

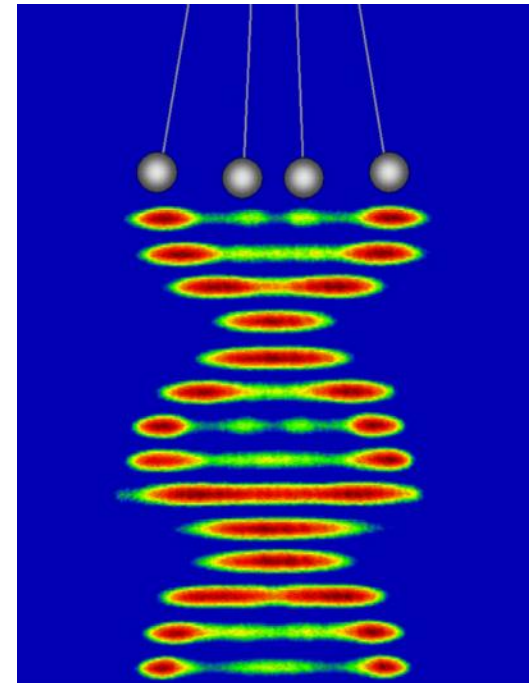
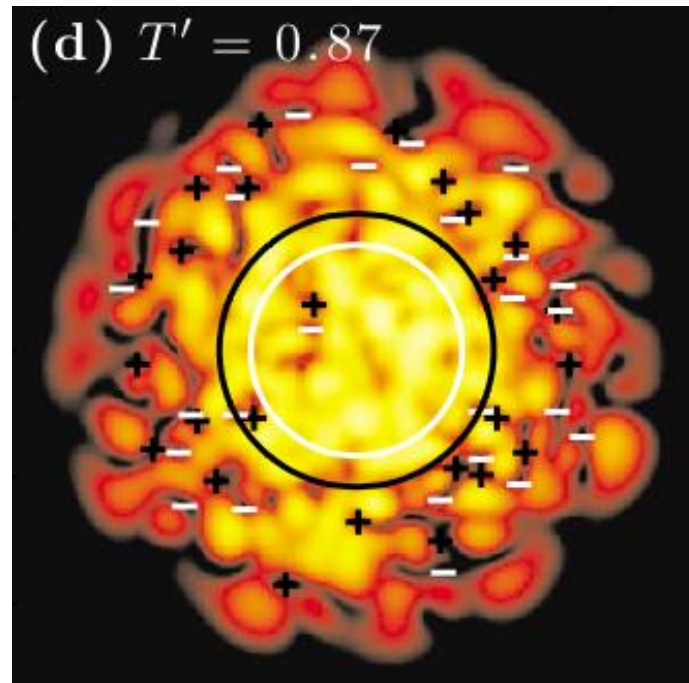
Low-dimensional quantum gases

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NARODOWE
CENTRUM
NAUKI

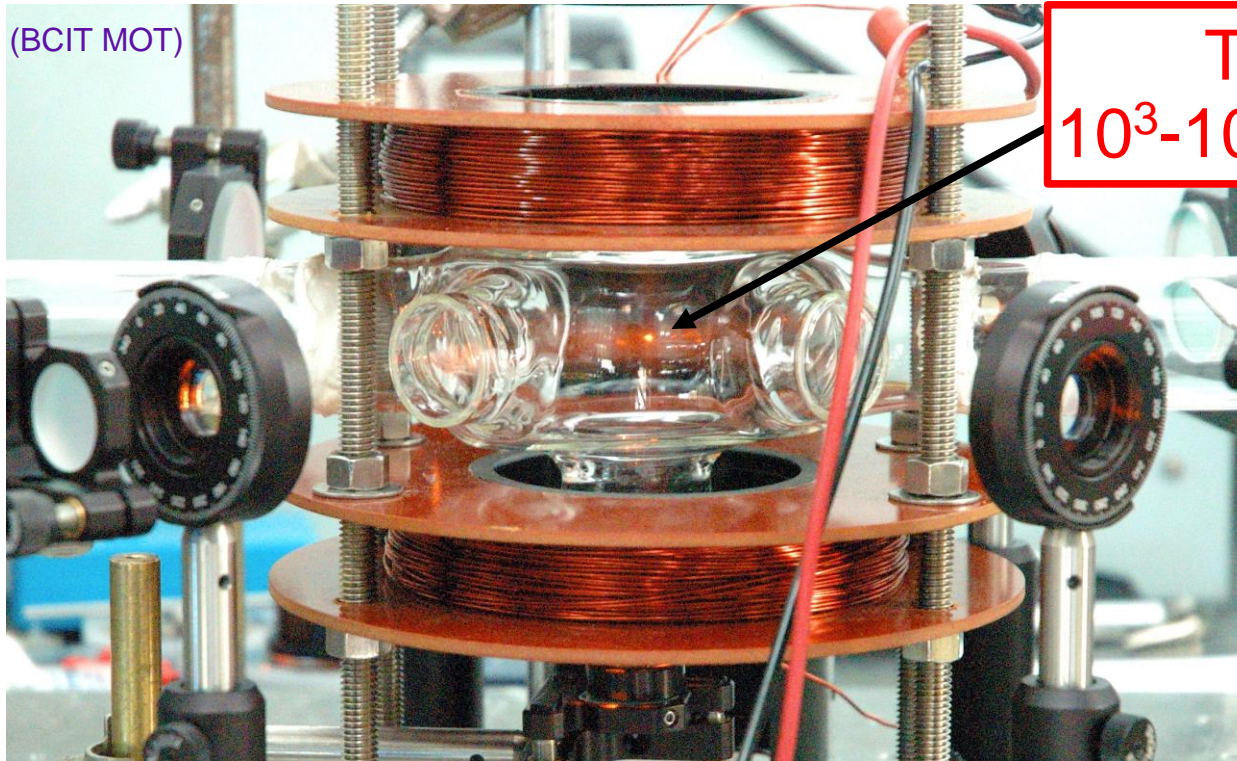


Outline

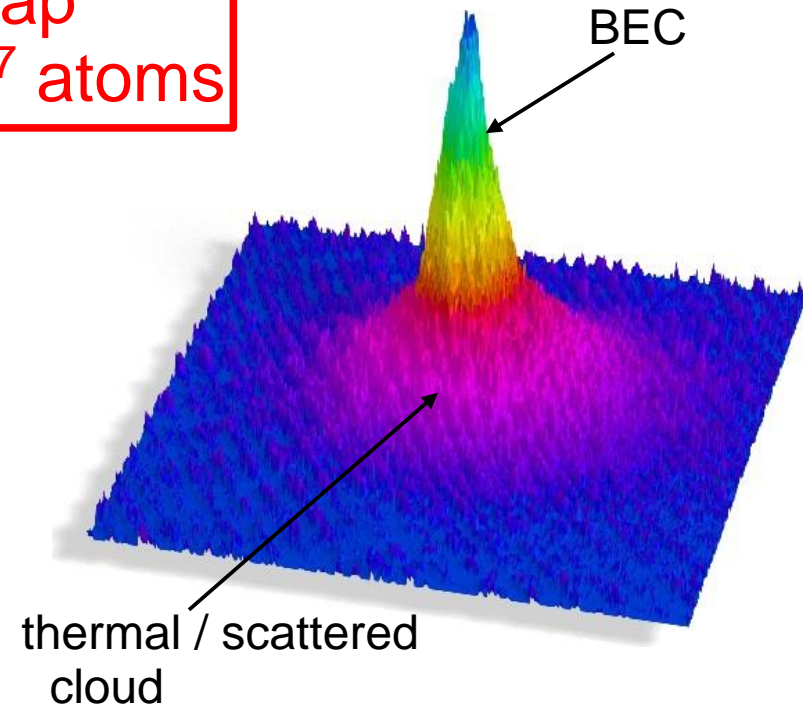
1. General features of the ultracold gas
2. Reduced dimensions
3. Intergability
4. Quasicondensate
5. Thermal solitons
6. Anderson localization
7. 2D gas – Berezinskii-Kosterlitz-Thouless transition

