

Bogoliubov quantum dynamics at $T \geq 0$ (even without a condensate)

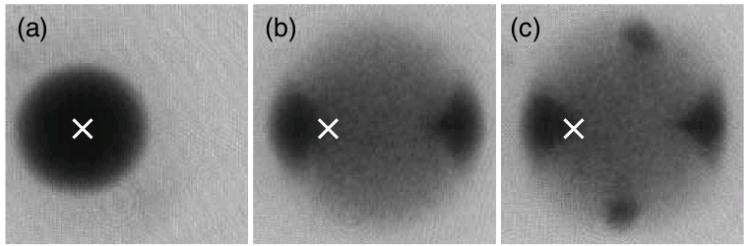
Piotr Deuar

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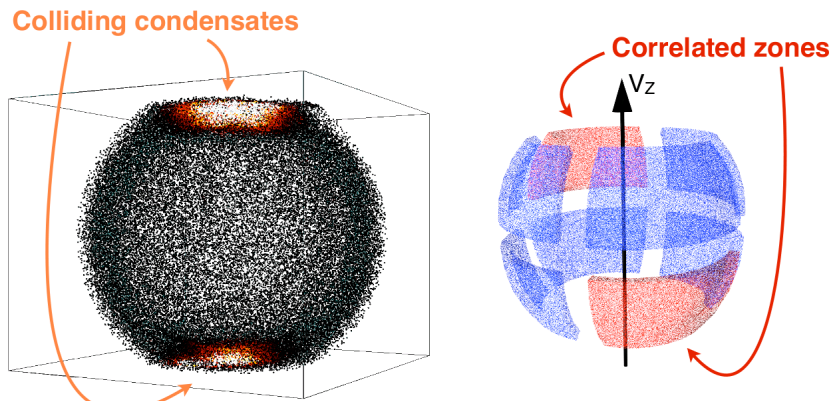


1. Supersonic pair creation
2. Palaiseau BEC collision experiment
3. Simulation of scattered pair dynamics at $T=0$
4. Quasicondensate $0 < T < T_c$
5. $T > T_c$?

Supersonic pair creation

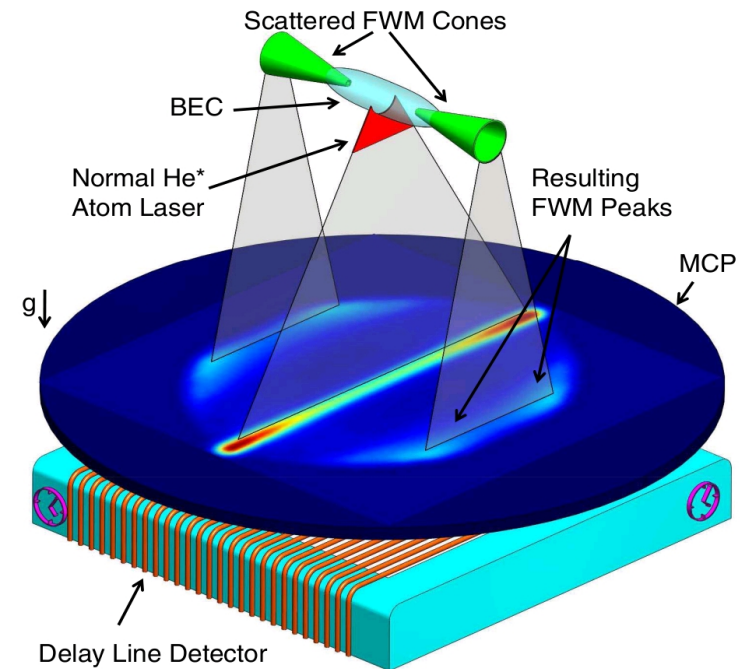


J.Vogels *et al*, PRL **89**, 020401 (2002)



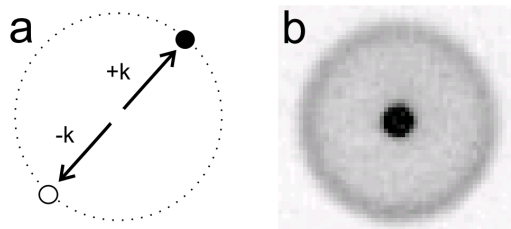
J-C.Jaskula *et al*, PRL **105**, 190402 (2010)

