Tractable Bogoliubov dynamics of non-equilibrium systems in a positive-P representation

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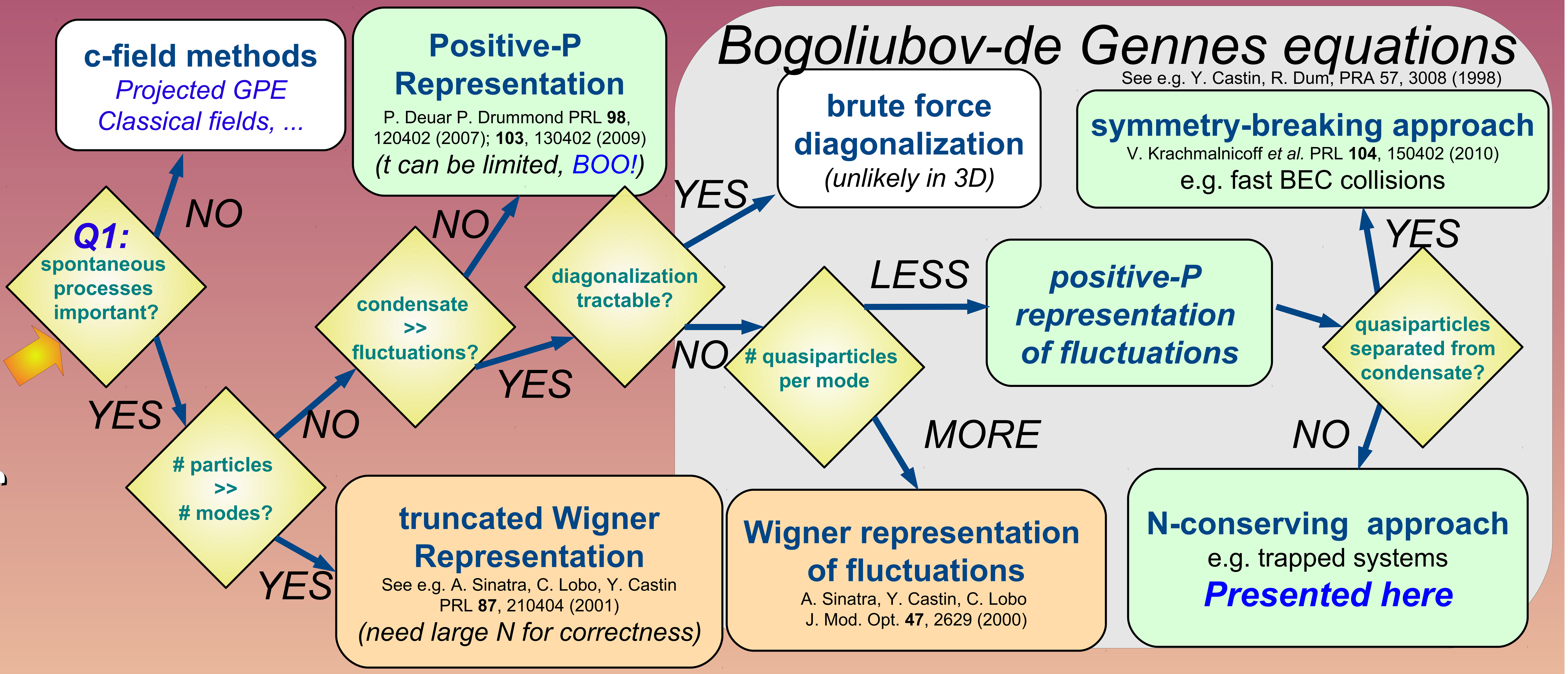
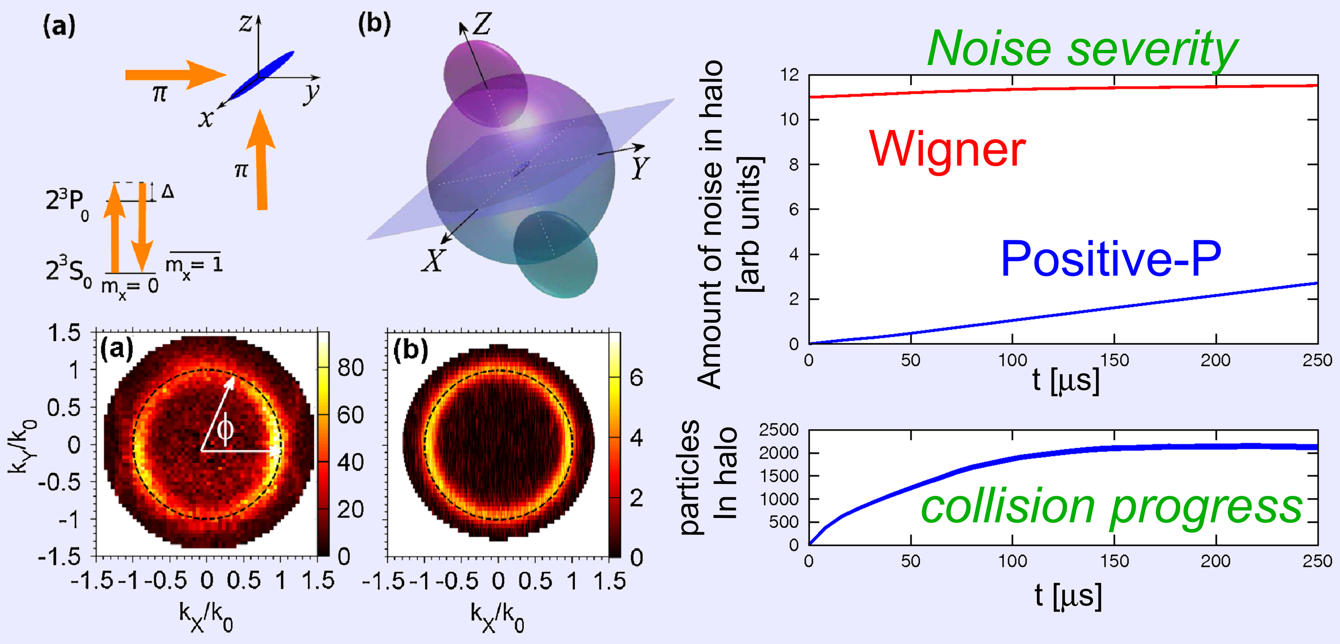
boson dynamics in 2D & 3D beyond mean-field



