First-principles quantum dynamics of a colliding BEC with 150 000 atoms
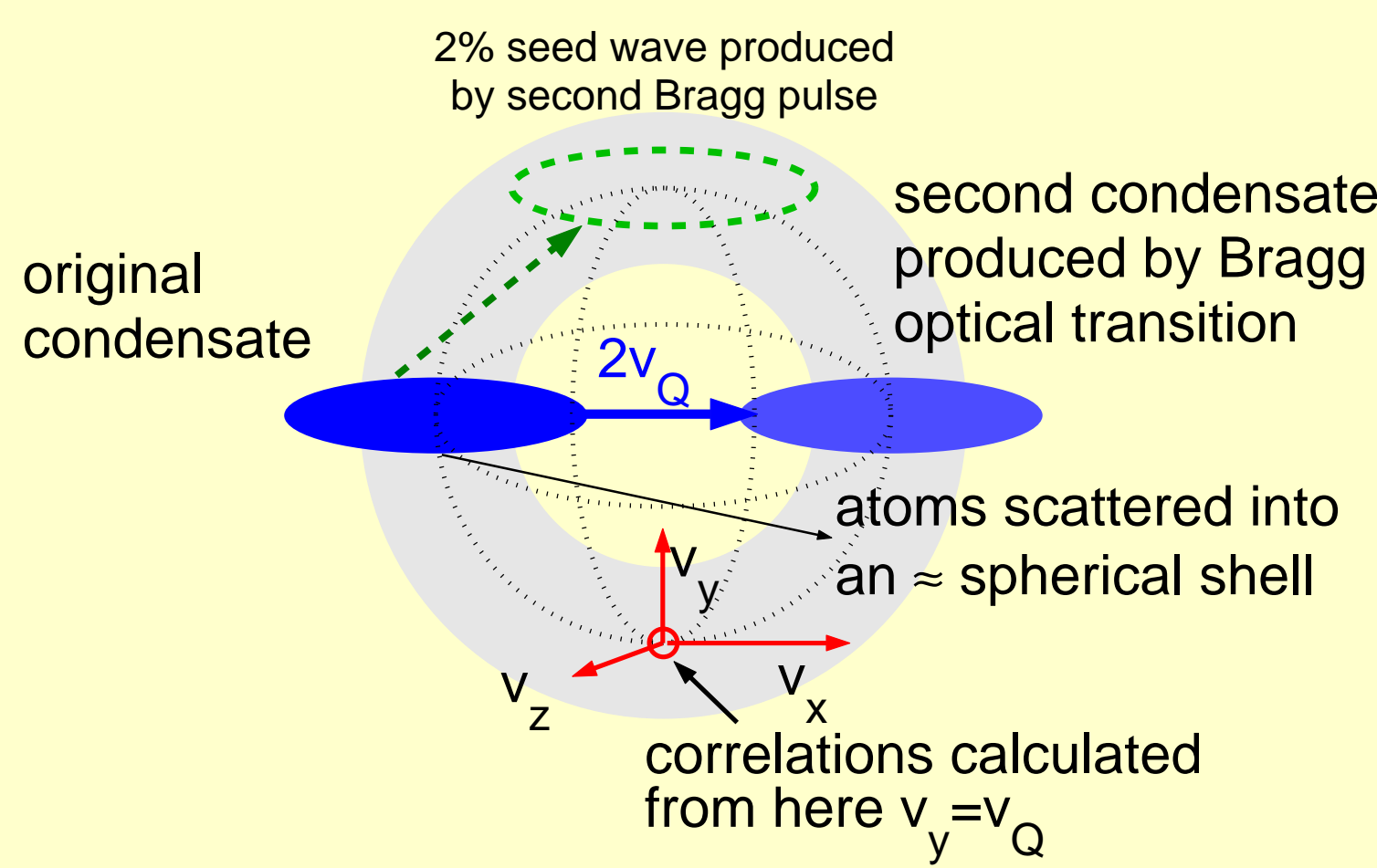
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System

- 4-wave mixing as per Vogels *et al*[1] experiment (but less atoms).
- Initial Na BEC formed in cigar-shaped 20x80x80Hz trap.
- Trap turned off at t=0.
- Collision along the axial direction "x".

