The superfluidity of dipolar Fermi gases

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Outline

superfluidity of dipolar fermions

1. Motivation

2. The simplest model

3. “Phase diagram” of low energy collective excitations

4. The new “aligned” superfluid

Motivation:

- Superfluidity predicted for single-species Fermi gas of dipoles
  Baranov et al. PRA **66**, 013606 (2002)
  (So far, superfluidity of fermion gases is of the two-species simple BCS sort)

- Recent progress in cooling of heteronuclear molecules with large dipole moment
  K.-K. Ni et al., arXiv:0808.2963
  $\Longrightarrow$ Experimental realisations of the new superfluid are possible

- Symmetry of BCS-like gap is unlike in the standard s-wave BCS gas
  $\Longrightarrow$ what effect does this have?

- Node structure like e.g. in *polar phase* of $^3\mathrm{He}$. (Never experimentally realized)
  Aoyama & Ikeda, PRB 73, 060504 (2006), Elbs *et al.*, arXiv:0707.3544
  $\Longrightarrow$ New effects not seen in condensed matter?

comparison to standard BCS

dipole–dipole potential

$$V(r,\theta) = \frac{d^2}{r^3}(1 - 3\cos^2\theta)$$

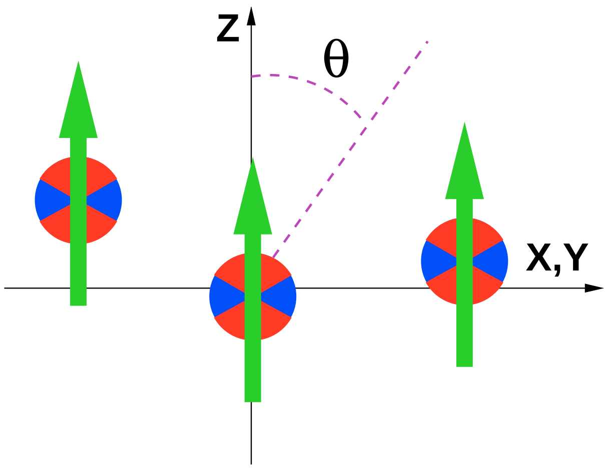
- long range interaction
  $\longrightarrow$ Needs 1 spin component
- always partly attractive
  BCS pairing if *polarised*
- Energy gap has nodes
- *Anisotropic*

contact s-wave $\uparrow\downarrow$ potential

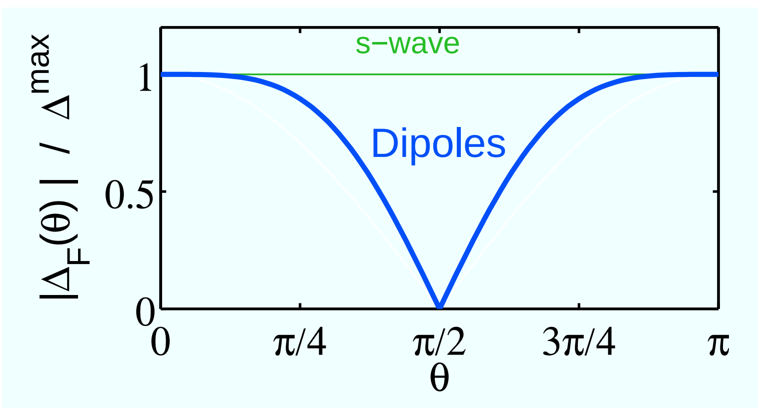
$$V(r) = g\,\delta(r)$$

- short range interaction
  $\longrightarrow$ Needs 2 spin components
  (Pauli blocking)
- arttractive or repulsive
  BCS pairing only if $a_s < 0$
- Energy gap always > 0
- *Isotropic*

Uniform 3D dipolar Fermi gas



BCS pairing gap on Fermi surface has zeros



Baranov et al. PRA **66**, 013606 (2002)

- Uniform & 3D
- Cold: $T < T_c^{BCS}$
- static external field (E or B)
  $\Longrightarrow$ full polarisation
- single-species (spin polarised)
- dilute
  $\Longrightarrow$ Energy dominated by Fermi sea
  $\Longrightarrow$ BCS-like model
- Cooper pairs are cheap near gap zero.

