Kolizje kondensatów BEC i symulacja mikroskop*owej*~~ijnej~~ dynamiki kwantowej

Piotr Deuar

(IF PAN)

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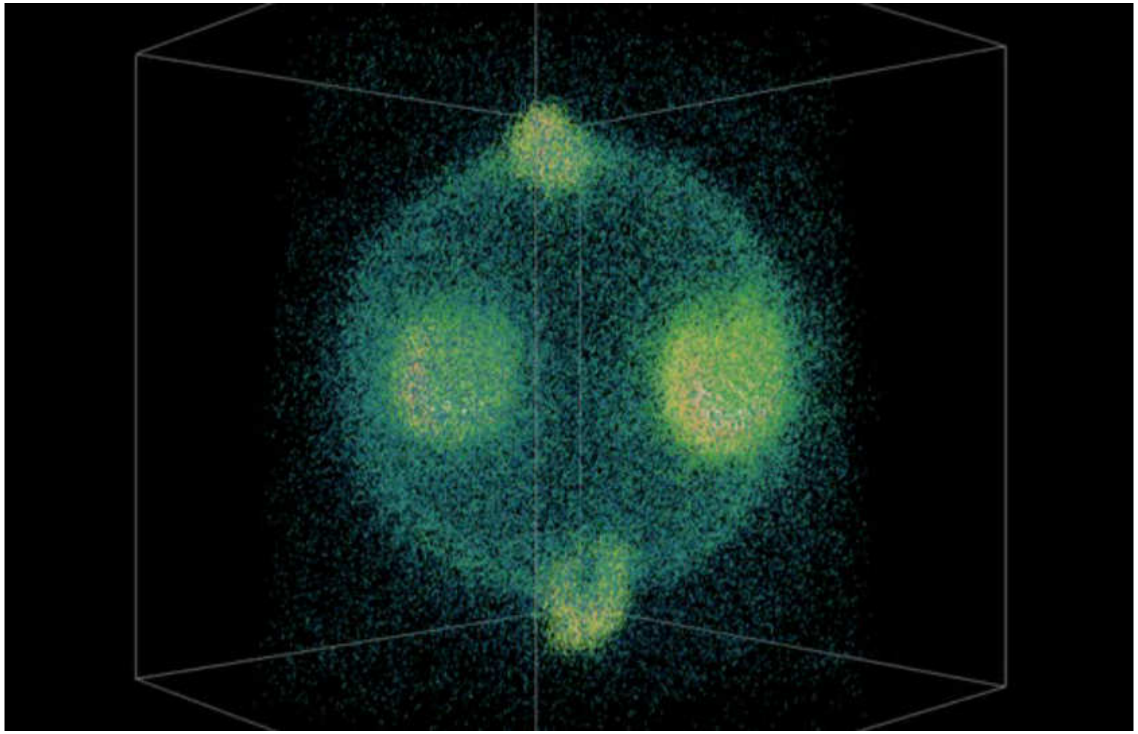
Collaborations on the topic:

Chris Westbrook *(Institut d'Optique)*
Denis Boiron
Valentina Krachmalnicoff
Jean-Christophe Jaskula
Marie Bonneau
Guthrie Partrigde

Marek Trippenbach *(Warsaw University)*
Paweł Ziń *(IPJ)*
Jan Chwedeńczuk *(INFM-BEC, Trento)*
Karen Kheruntsyan *(Uni. Queensland)*
Aurelién Perrin
Vanessa Leung *(Uni. Virginia)*

Issues I will try to touch on

- Some similarities and differences between ultracold atoms and quantum optics
- When is a mean-field description of BECs inadequate?
- How BEC collisions are an example of this inadequacy
- What BEC collision experiments look like
- How to go beyond a mean field description in a “straightforward” way
- Four wave mixing with atoms vs. with photons
- The growth of spontaneous condensates: “phase grains” *(maybe)*



Common description of a "BEC" – the Hamiltonian

Boson field operators $\widehat{\Psi}(x)$.

$$\widehat{H} = \int dx\, \widehat{\Psi}^{\dagger}(x) \left\{ -\frac{\hbar^2 \nabla^2}{2m} + V(x) + \frac{g}{2} \widehat{\Psi}^{\dagger}(x) \widehat{\Psi}(x) \right\} \widehat{\Psi}(x)$$

– In terms of mode occupations for a grid with mesh grid volume ΔV

$$\widehat{\Psi}(x) \sim \frac{\widehat{a}_x}{\sqrt{\Delta V}} \qquad ; \qquad [\widehat{a}_x, \widehat{a}_y^{\dagger}] = \delta_{xy}$$

– Get something that looks quantum-optics like

$$\widehat{H} = \sum_x \left\{ \widehat{a}_x^{\dagger} H_0 \widehat{a}_x + \chi \left(\widehat{a}_x^{\dagger} \right)^2 \widehat{a}_x^2 \right\}$$

with $\chi = g/2\Delta V$, and $H_0 = -\hbar^2/2m\nabla^2 + V(x)$.

– Similar to Kerr $\chi^{(3)}$ nonlinearity.

Common description of a "BEC": mean field GP equation

The Boson field $\widehat{\Psi}(x)$ is approximately a macroscopic wavefunction $\Psi(x)$ for a single mode occupied by all N atoms.

$\Psi(x)$ Obeys the (superfluid) Gröss-Pitaevskii (GP) equation:

$$i\hbar\frac{\partial\Psi(x,t)}{\partial t} = \left\{ -\frac{\hbar^2\nabla^2}{2m} + V(x) + g|\Psi(x,t)|^2 \right\} \Psi(x,t)$$

What does the GP description miss?

1. Incoherent atoms

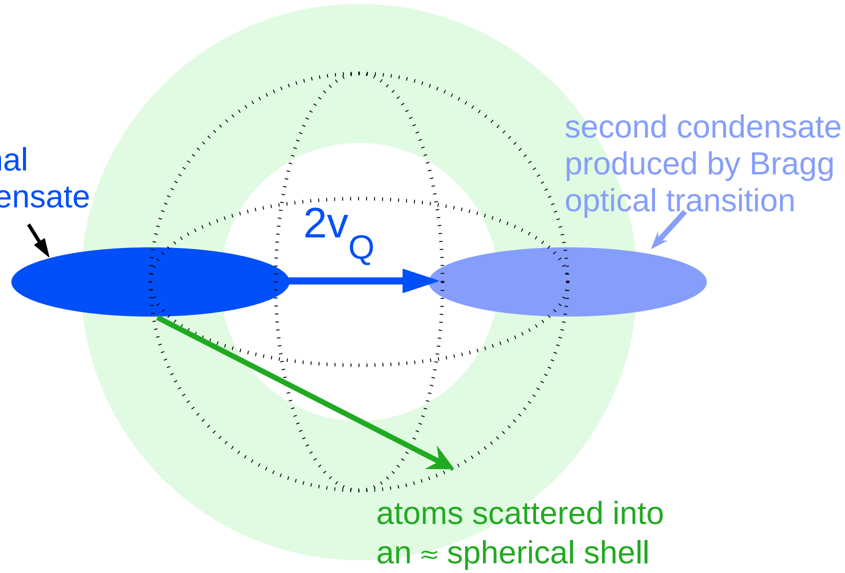
2. Supersonic effects
   — above the speed of sound $c(x) = \sqrt{gn(x)/m}$, motion is no longer superfluid.

