Excitations in superfluid dipolar Fermi gases

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Overview

1. Why are gases of fermionic dipoles interesting

    A comparison with the case of short-range interactions

2. Making a *superfluid* dipole gas

    physical realisations, critical temperature T_c

3. Model for the uniform 3D gas

    $\widehat{H}$, assumptions

4. Elementary excitations

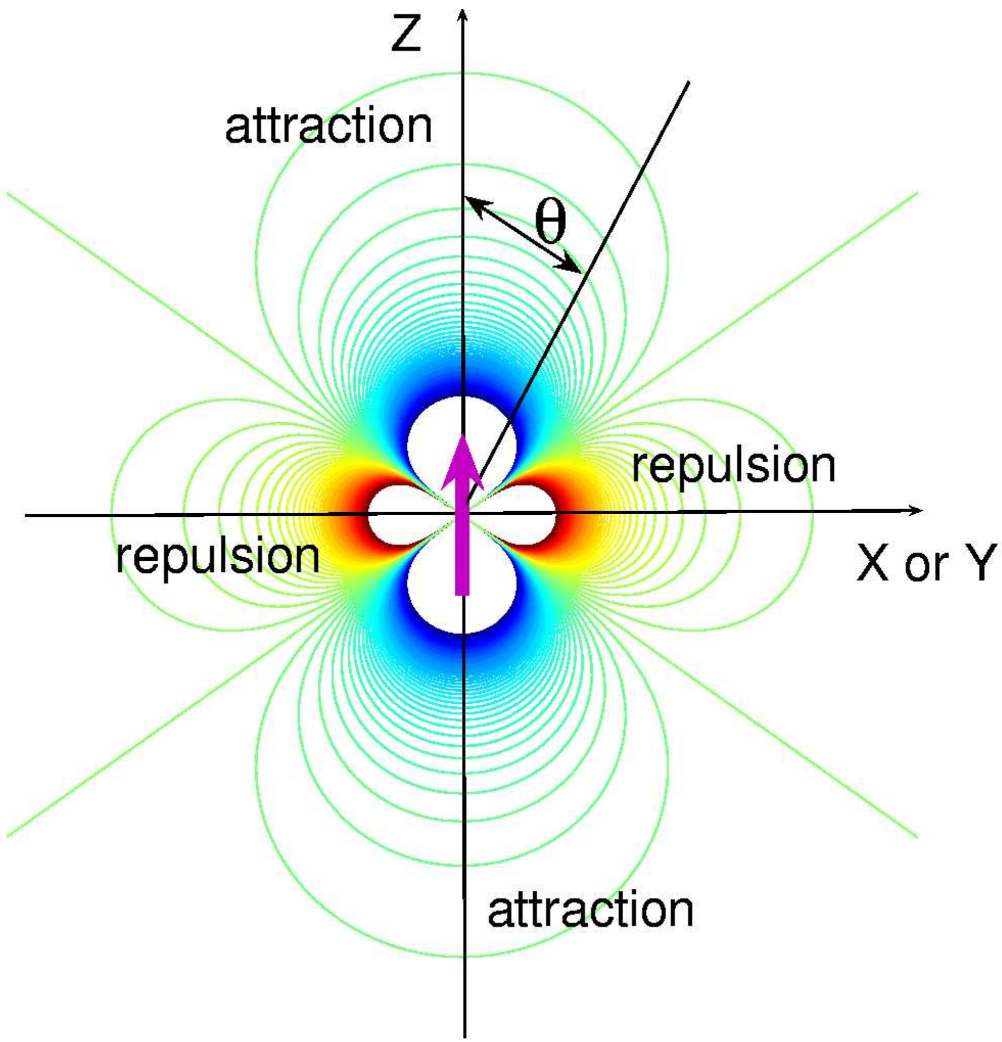
    Quasiparticles, breaking a BCS pair

5. Collective excitations & superfulidity

    Sound waves, hydrodynamics, superfluid current response

Interparticle Potential

$$V_D(R,\theta) = \frac{d^2}{R^3}\left(1 - 3\cos^2\theta\right)$$



dipole–dipole potential

- *LONG* range interaction
- *ANIsotropic*
- always partly attractive
  BCS pairing if *polarised*
- BCS with 1 component
- *p*,*d*,… -wave scattering
- much yet to explore

short-range potential

- *SHORT* range interaction
- *Isotropic*
- results in
  BCS pairing only if $a_s < 0$
- BCS needs 2 components
- only *s*-wave scattering