2% seed wave produced by second Bragg pulse

original condensate

second condensate produced by Bragg optical transition

atoms scattered into an ≈ spherical shell

correlations calculated from here $v_y = v_Q$

Hamiltonian density:

$$\widehat{H} = \frac{\hbar^2}{2m}\nabla\widehat{\Psi}^\dagger\nabla\widehat{\Psi} + V(x)\widehat{\Psi}^\dagger\widehat{\Psi} + \frac{g}{2}\widehat{\Psi}^{\dagger 2}\widehat{\Psi}^2$$

Boson creation operators $\widehat{\Psi}^\dagger(x)$ at x.

Method

positive P representation

$$\hat{\rho} = \int P(\vec{v}) \bigotimes_x |\alpha(x)\rangle \langle\beta^*(x)|\ d\vec{v}$$

- Probability distribution P of variables $\vec{v} = \{\vec{\alpha}(\vec{x}), \vec{\beta}(\vec{x})\}$ which specify LOCAL coherent state *projectors*.
- 2 complex variables per lattice point.
- Describes *any* quantum state.
- Correspondences:
1. Master equation for $\hat{\rho}$.
2. $\rightarrow$ Fokker-Planck equation for P.
3. $\longrightarrow$ Stochastic equations for $\vec{\alpha}, \vec{\beta}$
- Quantum observables correspond to appropriate averages of variables $\vec{v}$

$$\widehat{\Psi}(x) \leftrightarrow \alpha(x) \qquad \widehat{\Psi}^\dagger(x) \leftrightarrow \beta(x)$$

Dynamics
Just Gross-Pitaevskii equations plus Gaussian noise

$$\frac{d\alpha(x)}{dt} = -i\hbar\sum_y \omega_{xy}\alpha(y) - \frac{ig}{\Delta x}\alpha(x)^2\beta(x)$$
$$+ i\sqrt{\frac{ig}{\Delta x}}\alpha(x)\xi_1(x,t)$$

And $\frac{d\beta(x)}{dt} = \frac{d\alpha^*(x)}{dt}$ but with $\alpha^* \leftrightarrow \beta$ and new noises ξ_2.

- $\xi_j(x,t)$ are independent Gaussian noises of variance $1/\Delta t$ for each x, t, j.
- Linear couplings ω_{xy} between x and y contain kinetics and external potential.