3. Details on scales smaller than the healing length $\xi(x) = \hbar/mc\sqrt{2}$.

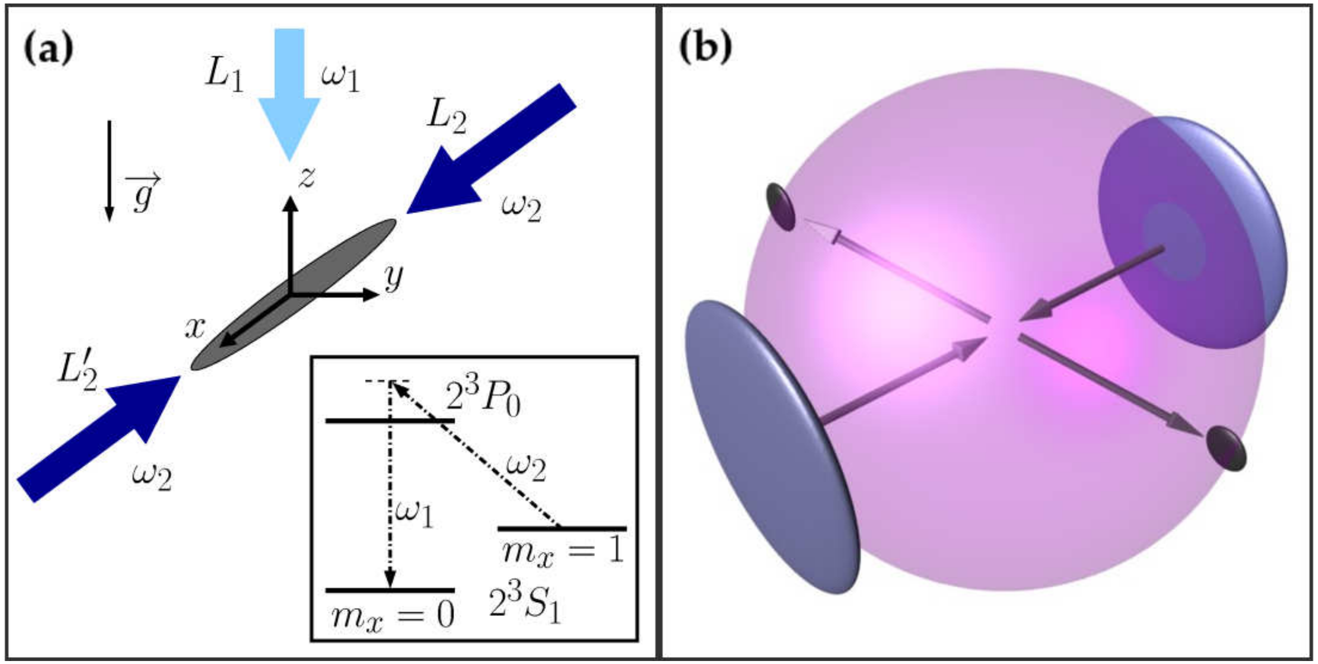
Example where this lack is serious: supersonic BEC collisions.

Supersonic BEC collision

Schematic in x-space:

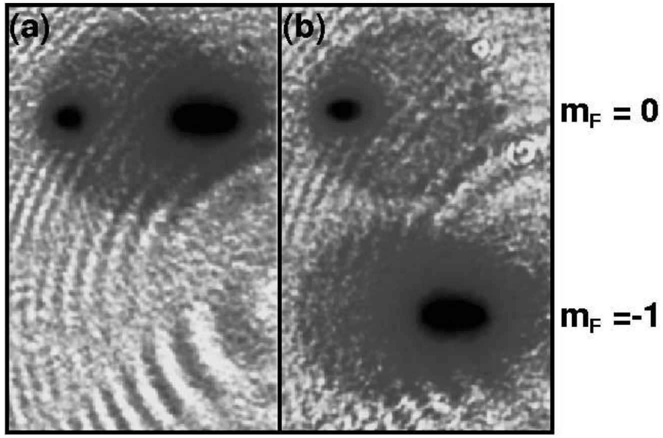


Lasers in x-space | View in k-space



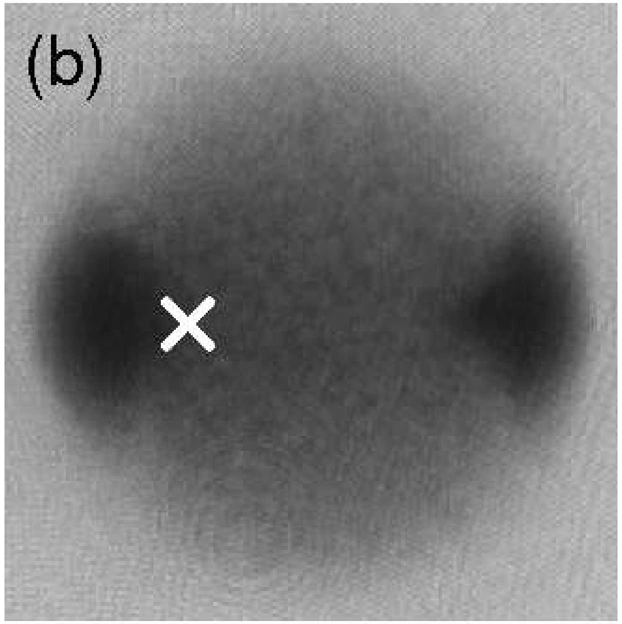
From A. Perrin *et al.*,
PRL **99** 150405 (2007)

Experimental examples

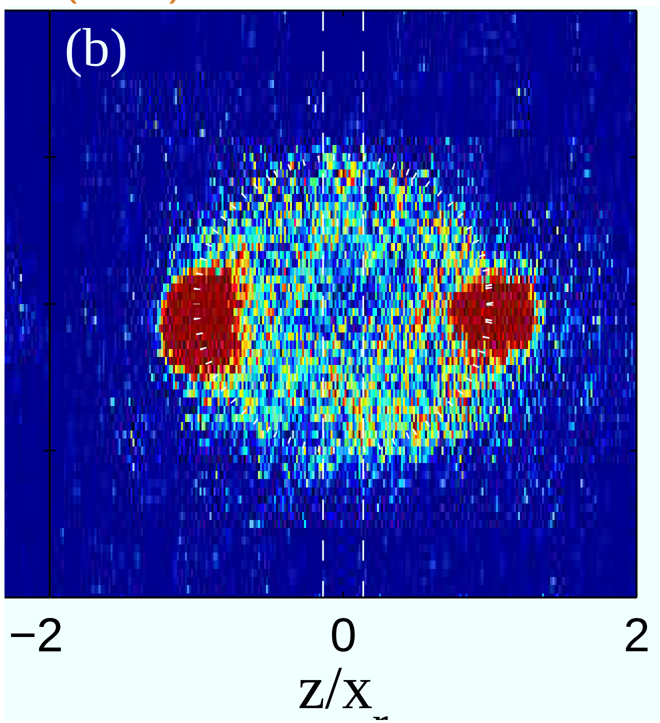


Integrated density

From A.P. Chikkatur *et al.*, PRL **85**, 483 (2000).



From J.M. Vogels *et al.*,
PRL **89**, 020401 (2002).