Ultracold gas - features

“most atoms are concentrated in a small subset of states”



Trap
 $10^3 - 10^7$ atoms



$$\hat{H} = \int d^3\mathbf{x} \left\{ \hat{\Psi}^\dagger(\mathbf{x}) \left[V(\mathbf{x}) - \frac{\hbar^2}{2m} \nabla^2 \right] \hat{\Psi}(\mathbf{x}) + \frac{g}{2} \hat{\Psi}^\dagger(\mathbf{x})^2 \hat{\Psi}(\mathbf{x})^2 \right\}$$

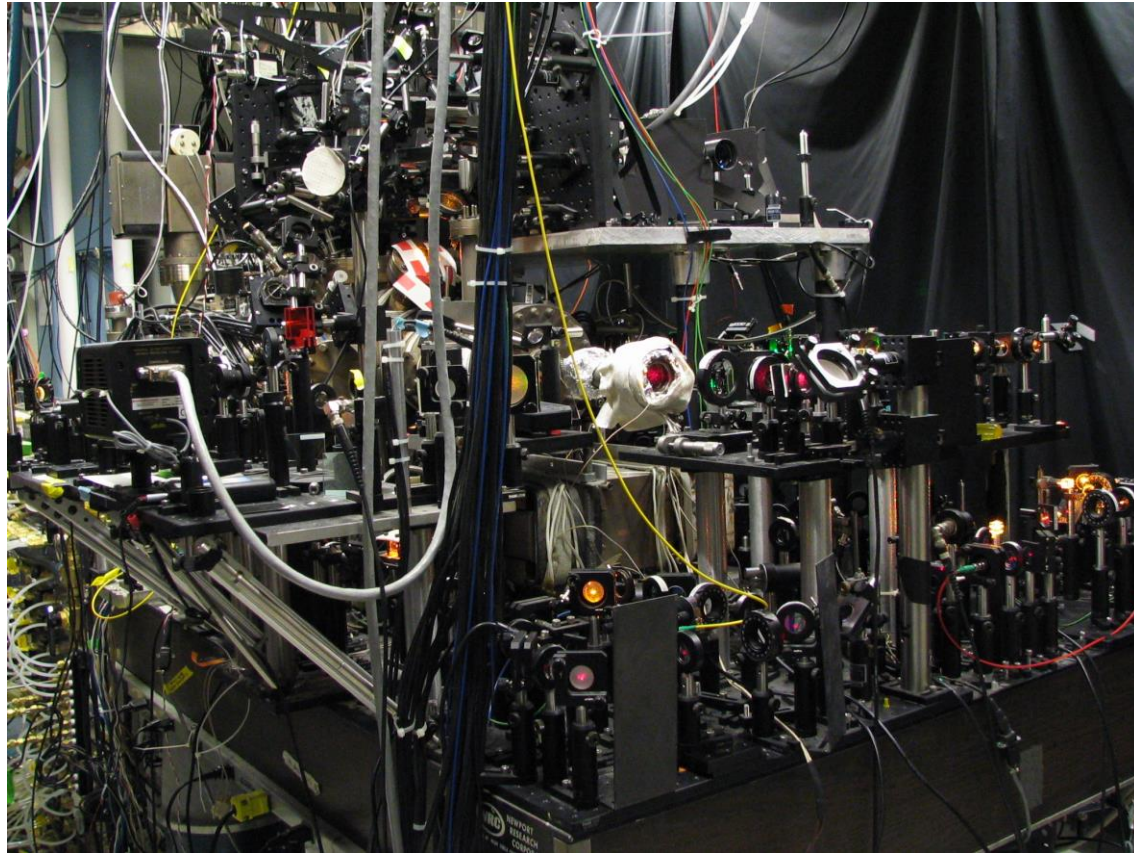
(2nd quantization)

Boson field $\left[\hat{\Psi}(\mathbf{x}), \hat{\Psi}^\dagger(\mathbf{x}') \right] = \delta^{(3)}(\mathbf{x} - \mathbf{x}')$