(2) Prospects for superfluidity

Possible Physical Realisations

- Heteronuclear ("polar") molecules

- Magnetic atomic dipoles
  - e.g. ^{52}Cr (6 parallel spins in valence electron shell)
  - BEC achieved A. Griesmaier *et al*, PRL **94**, 160401 (2005)
  - For Bosons, a strong short-range interaction is usually present
    $\Longrightarrow$ dipole effects seen were only perturbative
  - Fermions don't have s-wave $\Longrightarrow$ much purer dipole effects

- Induce electric dipoles in atoms with strong E fields

Critical Temperature for BCS

Short-range-interacting gas:

$$T_c = 0.28 E_F \exp\left(-\frac{\pi}{2|a_s|k_F}\right)$$

Dipole gas:

M. Baranov *et al*, PRA **66**, 013606 (2002)

$$T_c = 1.44 E_F \exp\left(-\frac{\pi}{2|a_D|k_F}\right)$$

$\Longrightarrow$ *Effective* scattering length a_D:

$$a_D = -2m\left(\frac{d}{\pi\hbar}\right)^2$$

T_c rises strongly with $a_D \propto md^2$

Candidates for BCS pairing

(large $|a_D|$ desirable)

Short-range interactions

- Two spin components. For example ^{6}Li : $a_s = -114$ nm

Dipoles

- Heteronuclear ("polar") molecules

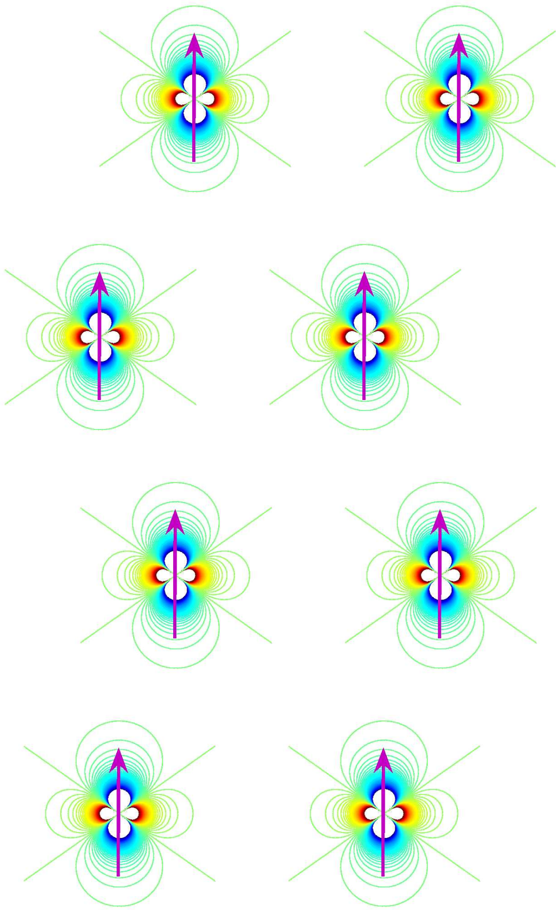
  ^{14}N^{16}O : $a_D = -2.4$ nm
  ^{15}ND^{3} : $a_D = -145$ nm

- Magnetic atomic dipoles (from electronic spin)
  - ^{52}Cr : $a_D = -0.5$ nm (weak compared to a_s)

- Atoms with induced electric dipole
  - $a_D \approx -1$ to -10 nm (need $\approx 10^6$ V/cm)

(3) Model

Assumptions



- uniform 3D gas
- static external field (E or B)
  $\Longrightarrow$ full polarisation
- single-species
- dilute $\Longrightarrow$ Energy dominated by Fermi sea to leading order
- short-range interaction (e.g. p-wave) negligible (Fermi exclusion)

Hamiltonian

$$\widehat{H} = \mathrm{K.E.} + \frac{1}{2}\int d^3x\,d^3y\left\{\,\widehat{\Psi}^\dagger_x\widehat{\Psi}_x\,V_D(x-y)\,\widehat{\Psi}^\dagger_y\widehat{\Psi}_y\,\right\}$$

- $\widehat{\Psi}_x$ is the anihilating Fermi field operator at point x.