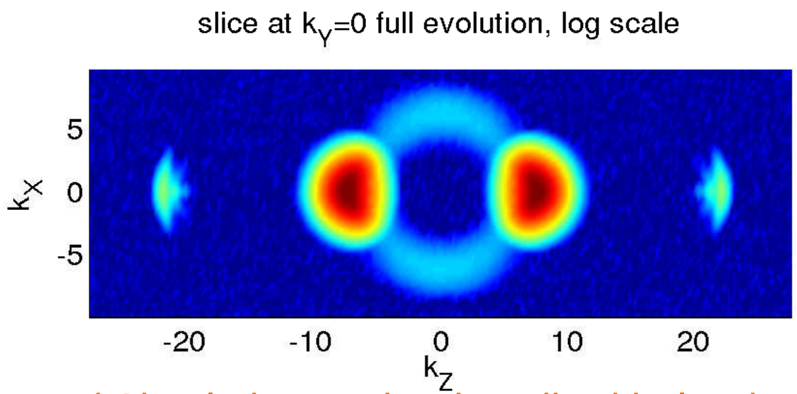
From N. Katz *et al.*, PRL **95**, 220403 (2005).

Why is mean field no good here?

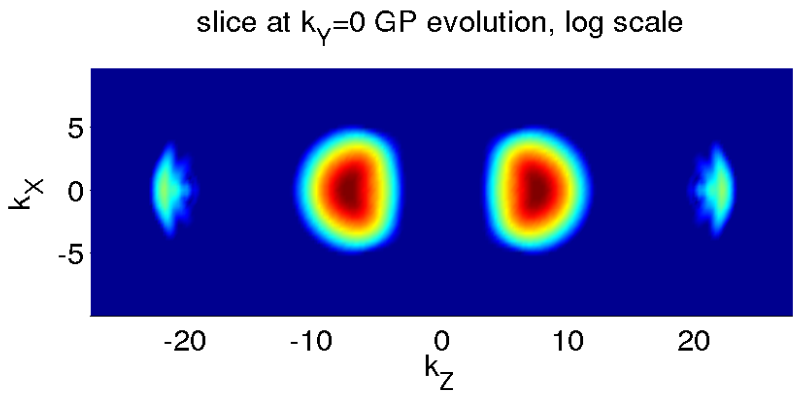
$$i\hbar\frac{\partial\Psi(x,t)}{\partial t}=\left\{-\frac{\hbar^2\nabla^2}{2m}+V(x)+g|\Psi(x,t)|^2\right\}\Psi(x,t)$$

In the halo, initial condensate field $\Psi(x,0)$ is zero, and so stays that way.

(It's a spontaneous process initially)

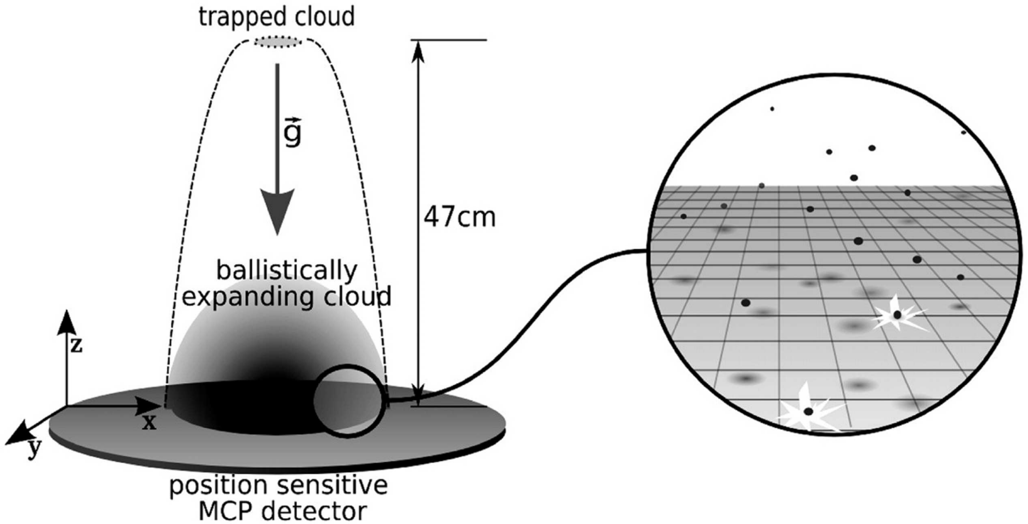


(Simulations to be described below)



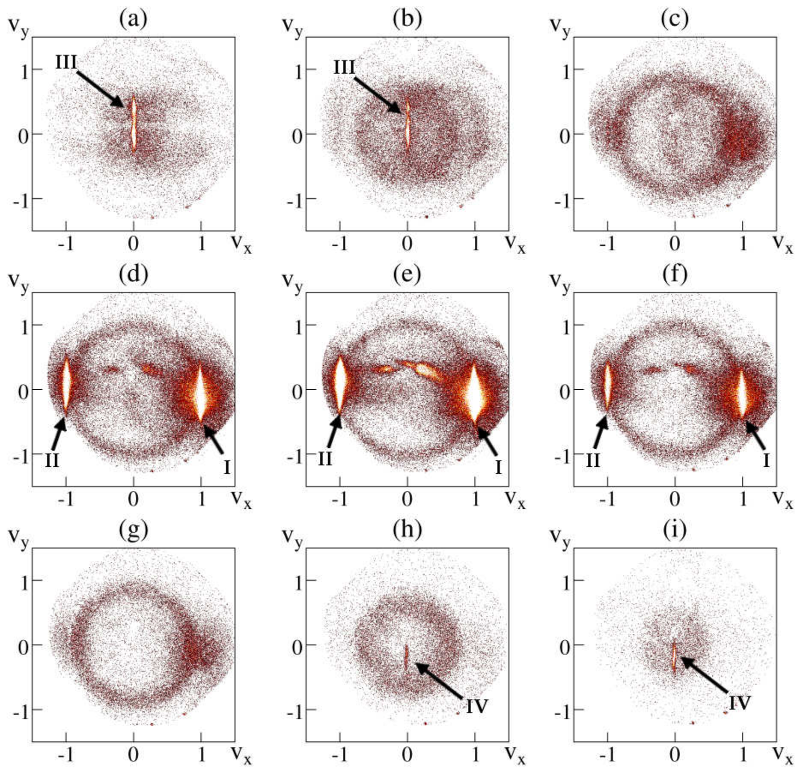
Metastable He* experiment

Uses multi-channel plate – Allows mapping of 3D atom distribution



From M. Schellekens *et al.*,

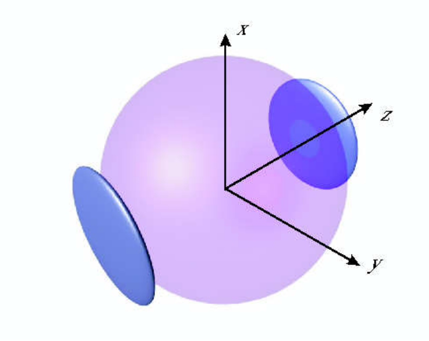
Science **310**, 648 (2005).