low $T \rightarrow$ low E
dilute
 \rightarrow contact, s-wave interactions

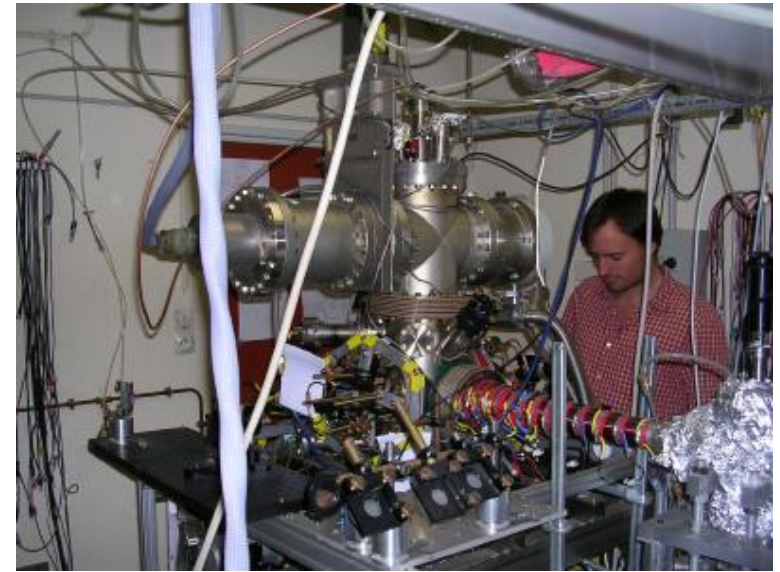
Experiments

Old-style look

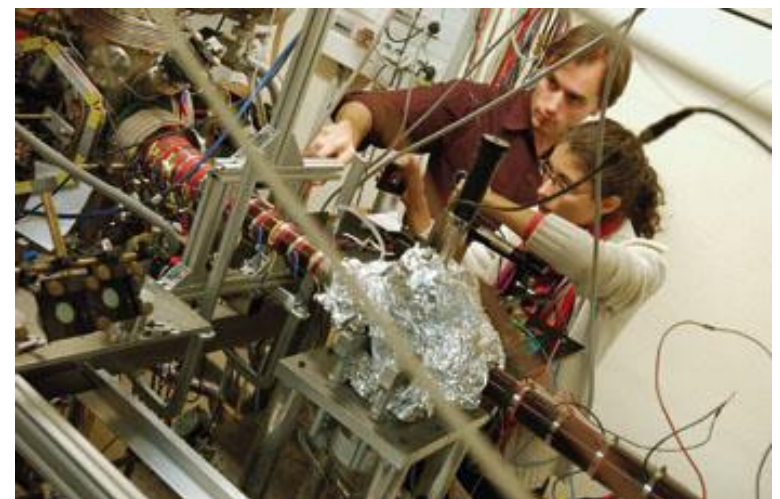


Ketterle expt. MIT

Newer look



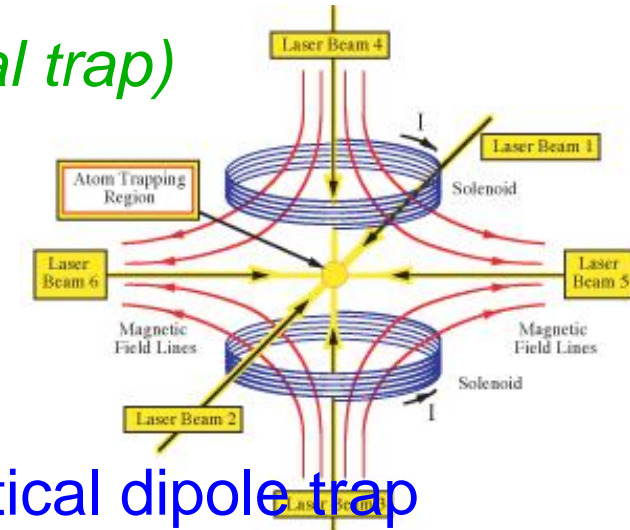
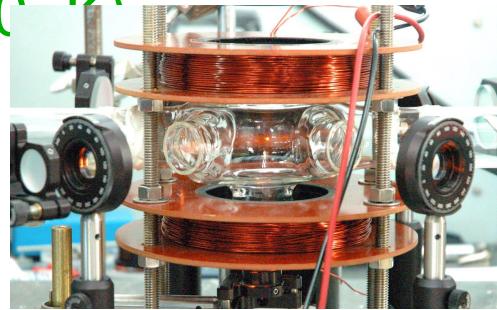
Westbrook expt. Institut d'Optique



Ultracold gas – cooling

1. Laser cooling + MOT (Magneto-optical trap) ($\approx 100 \mu\text{K}$)

MOT:



2. Evaporative cooling + magnetic or optical dipole trap ($\approx 100\text{nK}$)

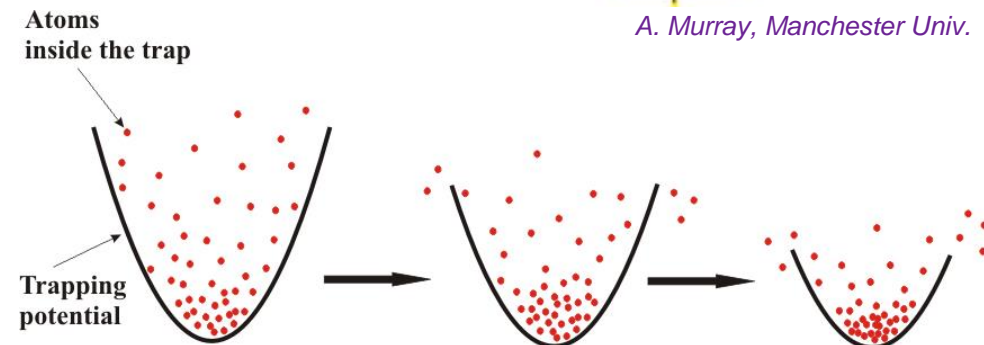
A. Murray, Manchester Univ.

Optical dipole trap



Atoms inside the trap

Trapping potential



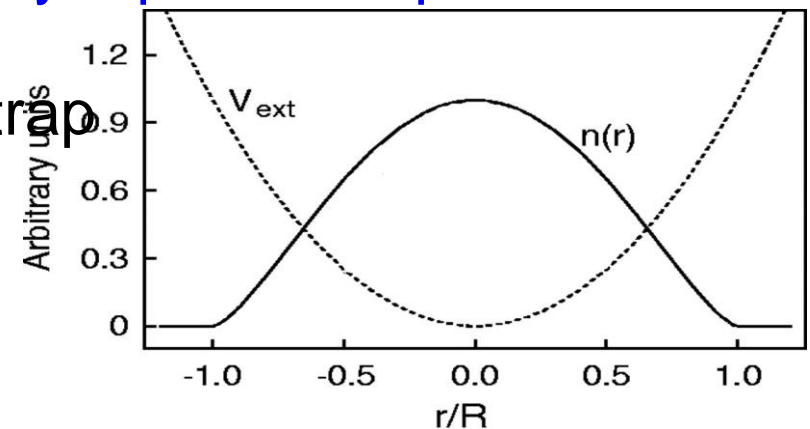
G. Raithel, Univ. Michigan

Traps usually well approximated by a parabolic potential

Typically, 10^5 - 10^7 atoms in a 3D trap

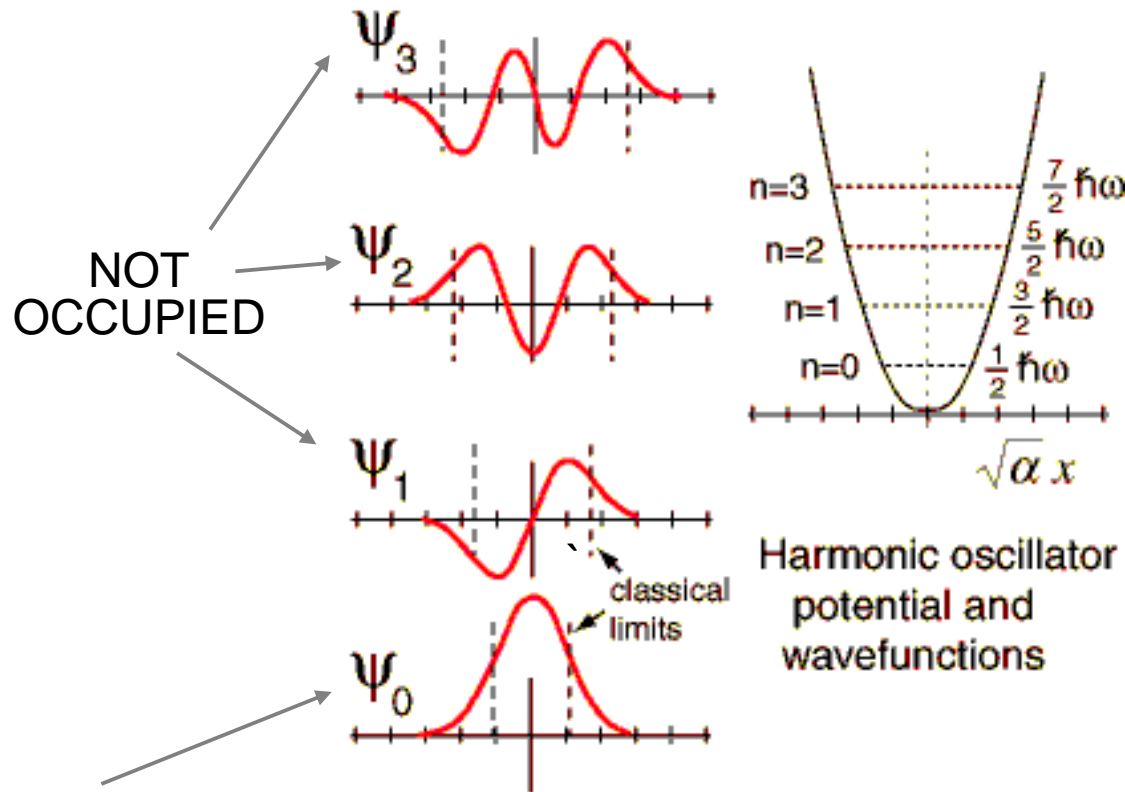
density 10^{12} - 10^{14} cm^{-3}

[air : $3 \times 10^{19} \text{ cm}^{-3}$]