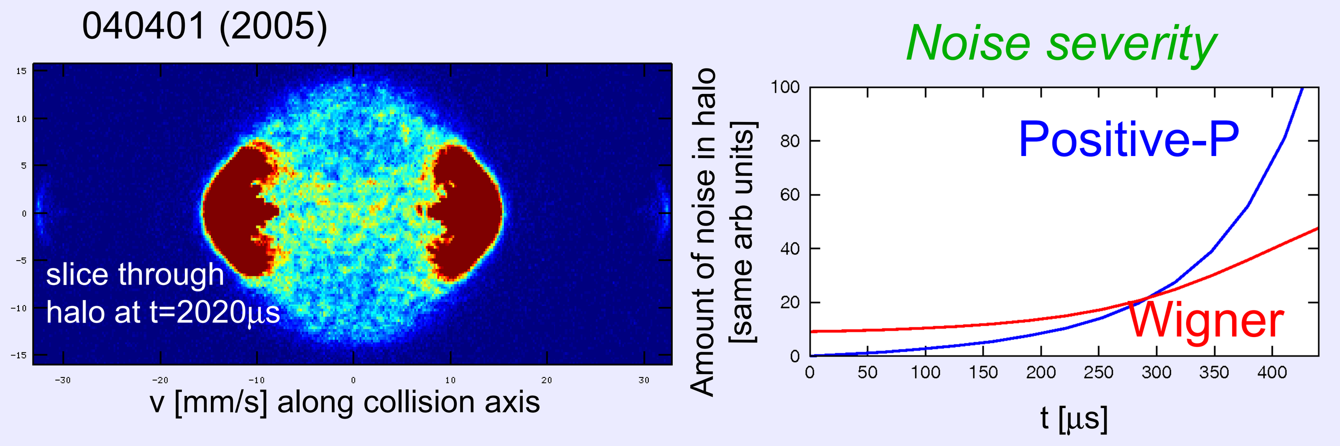
Wigner vs positive-P noise properties in treatment of Bogoliubov theory

