A controlled transition from semiclassical to complete quantum dynamics for atomic gases

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Motivation

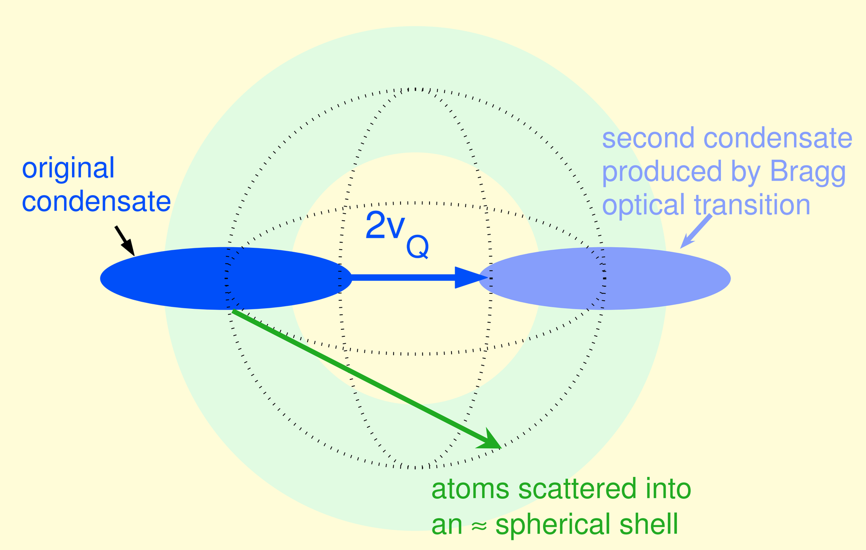
Methods for interacting many-body quantum dynamics of cold bosons all have a variety of caveats:

- MEAN FIELD (GP) – insufficient for various experiments in BEC dynamics (see below for 1 example).
- EXACT DIAGONALIZATION – Hilbert space too large $\sim e^N$.
- BOGOLIUBOV EXPANSION – OK only for small condensate depletion; also, the diagonalization of $\widehat{H}_{\rm eff}$ is very onerous when the BEC is evolving.
- POSITIVE-P – complete quantum dynamics but time limited by nonlinear amplification of noise. PD&P.D.Drummond,J.Phys.A 39,1163 (2006).
- "TRUNCATED" WIGNER - superior to Bogoliubov, but when N is less than the required lattice size, has bogus dynamics and very poor SNR due to $1/2$ virtual particle per mode in initial conditions (see below).

A controlled continuous transition between methods allows one to **QUANTITATIVELY** analyze trends towards complete quantum dynamics when exact methods fail.