Scattering Dynamics

- No seed wave for now.
- Coherent and incoherent evolution coupled together.
- Initially: GP ground state of trap.
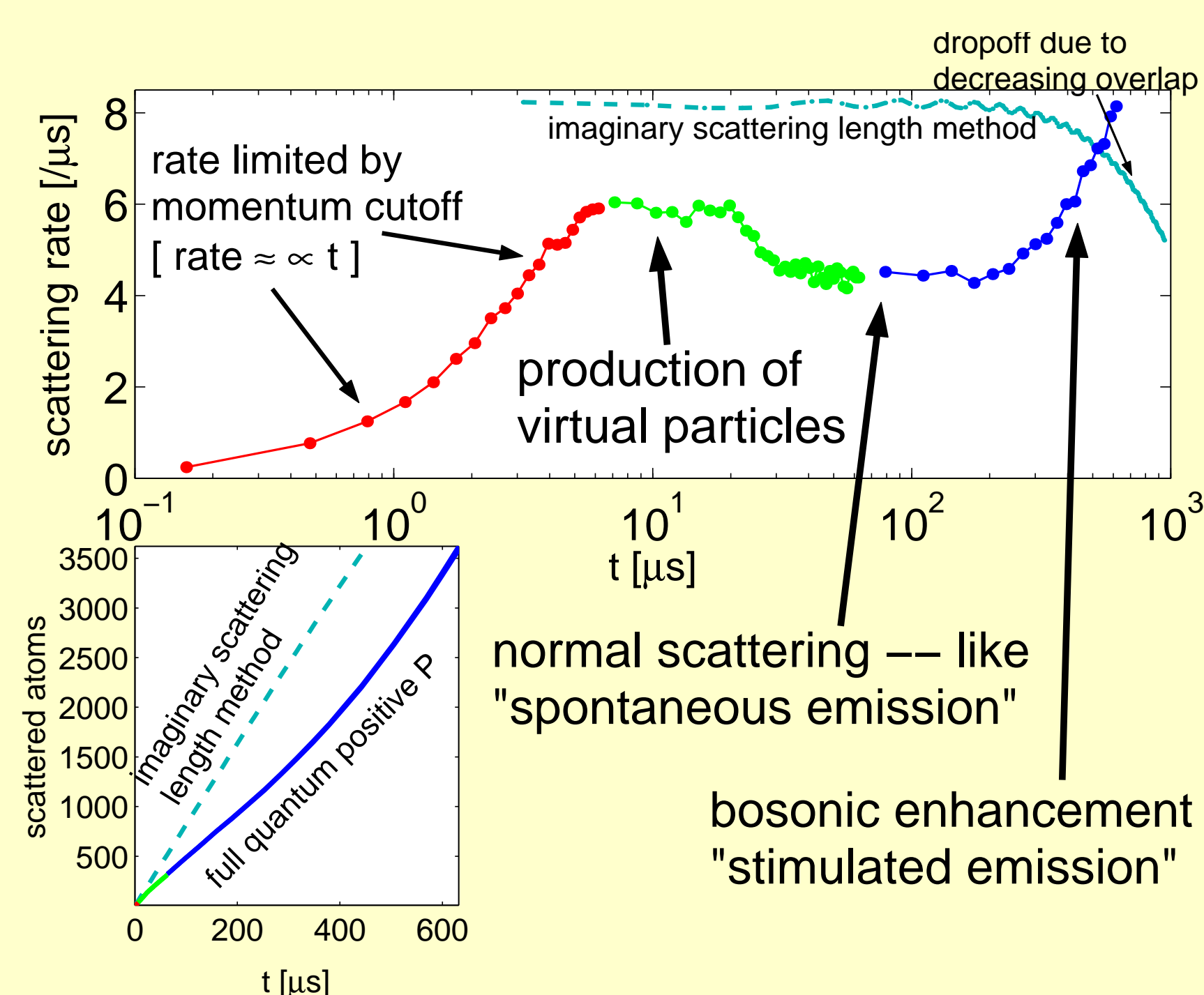
Velocity distribution dynamics