BEC Collisions

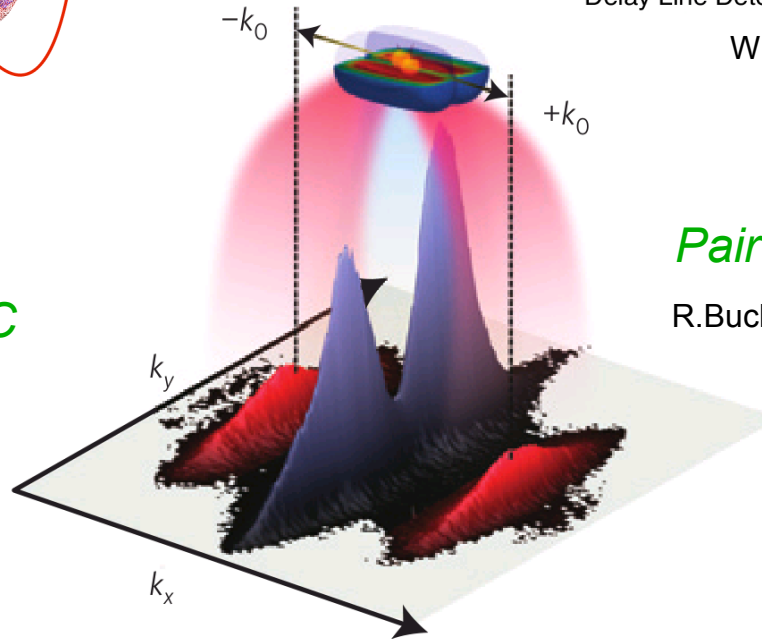


W. RuGway *et al*, PRL **107**, 075301 (2011)

Dissociation of molecular BEC



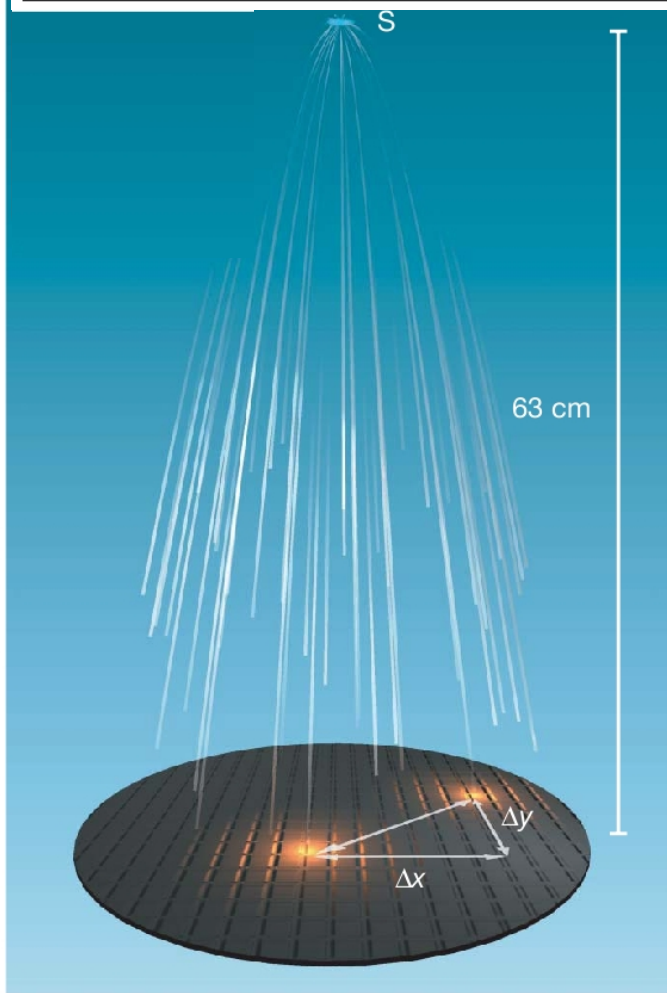
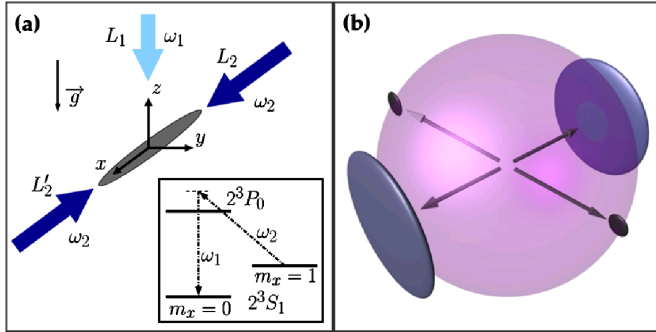
M. Greiner *et al*, PRL **94**, 110401 (2005)



Pair emission from a 1D gas

R.Bucker *et al*, Nature Phys. **7**, 608 (2011)

BEC collision – Palaiseau experiment



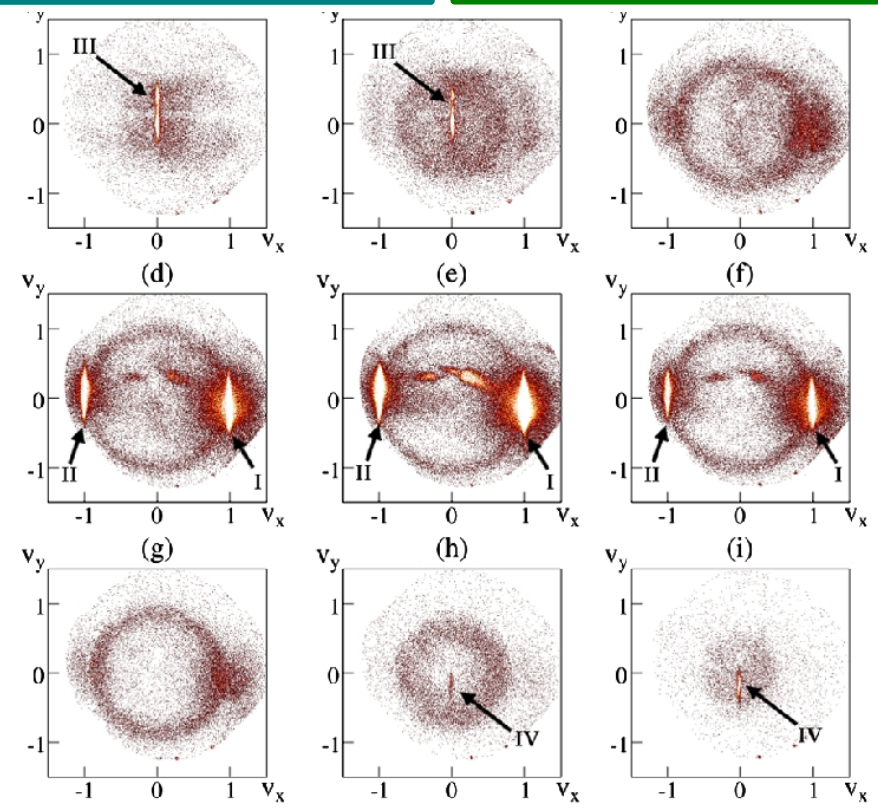
Experiment:

INSTITUT
d'OPTIQUE
GRADUATE SCHOOL

Chris Westbrook
 Denis Boiron
 Jean-Christophe Jaskula
 Valentina Krachmalnicoff
 Marie Bonneau
 Vanessa Leung
 Guthrie Partridge
 Alain Aspect

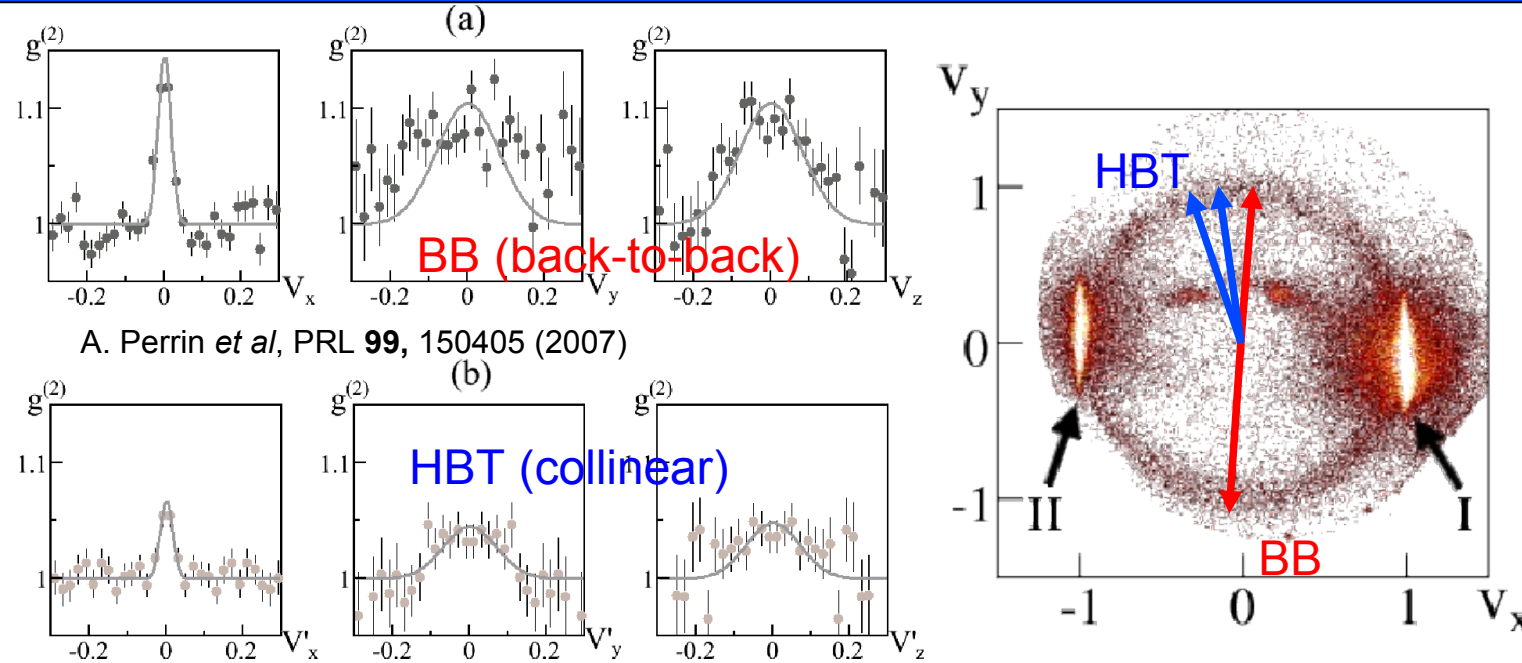
Theory:

Piotr Deuar (Warsaw)
Karen Kheruntsyan (Queensland)
Marek Trippenbach (Warsaw)
 Jan Chwedeńczuk
 Tomasz Wasak
 Paweł Ziń