Example system

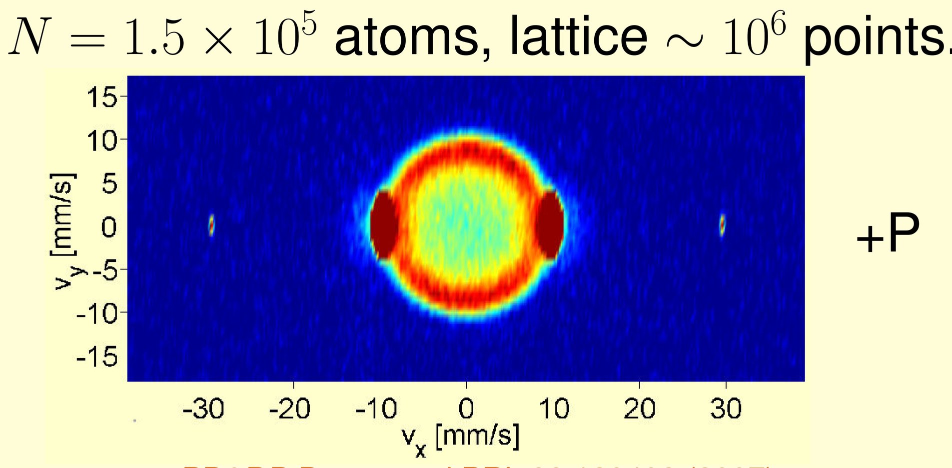
BEC collision



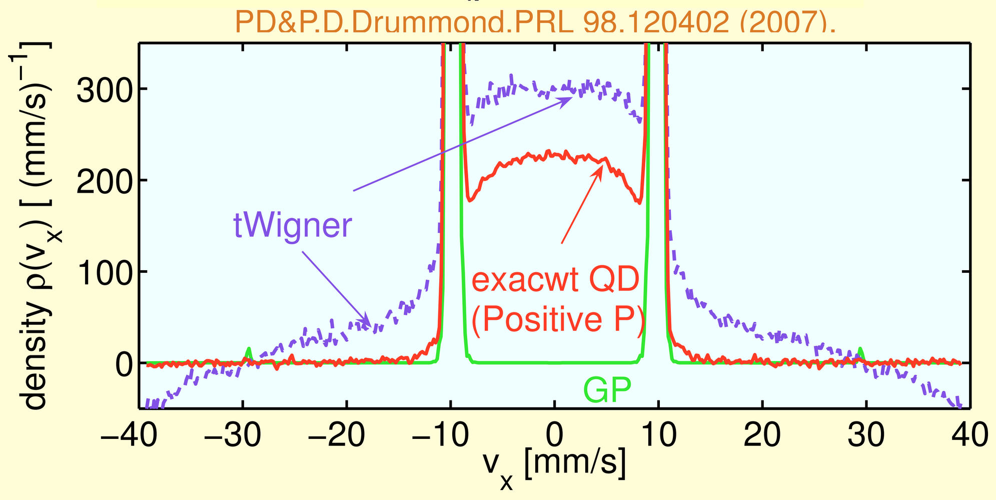
The halo of scattered atoms is of much interest and poorly understood by theory. Some experiments:

- He^* A.Perrin et al., PRL 99, 150405 (2007).
- Na A.P.Chikkatur et al, PRL 85, 483 (2000). J.M.Vogels et al, PRL 89, 020401 (2007).
- Rb N.Katz, PRL 95, 220403 (2005).

Simulation example (Na):
$N=1.5\times10^5$ atoms, lattice $\sim10^6$ points.



PD&P.D.Drummond.PRL 98.120402 (2007).



Complete dynamics: Positive P distribution

Off-diagonal expansion in local coherent states $|\psi(x)\rangle = e^{\psi(x)\widehat{\Psi}^\dagger(x)}|0\rangle$.

$$\hat{\rho} = \int P(\psi(x),\widetilde{\psi}(x)) \bigotimes_x \frac{|\psi(x)\rangle\langle\widetilde{\psi}(x)|}{\langle\widetilde{\psi}(x)|\psi(x)\rangle}\mathcal{D}\psi\,\mathcal{D}\widetilde{\psi}$$

Quantum dynamics equivalent to a random walk of samples of P §

§ In the ∞ samples limit.

$$i\hbar\frac{d\psi(x)}{dt} = \left[-\frac{\hbar^2}{2m}\nabla^2 + g\,\psi(x)\widetilde{\psi}^*(x) + \sqrt{\frac{g}{i\hbar}}\,\xi(x,t)\right]\psi(x)$$
$$i\hbar\frac{d\widetilde{\psi}(x)}{dt} = \left[-\frac{\hbar^2}{2m}\nabla^2 + g\,\psi(x)\widetilde{\psi}^*(x) + \sqrt{\frac{-g}{i\hbar}}\,\widetilde{\xi}(x,t)\right]\widetilde{\psi}(x)$$

= mean field GP + noise fields $\xi,\widetilde{\xi}$

$$\overline{n}(x) = \left\langle \widehat{\Psi}^\dagger(x)\widehat{\Psi}(x)\right\rangle = \left\langle \widetilde{\psi}^*(x)\psi(x)\right\rangle_{\rm ensemble}$$

Truncated Wigner

GP evolution + $\frac{1}{2}$ virtual particle in IC

$$\psi(x,0) = \phi_{GP}(x,0) + \frac{\eta(x)}{\sqrt{2}} \qquad \eta(x):\ \text{gaussian noise}$$

$$i\hbar\frac{d\psi(x)}{dt} = \left[-\frac{\hbar^2}{2m}\nabla^2 + g\,|\psi(x)|^2\right]\psi(x)$$

$$\overline{n}(x) = \left\langle |\psi(x)|^2 - \frac{1}{2}\right\rangle_{\rm ensemble}$$