Postulate interaction to be primarily with an effective "mean" field

$$\begin{aligned}\widehat{H}_{\mathrm{eff}} = \frac{1}{2}\int d^3x\,d^3y\Big\{ \quad & \frac{\hbar^2}{m}\,\widehat{\Psi}^\dagger_x\,\nabla^2\widehat{\Psi}_x\,\delta(x-y) && \textit{Kinetic}\\ & \Delta^*(x-y)\,\widehat{\Psi}_x\widehat{\Psi}_y - \Delta(x-y)\,\widehat{\Psi}^\dagger_x\widehat{\Psi}^\dagger_y && \textit{BCS}\\ & +W(x-y)\widehat{\Psi}^\dagger_x\widehat{\Psi}_y \Big\} && \textit{Hartree}\end{aligned}$$

- With "appropriate" $\Delta(x-y)$ and $W(x-y)$

Gap equation

Choose $\Delta(x-y)$ and $W(x-y)$ to minimise the full Free energy

$$F = \langle \widehat{H} \rangle_{\text{eff}} - \mu N - TS$$

when calculated with eigenstates of $\widehat{H}_{\text{eff}}$.

Obtain:

$$\Delta(x-y) = V_D(x-y) \left\langle \widehat{\Psi}_x \widehat{\Psi}_y \right\rangle_{\text{eff}}$$

$$W(x-y) = -V_D(x-y) \left\langle \widehat{\Psi}_x^\dagger \widehat{\Psi}_y \right\rangle_{\text{eff}}$$

Final Hamiltonian

In k–space

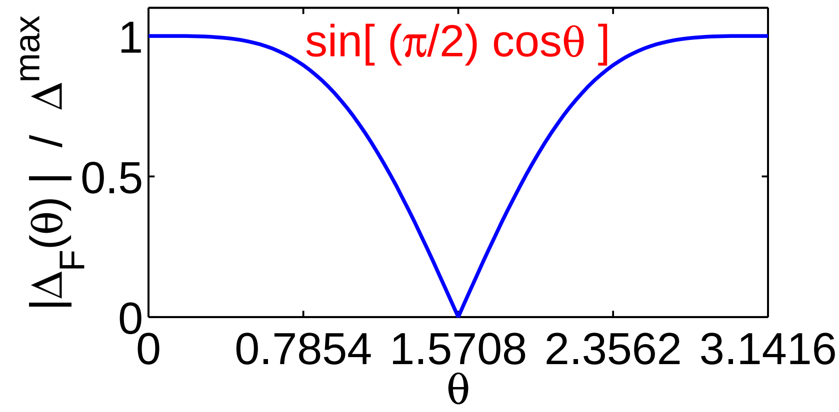
$$\widehat{H}_{\rm eff} = \frac{1}{2}\int d^3k \left\{ \left(\frac{\hbar^2 k^2}{m} - W(k) \right) \widehat{\Psi}_k^\dagger \widehat{\Psi}_k + \Delta^*(k) \widehat{\Psi}_k \widehat{\Psi}_{-k} - \Delta(k) \widehat{\Psi}_k^\dagger \widehat{\Psi}_{-k}^\dagger \right\}$$

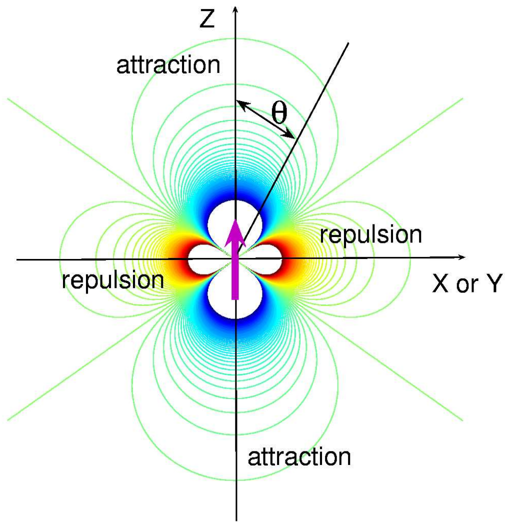
- $W(k)$ is a minor energy shift of Fermi surface $\Longrightarrow$ ignore it
- Order parameter $\Delta(k) \neq 0$ corresponds to BCS pairing of k and $-k$ atoms.
- Important difference: $\Delta(k)$ anisotropic