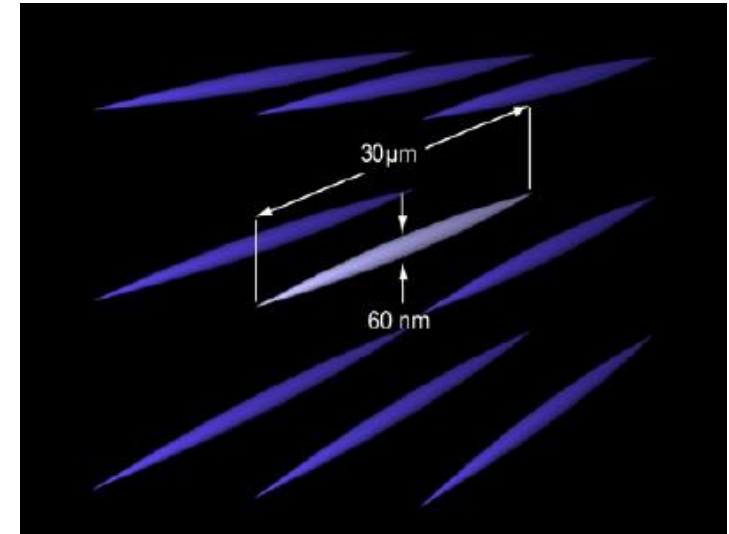
Reduction of dimensions

Requirements in the narrow directions



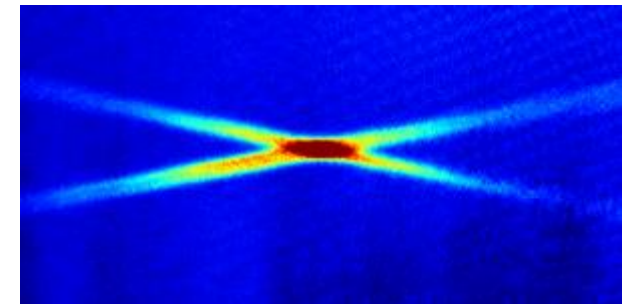
OCCUPIED - Ground state of transverse motion ONLY

Need $\hbar\omega_{\text{long}} \ll k_B T, \mu \ll \hbar\omega_{\text{narrow}}$



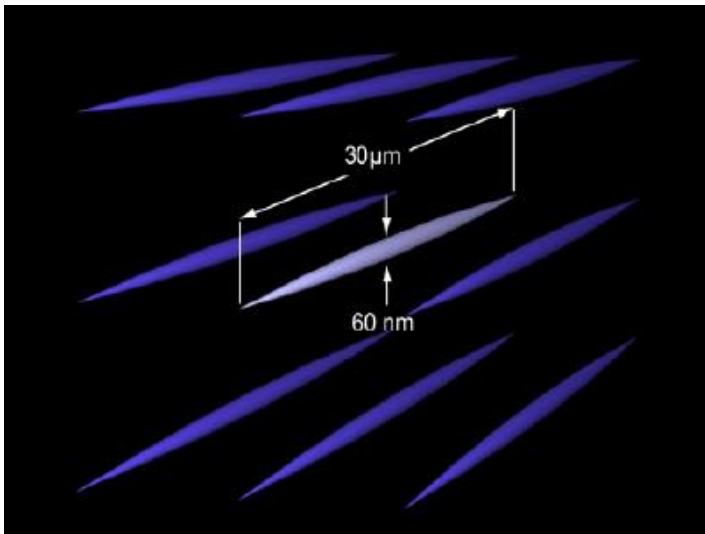
Moritz, Kohl, Esslinger, PRL **91**, 250402 (2003)

“Crossed” dipole trap

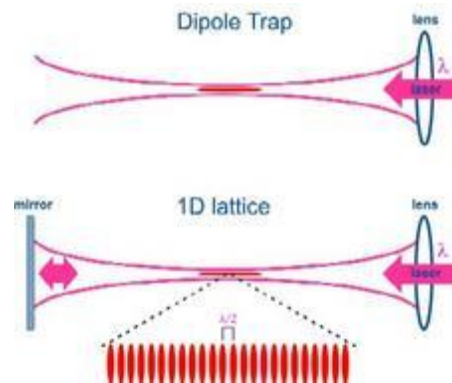


Noether expt. Stuttgart

Lattices

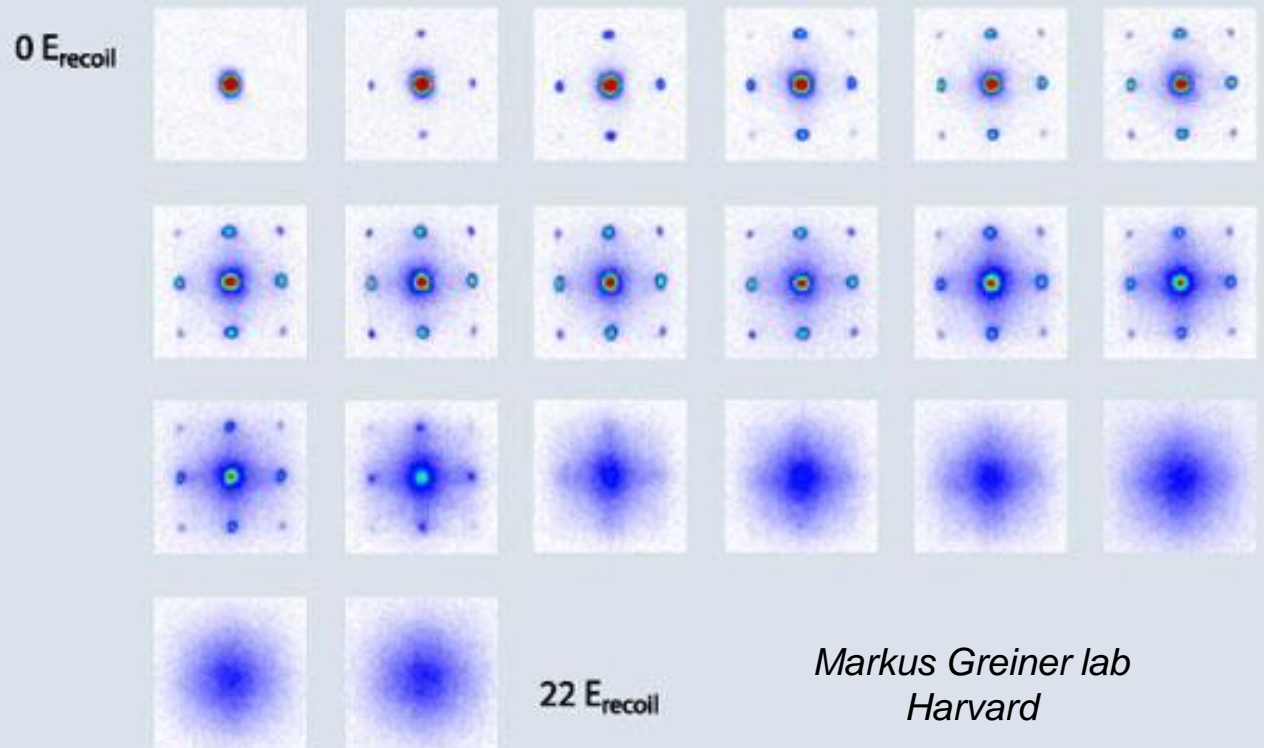


Moritz, Kohl, Esslinger, PRL **91**, 250402 (2003)



