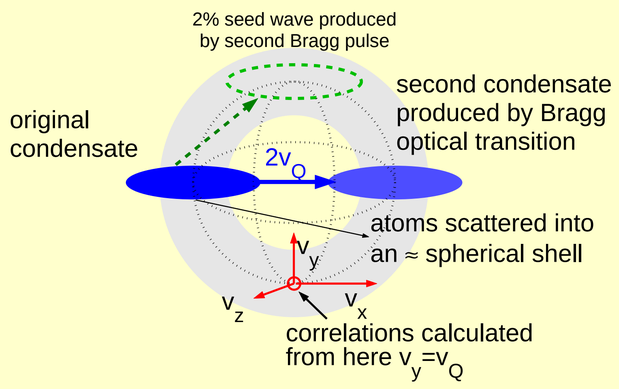
First-principles quantum dynamics with 150,000 atoms: Correlations in a BEC collision

P. Deuar[1,2,3] and P. D. Drummond[1]

1. Australian Centre for Quantum Atom Optics, *The University of Queensland*, Brisbane, Australia
2. Laboratoire de Physique Théorique et Modèles Statistiques, *Université Paris-Sud*, Orsay, France
3. Van der WaalsZeeman Institute, *Universiteit van Amsterdam*, Amsterdam, The Netherlands

System

- 4-wave mixing as per Vogels *et al*[1] experiment (but less atoms).
- Initial Na BEC formed in cigar-shaped 20x80x80Hz trap.
- Trap turned off at t=0.
- Collision along the axial direction ”x”.



Hamiltonian density:

$$\widehat{H} = \frac{\hbar^2}{2m}\nabla\widehat{\Psi}^\dagger\nabla\widehat{\Psi} + V(x)\widehat{\Psi}^\dagger\widehat{\Psi} + \frac{g}{2}\widehat{\Psi}^{\dagger 2}\widehat{\Psi}^2$$

Boson creation operators $\widehat{\Psi}^\dagger(x)$ at x.

Method

positive P representation

$$\hat{\rho} = \int P(\vec{v}) \bigotimes_x |\alpha(x)\rangle \langle\beta^*(x)| \; d\vec{v}$$

- Probability distribution P of variables $\vec{v} = \{\vec{\alpha}(\vec{x}), \vec{\beta}(\vec{x})\}$ which specify LOCAL coherent state *projectors*.
- 2 complex variables per lattice point.
- Describes *any* quantum state.
- Correspondences:
  1. Master equation for $\hat{\rho}$.
  2. $\longrightarrow$ Fokker-Planck equation for P.
  3. $\longrightarrow$ Stochastic equations for $\vec{\alpha}$, $\vec{\beta}$
- Quantum observables correspond to appropriate averages of variables $\vec{v}$

$$\widehat{\Psi}(x) \leftrightarrow \alpha(x) \qquad \widehat{\Psi}^\dagger(x) \leftrightarrow \beta(x)$$

Dynamics

Just Gross-Pitaevskii equations plus Gaussian noise

$$\frac{d\alpha(x)}{dt} = -i\hbar\sum_y \omega_{xy}\,\alpha(y) - \frac{ig}{\Delta x}\alpha(x)^2\beta(x) + i\sqrt{\frac{ig}{\Delta x}}\,\alpha(x)\,\xi_1(x,t)$$

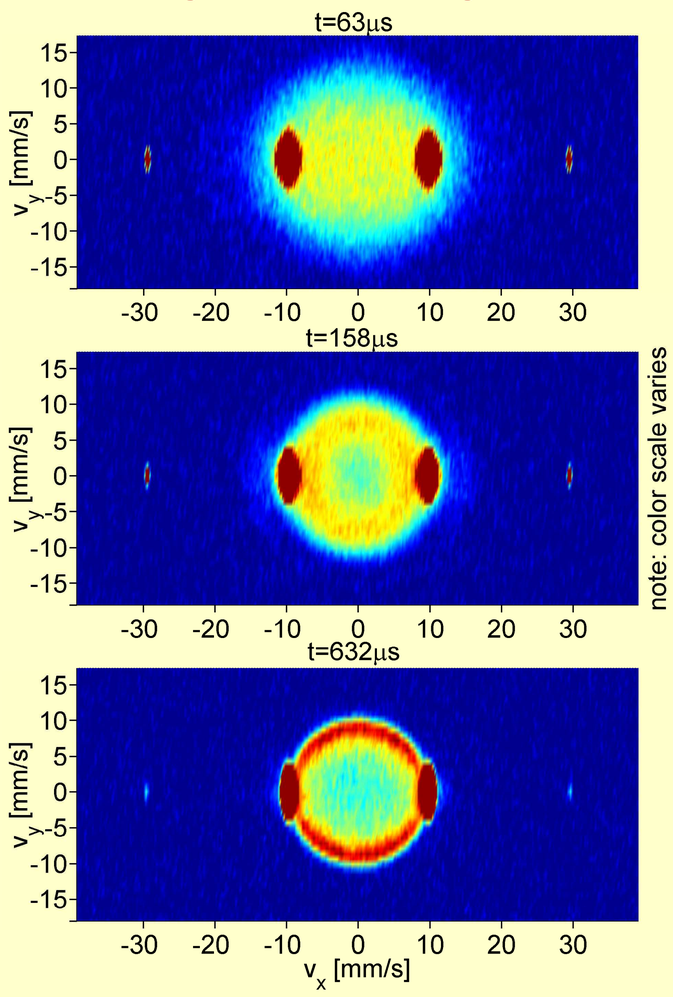
And $\frac{d\beta(x)}{dt} = \frac{d\alpha^*(x)}{dt}$ but with $\alpha^* \leftrightarrow \beta$ and new noises ξ_2.

