A CONTINUUM OF C-FIELD ENSAMBLES FROM CE TO GCE AND THE ONSET OF THEIR EQUVALENCE

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Abstract: The canonical (CE) and grand-canonical (GCE) ensembles are the usual marginal cases for ultracold Bose gases, but real collections of experimental runs commonly have intermediate properties. Here we study the continuum of intermediate cases, and look into the appearance of ensemble equivalence as interaction rises for mesoscopic 1d systems. To make this study, a straightforward approach for generating canonical and intermediate classical field ensembles using a modified stochastic Gross-Pitaevskii equation (SGPE) is developed.

Bose field operator → Classical fields

$$\hat{\Psi}(\mathbf{x}) = \sum_k \hat{a}_k \, \psi_k(\mathbf{x}) \;\rightarrow\; \left\{ \sum_{k \epsilon C} \xi_k \, \psi_k(\mathbf{x}) \right\}$$

Assume highly occupied modes
Replace mode amplitude operators $\hat{a}_k$
With complex numer amplitudes ξ_k
„Quantum field theory, without discretized particles"

$$\hbar \frac{\partial \psi(x,t)}{\partial t} = \quad \text{SGPE equation with restriction of atom number}$$

$$= -i(1-i\gamma)\left(-\frac{\hbar^2}{2m}\frac{\partial^2}{\partial x^2} + V(x) - \mu + g_0|\psi(x,t)|^2\right)\psi(x,t) \;\; + \sqrt{2k_B T\,\gamma}\,\eta(x,t)$$

$$+ \left(-\frac{k_B T\,\gamma}{\sigma^2}\right)\left(N(\psi) - \bar{N}\right)\,\psi(x,t)$$

Observable whose fluctuactions are controlled should commute with the Hamiltonian!

Probability distribution:

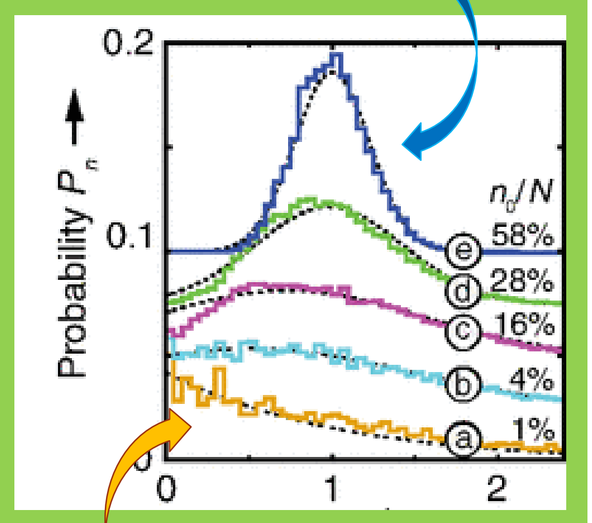
$$P_\sigma(\psi) \propto exp\left(-\frac{E(\psi) - \mu\, N(\psi)}{k_B T} - \frac{[N(\psi) - \bar{N}\,]^2}{2\,\sigma^2}\right)$$

Using the Fokker Planck equation which corresponds to the stochastic SGPE equation one can derive conditions for a stationary probability distribution P_σ .

AIM:
GENERATION OF AN ENSEMBLE WITH INTERMEDIATE, CONTROLLED STATISTICS

Experimental example:
A condensate of light[1]:

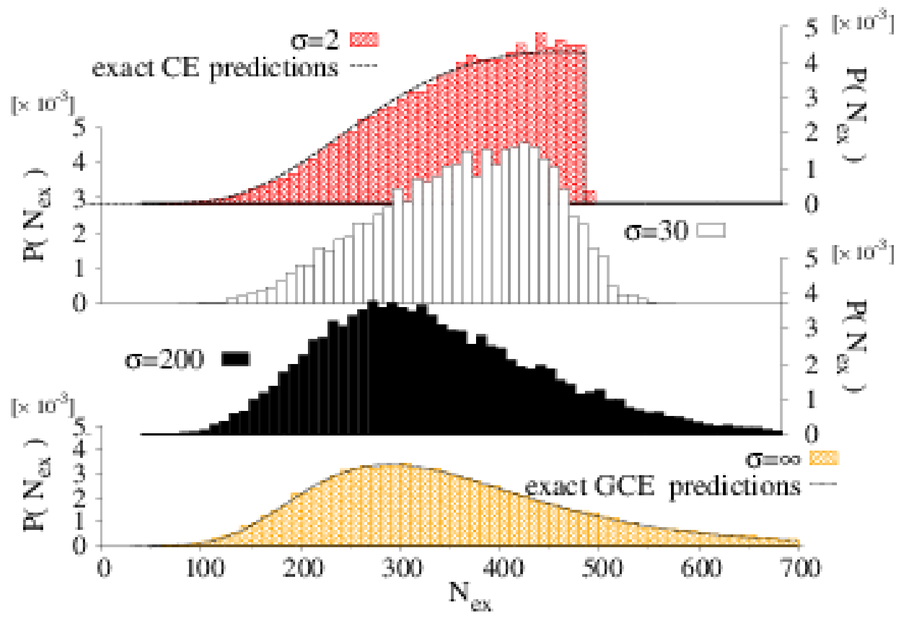
small reservoir
(restricted ensemble-tending to canonical ensemble)

