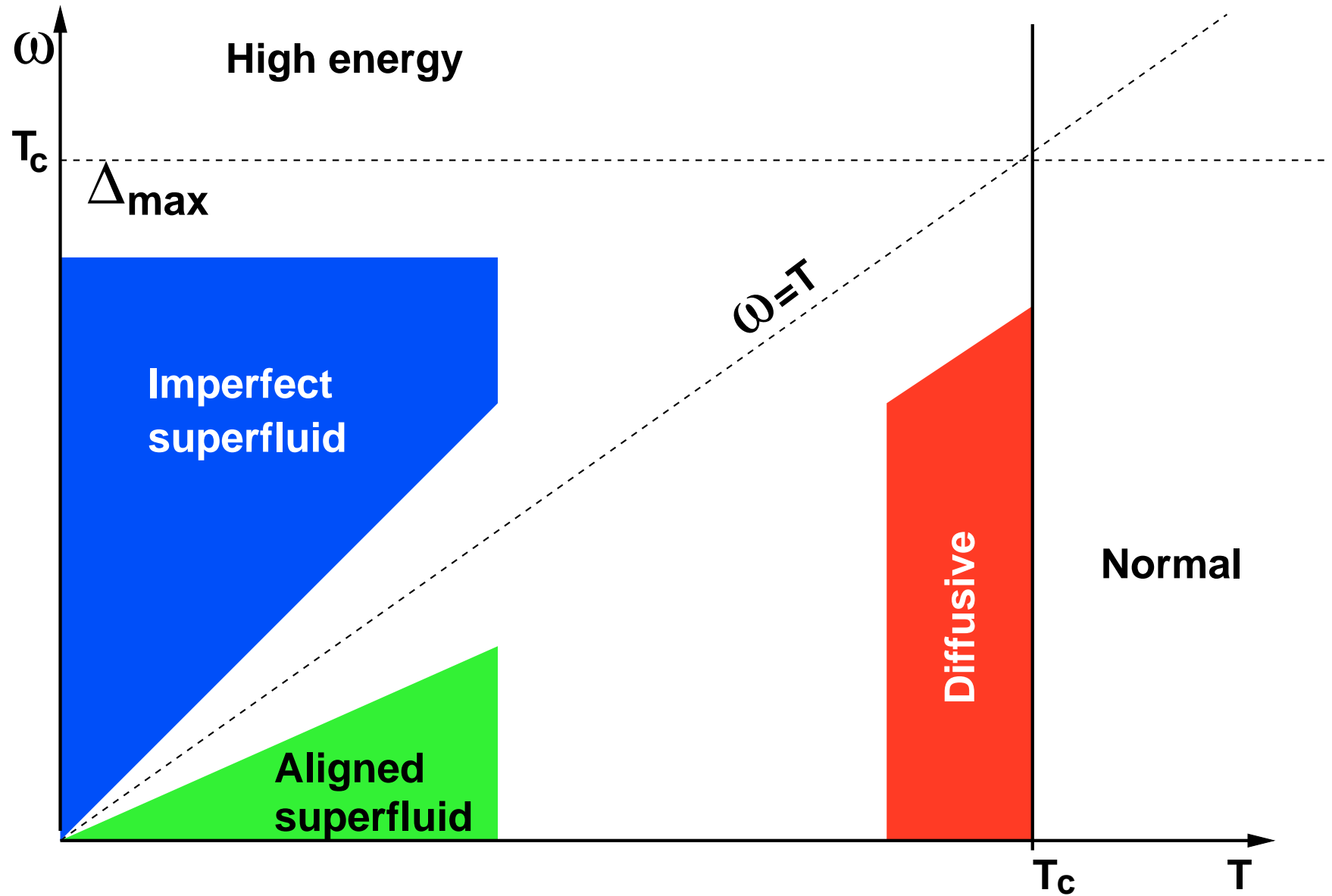
# Effective consistency equation

1. Want to obtain results only in the observable long wavelength  $\mathbf{k}$   
NOT in  $\mathbf{M} \sim 1/k_F$
2. Integrate out short-wavelength degrees of freedom in the Lagrangian
3. Effective lagrangian  $\mathcal{L}(\omega, \mathbf{k})$  only.
4. Effective consistency equation for  $\omega(\mathbf{k})$  is a weighted integral