(4) Quasiparticle excitations

BCS gap $\Delta_F(\theta)$ on Fermi surface

$T < T_c$





- ANISOTROPIC, unlike short-range gas, and ZERO in plane $\perp$ to polarisation.

  $\Longrightarrow$ From these, all interesting features follow.

- $T = T_c$ known (Baranov *et al* (2002))

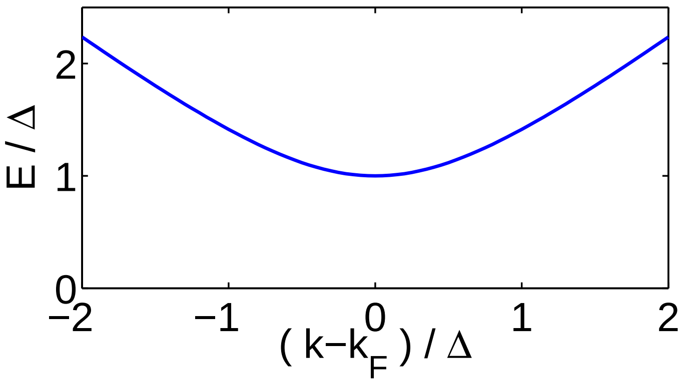
- Found that for $T < T_c$, $\dfrac{\Delta(T)}{\Delta(T_c)} = \text{const}$ to $O(\Delta^{\max}/E_F)$

Single-particle excitations $T < T_c$

Breaking a pair costs $2\times$

$$E(|k|,\theta_k) = \sqrt{(\text{K.E.} - E_F)^2 + \Delta^2}$$

in both cases



dipoles	**short-range**
$\Delta(\|k\|,\theta_k)$	Δ constant
easy to excite a pair in plane $\perp$ to polarisation because energy cost is small	appreciable energy cost always

(5) Collective excitations and superfuidity

Collective excitations (Sound)

Phase perturbations of the order parameter

$$\boxed{\Delta_0(x-y) \quad\longrightarrow\quad \Delta(x,y) = \Delta_0(x-y)\, e^{2i\phi(x,t)}}$$

Assumptions:

- Low energy $\quad (\, \omega \ll \Delta_0^{\max} \,)$
- Phase perturbations only (ϕ real) because amplitude perturbations require splitting of pairs (and so are gapped)
- Low $\omega \quad \Longrightarrow \quad$ long wavelength ($k \ll k_F$)
  $\Longrightarrow \quad$ insensitive to small-scale of $|x-y| \quad \Longrightarrow \quad \phi \approx \phi(x \text{ only })$
- Weak perturbation $\quad \Longrightarrow \quad$ lowest order in ϕ

$T = 0$ **Superfluid**

Find Bogoliubov sound, same as for the short-range-interacting gas

$$\boxed{\omega = \left(\frac{v_F}{\sqrt{3}}\right)\, k}$$

To lowest order in $\omega \ll E_F/\hbar$ and $k \ll k_F$.

Not too surprising from hydrodynamics . . .

$T = 0$ Hydrodynamics

Relies on the hydrodynamic Hamiltonian for superfluid velocity v_s

$$H \approx \int d^3x \left\{ \frac{1}{2} m\rho v_s(x)^2 + U(\rho) \right\}$$

and the continuity and current equations

$$\vec{v}_s = \frac{\vec{J}_s(x)}{\rho} = \frac{\hbar}{m}\, \rho\, \vec{\nabla}\phi(x) \qquad \text{and} \qquad \vec{\nabla}\cdot\vec{J}_s(x) = -\frac{\partial\rho}{\partial t}$$

which are found to be the same for dipoles and short-range gases to order $O(\Delta^{\max}/E_F)$.