Katori *et al*, PRL **91**, 173005 (2003)

Momentum distribution for different potential depths of a 3D lattice:



Markus Greiner lab
Harvard

Ideal 1D gas

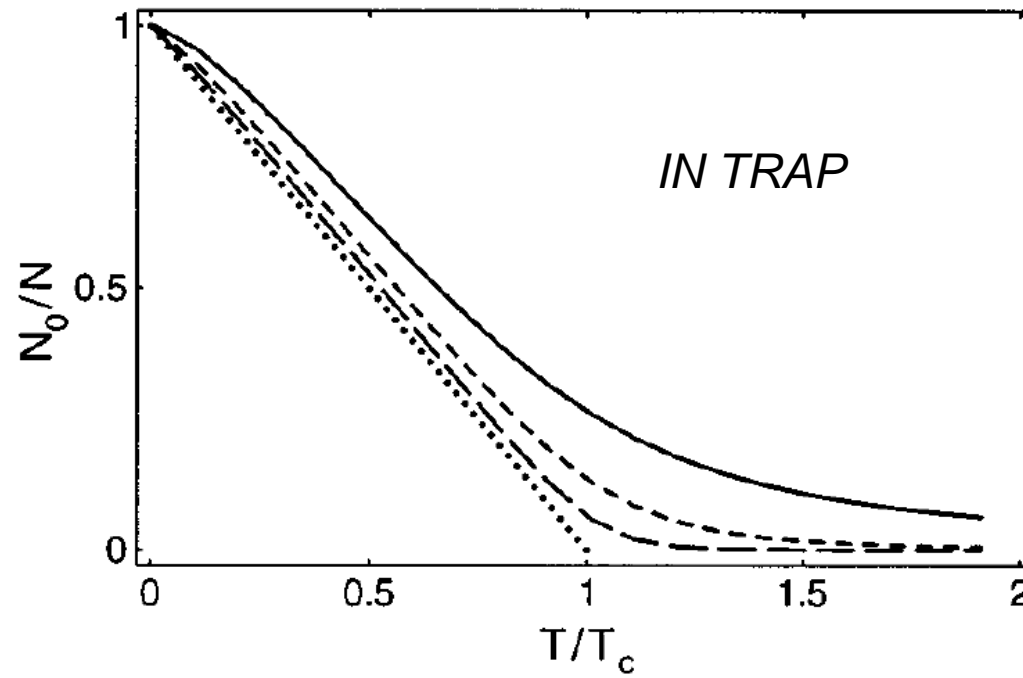
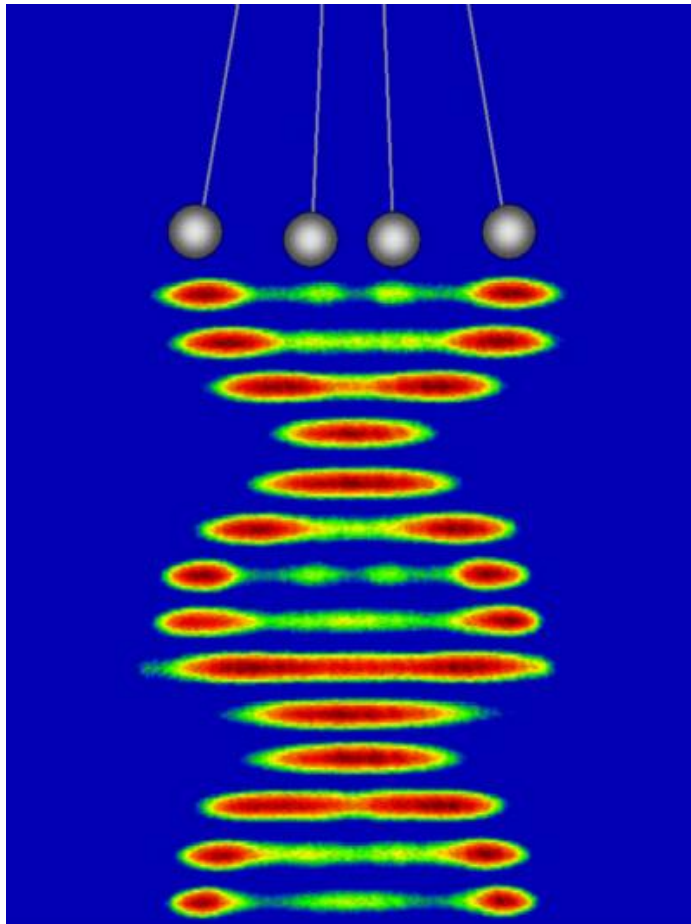


FIG. 4. The condensate fraction for a finite number N of atoms in a one-dimensional harmonic potential versus temperature. Plots are shown for $N = 100$ (solid line), 10^4 , 10^8 , and infinite (dotted).

Ketterle, van Druten, PRA **54**, 656 (1996)

Integrability

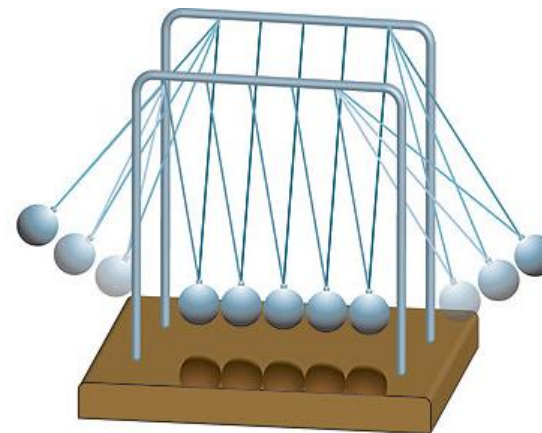
Quantum “Newton's cradle”



Kinoshita, Wenger, Weiss, Nature 440, 900 (2006)

Even in the *INTERACTING* uniform gas,
the momentum distribution is invariant
Because collisions create no dispersion