- $\xi_j(x,t)$ are independent Gaussian noises of variance $1/\Delta t$ for each x, t, j.
- Linear couplings ω_{xy} between x and y contain kinetics and external potential.

Scattering Dynamics

- No seed wave for now.
- Coherent and incoherent evolution coupled together.
- Initially: GP ground state of trap.

Velocity distribution dynamics



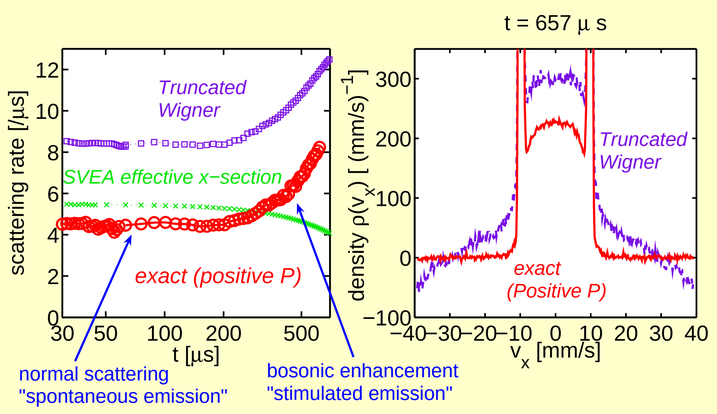
velocities relative to COM moving at $\mathrm{v_Q}$

FIRST-PRINCIPLES DYNAMICS TRACTABLE IN MANY CASES

e.g. four wave mixing:

- 150 000 atoms.
- 432x105x50 lattice. That’s over two million points.
- 1024 trajectories
- About a week on a single PC.
- No truncation or linearization.

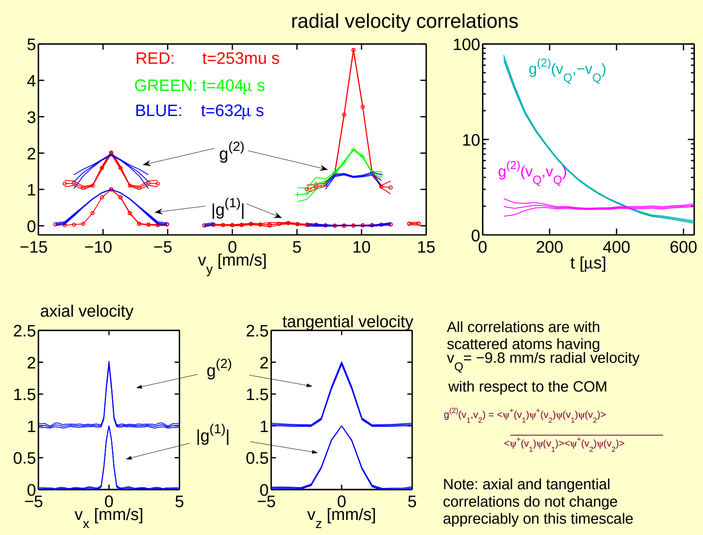
Bosonic enhancement



- Truncated Wigner is in error in such dilute regimes.
- This is due to the spurious virtual particles meant to mimic quantum noise.

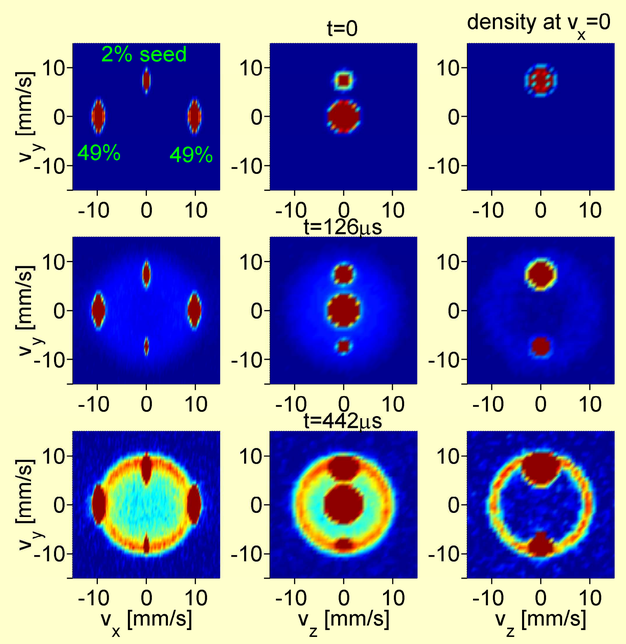
Correlation Dynamics

Correlations between scattered atoms at different velocities



- Further effects seen beyond previous truncated Wigner treatments [2]:
- Marked *radial–only* growth of phase grains with time.
- At short times $g^{(2)}(\mathrm{v_Q}, -\mathrm{v_Q}) \gg 2$.
- Coherence ($g^{(1)}$) and correlation ($g^{(2)}$) lengths consistent with analytic estimates [3].

Four Wave Mixing



Acknowledgments

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References

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[3] P. Ziń, J. Chwedeńczuk, A. Veitia, K. Rzążewski, and M. Trippenbach, Phys. Rev. Lett. **94**, 200401 (2005).