Since $U(\rho)$ arises overwhelmingly from the filled Fermi sphere,

$\Longrightarrow$ no appreciable dependence on interaction details

Beyond hydrodynamics

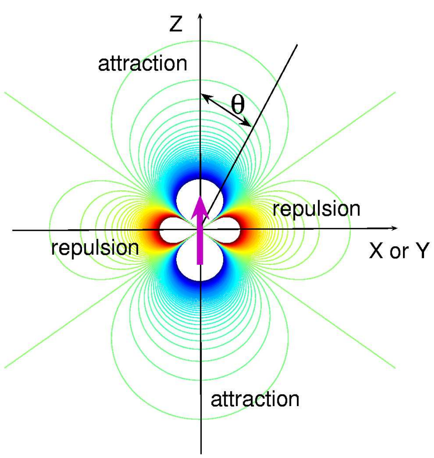
$T = 0$ Anisotropic damping of sound

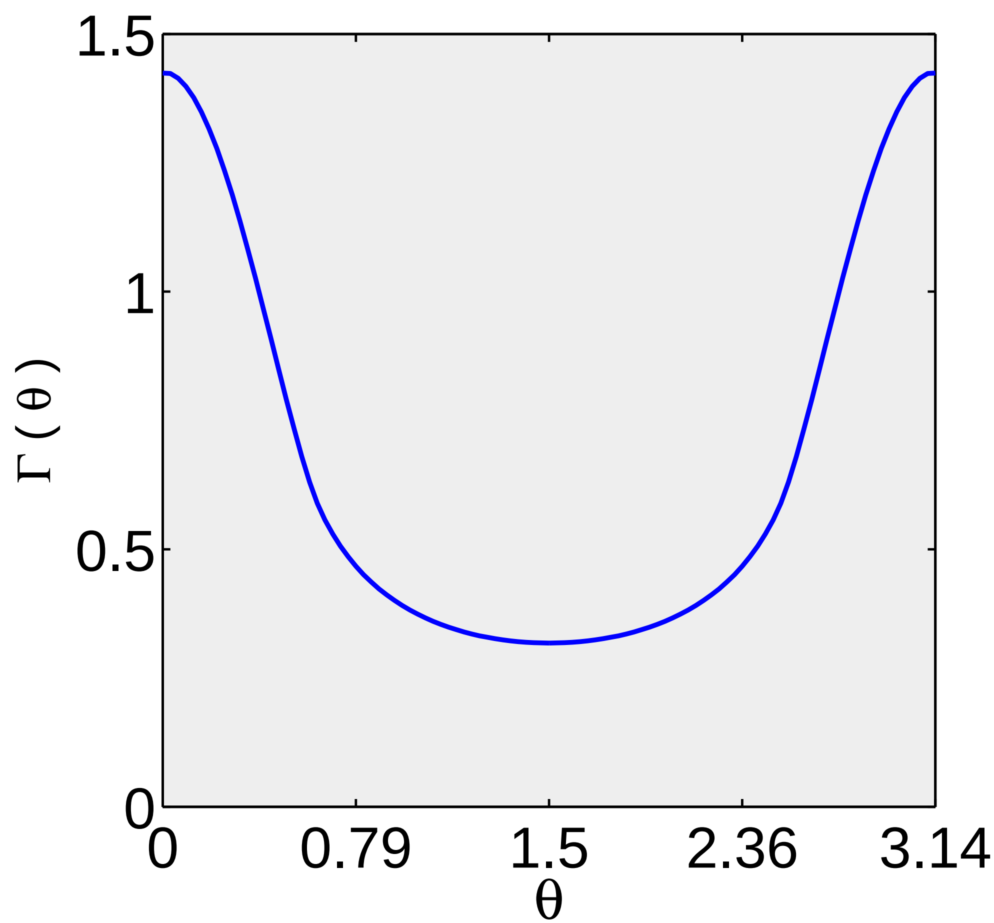
$$\omega = \left(\frac{v_F}{\sqrt{3}}\right) k \left\{1 - i\,k\left(\frac{\hbar v_F}{\sqrt{3}\,\Delta_{\max}}\right)\Gamma(\theta)\right\}$$

absent for short-range gas

collective exc.