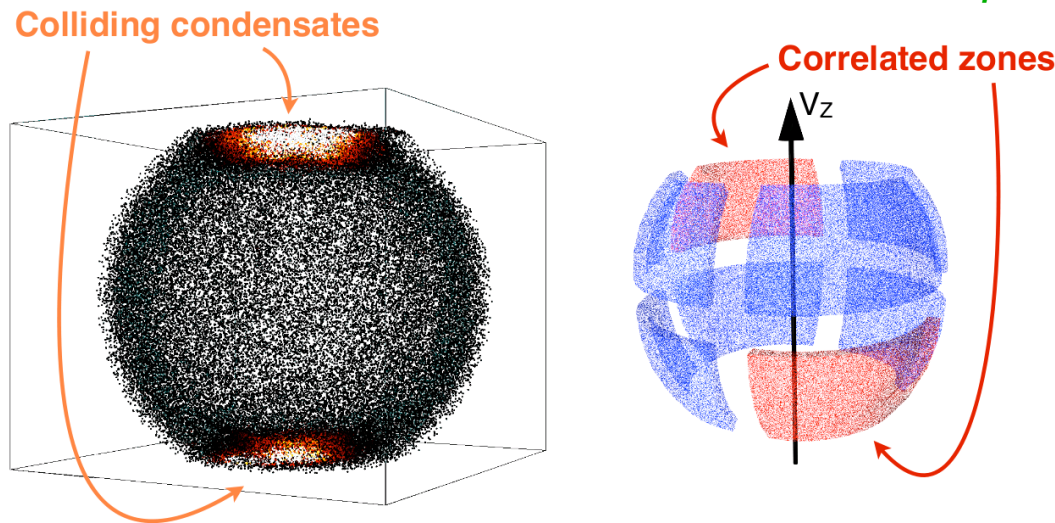


A. Perrin *et al*,
 PRL **99**, 150405
 (2007)

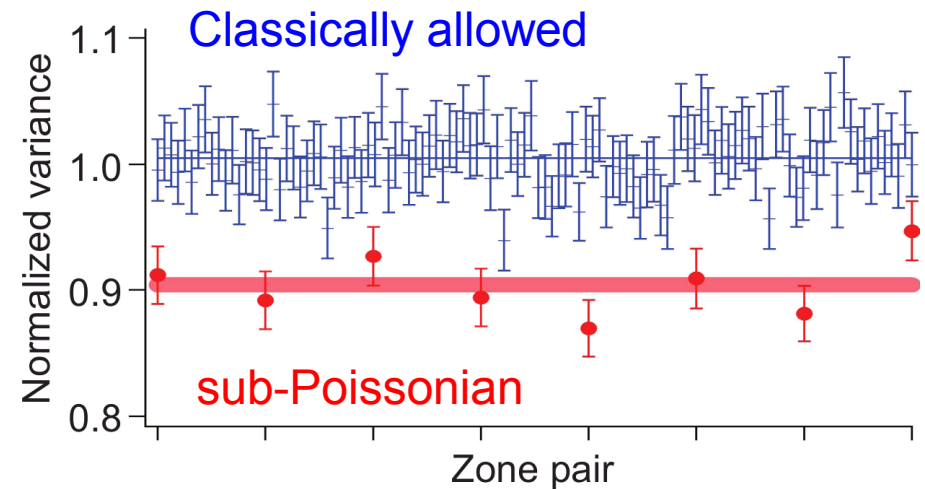
Pairing and density correlations



Squeezing of relative particle number between zones



J-C.Jaskula *et al*, PRL **105**, 190402 (2010)



Positive-P representation

$$\hat{H} = \int dx \left\{ \hat{\Psi}^\dagger(x) \left[V(x) - \frac{\hbar^2}{2m} \nabla^2 \right] \hat{\Psi}(x) + \frac{g}{2} \hat{\Psi}^\dagger(x)^2 \hat{\Psi}(x)^2 \right\}$$

$H_0(x)$

$$\hat{\rho} = \int P[\psi, \tilde{\psi}] |\psi\rangle \langle \tilde{\psi}| \mathcal{D}^2\psi(\vec{x}) \mathcal{D}^2\tilde{\psi}(\vec{x})$$

Probability distribution of
bra & ket coherent fields

$$\psi(x), \tilde{\psi}(x)$$

dynamics

$$i\hbar \frac{d\psi(x)}{dt} = \left\{ H_0(x) + g\tilde{\psi}^*(x)\psi(x) + \sqrt{i\hbar g} \xi(x, t) \right\} \psi(x)$$

$$i\hbar \frac{d\tilde{\psi}(x)}{dt} = \left\{ H_0(x) + g\psi^*(x)\tilde{\psi}(x) + \sqrt{i\hbar g} \tilde{\xi}(x, t) \right\} \tilde{\psi}(x)$$

Gaussian real white noise $\xi(x, t), \tilde{\xi}(x, t)$

However:

**intractable
after an
inconvenient
time**

$$\widehat{\Psi}(\mathbf{x}, t) = \phi(\mathbf{x}, t) + \widehat{\delta}(\mathbf{x}, t) \quad (\text{symmetry breaking version})$$

condensate

Bogoliubov fluctuation field – *MUST BE “small”*

Treat only $\widehat{\delta}(\mathbf{x}, t)$ using positive-P representation

$$i\hbar \frac{d\phi(x)}{dt} = [H_0(x) + g|\phi(x)|^2] \phi(x) \quad \text{Mean field}$$

$$i\hbar \frac{d\psi(x)}{dt} = \{H_0(x) + 2g|\phi(x)|^2\} \psi(x) + g\phi(x)^2 \widetilde{\psi}(x)^* + \sqrt{i\hbar g} \phi(x) \xi(x, t)$$

$$i\hbar \frac{d\widetilde{\psi}(x)}{dt} = \{H_0(x) + 2g|\phi(x)|^2\} \widetilde{\psi}(x) + g\phi(x)^2 \psi(x)^* + \sqrt{i\hbar g} \phi(x) \widetilde{\xi}(x, t)$$

Now equations are linear -----> no blow-up of noise :)

Can use plane wave basis ---> no diagonalizing of $10^6 \times 10^6$ matrices :)