Some examples of BEC collisions treated by Bogoliubov theory

Few particle case: He* collision as per the Palaiseau experiment V. Krachmalnicoff *et al.*, PRL **104**, 150402 (2010)

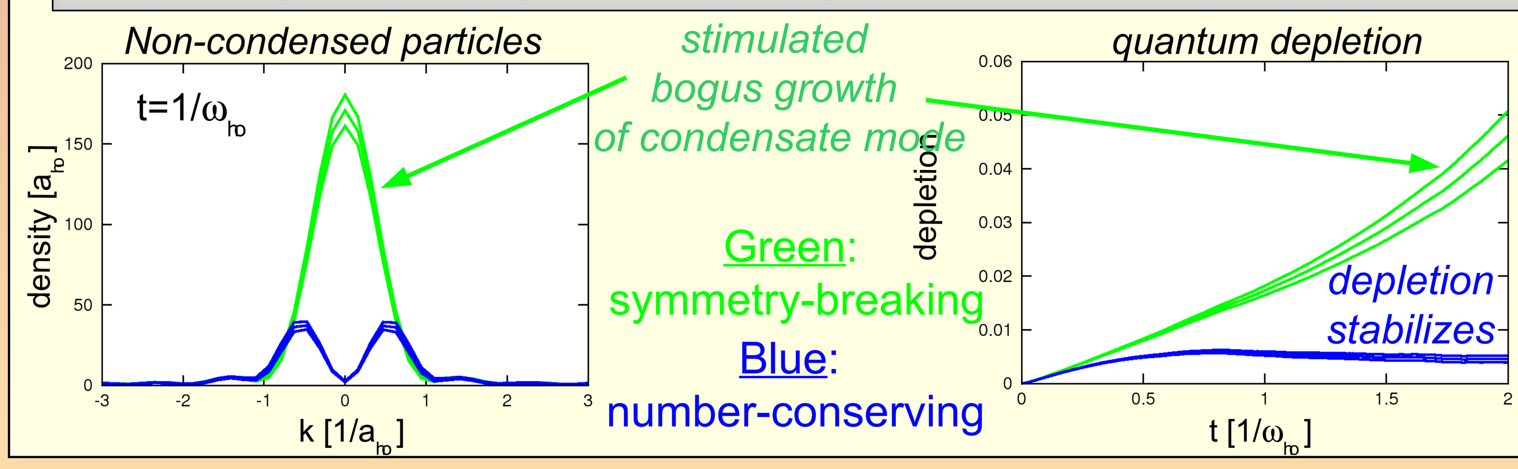


Case with more particles: ^{23}Na collision as per calculations in A.A. Norrie, R.J. Ballagh, C.W. Gardiner PRL **94**, 040401 (2005)



Conclusion:
modes > # particles : positive-P better
modes < # particles : Wigner better