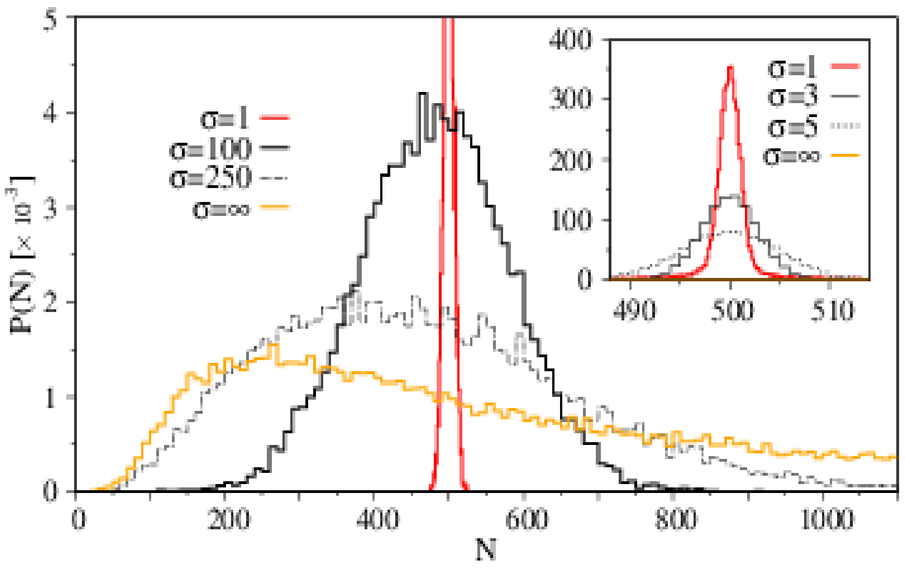


large reservoir
(Poissonian distribution in grand canonical ensemble)

Behaviour of 1D Bose gas in a BOX:

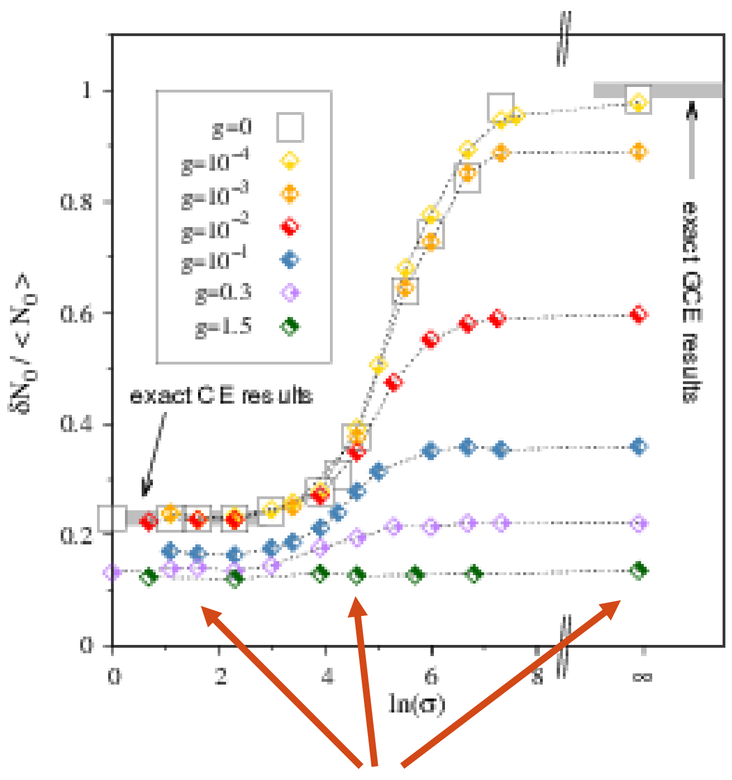
A) IDEAL GAS CASE (comparison with [2])



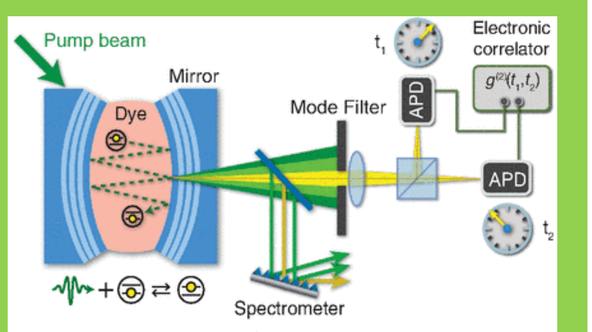


B) INTERACTING CASE

$$\frac{dN_0}{<N_0>} = \frac{[<N_0^2> - <N_0>^2]}{<N_0>}$$



Ensemble equivalence



Summary:

- Our method offers a way to generate **intermediate ensamble**
- *SGPE with additional term* allows to describe real experimental situations (by matching to the observable fluctuations)
- Applications to many cases of different dimensionality, external potential, interactions or multiple components [3]

[1] J. Schmitt, T. Damm, D. Dung, F. Vewinger, J. Klaers, M. Weitz, *PRL* **112**, 030401 (2014)
[2] E. Witkowska, M.Gajda, K. Rzążewski, *Optic Communications*, 283,671 (2010)
[3] C. Frapolli, T. Zibold, A. Invernizzi, K. Jimenez-Garcia, J. Dalibard, F. Gerbier, *PRL* **119**, 050404 (2017)

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