Experimental prospects for superfluidity

$$T_c^{\mathrm{BCS}} = 1.44\, E_F \exp\left(-\frac{\pi}{2|a_D|k_F}\right); \qquad |a_D| = \frac{2m|\mathbf{d}|^2}{\pi^2\hbar^2}$$

Baranov et al., PRA **66**, 013606 (2002)

Comparison to RECENT VALUES

K.-K. Ni et al., arXiv:0808.2963

$$\begin{aligned} |\mathbf{d}| &= 0.566\,\mathrm{D} \\ n &\sim 10^{12}/\mathrm{cm}^3 \end{aligned} \Longrightarrow \boxed{T_c^{\mathrm{BCS}} \approx 1.6\mathrm{nK}}$$

small! :-(

However, with $10\times$ more density (plausible?), one would have

$$\boxed{T_c^{\mathrm{BCS}} \approx 40\mathrm{nK} \sim T_F \qquad :-)}$$

Hamiltonian

$$\widehat{H} = \text{K.E.} + \frac{1}{2}\int d^3x\, d^3y \left\{ \widehat{\Psi}_x^\dagger \widehat{\Psi}_x\, V_D(x-y)\, \widehat{\Psi}_y^\dagger \widehat{\Psi}_y \right\}$$

Resulting effective BCS mean-field Hamiltonian

$$\widehat{H}_{\text{eff}} = \frac{1}{2}\int d^3x\, d^3y \Big\{ \frac{\hbar^2}{m}\, \widehat{\Psi}_x^\dagger\, \nabla^2 \widehat{\Psi}_x\, \delta(x-y) \quad \textit{Kinetic}$$

$$\Delta^*(x-y)\, \widehat{\Psi}_x \widehat{\Psi}_y - \Delta(x-y)\, \widehat{\Psi}_x^\dagger \widehat{\Psi}_y^\dagger \quad \textit{BCS}$$

$$W(x-y)\widehat{\Psi}_x^\dagger \widehat{\Psi}_y \Big\} \quad \textit{Hartree}$$

Gap consistency equation

$$\Delta(x-y) = V_D(x-y) \left\langle \widehat{\Psi}_x \widehat{\Psi}_y \right\rangle_{\text{eff}}$$

Low energy superfluidity

Phase perturbations of the ground state order parameter

$$\Delta_0(x-y) \quad \longrightarrow \quad \Delta(x,y) = \Delta_0(x-y)\, e^{2i\phi(x,t)}$$

Goldstone mode

Assumptions:

- Low energy $\quad (\hbar\omega \ll \Delta_0^{\mathrm{max}} \sim T_c)$
- Phase perturbations only (amplitude perturbations are gapped)
- Low ω $\quad \Longrightarrow \quad$ long wavelength ($k \ll k_F$)
  $\Longrightarrow$ insensitive to small-scale of $|x-y|$ $\quad \Longrightarrow \quad$ $\phi \approx \phi(x$ only $)$
- Weak perturbation $\quad \Longrightarrow \quad$ lowest order in ϕ

Bogoliubov–de Gennes Equations

Single-particle wavefunctions $U_\nu(\mathbf{r},t)$ and $V_\nu(\mathbf{r},t)$ from a Bogoliubov diagonalization

$$\widehat{\Psi}(\mathbf{r}) = \sum_\nu \left[U_\nu(\mathbf{r})\widehat{b}_\nu + V_\nu(\mathbf{r})^* \, \widehat{b}_\nu^\dagger\right]$$

obey BDG equations

$$i\hbar\frac{\partial}{\partial t}\left[\begin{array}{c} U_\nu(\mathbf{r}) \\ V_\nu(\mathbf{r}) \end{array}\right] = H_0(\mathbf{r})\left[\begin{array}{c} U_\nu(\mathbf{r}) \\ -V_\nu(\mathbf{r}) \end{array}\right] - \int d^3\mathbf{r}'\left[\begin{array}{c} \Delta(\mathbf{r},\mathbf{r}')V_\nu(\mathbf{r}') \\ \Delta^*(\mathbf{r},\mathbf{r}')U_\nu(\mathbf{r}') \end{array}\right]$$

