Application of the ultrametric distance to portfolio taxonomy. Critical approach and comparison with other methods

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Abstract

We calculate the ultrametric distance between the pairs of stocks that belong to the same portfolio. The ultrametric distance allows us to distinguish groups of shares that are related. In this way, we can construct a portfolio taxonomy that can be used for constructing an efficient portfolio. We also construct a portfolio taxonomy based not only on stock prices but also on economic indices such as liquidity ratio, debt ratio and sales profitability ratio. We show that a good investment strategy can be obtained by applying to the portfolio chosen by the taxonomy method the so-called Constant Rebalanced Portfolio.

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1. Introduction

Taxonomy is a science that deals with classifying and ordering objects. It is the first domain, apart from mathematics, where the notion of ultrametricity appeared.

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This notion is strictly connected with classification of objects, which is usually illustrated with the help of hierarchical trees [1,2]. Our goal is to apply the taxonomical methods to financial instruments as proposed in Ref. [3]. There are many models that allow to construct an efficient portfolio, e.g., multicriteria model, taxonomic model based on stock prices and economic indices, CAPM, stochastic dominance, buy and keep strategy, strategy on line, CRP or CUP (see Refs. [4–6]). First of all it is worth noticing that we can apply ultrametric distance taxonomy to financial instruments in various cases. The first case is a comparison of stocks based on one criterion, e.g., their daily closing prices. The second case requires a more advanced tool—a multicriteria comparison analysis, which at the end also leads to determining an ultrametric distance between a pair of stocks. We have made calculations for daily closing prices for Warsaw Stock Exchange index WIG20 shares for the year 2002 (see Refs. [7,8]).

2. Application of an ultrametric distance to portfolio taxonomy

Using the method proposed in Ref. [3] we have obtained a minimum spanning tree for WIG20 shares for the year 2002 [7,8]. Results are summarized in Fig. 1. The calculations revealed a common law that companies from the same sector undergo the same processes on the Warsaw Stock Exchange (Prokom, Compland, Softbank and Comarch are computer companies). The ultrametric distance in portfolio taxonomy allowed us to distinguish clusters of shares that are connected in such a way that their prices change simultaneously. By distinguishing the groups of shares that are strongly related we have made the portfolio more clear. The taxonomy can be successfully used in constructing an efficient portfolio.

As an example we choose a portfolio that consists of shares not strongly related, i.e., those for which the ultrametric distances are large: Świecie, Kęty, Budimex, Debica, PGF. We buy on the 3rd of January 2003 one share of each and spend 200.2 PLN. On the 30th of April 2003 we can sell the stocks for 248.7 PLN. This shows that the ultrametric distance method can be successfully applied for constructing an efficient portfolio.

3. Portfolio taxonomy for multicriteria factors

We have $N$ objects. Each object can be described by $m$ different indices that give us the observation matrix. In order to make use of the taxonomic methods we have got to standardize the variables, i.e., make values stated in various scales comparable. This means that we have to make the variables homogeneous, eliminate negative values, replace different ranges of variation of variables by the same range.

Each firm can be characterized by several coefficients, e.g., the liquidity ratio coefficient, the debt ratio coefficient and sales profitability ratio coefficient. The stock price is a derivative of those indices and the demand and supply. To describe a stock we take: coefficient of profitability of share for years 2001 and 2002, liquidity
ratio, debt ratio, sales profitability ratio. We characterize companies outside a banking sector with WIG20. The coefficient of profitability was calculated for Friday quotations of close in the years 2001 and 2002. Coefficients of liquidity and profitability of sale are stimulating variables, i.e., their high values are desirable from the point of general characterization of company. Coefficient of debt is a dissimulating variable, i.e., its high values are undesirable from the point of view of a general characterization of a studied company. The coefficient of liquidity should take its values in the interval 1.2 to 2.

We have received the hierarchical tree that is presented in Fig. 2. We will construct a portfolio consisting of the stocks with large ultra metric distances, i.e., Orbis,
Świecie, Agora, Budimex. We buy one share of each and invest 136.1 PLN. We invest the money at the beginning of the year. The amount invested at the beginning of the year will grow for a short time but then will decrease. It will reach the invested level only on the 13th of March but if we sell the shares without transaction costs on the 30th of April we will obtain only 132.8 PLN. Our loss was caused by a rapid decrease of Agora trading prices. If we eliminate Agora from our portfolio and invest in January 86.1 PLN in remaining shares, by the 14th of April we will be able to get over 100 PLN. The example of Agora can be a model example of rapid change in price caused by information. In that way we can see that taxonomy alone is not a
sufficient tool for constructing an efficient portfolio. In order to build an efficient portfolio we have got to follow the information available and adjust our portfolio in time with it.

The ultrametric distance in portfolio taxonomy allowed us to distinguish groups of shares that are connected is such a way that their prices change simultaneously. The taxonomy method is one of many methods that allow us to classify the shares and in that way decrease the number of shares to be concerned. Moreover, instead of many values that describe one share we have one value—the ultrametric distance. By distinguishing the groups of shares that are strongly related we have also made the portfolio clearer. The taxonomy can be successfully used in constructing an efficient portfolio.

4. Algorithm constant rebalanced portfolio (CRP)

The CRP algorithm can be applied for construction of the portfolio in the following way. We have picked $N$ stocks and we want to invest our money in those stocks in a given time period. The method answers the question as to how to divide our money between the stocks to achieve maximal profit. The CRP says that at the end of each trading day the wealth should be redistributed into all stocks based on a given distribution. For example, we can split the money evenly at the end of each trading day.

We will use the CRP method to construct a portfolio consisting of shares that are in different groups in our hierarchical tree, e.g., Orbis, Świecie, Budimex, Softbank, TP S.A. Let us assume that our initial capital is 1000 PLN. We divide the amount on the 3rd of January 2003 evenly among the shares of those five companies. That means that we invested 200 PLN in each share. After making calculations based on stock prices in the period from 3rd of January till the 30th of April we obtained 1156.037 PLN. This means that the CRP preceded by a taxonomy method is a very effective tool in investment strategies. We have earned 156.037 PLN despite the general fall tendency on Warsaw Stock Exchange (WGPW).

5. Concluding remarks

We have presented three methods that can be applied for constructing an investment portfolio. The first two methods were based on taxonomy. The first taxonomy of shares took as its basis only share prices. The second taxonomy method was based on more factors. The third method (CRP) focused on determining the percentage of money invested in each stock when the portfolio was fixed. The calculations that we have made allowed us to compare the methods and showed that good results can be obtained by simultaneous application of taxonomy method and CRP. Taxonomic methods allow us to choose shares to be considered in our portfolio while the percentage of share is determined by CRP method. We have also noticed that in order to earn money by investing on the WGPW one has to verify the
portfolio and track all available information that could influence the behavior of the WGPW.

References