t=63μs

t=158μs

t=632μs

note: color scale varies
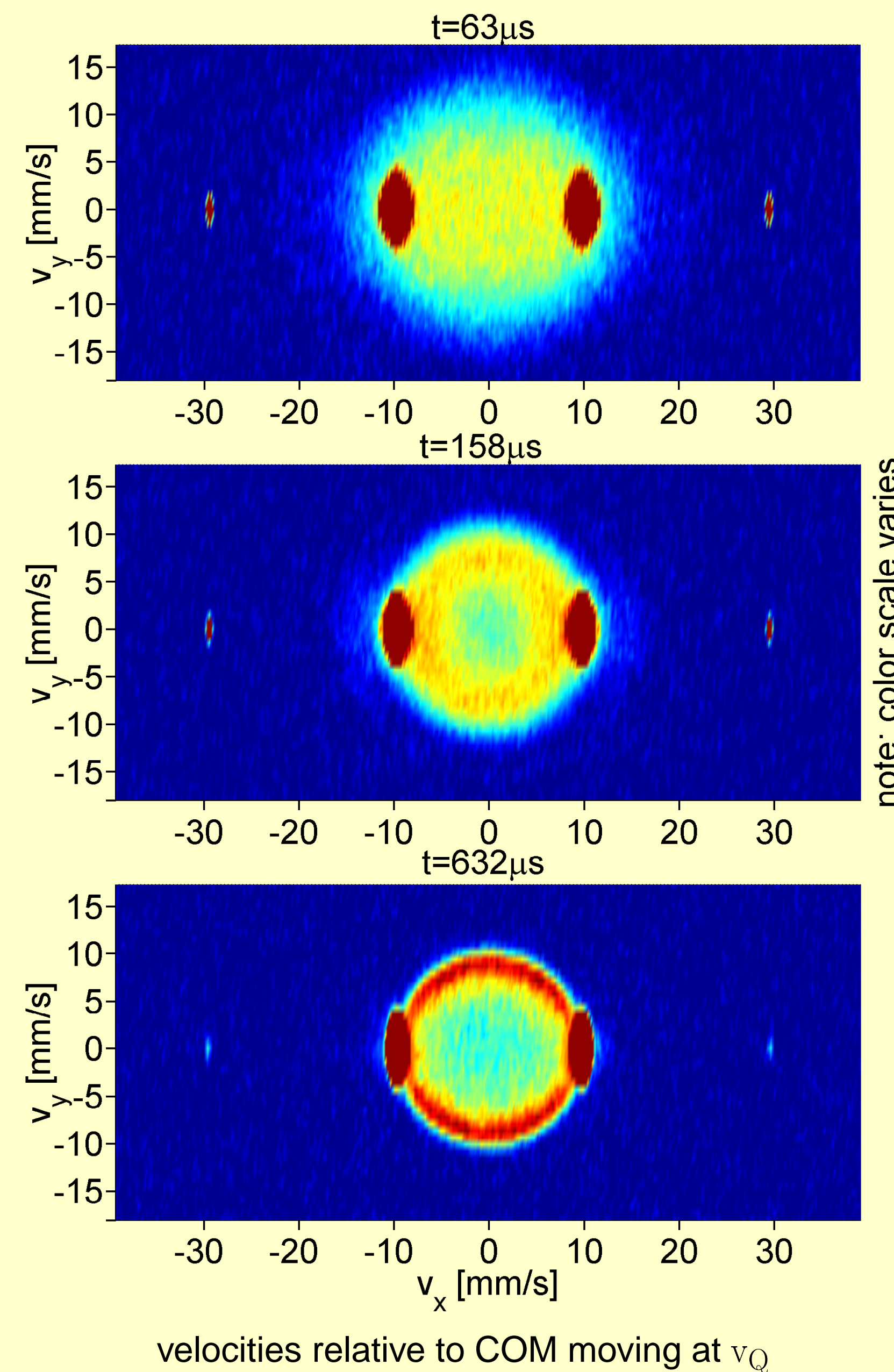
velocities relative to COM moving at v_Q