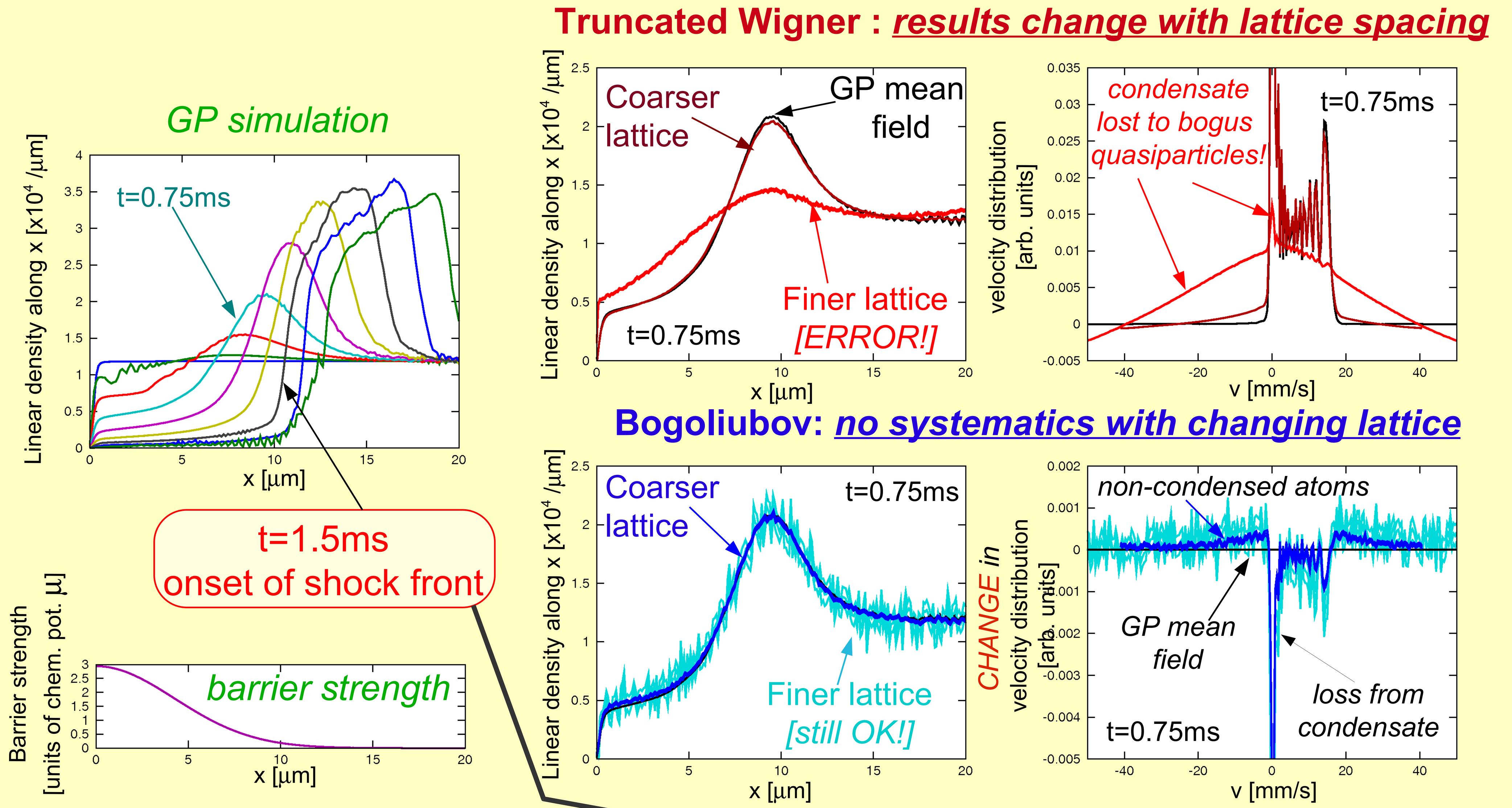
Symmetry-breaking vs number conserving Bogoliubov



Time-evolution of a 1D trapped system like the Wigner-Bogoliubov test case in A. Sinatra, Y. Castin, C. Lobo, J. Mod. Opt. **47**, 2629 (2000)

Truncated Wigner vs Bogoliubov

Example: *Formation of a shock wave in a BEC after insertion of a repulsive barrier at t=0*
As per the WSU experiment J.J. Chang, P. Engels, M.A. Hoefer, PRL **101**, 170404 (2008)



Equations (number-conserving Bogoliubov theory using positive-P representation)

(1) We start from the positive-P formulation of the "exact" / full boson field evolution

$$i\hbar\frac{\partial\Psi(x)}{\partial t} = \left\{H_0(x) + g\Psi(x)\widetilde{\Psi}(x)^* + \sqrt{ig}\,\xi(x)\right\}\Psi(x)$$
$$i\hbar\frac{\partial\widetilde{\Psi}(x)}{\partial t} = \left\{H_0(x) + g\widetilde{\Psi}(x)\Psi(x)^* + \sqrt{ig}\,\widetilde{\xi}(x)\right\}\widetilde{\Psi}(x)$$