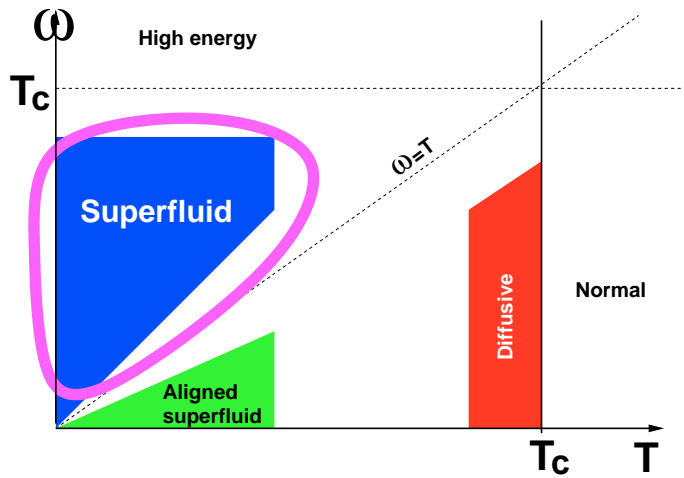
$$\int d\mathbf{M} \Delta_{\mathbf{M}}^0(\dots) = 0$$

of the previous microscopic equation  $(\dots) = 0$ .

# Collective excitation regimes

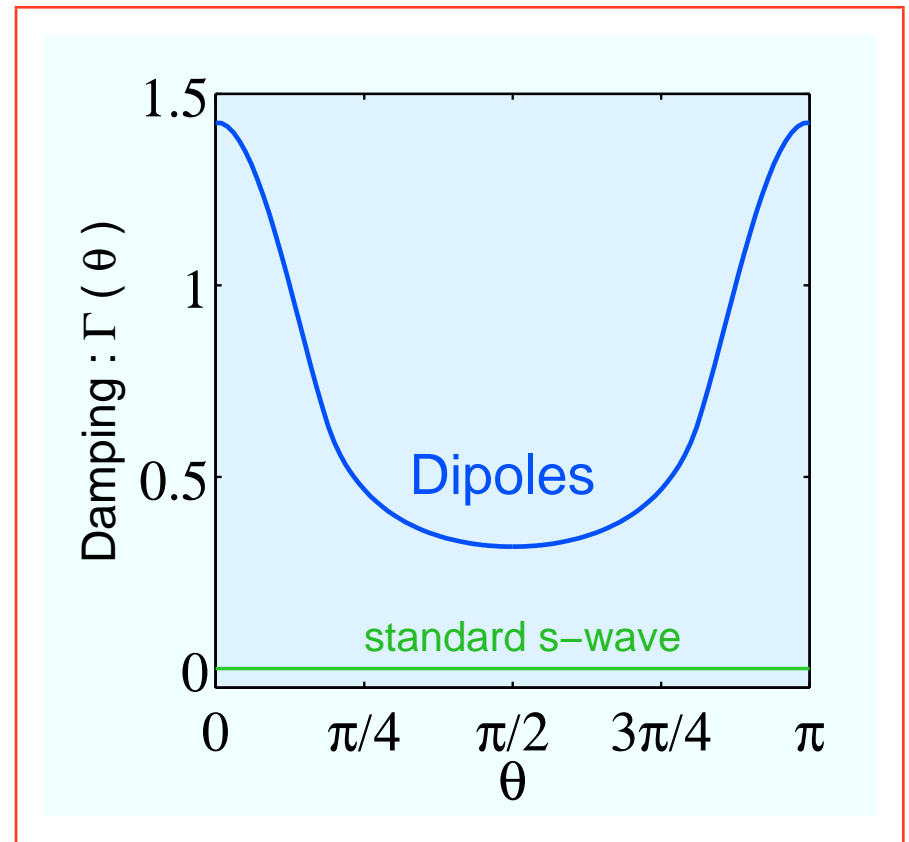
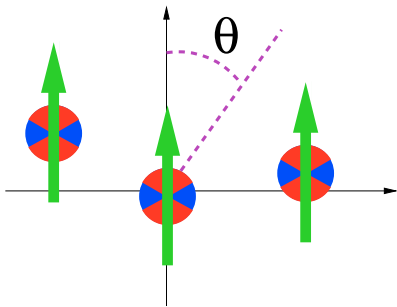