FIRST-PRINCIPLES DYNAMICS TRACTABLE IN MANY CASES
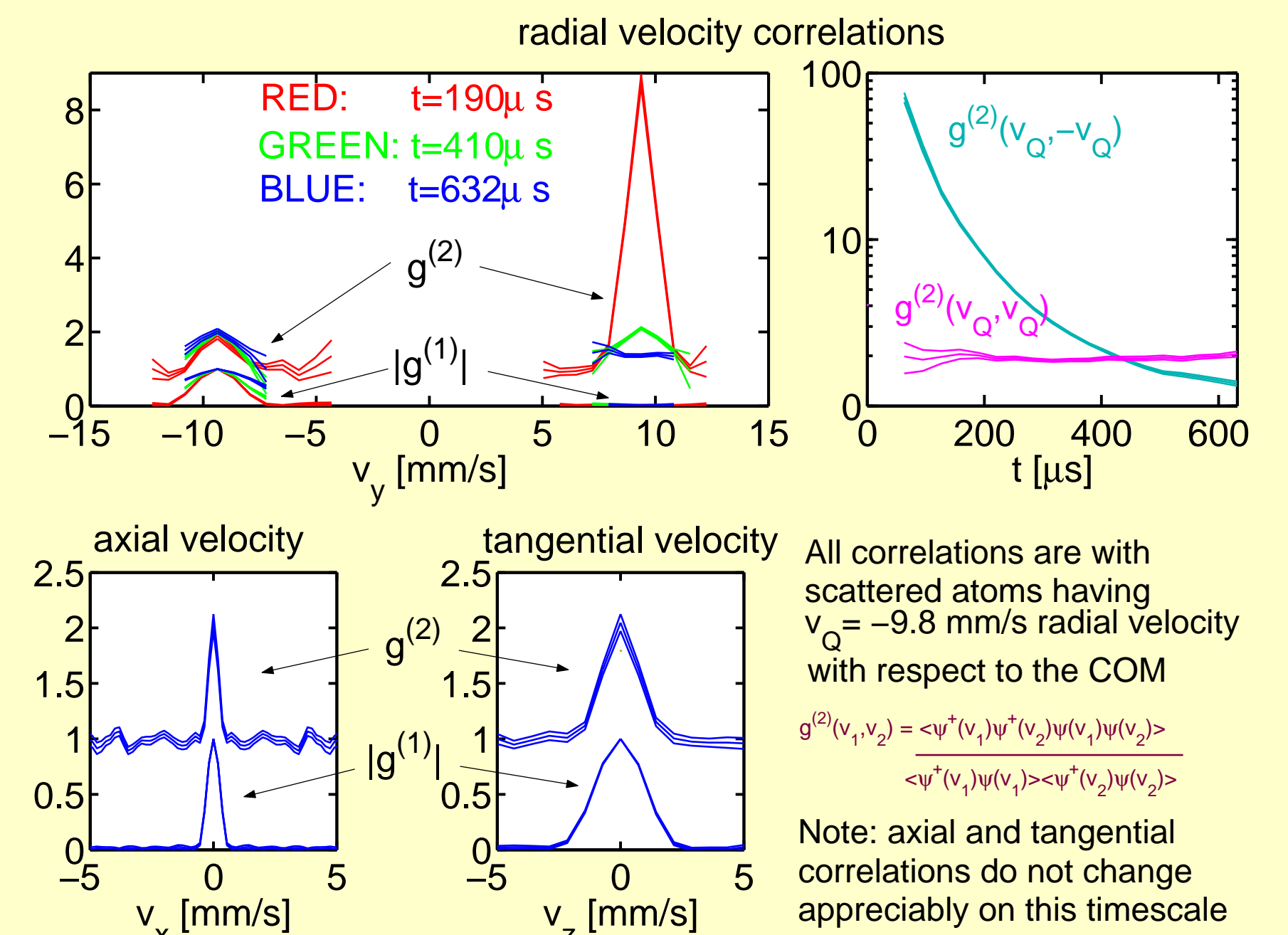e.g. four wave mixing:
- 150 000 atoms.
- 432x105x50 lattice. That's over two million points.
- 1024 trajectories
- About a week on an oldish PC.
- No truncation or linearization.

Scattering rate

dropoff due to decreasing overlap

imaginary scattering length method

rate limited by momentum cutoff [rate ≈ ∝ t]

production of virtual particles

normal scattering –– like "spontaneous emission"

bosonic enhancement "stimulated emission"

imaginary scattering length method

full quantum positive P

Correlation Dynamics

Correlations between scattered atoms at different velocities

radial velocity correlations

RED: t=190μs
GREEN: t=410μs
BLUE: t=632μs

$g^{(2)}$

$|g^{(1)}|$

$g^{(2)}(v_Q, -v_Q)$

$g^{(2)}(v_Q, v_Q)$

axial velocity

tangential velocity

$g^{(2)}$

$|g^{(1)}|$

All correlations are with scattered atoms having $v_Q = -9.8$ mm/s radial velocity with respect to the COM

$g^{(2)}(v_1, v_2) = \frac{\langle\psi^*(v_1)\psi^*(v_2)\psi(v_1)\psi(v_2)\rangle}{\langle\psi^*(v_1)\psi(v_1)\rangle\langle\psi^*(v_2)\psi(v_2)\rangle}$

Note: axial and tangential correlations do not change appreciably on this timescale

- At short times $g^{(2)}(v_Q, -v_Q) \gg 2$, which appears to differ from recent Wigner method estimates[3].
- This is due to truncation of high order FPE terms in the Wigner method.
- Coherence ($g^{(1)}$) and correlation ($g^{(2)}$) lengths consistent with analytic estimates in Zin *et al* [2].