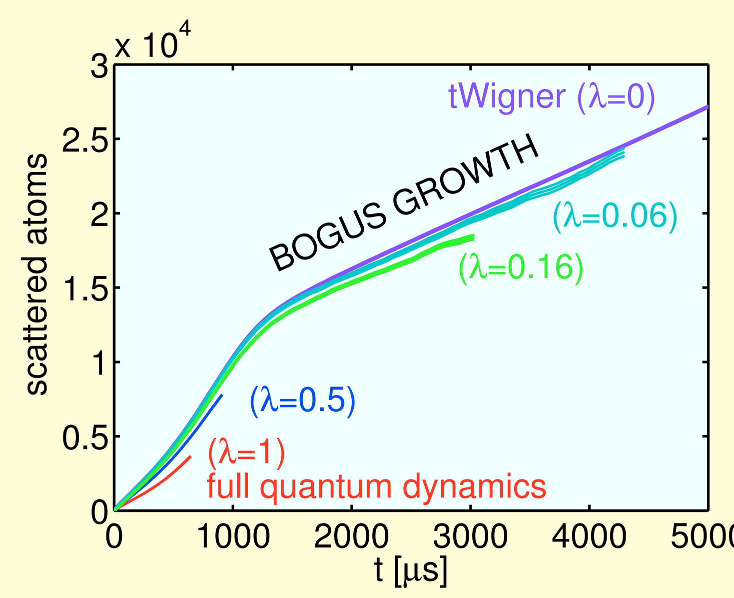
tWigner / Positive-P blend

$\lambda=0$: Wigner, $\lambda=1$: complete QD

$$\psi(x,0) = \widetilde{\psi}(x,0) = \phi_{GP}(x,0) + \sqrt{\frac{1-\lambda}{2}}\,\eta(x)$$

$$i\hbar\frac{d\psi(x)}{dt} = \left[-\frac{\hbar^2}{2m}\nabla^2 + g\,\psi(x)\widetilde{\psi}^*(x) + \sqrt{\frac{\lambda g}{i\hbar}}\,\xi(x,t)\right]\psi(x)$$
$$i\hbar\frac{d\widetilde{\psi}(x)}{dt} = \left[-\frac{\hbar^2}{2m}\nabla^2 + g\,\psi(x)\widetilde{\psi}^*(x) + \sqrt{\frac{-\lambda g}{i\hbar}}\,\widetilde{\xi}(x,t)\right]\widetilde{\psi}(x)$$

$$\overline{n}(x) = \left\langle \widetilde{\psi}^*(x)\psi(x) - \frac{1-\lambda}{2}\right\rangle_{\rm ensemble}$$



Bad features of both methods BUT STILL USEFUL

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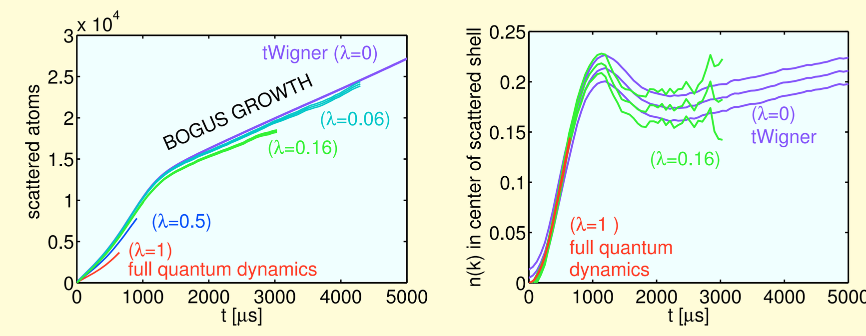
USE: verification

Not all observables are as sensitive to bogus Wigner evolution. E.g. halo peak density:



When there is no significant difference in an observable between Wigner and $\lambda<1$, then the Wigner can be assumed accurate.