# Imperfect BCS superfluid



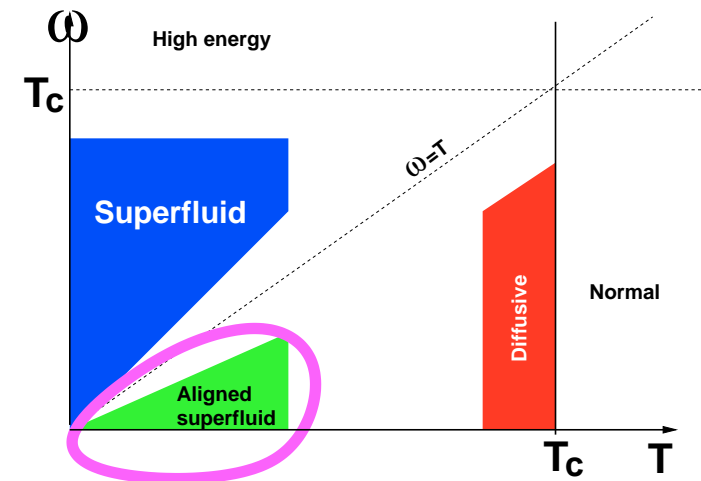
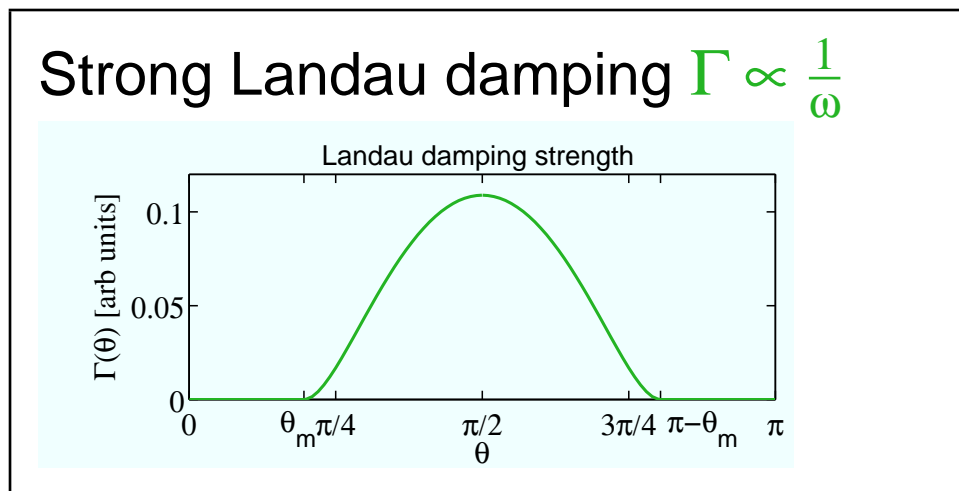
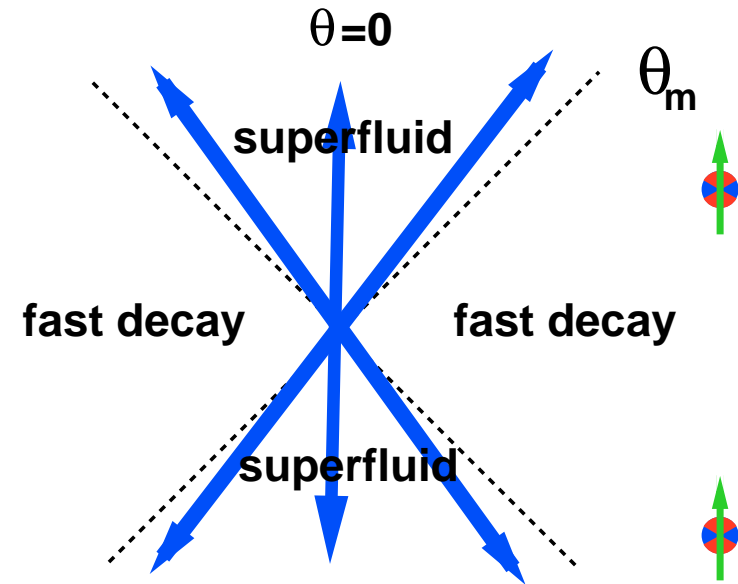
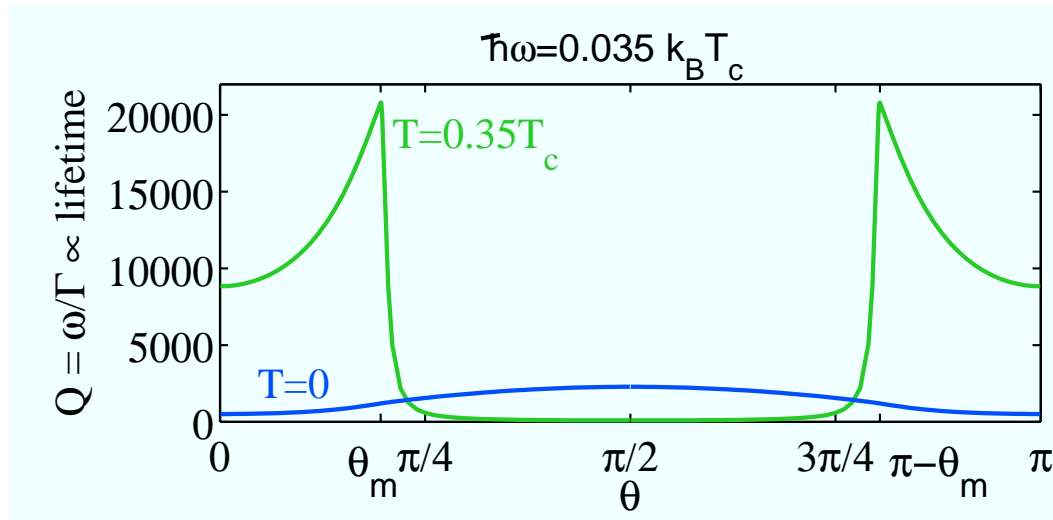
$$\omega = \left( \frac{v_F}{\sqrt{3}} \right) k \left\{ 1 - i \left( \frac{\hbar \omega_{\text{Bog}}}{\Delta_{\text{max}}} \right) \Gamma(\theta) \right\}$$