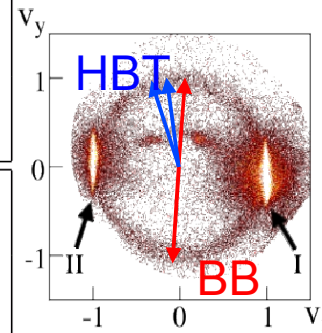
1st generation experiment:

good agreement

Second generation experiment: something suspicious

Pair correlations along long axis $g^{(2)}(\Delta k_z)$

	experiment		numerics	
	$\frac{\text{BB width}}{\text{HBT width}}$	$\frac{\text{BB height}}{\text{HBT height}}$	$\frac{\text{BB width}}{\text{HBT width}}$	$\frac{\text{BB height}}{\text{HBT height}}$
1st generation	1.1	2.1	1	2.2
2nd generation	18	0.5	1	2.2



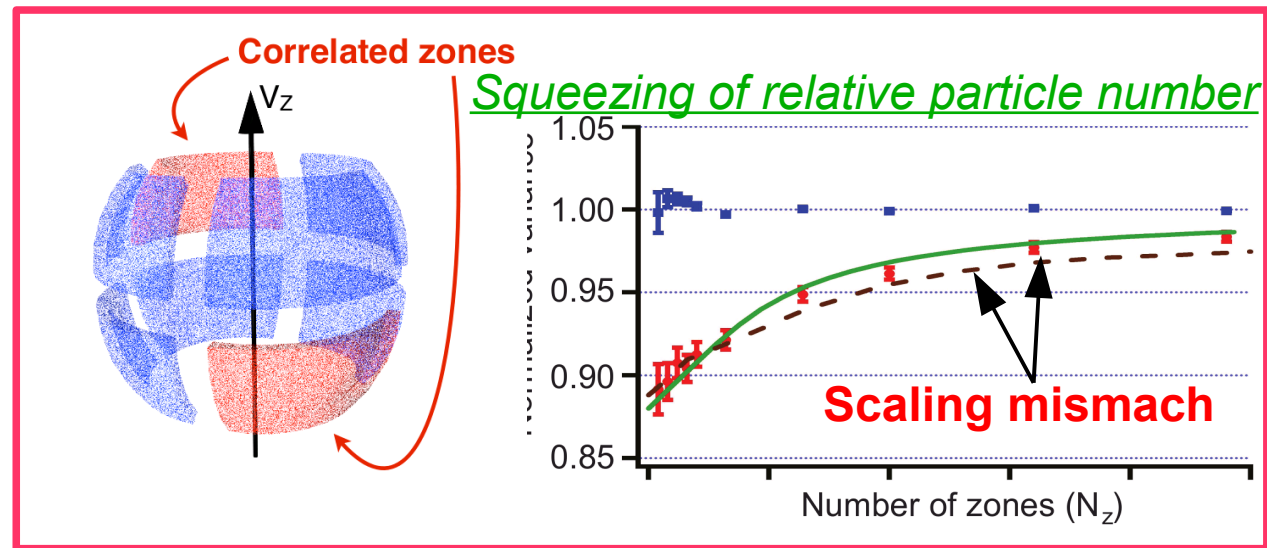
The suspect:

A quasicondensate

experiment	ω_z	ω_r	aspect
1st generation	47	1150	24.5
2nd generation	7.5	1500	200

← ~ elongated 3d bec

← Quasicondensate at our temperatures
condensate fraction ~5%



Classical field /PGPE/SGPE/... for quasiBEC

e.g. free space : plane wave basis

Full quantum field

$$\Psi(\mathbf{r}) = \sum_{\mathbf{k}} a_{\mathbf{k}} \frac{1}{\sqrt{V}} e^{i\mathbf{k}\mathbf{r}}$$

c-fields

$$\Phi(\mathbf{r}) = \sum_{|\mathbf{k}| \leq \mathbf{K}_{\max}} \alpha_{\mathbf{k}} \frac{1}{\sqrt{V}} e^{i\mathbf{k}\mathbf{r}}$$

Replace mode amplitude operators $a_{\mathbf{k}}$
with complex number amplitudes $\alpha_{\mathbf{k}}$

Thermal initial state:

- $|\alpha_{\mathbf{k}}|^2$ Distributed according to Bose-Einstein distribution
- Phase of $\alpha_{\mathbf{k}}$ is random
- Use many realizations to get thermal ensemble