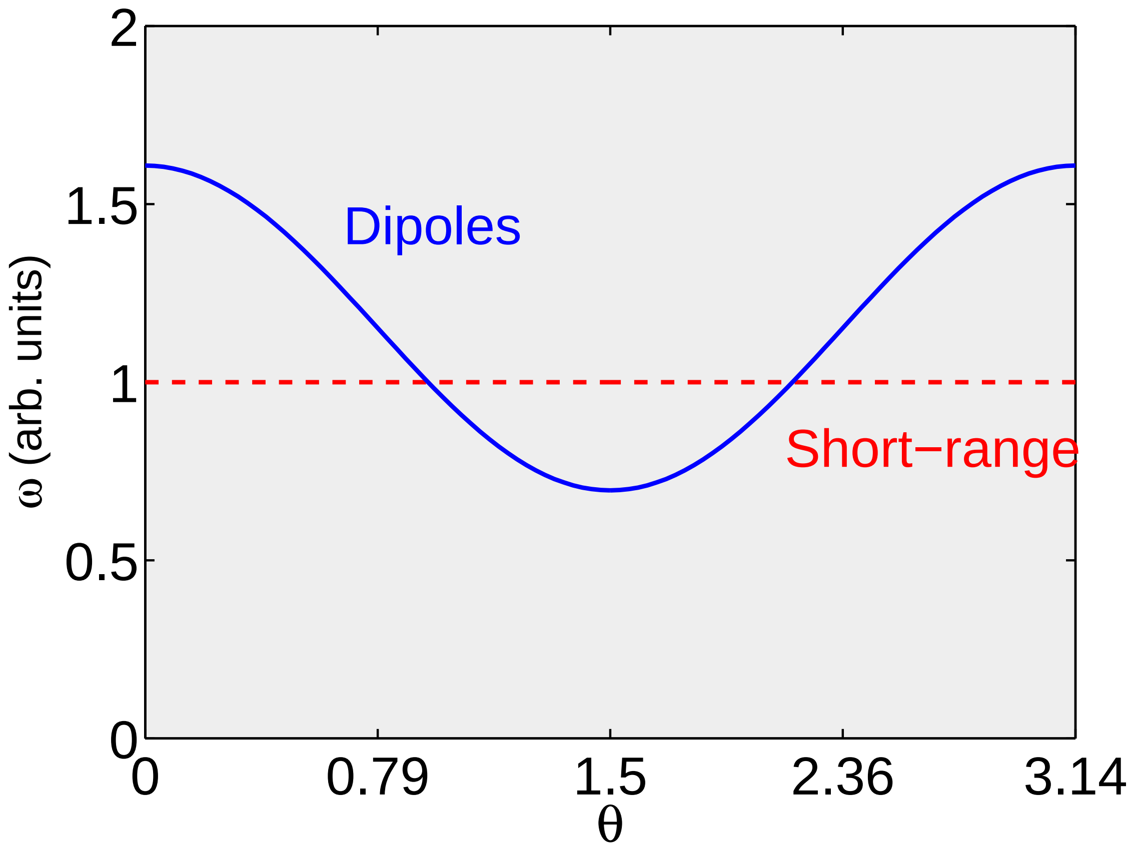
$\Longrightarrow \quad 2\times$ 1-particle exc.