Expand them in terms of the uniform-gas wavefunctions $U^0(\mathbf{r})$ and $V^0(\mathbf{r})$ and coefficients $C^{(\eta)} \sim O(\phi)$

$$\left[\begin{array}{c} U_\nu(\mathbf{r}) \\ V_\nu(\mathbf{r}) \end{array}\right] = \sum_j \left\{(\delta_{j\nu} + C_{j\nu}^{(1)})\left[\begin{array}{c} U_\nu^0(\mathbf{r}) \\ V_\nu^0(\mathbf{r}) \end{array}\right] + C_{j\nu}^{(2)}\left[\begin{array}{c} V_\nu^0(\mathbf{r})^* \\ -U_\nu^0(\mathbf{r})^* \end{array}\right]\right\},$$

find $C^{(\mu)}$ from BDG equation, and substitute it all into Gap equation, which must be satisfied up to $O(\phi)$.

$$\Delta(\mathbf{r},\mathbf{r}') = \frac{V_D(\mathbf{r}-\mathbf{r}')}{2}\sum_\nu \tanh\left(\frac{E_\nu}{2k_BT}\right)\left[U_\nu(\mathbf{r})V_\nu^*(\mathbf{r}') - U_\nu(\mathbf{r}')V_\nu^*(\mathbf{r})\right]$$

Fun #1:
Full consistency equation in k-space

$$-\frac{\phi_{\mathbf{k}}\Delta^0_{\mathbf{M}}\tau^0_{\mathbf{M}}}{2E^0_{\mathbf{M}}} = \frac{\phi_{\mathbf{k}}\Delta^0_{\mathbf{M}}}{4E^0_{\mathbf{m}}E^0_{\mathbf{n}}}\left\{\left(\frac{\tau^0_{\mathbf{n}}-\tau^0_{\mathbf{m}}}{2}\right)\left[\frac{(E^0_{\mathbf{n}}+\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}-\varepsilon_{\mathbf{m}})+\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega-E^0_{\mathbf{n}}+E^0_{\mathbf{m}}+i0}-\frac{(E^0_{\mathbf{n}}-\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}+\varepsilon_{\mathbf{m}})+\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega+E^0_{\mathbf{n}}-E^0_{\mathbf{m}}+i0}\right]\right.$$
$$\left.+\tau^0_{\mathbf{n}}\left[\frac{(E^0_{\mathbf{n}}+\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}+\varepsilon_{\mathbf{m}})-\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega-E^0_{\mathbf{n}}-E^0_{\mathbf{m}}+i0}\right]-\tau^0_{\mathbf{m}}\left[\frac{(E^0_{\mathbf{n}}-\varepsilon_{\mathbf{n}})(E^0_{\mathbf{m}}-\varepsilon_{\mathbf{m}})-\Delta^0_{\mathbf{n}}\Delta^0_{\mathbf{m}}}{\hbar\omega+E^0_{\mathbf{n}}+E^0_{\mathbf{m}}+i0}\right]\right\}.$$

where $\mathbf{n}=\mathbf{M}+\mathbf{k}/2$, $\mathbf{m}=-\mathbf{M}+\mathbf{k}/2$,
$\varepsilon_{\mathbf{k}}=\hbar^2k^2/2m-E_F$, $E^0_{\mathbf{k}}=\sqrt{\varepsilon^2_{\mathbf{k}}+(\Delta^0_{\mathbf{k}})^2}$, and $\tau^0_{\mathbf{k}}=\tanh(E^0_{\mathbf{k}}/2T)$

$\Delta^0_{\mathbf{n}}=\sin\left(\frac{\pi}{2}\cos\theta_{\mathbf{n}}\right)$ on Fermi surface.

- LONG wavelength $\mathbf{k}$, SHORT wavelength $\mathbf{M}\sim 1/k_F$.

$\Longrightarrow$ $\boxed{\omega(\mathbf{k},\mathbf{M})}$. What can you do with THAT?