Apparently, this remains
a good approximation
in the trap

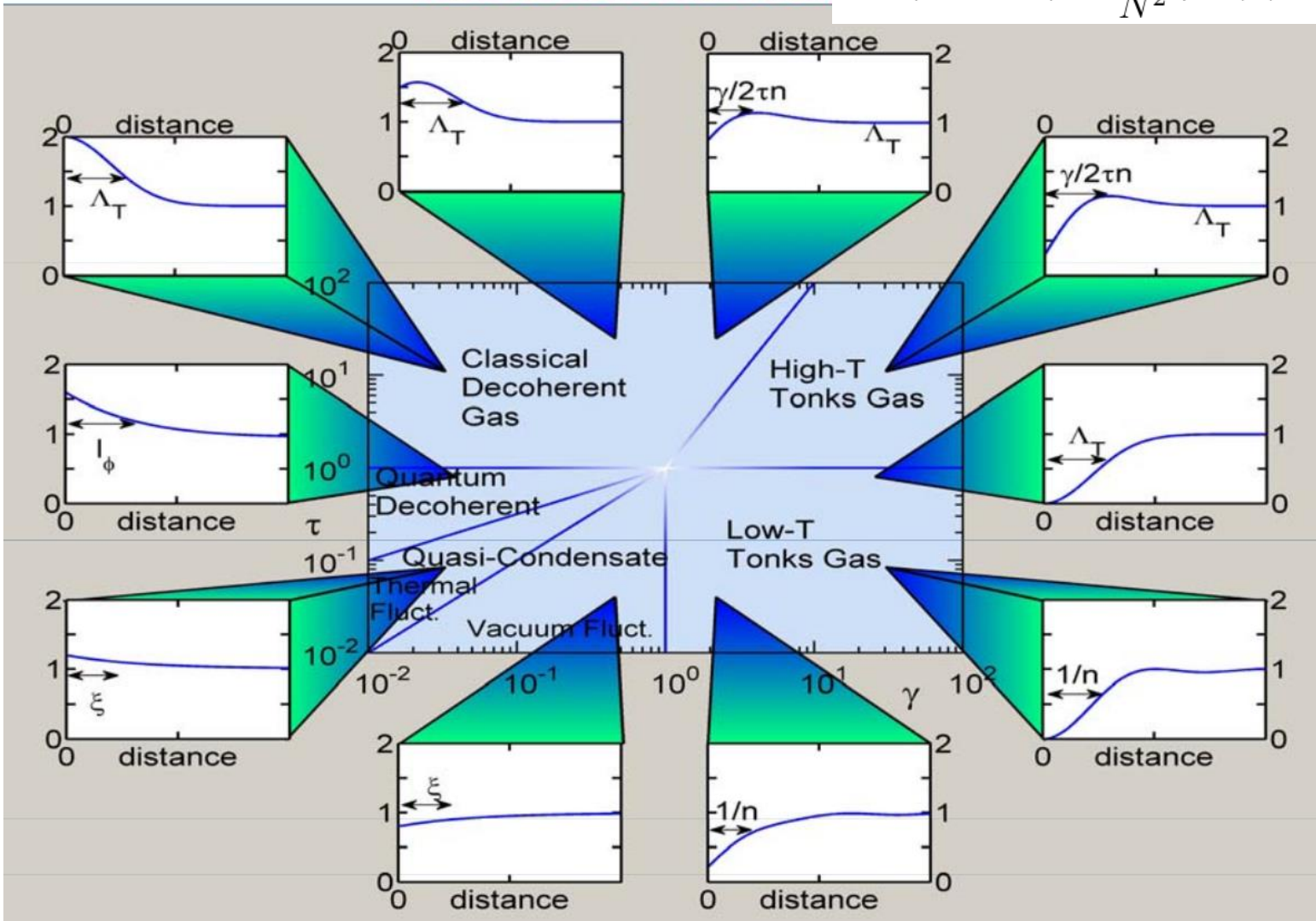


Interacting Bose gas

Uniform case is exactly solvable:

$T = 0$: Lieb & Liniger, Phys. Rev. **130**, 1605 (1963)
 $T > 0$: Yang & Yang, J. Math. Phys. **10**, 1115 (1969)

Pair correlation function $g^{(2)}(x, x + y) = \frac{1}{N^2} \langle \hat{\Psi}^\dagger(x) \hat{\Psi}^\dagger(x + y) \hat{\Psi}(x) \hat{\Psi}(x + y) \rangle$



$$\gamma = \frac{mg}{\hbar^2 n}$$

Deuar, Sykes, Gangardt, Davis, Drummond, Kheruntsyan, PRA **79**, 043619 (2009)

Quasicondensate

- In the uniform 1D gas, there is no true condensate for $T > 0$
- **However:** finite coherence length l_ϕ and small density fluctuations

$$g^{(1)}(x, x') \sim \exp \left[-\frac{|x - x'|}{l_\phi} \right]$$

$$; \quad l_\phi \sim \frac{N^{2/3}}{T}$$

$$g^{(1)}(0, x) = \frac{\langle \hat{\Psi}^\dagger(0) \hat{\Psi}(x) \rangle}{[\rho(0)\rho(x)]^{1/2}}$$