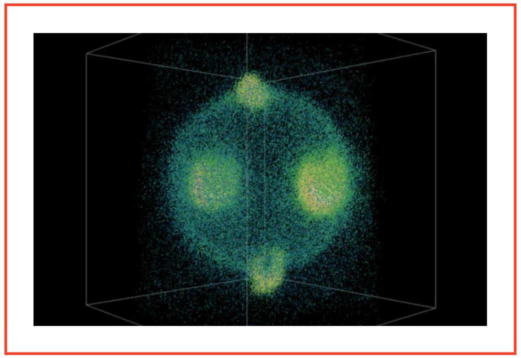


From A. Perrin *et al.*, PRL **99** 150405 (2007)

After long time of flight,

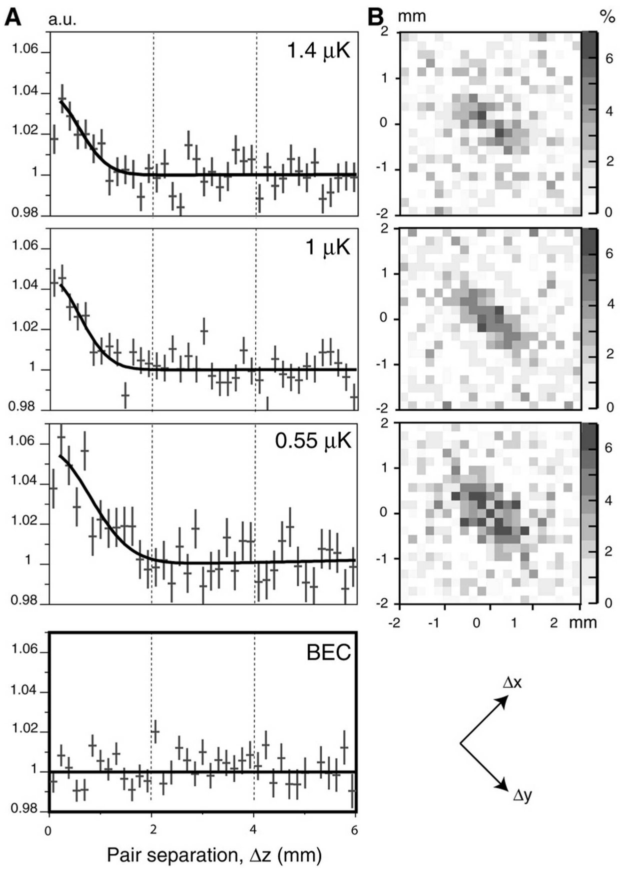
$$n(x,t) \rightarrow n(k,0)$$





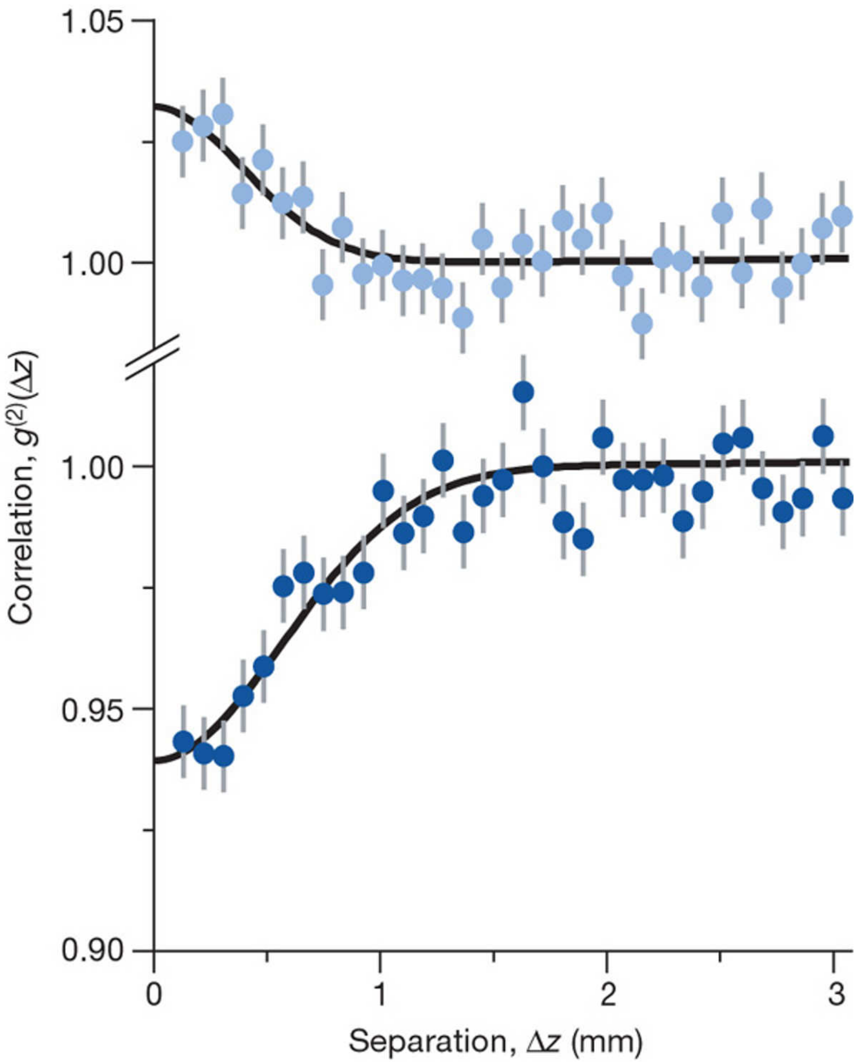
Allows measurement of HBT correlations - density $g^{(2)}$

$$g^{(2)}(x,y) = \frac{\langle \hat{a}_x^\dagger \hat{a}_y^\dagger \hat{a}_y \hat{a}_x \rangle}{\langle \hat{n}(x) \rangle \langle \hat{n}(y) \rangle}$$



Hanbury Brown-Twiss short-range correlations $g^{(2)}(z, z+\Delta z)$ in a single cloud

Schellekens *et al.*, Science 310, 648 (2005)

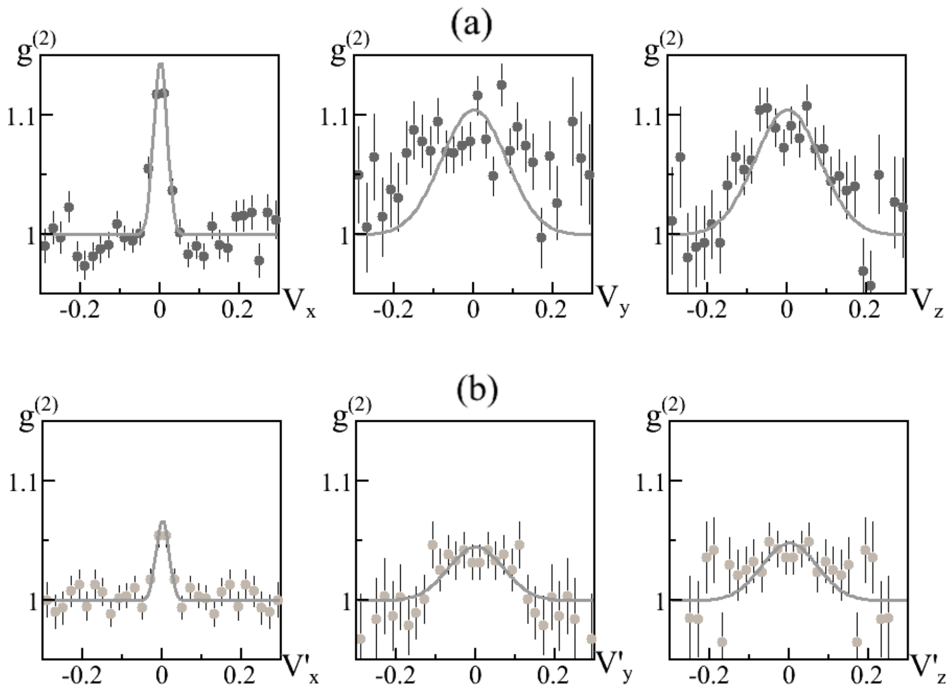


in a Boson He^4 and a Fermion He^3 cloud. Note how healing length $\xi(x) = \hbar / mc\sqrt{2} \propto 1/m$