Effective consistency equation

1. Want to obtain results only in the observable long wavelength $\mathbf{k}$

   NOT in $\mathbf{M} \sim 1/k_F$

2. Integrate out short-wavelength degrees of freedom in the Lagrangian

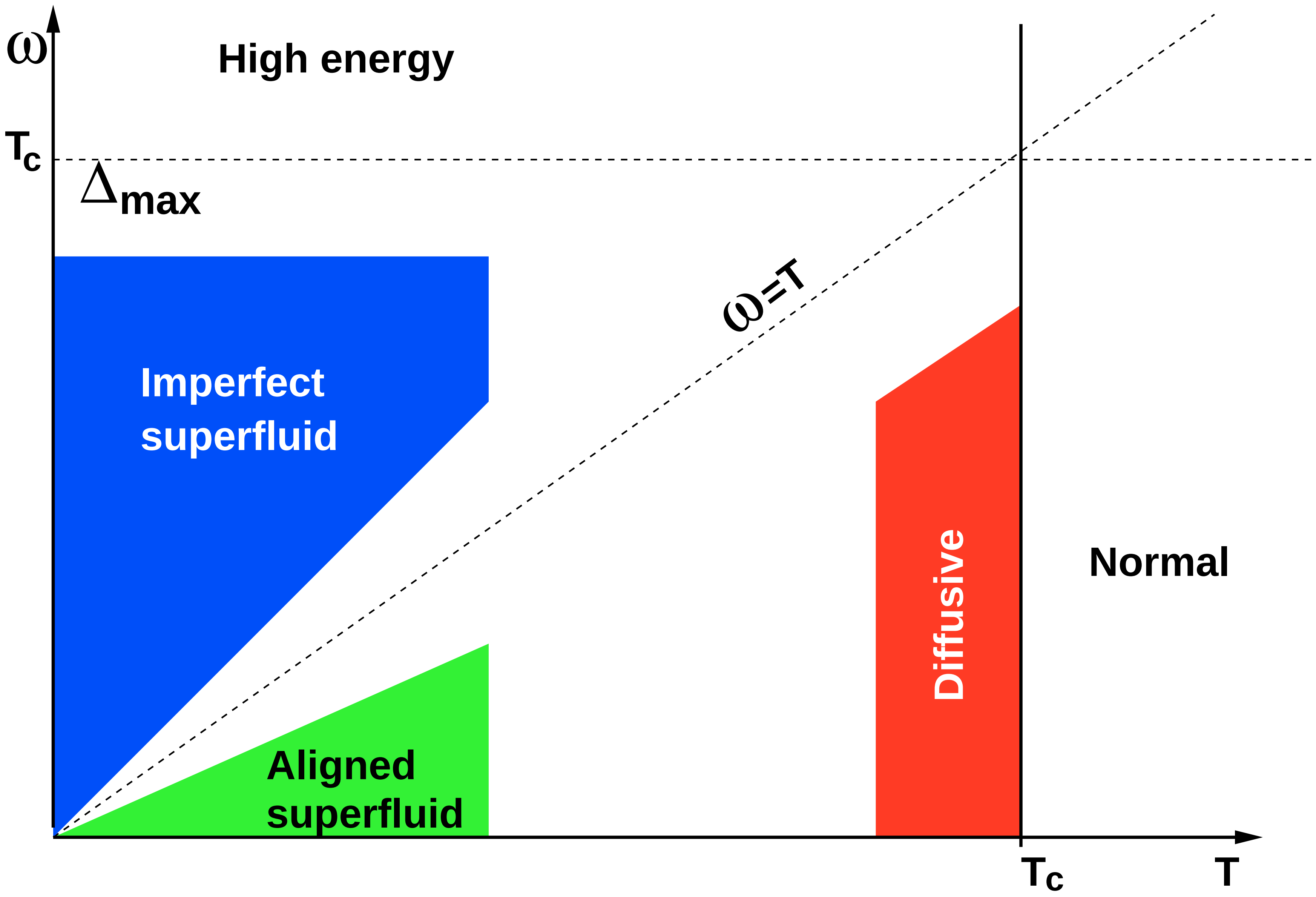
3. Effective lagrangian $\mathcal{L}(\omega, \mathbf{k})$ only.

4. Effective consistency equation for $\omega(\mathbf{k})$ is a weighted integral

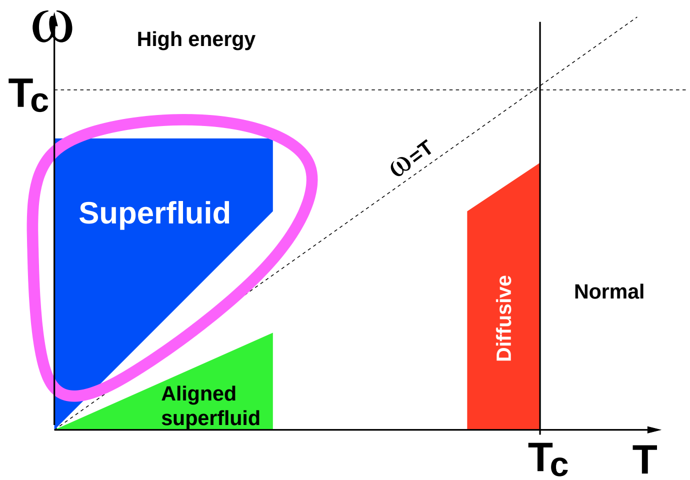
$$\int d\mathbf{M}\Delta^0_{\mathbf{M}}(\cdots) = 0$$

of the previous microscopic equation $(\cdots) = 0$.