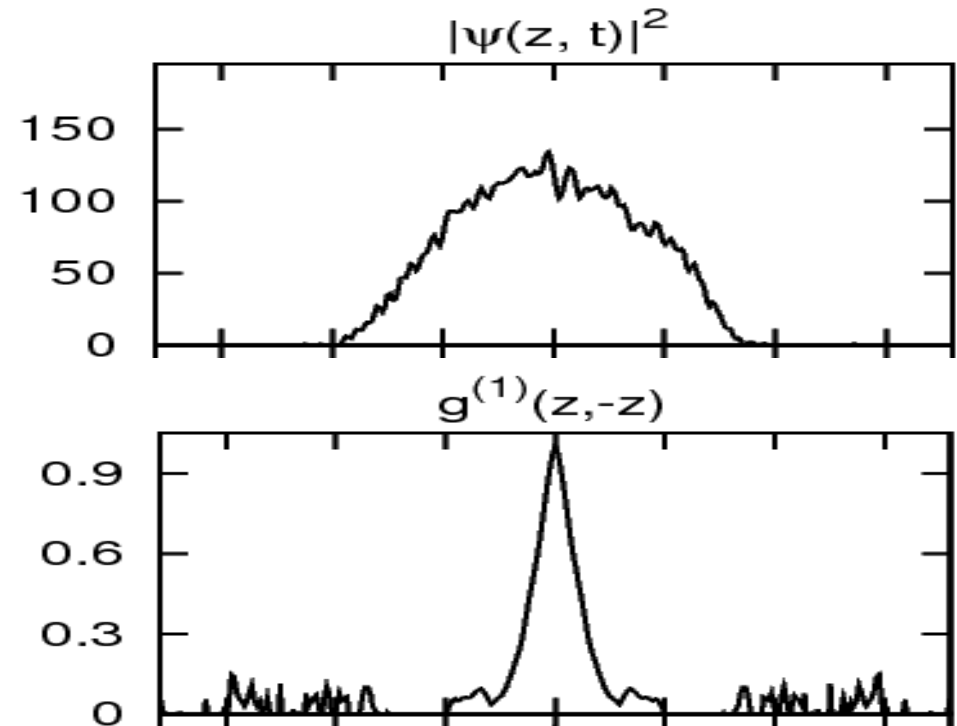
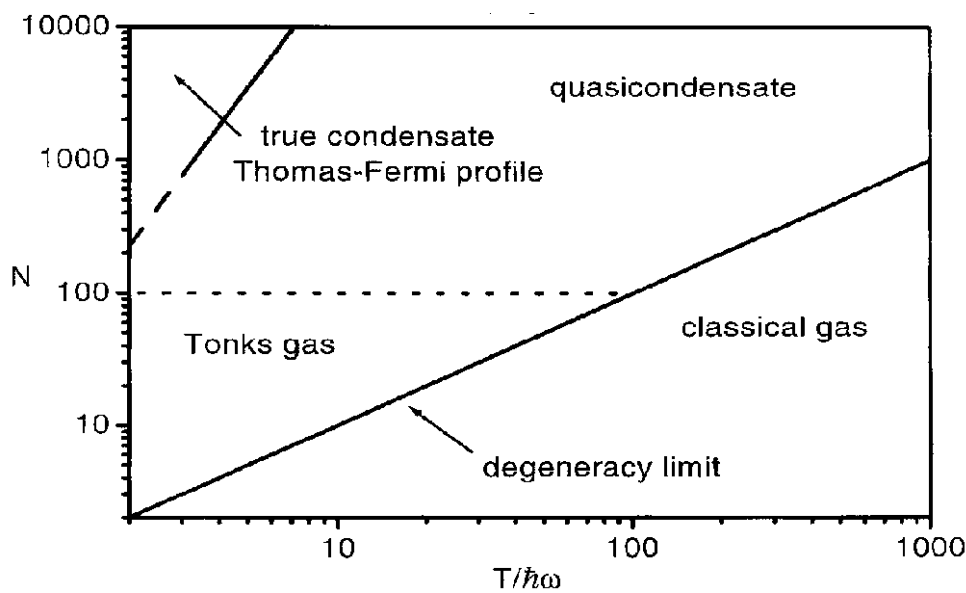
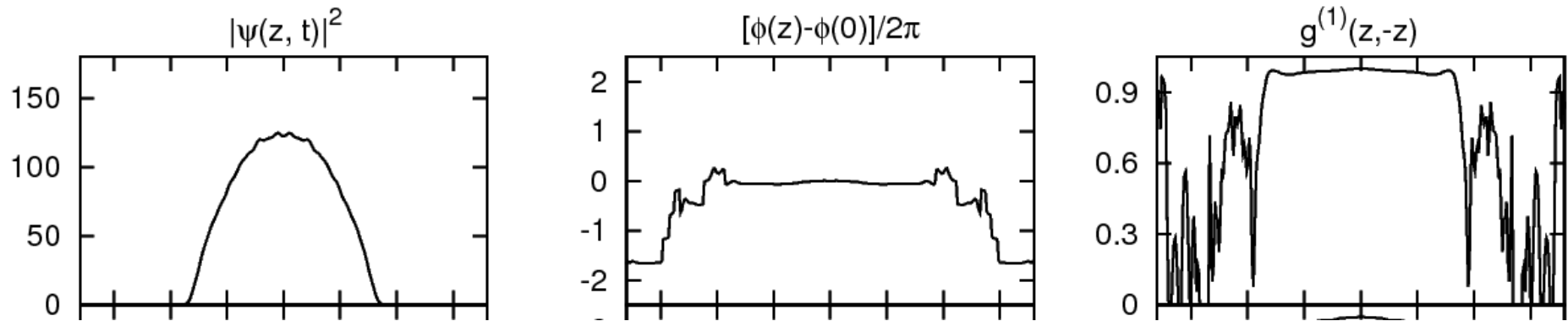


FIG. 1. Diagram of states for a trapped 1D gas.

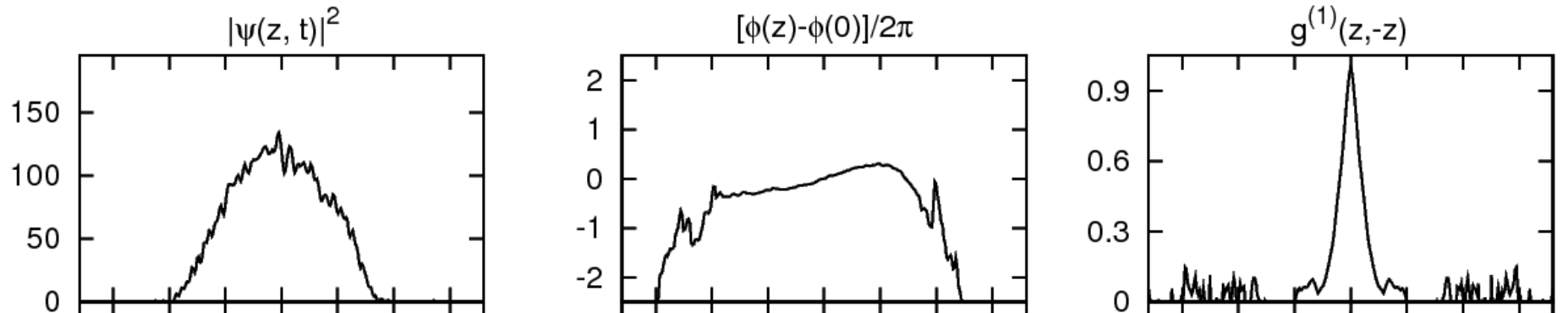
D. Petrov, G. Shlyapnikov, J. Walraven, PRL **85**, 3745 (2000)