Jeltes *et al.*, Nature 445, 402 (2007)

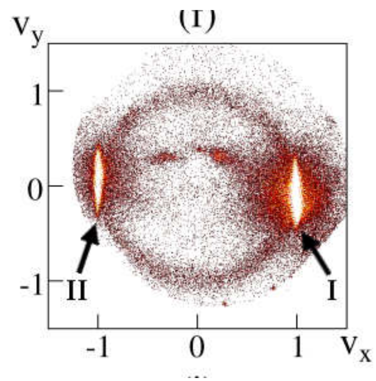
Density correlations in the scattered halo

$$g^{(2)}(k,k') = \frac{\langle \widehat{n}(k)\,[\widehat{n}(k') - \delta_{kk'}]\,\rangle}{\langle \widehat{n}(k)\rangle\langle \widehat{n}(k')\rangle}$$



Back-to-Back $k' \sim -k$
(pairing)

Collinear $k' \sim k$
(HBT)



From A. Perrin *et al.*, PRL **99** 150405 (2007)

Another interesting issue – Phase grains

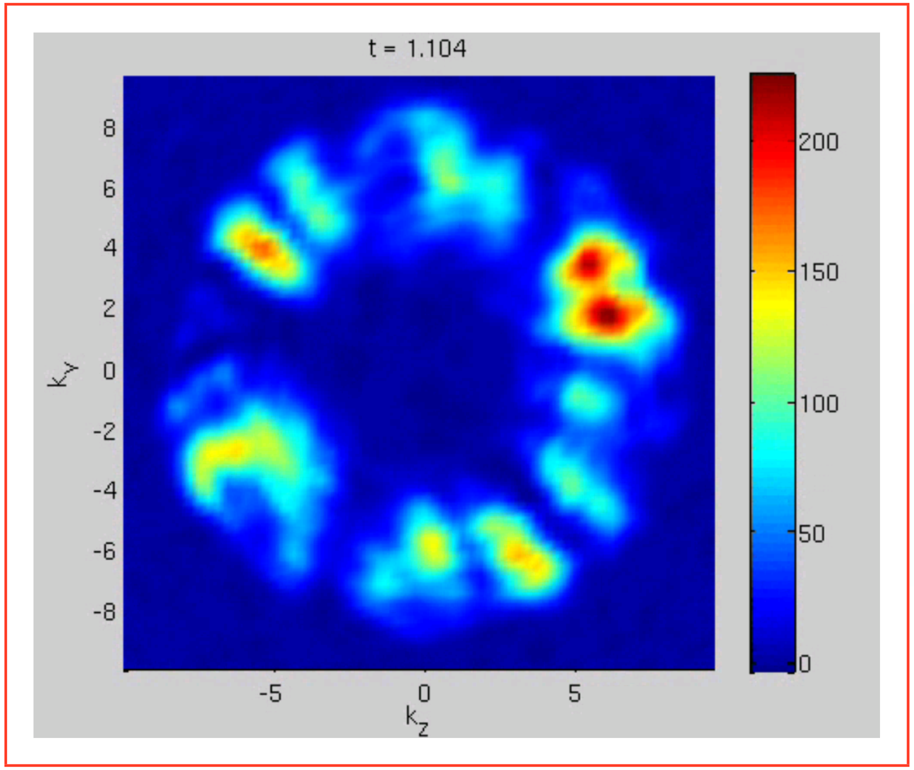
Coherence: $$g^{(1)}(k, k+\delta k) = \frac{\langle \widehat{\Psi}^\dagger(k)\widehat{\Psi}(k+\delta k)\rangle}{\sqrt{\langle \widehat{n}(k)\rangle\langle \widehat{n}(k+\delta k)\rangle}}$$

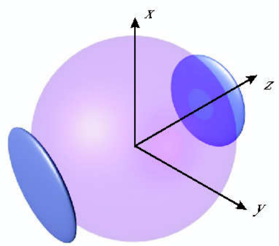
– Locally coherent regions. $|g^{(1)}| \gg 0$ Norrie *et al.*, PRL **94**, 040401 (2005)

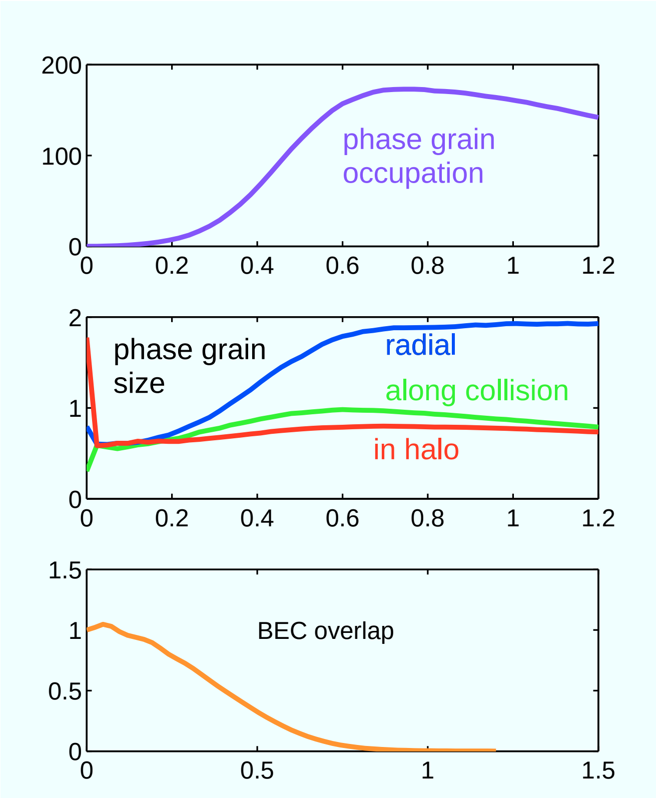
– Scattering rate into such a coherent region with n atoms is $\propto (1+n)$

$\longrightarrow$ Bose stimulation if $n \gtrsim 1$ leads to rapid coherent growth of occupation of the phase grain

$\longrightarrow$ Mini condensates formed.







Simulation beyond mean field: Bogoliubov Hamiltonian

1. Write $\widehat{\Psi}(x,t) = \phi(x,t) + \widehat{\psi}_B(x,t)$

2. Substitute into full $\widehat{H} = \int dx\, \widehat{\Psi}^\dagger(x) \left\{ -\frac{\hbar^2\nabla^2}{2m} + \frac{g}{2}\, \widehat{\Psi}^\dagger(x)\widehat{\Psi}(x) \right\} \widehat{\Psi}(x)$

3. <u>Assume</u> $\widehat{\psi}_B(x,t)$ is orthogonal to $\phi(x,t)$.

4. <u>Assume</u> δN the number of particles contained in $\widehat{\psi}_B$ is $\ll N$, the total number.