Collective excitation regimes



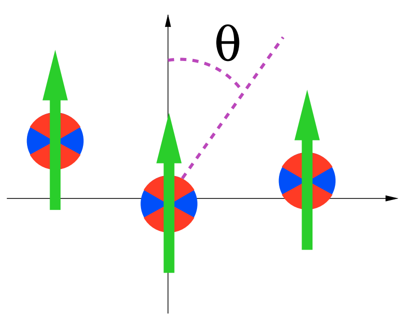
Imperfect BCS superfluid

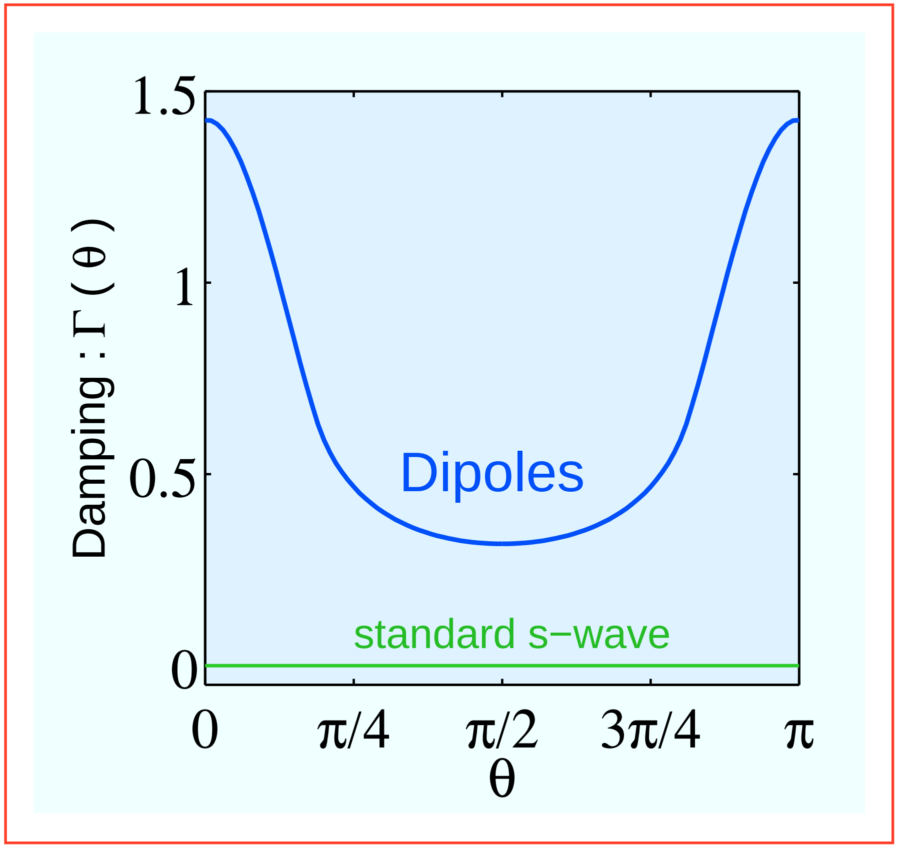


$$\omega = \left(\frac{v_F}{\sqrt{3}}\right)\ k\ \left\{1 - i\left(\frac{\hbar\omega_{\mathrm{Bog}}}{\Delta_{\max}}\right)\ \Gamma(\theta)\right\}$$

Beliaev process:

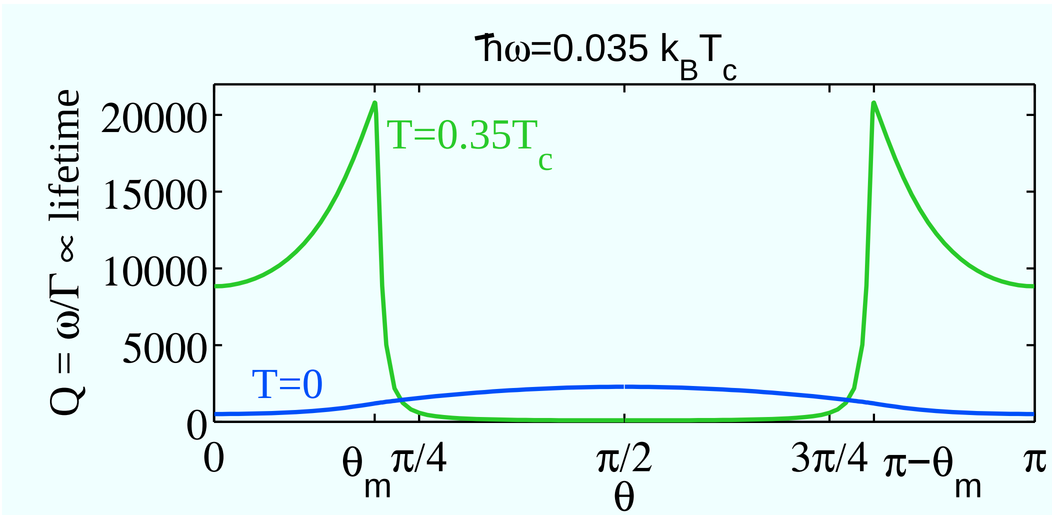
collective $\Longrightarrow$ $2\times$quasipart.

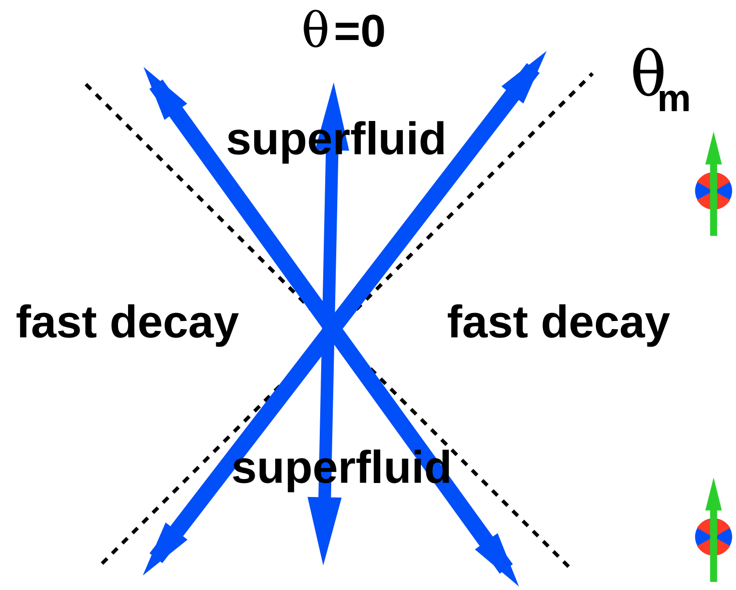




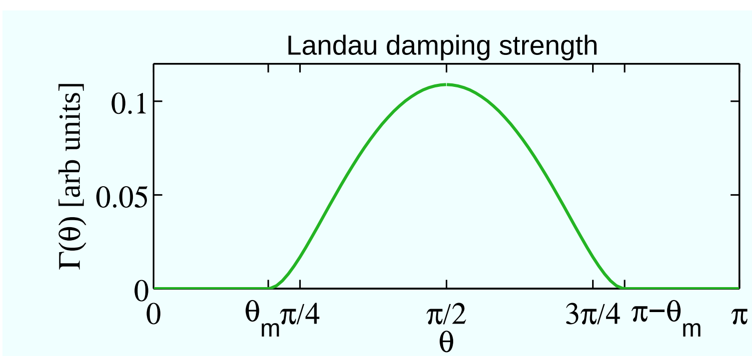
New "Aligned superfluid" regime

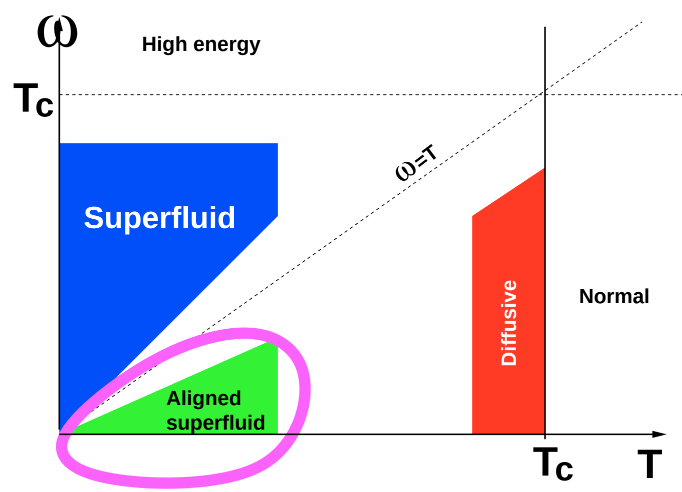
(No s-wave BCS analogue)