Real white noise fields $\xi(x,t), \widetilde{\xi}(x,t)$ obey

$$\langle\xi(x,t)\widetilde{\xi}(x',t')\rangle = \delta(x-x')\delta(t-t')$$

(2) Isolate condensate and incoherent parts by defining new variables

$$\Psi(x) = (1-\varepsilon)\phi(x) + \alpha(x)$$
$$\widetilde{\Psi}(x) = (1-\widetilde{\varepsilon})\phi(x) + \widetilde{\alpha}(x)$$

(3) Remove terms high order in ε and/or quasiparticle fields α ~ O(φε) as per e.g. Y.Castin, R. Dum, PRA **57**, 3008 (1998)

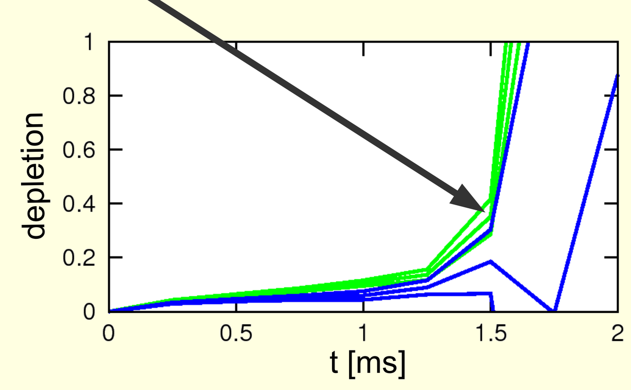
(4) Resulting evolution equations:

$$i\hbar\frac{\partial\phi(x)}{\partial t} = \left\{H_0(x) + g|\phi(x)|^2\right\}\phi(x)$$
$$i\hbar\frac{\partial\alpha(x)}{\partial t} = \left\{H_0(x) + g|\phi(x)|^2\right\}\alpha(x) + Q\left[v(x)\right]$$
$$i\hbar\frac{\partial\widetilde{\alpha}(x)}{\partial t} = \left\{H_0(x) + g|\phi(x)|^2\right\}\widetilde{\alpha}(x) + Q\left[\widetilde{v}(x)\right]$$
$$i\hbar\frac{\partial\varepsilon}{\partial t} = \frac{1}{N}\int dx\,\phi(x)^*v(x) = \langle\phi|v\rangle$$
$$i\hbar\frac{\partial\widetilde{\varepsilon}}{\partial t} = \frac{1}{N}\int dx\,\phi(x)^*\widetilde{v}(x) = \langle\phi|\widetilde{v}\rangle$$
$$v(x) = g\phi(x)^2\widetilde{\alpha}^*(x) + g|\phi(x)|^2\alpha(x) + \sqrt{ig}\,\phi(x)\xi(x) - g(\varepsilon+\widetilde{\varepsilon}^*)|\phi(x)|^2\phi(x)$$
$$\widetilde{v}(x) = g\phi(x)^2\alpha(x)^* + g|\phi(x)|^2\widetilde{\alpha}(x) + \sqrt{ig}\,\phi(x)\widetilde{\xi}(x) - g(\varepsilon^*+\widetilde{\varepsilon})|\phi(x)|^2\phi(x)$$
$$Q[v(x)] = v(x) - \frac{\phi(x)}{N}\int dy\,\phi(y)^*v(y) = \left[1-|\phi\rangle\langle\phi|\right]|v\rangle$$

(5) Observables take into account the depletion

$$\langle\widehat{n}(x)\rangle = (1-\delta N)|\psi(x)|^2 + \langle\mathrm{Re}\left[\alpha(x)\widetilde{\alpha}(x)^*\right]\rangle \qquad \delta N = \int dx\,\langle\mathrm{Re}\left[\alpha(x)\widetilde{\alpha}(x)^*\right]\rangle = \langle\widetilde{\alpha}|\alpha\rangle$$

Non-equilibrium Bogoliubov caveats



The suspect:
Violent changes in the condensate as it evolves "underneath" the quasi-particle field leads to large depletions in an attempt to compensate?

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