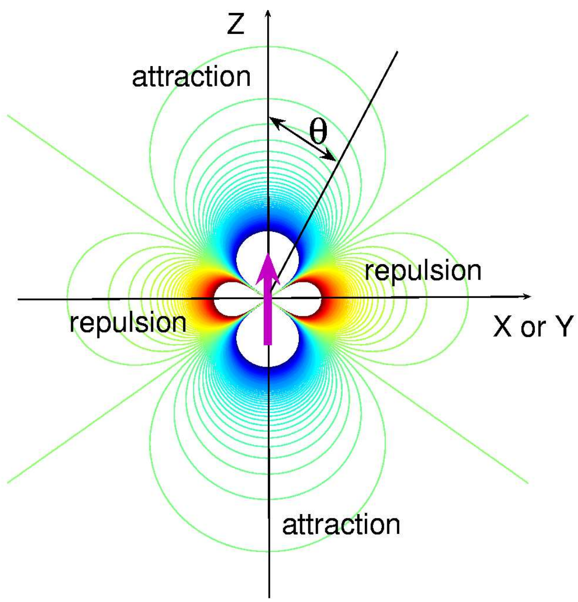




$T \approx T_c$ behaviour

$$\omega = -i\left(\frac{7\zeta(3)}{6\pi^3}\right)\left(\frac{\hbar v_F^2}{T_c}\right) k^2 \left(1+\frac{3}{2\pi^2}(1+3\cos 2\theta)\right)$$



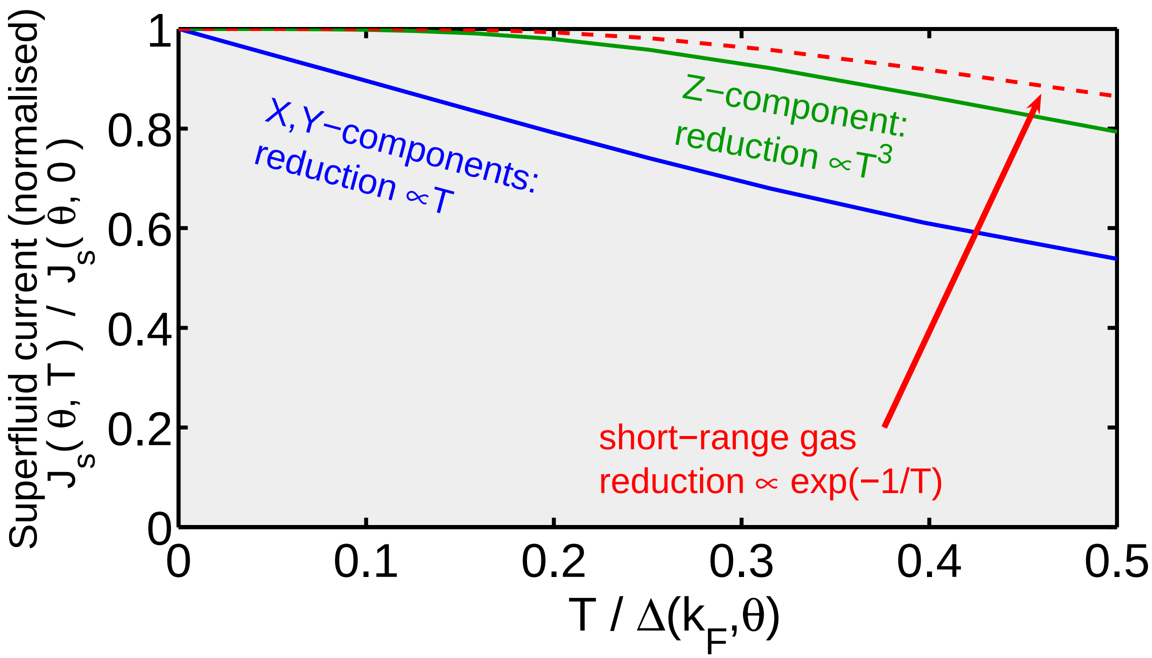


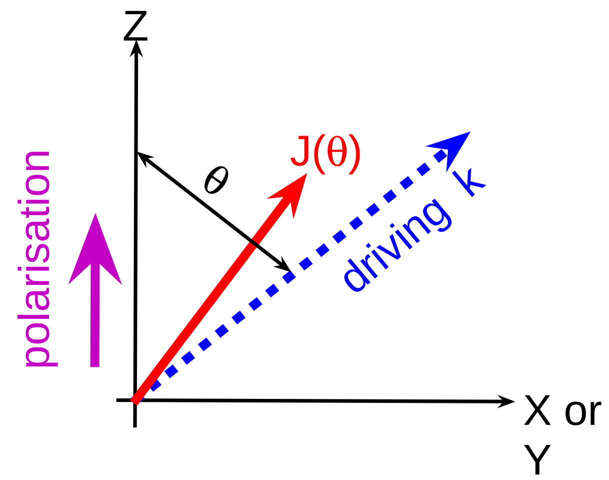
- Purely diffusive (as for short-range interactions)
- Anisotropic (differently to short-range interactions)