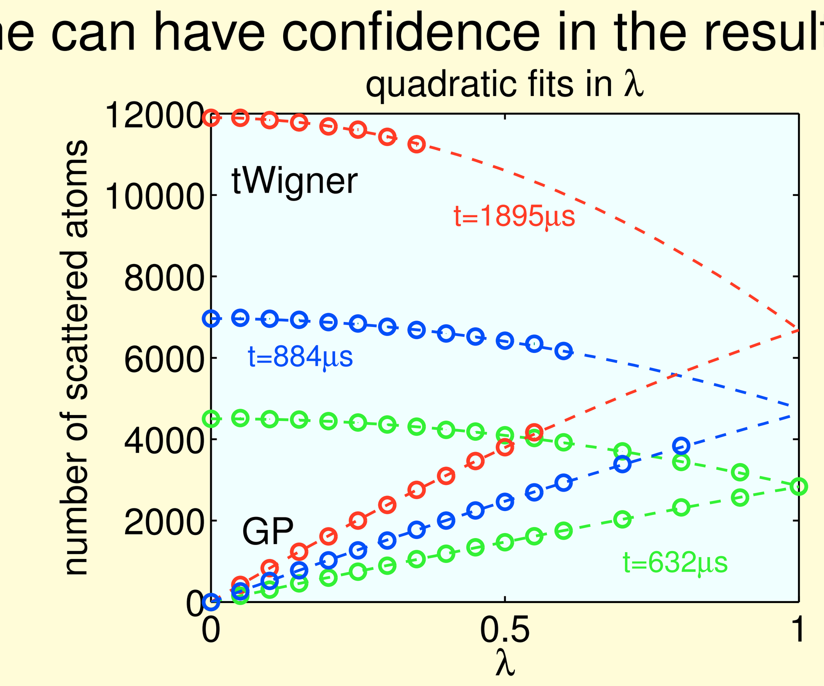
USE: interpolation of long time observables

One is tempted to extrapolate from variation in λ to QD at $\lambda=1$. However – CAVEATS:

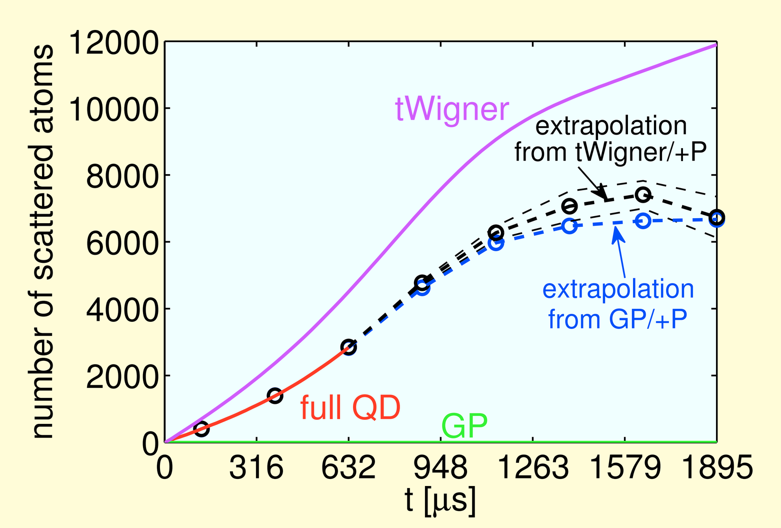
- Extrapolations can be misleading.
- It can be difficult to decide on an empirical guiding function for extrapolation - linear? quadratic? etc?

TECHNICAL SOLUTION

- Compare to a blend of complete QD with a different semiclassical theory.
- For example, with GP. (One obtains dynamics equations as Wig/+P blend, but initial conditions and observables as pure +P)
- Akin to usage of different summation techniques in diagrammatic Monte Carlo. See N.V.Prokof'ev&B.V.Svistunov, PRB 77, 125101 (2008).
- When independent extrapolations agree one can have confidence in the result.



A "controlled **IN** terpolation" between different semiclassical methods.



Outlook

- Implement Bogoliubov/+P transition. (Bogoliubov is much closer to full quantum dynamics than GP)
- Can a scaling law with λ for observables be derived analytically?
- Fermions?