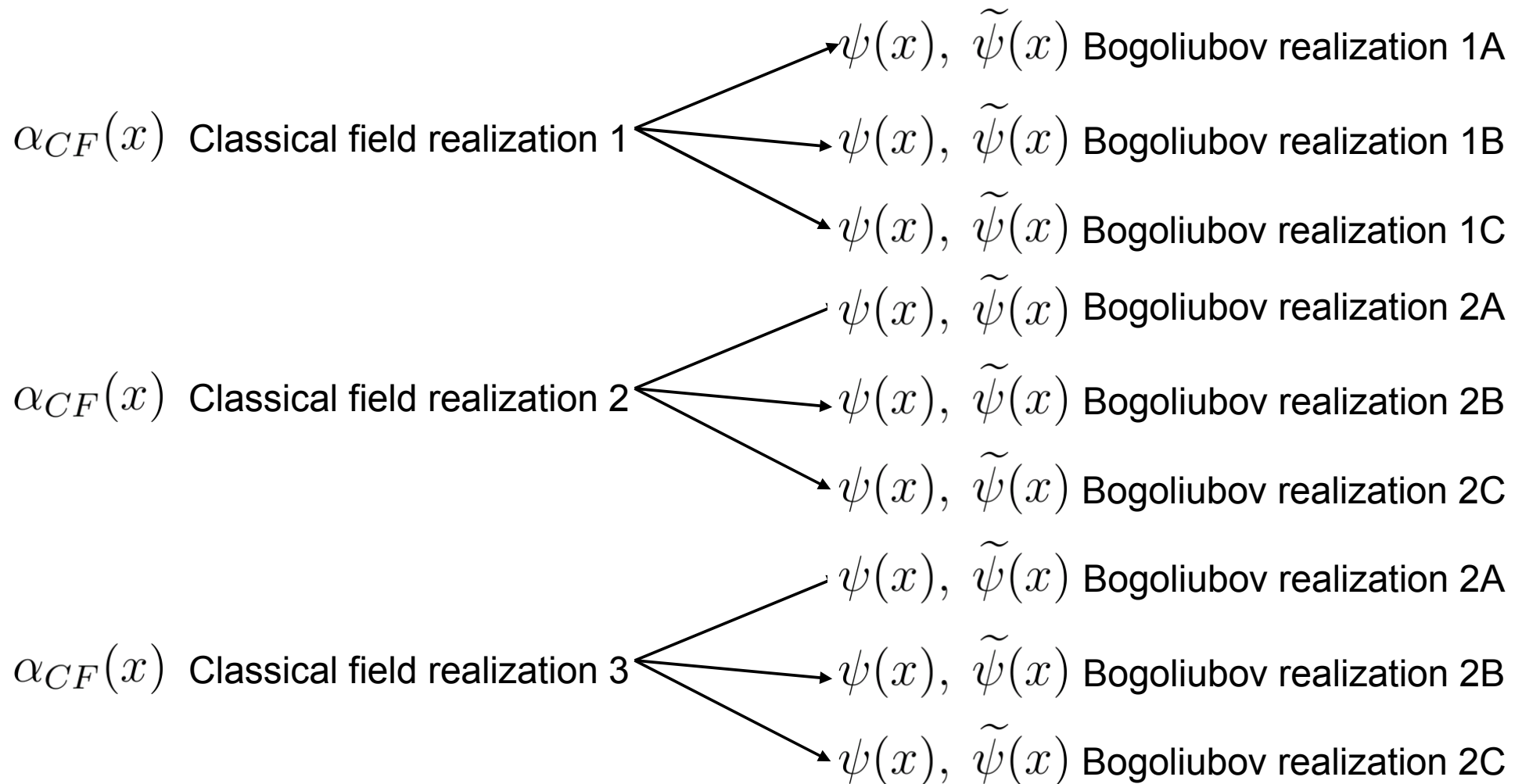
Used to model our quasicondensate

Useful papers: [D. Petrov et al, PRL 87, 050404 \(2001\)](#)
[M. Brewczyk et al, J. Phys B 40, R1 \(2007\)](#)
[P. Blakie et al. Adv. Phys. 57, 363 \(2008\)](#)
[N. Proukakis, B. Jackson, J. Phys A 41, 203002 \(2008\)](#)

First Trick: each realization is independent

INITIAL STATE $t=0$

Treat $\alpha_{CF}(x)$ as condensates
 $\phi(x) = \alpha_{CF}(x)$



2nd Trick: each realization is independent

INITIAL STATE $t=0$

Treat $\alpha_{CF}(x)$ as condensates
 $\phi(x) = \alpha_{CF}(x)$

$\alpha_{CF}(x)$ Classical field realization 1 $\longrightarrow \psi(x), \tilde{\psi}(x)$ Bogoliubov realization 1

$\alpha_{CF}(x)$ Classical field realization 2 $\longrightarrow \psi(x), \tilde{\psi}(x)$ Bogoliubov realization 2

$\alpha_{CF}(x)$ Classical field realization 3 $\longrightarrow \psi(x), \tilde{\psi}(x)$ Bogoliubov realization 2

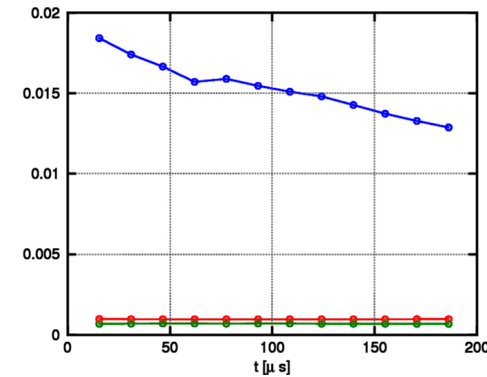
Look mum, no condensate! ($n_0 \sim 0.05$)