Superfluid current $0 < T < T_c$

- Consider current response to the external driving perturbation $\delta\Delta(\phi)$.
- Driving frequency ω, wave-vector k, in direction θ.

$$\vec{J}_s^{(T)}(\omega,k,\theta) = \vec{J}_s^{(T=0)}(\omega,k)\left[1-\delta(\theta)\right]$$





Direction-dependent superfluid

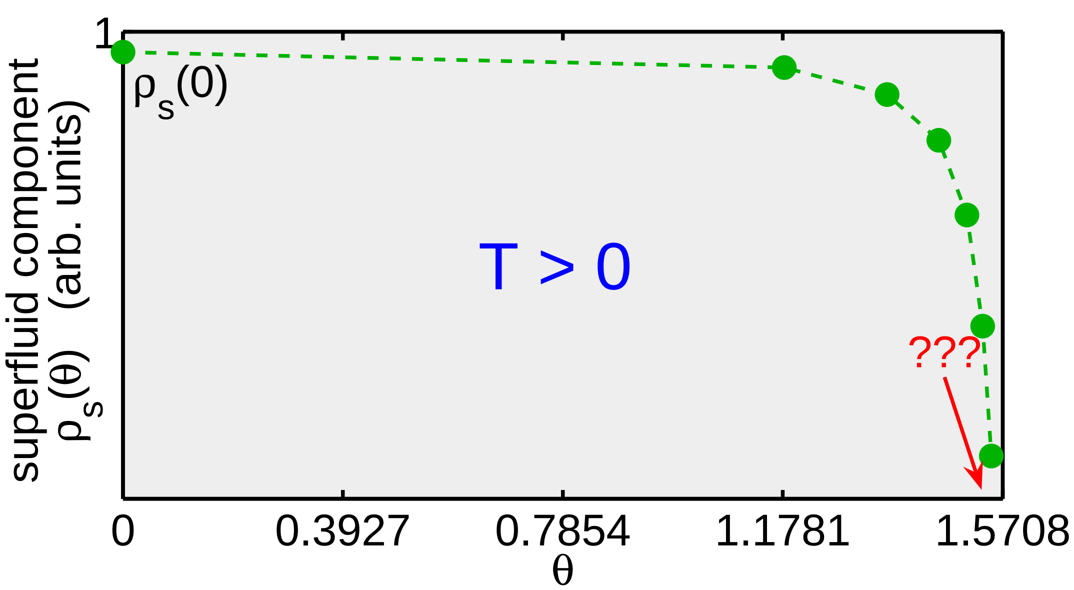
(preliminary and tentative)

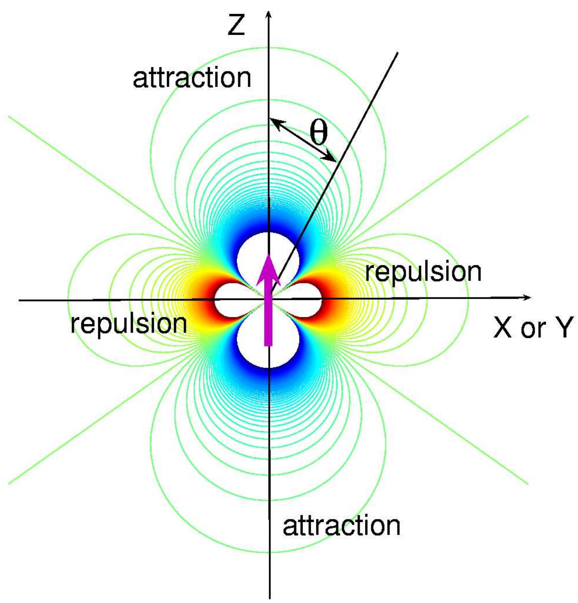
Can define direction-dependent "normal" and "superfluid" components

$$\rho = \rho_n(\theta) + \rho_s(\theta)$$

so that

$$\vec{J}_s = \frac{\hbar}{m}\rho_s \vec{\nabla}\phi$$





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