Quasicondensate

BEC



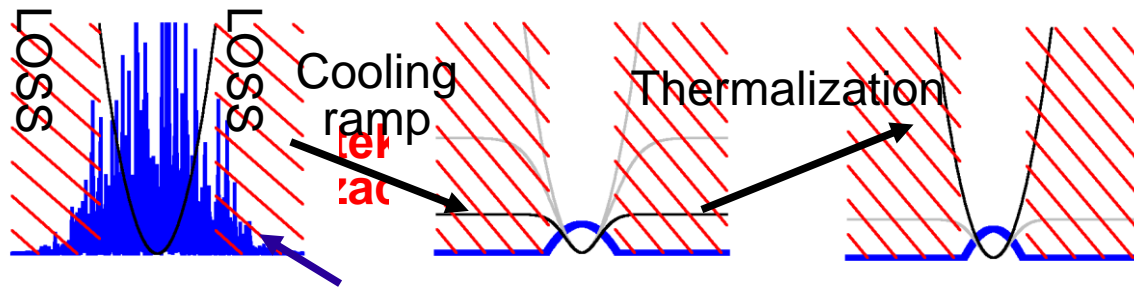
Quasicondensate



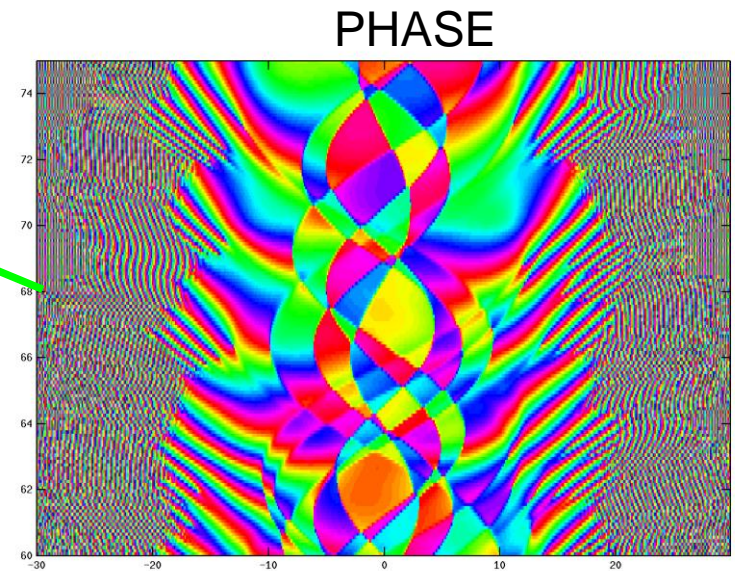
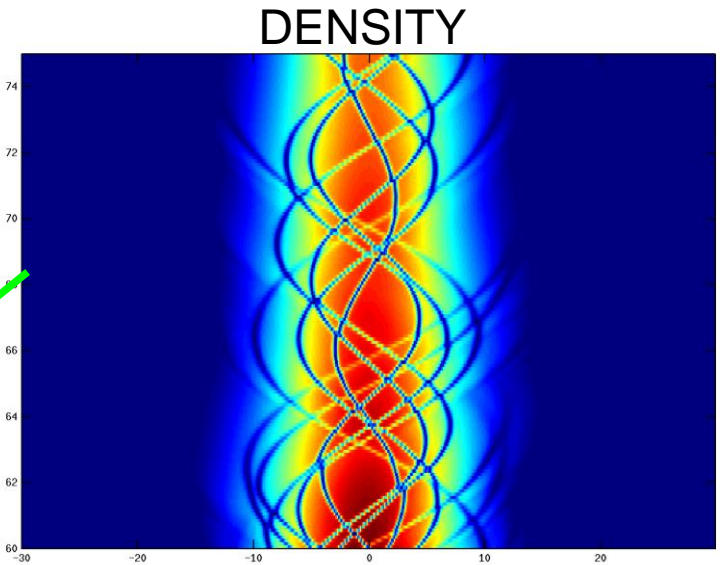
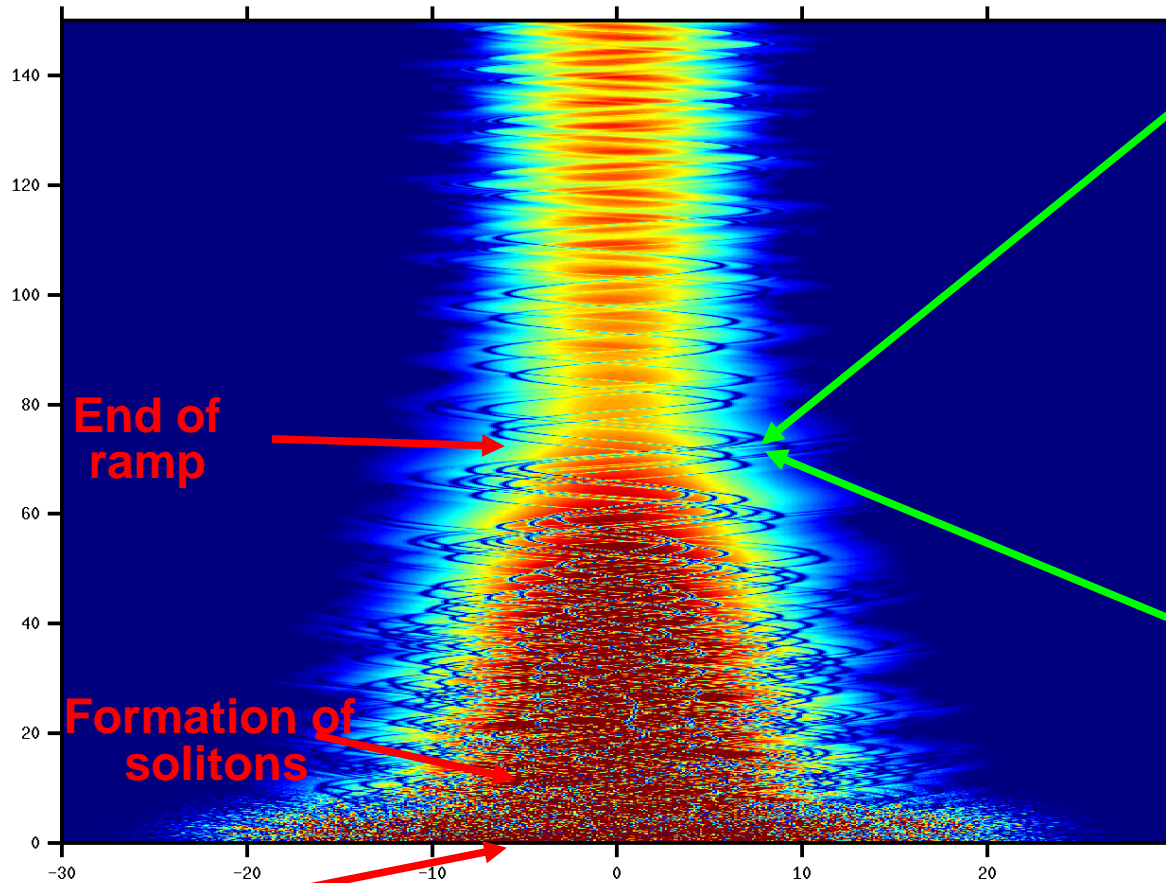
E. Witkowska, PD, M. Gajda, K. Rzażewski PRL **106**, 135301 (2011)

1D Bose gas cooling

E. Witkowska, PD, M. Gajda, K. Rzażewski PRL **106**, 135301 (2011)



Creation of defects via Kibble-Zurek mechanism



Ramp beginning

Long time after cooling, thermalization

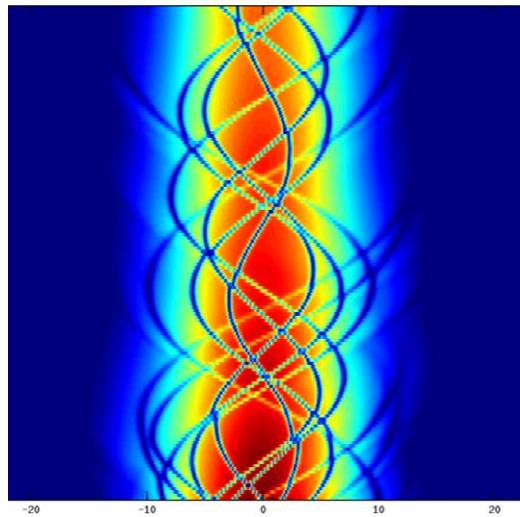
E. Witkowska, PD, M. Gajda, K. Rzażewski. PRL **106**, 135301 (2011)

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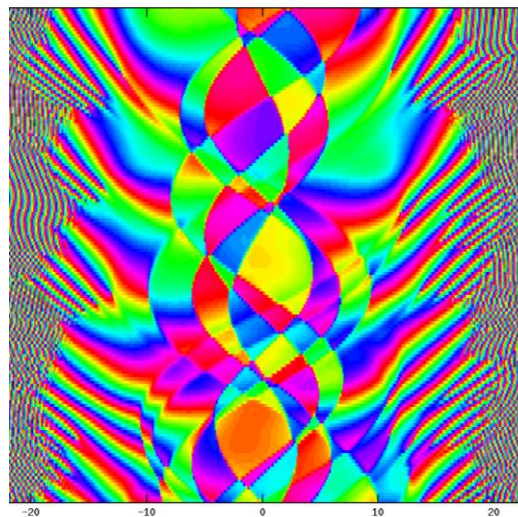
quasicondensate: $t_r = 75$

END OF COOLING

DENSITY



PHASE



THERMALISED