Four Wave Mixing
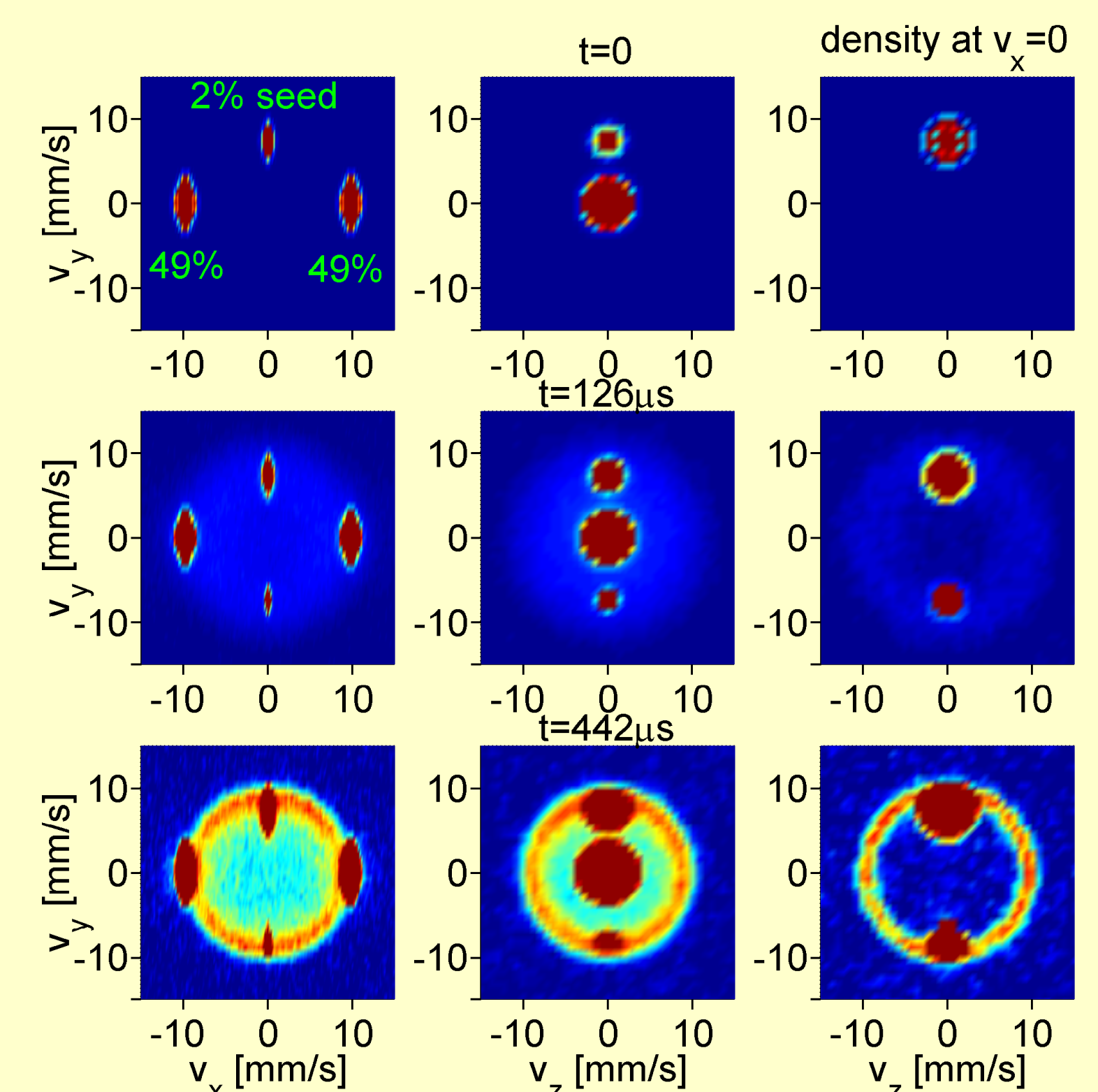
2% seed

density at $v_x = 0$

t=0

49%   49%

t=126μs

t=442μs

References

[1] J. M. Vogels, K. Xu, and W. Ketterle, Phys. Rev. Lett. **89**, 020401 (2002).

[2] P. Ziń, J. Chwedeńczuk, A. Veitia, K. Rzążewski, and M. Trippenbach, Phys. Rev. Lett. **94**, 200401 (2005).

[3] A. A. Norrie, R. J. Ballagh, and C. W. Gardiner, Phys. Rev. Lett. **94**, 040401 (2005).