Beliaev process:  
collective  $\implies 2 \times$  quasipart.

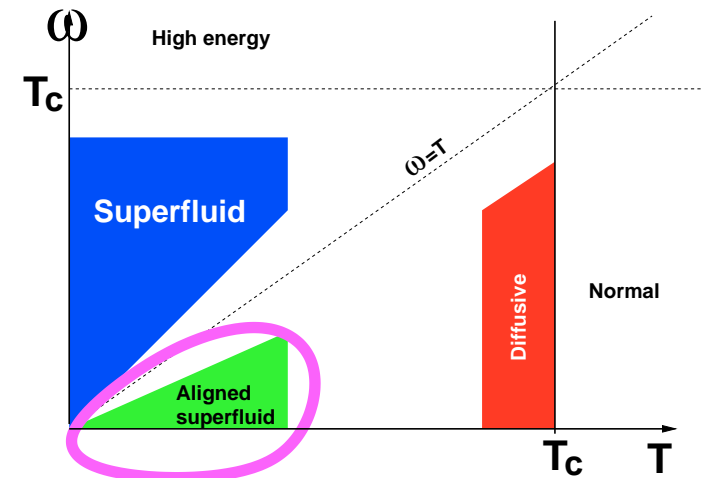
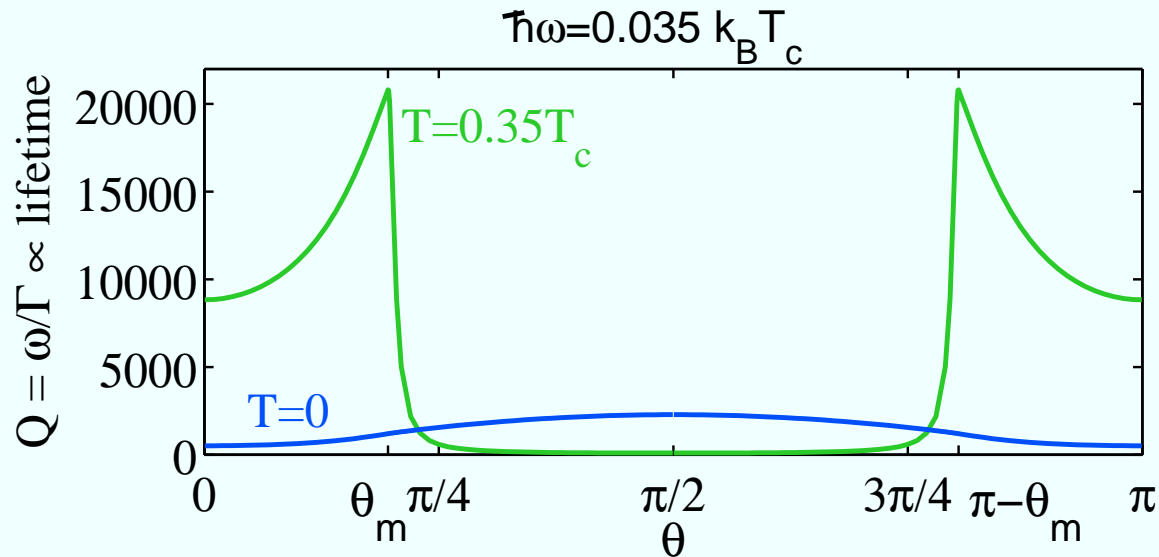


# New “Aligned superfluid” regime

(No s-wave BCS analogue)



# Thermally assisted superfluidity



- **MYSTERY:** How come the superfluid is better at **higher  $T$** ?
- Occurs when  $k_B T \gg \hbar\omega$
- Quasiparticles are fermionic, and low energy pairs are already filled.
- $\implies$  **Beliaev decay into quasiparticle pairs is blocked**, unlike  $T = 0$