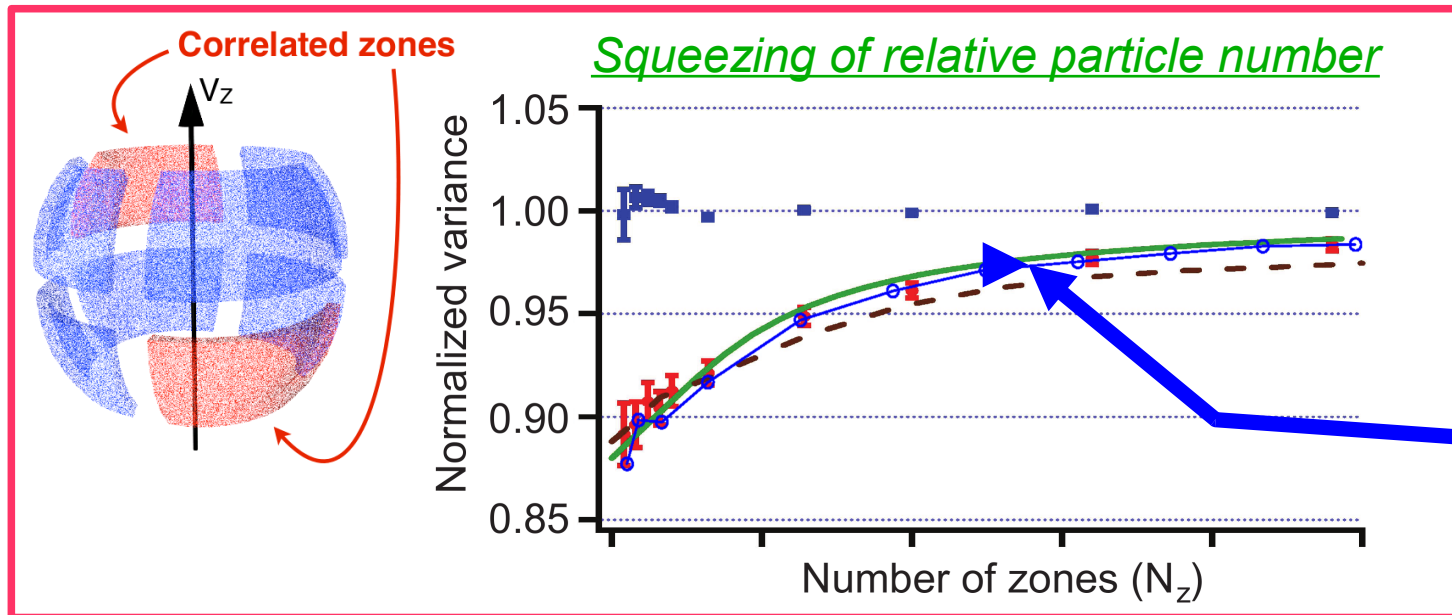
Look mum, no hands!

Pair correlations $g^{(2)}(\Delta k_z)$

	experiment		numerics	
	$\frac{\text{BB width}}{\text{HBT width}}$	$\frac{\text{BB height}}{\text{HBT height}}$	$\frac{\text{BB width}}{\text{HBT width}}$	$\frac{\text{BB height}}{\text{HBT height}}$
1st generation	1.1	2.1	1	2.2
2nd generation	18	0.5	16	0.3



Visible effect of Quasicondensate on pairing



Surprise!
improved match

Caveats

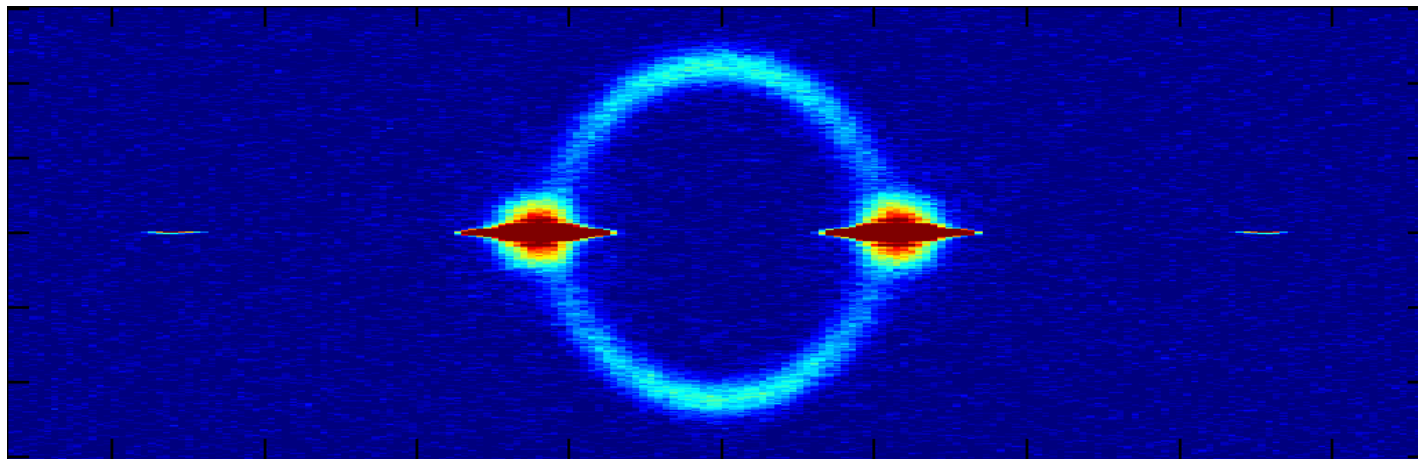
Caveat 1: need *additional* $t > 0$ depletion to be small

(initial depletion is apparently irrelevant)

Caveat 2: don't look at the condensate regions

(plane waves are not orthogonal to the condensate)