5. Remove terms of high order in $\delta N/N$ (quantum depletion) from $\widehat{H}$ to obtain $\widehat{H}_B$

6. For later convenience, separate right- and left-moving condensates (velocities $\approx \pm k_C$) into $\phi(x,t) = \phi_L(x,t) + \phi_R(x,t)$.

time-dependent Bogoliubov Hamiltonian

$$\begin{aligned}
\hat{H}_B \;=\; \int dx \Big\{ & \hat{\psi}_B^\dagger \left(-\frac{\hbar^2\nabla^2}{2m}\right)\hat{\psi}_B && \text{K.E.} \\
& +2g|\phi(t)|^2 \hat{\psi}_B^\dagger \hat{\psi}_B && \text{collective potential} \\
& +2g\phi_L(t)\phi_R(t)(\hat{\psi}_B^\dagger)^2 + \text{h.c.} && \text{halo pair production} \\
& +g\left[\phi_L(t)^2+\phi_R(t)^2\right](\hat{\psi}_B^\dagger)^2 + \text{h.c.} \Big\} && \text{pair production near BECs}
\end{aligned}$$

GP equations for condensates:

$$i\hbar\frac{d\phi_R(x,t)}{dt} = \left\{-\frac{\hbar^2}{2m}\nabla^2 + g\left[|\phi_R(x,t)|^2 + 2|\phi_L(x,t)|^2 + \phi_L^*(x,t)\phi_R(x,t)\right]\right\}\phi_R(x,t)$$

$$i\hbar\frac{d\phi_L(x,t)}{dt} = \left\{-\frac{\hbar^2}{2m}\nabla^2 + g\left[|\phi_L(x,t)|^2 + 2|\phi_R(x,t)|^2 + \phi_R^*(x,t)\phi_L(x,t)\right]\right\}\phi_L(x,t)$$

<u>Cute and useful feature:</u> can remove terms to see what process affects what observation.

A "TECHNICAL" DIFFICULTY:

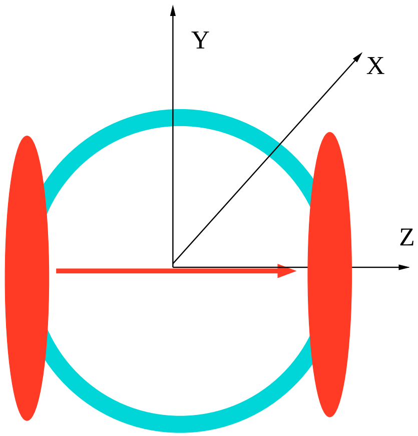
- Experimentally realistic situations require $10^5 - 10^7$ lattice points.
- Standard Bogoliubov quasiparticle evolution procedure requires diagonalization of $\widehat{H}_B$, finding of eigenstates, etc. *This is unlikely given the size of the space!*

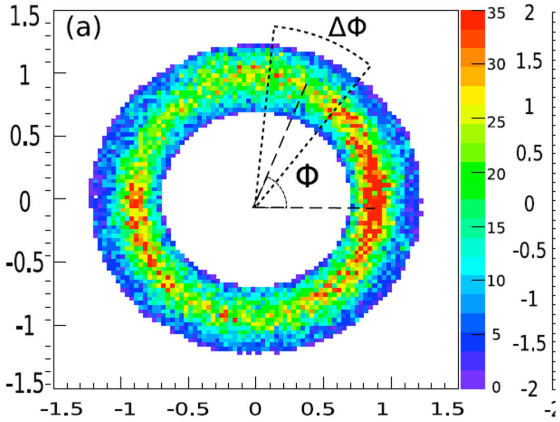
SOLUTION: Instead, the dynamics of $\widehat{\psi}_B$ can be treated stochastically using a method straight from quantum optics – the positive-P representation...

$$
\begin{aligned}
i\hbar\frac{d\psi_1(x,t)}{dt} &= \left[-\frac{\hbar^2}{2m}\nabla^2 + 2g|\phi(x,t)|^2\right]\psi_1(x,t) \\
&= +g\phi(x,t)^2\psi_2(x,t)^* + i\sqrt{ig}\psi_1(x,t)\,\xi_1(x,t) \\
i\hbar\frac{d\psi_2(x,t)}{dt} &= \left[-\frac{\hbar^2}{2m}\nabla^2 + 2g|\phi(x,t)|^2\right]\psi_2(x,t) \\
&= +g\phi(x,t)^2\psi_1(x,t)^* + i\sqrt{ig}\psi_2(x,t)\,\xi_2(x,t)
\end{aligned}
$$

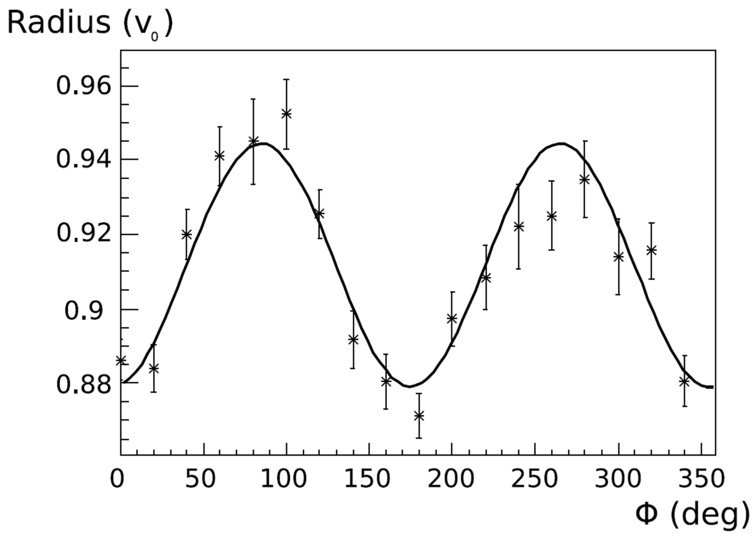
Here, $\xi_j(x,t)$ are independent Gaussian random variable fields with mean zero and variances $\langle\xi_i(x,t)\xi_j(x',t')\rangle = \delta_{ij}\delta(x-x')\delta(t-t')$. And e.g. $\langle\widehat{\Psi_B}^\dagger\widehat{\Psi}_B\rangle = \langle\psi_2^*\psi_1\rangle_{\mathrm{stoch}}$

Four wave mixing with ATOMS vs with PHOTONS





XY-plane



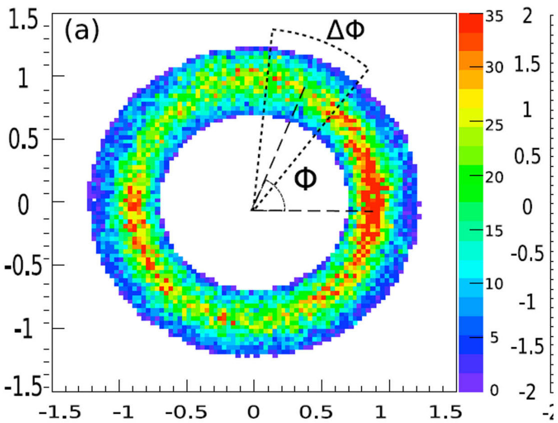
- Spontaneous scattering into empty “halo” modes as with light
- ATOMS: Halo radius NOT at expected $|v| = v_0$ collision velocity, but smaller
- ATOMS: Halo ellipsoidal if condensates non-spherical
- ATOMS: Superradiance-like effect, but small.