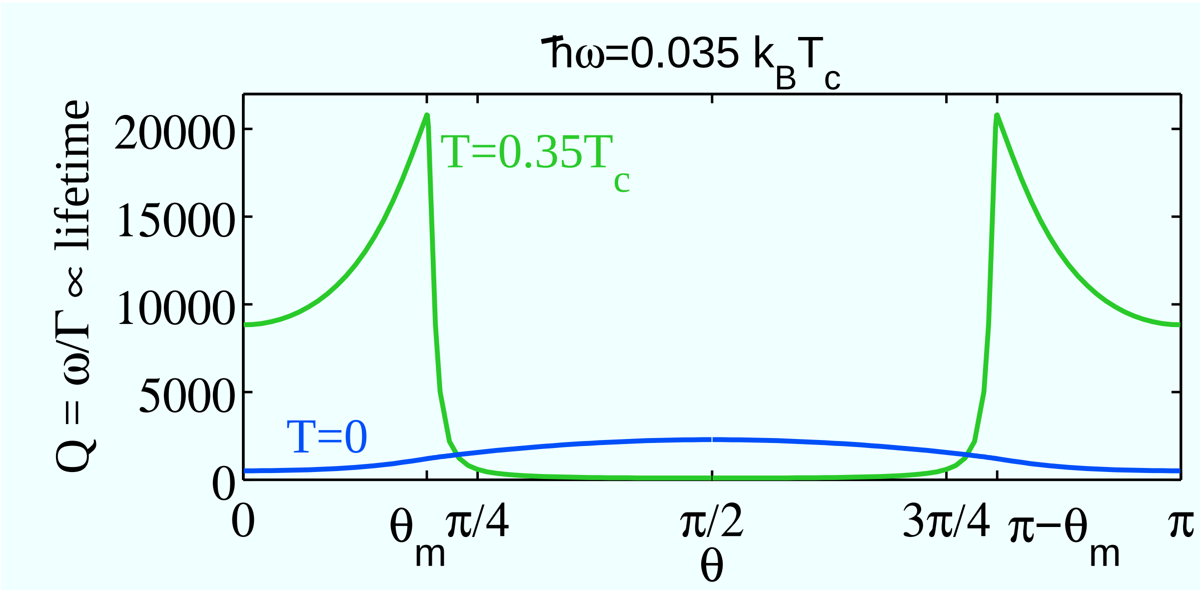


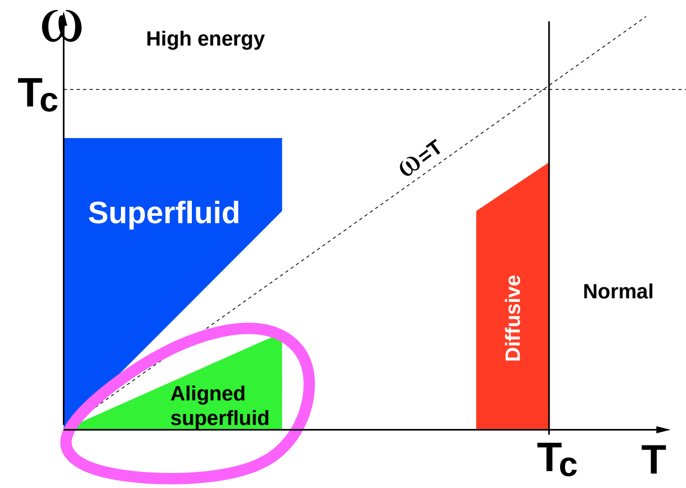
Strong Landau damping $\Gamma \propto \frac{1}{\omega}$





Thermally assisted superfluidity





- MYSTERY: How come the superfluid is better at higher T?
- Occurs when $k_B T \gg \hbar\omega$
- Quasiparticles are fermionic, and low energy pairs are already filled.
- $\Longrightarrow$ Beliaev decay into quasiparticle pairs is blocked, unlike $T = 0$