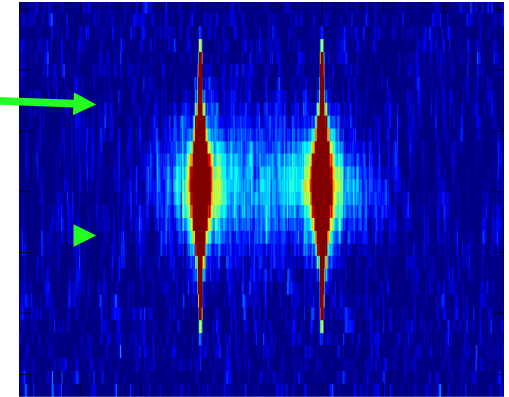
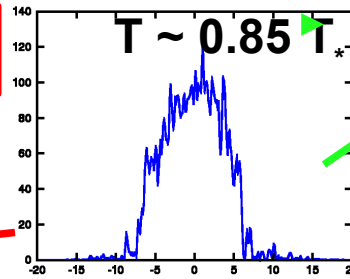
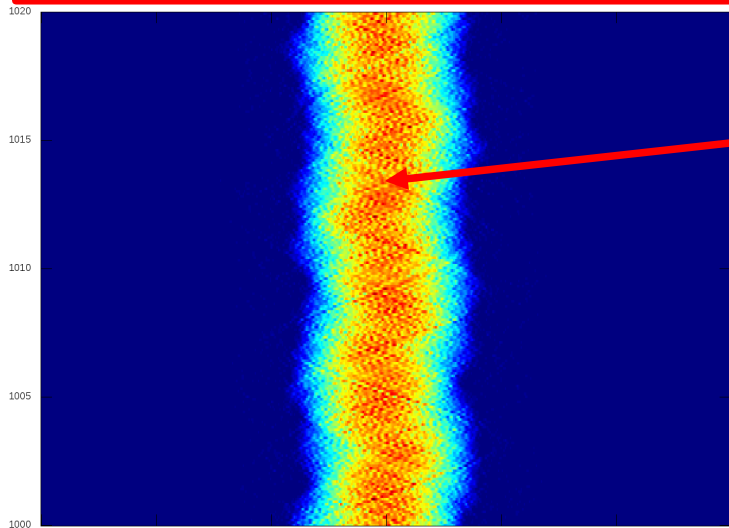
-----> mix-up of Bogoliubov modes and condensate there



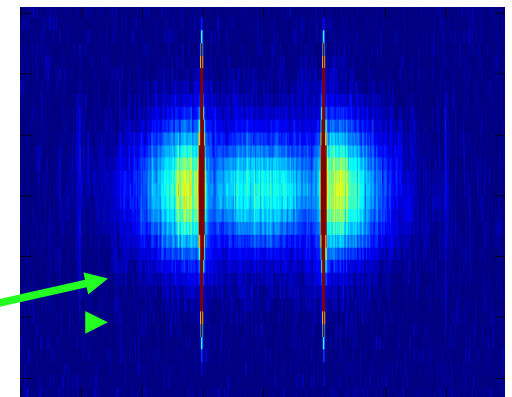
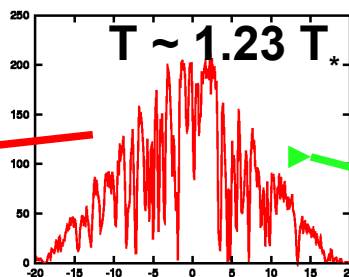
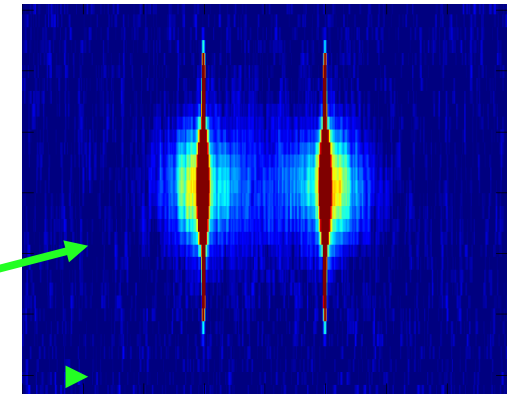
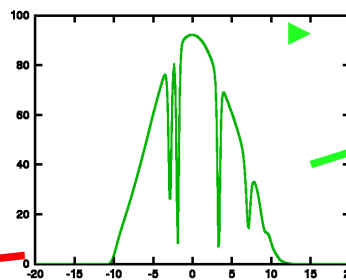
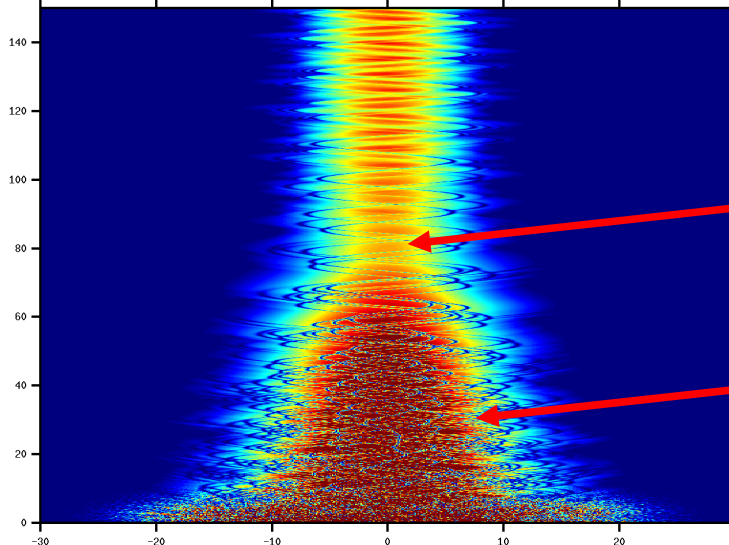
Even less condensate?

Classical field simulation of
1D evaporative cooling

See poster: Emilia Witkowska



E. Witkowska *et al*, PRL **106**, 135301 (2011)



RELEASE 1D CONFINEMENT AND COLLIDE

Summary

- Quantitative simulation of dynamics of pair scattering
With positive- P treatment of Bogoliubov field
- Bogoliubov dynamics made tractable for large systems
By use of plane wave modes + stochastic noise
- Treat classical field realizations as condensates
Apparently works even with no true condensate
- Need to work on number-conserving Bogoliubov version