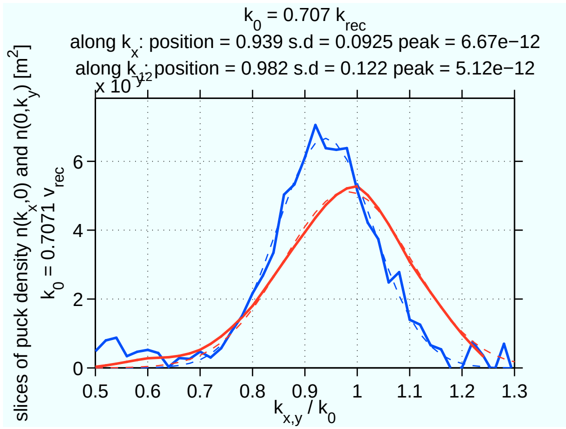
Why is Halo radius smaller

It costs $\mu = \frac{3}{2} g n(x)$ to remove a particle from the condensate (the mean-field energy from the repluslion of the remaining particles), but $2gn(x)$ to place one in a non-condensate mode. The energy balance is

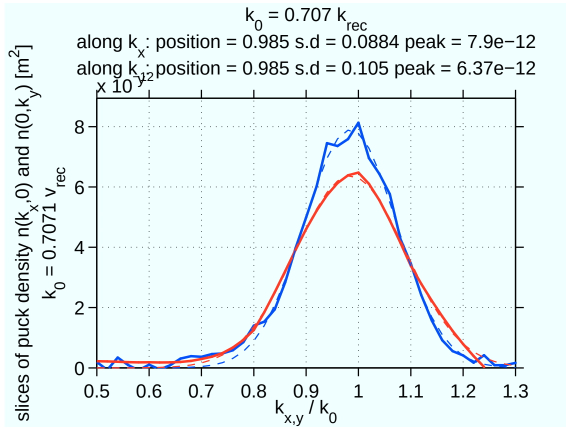
$$\frac{\hbar^2 k_0^2}{2m} + \frac{3}{2} g n = \frac{\hbar^2 k^2}{2m} + 2gn$$

When the mean-field energy is removed from the Hamiltonian $\widehat{H}_B$, the radius reverts to v_0 (and ellipticity disappears)





Full calculation

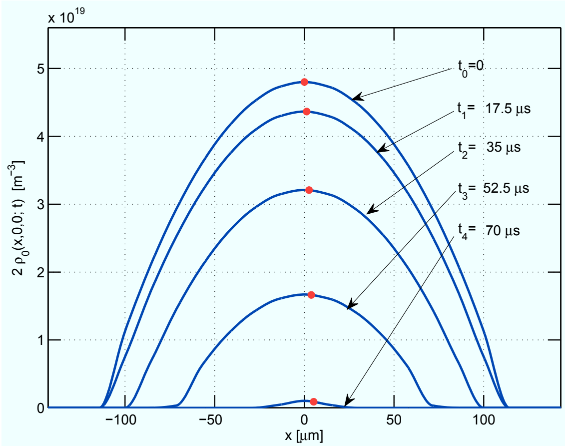


No MF energy $g|\phi|^2$ terms

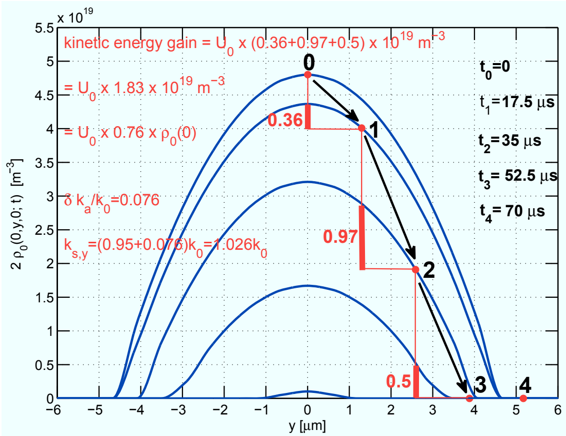
Why is the Halo an ellipse?

REASON 1: Particles can roll off the condensate to recover some or all of the lost mean-field energy $\propto gn(x)$.

BUT - in the long condensate direction, this does not happen because the density "falls out" from under the particles as the condensates move away before the particle can roll far.

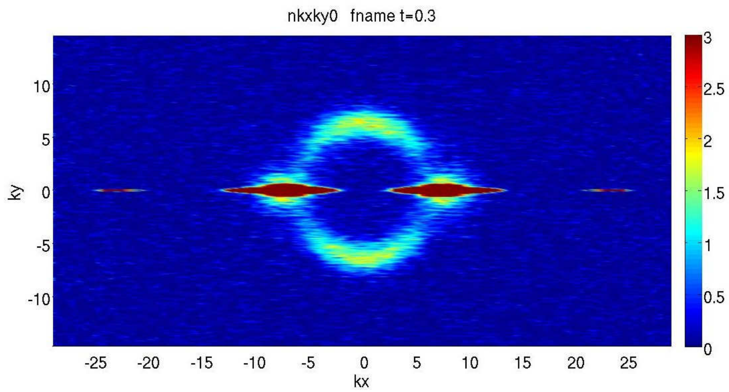


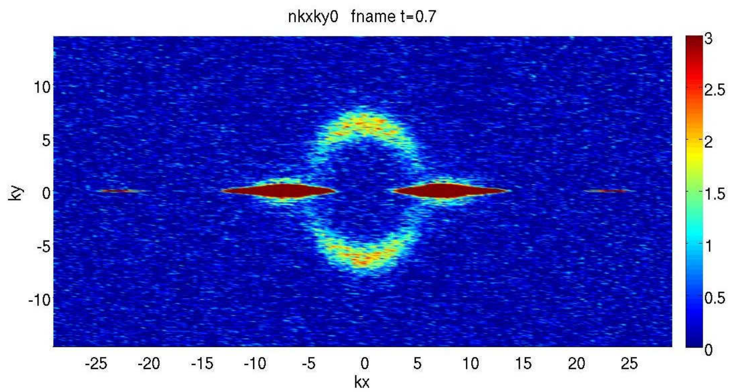
WHILE - in the short directions, a halo particle moves fast and rolls off before the condensate can change much.

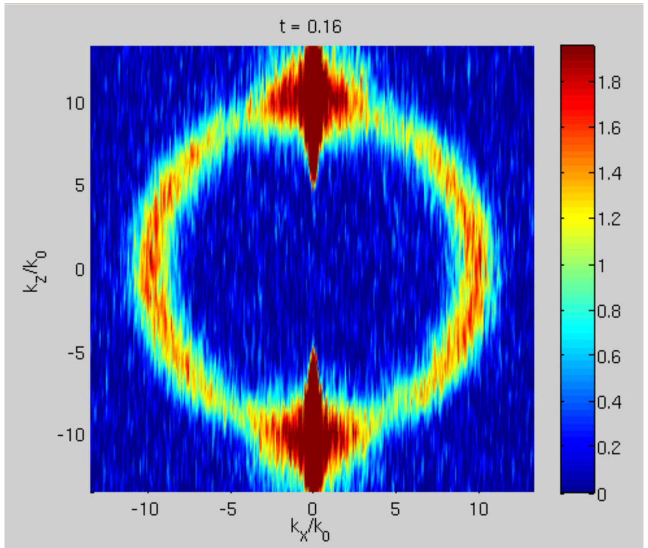


Why is the Halo an ellipse?

<u>REASON 2:</u> Along the collision direction, halo particles become bogged in the potential valley forming between the condensates (because they are slightly slower due to the halo radius shift mentioned before), and become *deccelerated* in this valley.

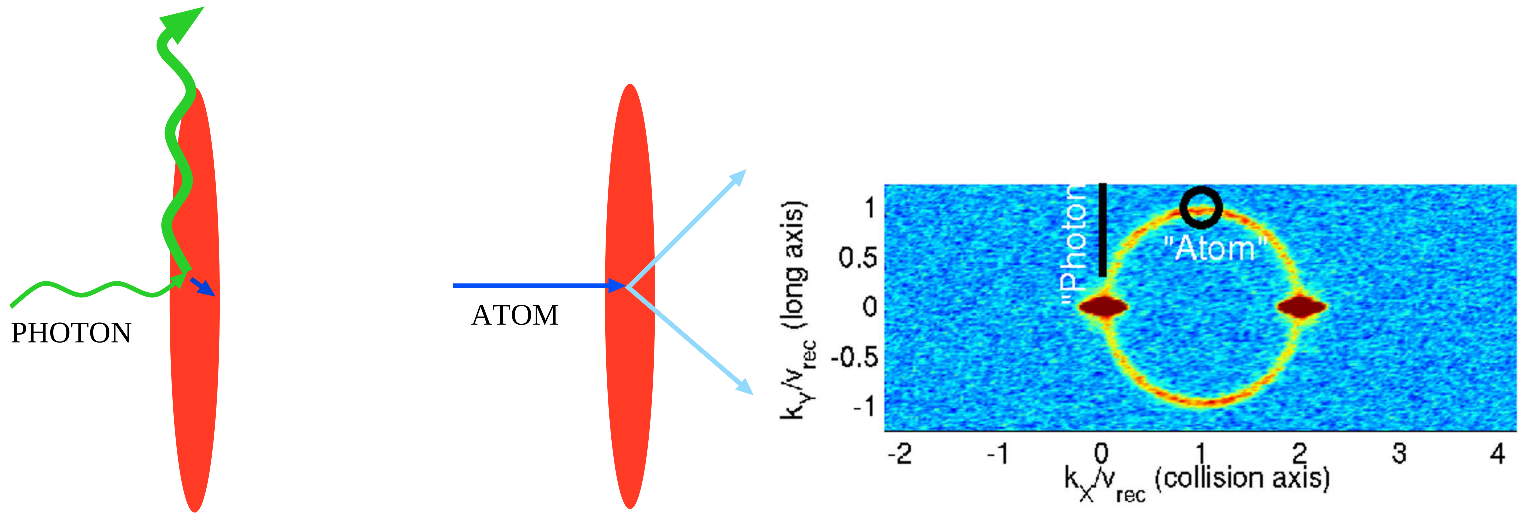






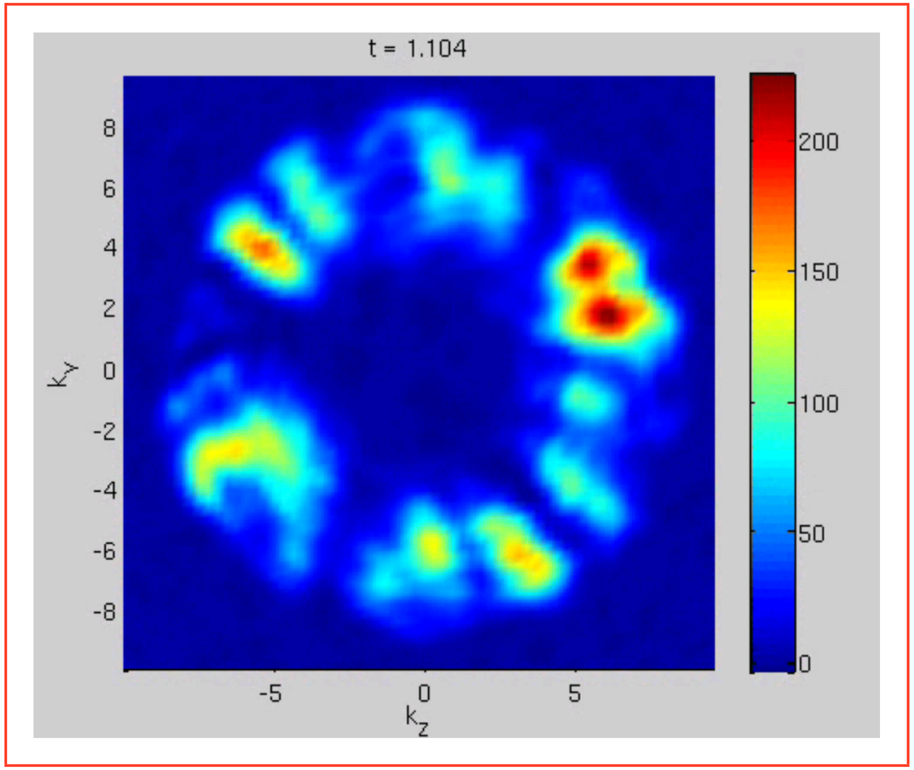
Why is the superradiance along the condensate small?

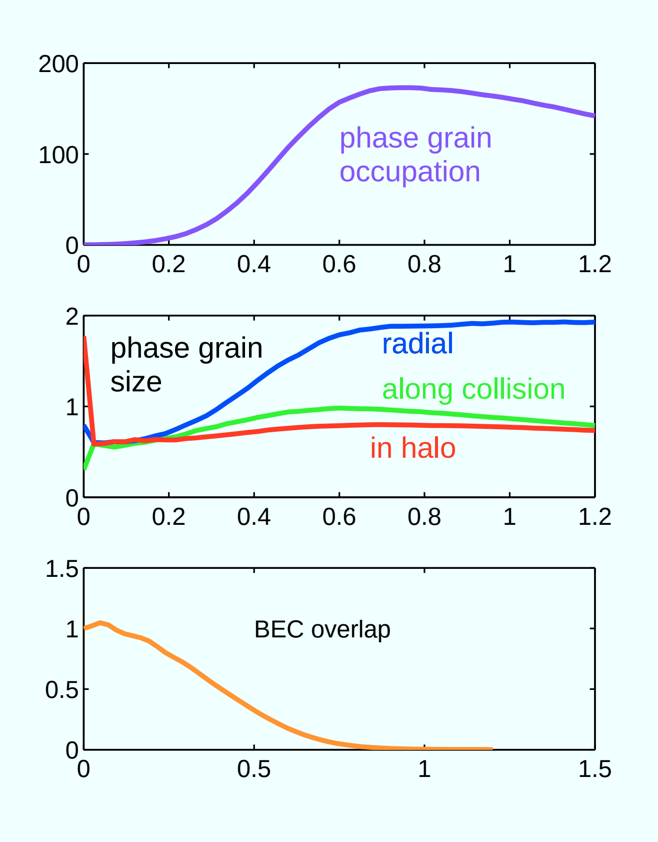
In standard photon-atom superradiance the atom can take on a lot of momentum but little energy
$\longrightarrow$ photon can be scattered at almost a right angle, and along long axis of BEC.



in atom-atom superradiance all particles have the same mass, changing the allowed geometry $\longrightarrow$ high-speed scattering along long axis of BEC not possible

Why do phase-grains elongate radially?





- At late times only lagging tails of BEC wavepackets remain overlapping
- These produce scattered atoms with lower velocity
- Because significant phase space density has already built up in the largest phase grains, they stimulate strong, coherent scattering out of the lagging BEC tails *preferentially next to the existing phase grain*.

Thank you :)