Proposal for exploring macroscopic entanglement with a single photon and coherent states

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Entanglement between macroscopically populated states can easily be created by combining a single photon and a bright coherent state on a beamsplitter. Motivated by the simplicity of this technique, we report on a method using displacement operations in the phase space and basic photon detections to reveal such an entanglement. We show that this eminently feasible approach provides an attractive way for exploring entanglement at various scales, ranging from 1 to 1000 photons. This offers an instructive viewpoint to gain insight into why it is hard to observe quantum features in our macroscopic world.

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I. INTRODUCTION

Why do we not easily observe entanglement between macroscopically populated (macro) systems? Decoherence is widely accepted as being responsible [1]. Loss or any other form of interactions with the surroundings destroys more and more rapidly the quantum features of physical systems as their size increases. Technologically demanding experiments, involving Rydberg atoms interacting with the electromagnetic field of a high-finess cavity [2] or superconducting devices, in the absence of interactions with the surroundings destroys more and more rapidly the quantum features of physical systems as their size increases. Technologically demanding experiments, involving Rydberg atoms interacting with the electromagnetic field of a high-finess cavity [2] or superconducting devices cooled down to a few tens of mK [3], have strengthened this idea. Measurement precision is likely another issue [4]. In a recent experiment [5], a phase covariant cloner was used to produce tens of thousands of clones of a single photon belonging initially to an entangled pair. In the absence of loss, this leads to a micro-macro entangled state [6]. Nobody knows, however, how the entanglement degrades with a lossy amplification [7]. This led to a lively debate [8] concerning the presence of entanglement in the experiment reported in Ref. [5]. What is known is that under moderate coarse-grained measurements, the micro-macro entanglement resulting from a lossless amplification leads to a probability distribution of results that is very close to the one coming from a separable micro-macro state [9]. This suggests that even if a macro system could be perfectly isolated from its environment, its quantum nature would require very precise measurements to be observed.

Both for practical considerations and from a conceptual point of view, it is of great interest to look for ways that are as simple as possible to generate and measure macro entanglement, so that the effects of decoherence processes and the requirements on the measurements can all be studied together. In this paper, we focus on an approach based on linear optics only, where a single photon and a coherent state are combined on a mere beamsplitter. We show that the resulting path-entangled state [10] could be useful in quantum metrology for precision phase measurement. More importantly, it allows one to easily explore entanglement over various photon number scales, spanning from the micro to the macro domain, by simply tuning the intensity of the laser producing the input coherent state. We show that the entanglement is more and more sensitive to phase fluctuations between the paths when it grows. However, it features surprising robustness against loss, making it well suited to travel over long distances and to be stored in atomic ensembles. We further present a simple and natural method relying on local displacement operations in the phase space and basic photon detections to reveal the entanglement. Our analysis shows that the precision of the proposed measurement is connected to the limited ability to control the phase of the local oscillator that is used to perform the phase-space displacements. We also report on preliminary experimental results demonstrating that entanglement containing more than 1000 photons could be created and measured with currently available technologies.

II. CREATING MACRO ENTANGLEMENT BY COMBINING A SINGLE PHOTON WITH A BRIGHT COHERENT STATE ON A BEAMSPILLTER

A particularly simple way of generating entanglement is to use a beamsplitter. Consider a single photon sent through a 50:50 beamsplitter. It occupies the two output modes A and B with the same probability and creates a simple form of entanglement between spatial modes (path entanglement),

\[
\frac{1}{\sqrt{2}} (|1\rangle_A |0\rangle_B - |0\rangle_A |1\rangle_B),
\]

known as single-photon entanglement [11]. In fact, any product input state of the form \(\rho_A \otimes |\beta\rangle\langle \beta|_B\), where \(|\beta\rangle\) is a coherent state, leads to entanglement after the beamsplitter if and only if \(\rho_A\) is nonclassical [12,13], i.e., it cannot be written as a mixture of coherent states [14]. Hence, a mere beamsplitter links two fundamental concepts of quantum physics: nonclassicality and entanglement. It also provides an attractive way to bring entanglement to a macroscopic level, as explained below.

Let us focus on the case in which the beamsplitter inputs are a single photon \(|1\rangle\) and a coherent state with \(2\alpha^2\) photons on average (see Fig. 1),

\[
|\psi_{in}\rangle = a^\dagger |0\rangle_A \otimes D_b(\sqrt{2}\alpha) |0\rangle_B.
\]

The displacement operator generating a coherent state \(D_b(\sqrt{2}\alpha)\) from vacuum (in mode B), and a and b are bosonic annihilation operators associated with modes A and B, respectively. A 50:50 beamsplitter transforms \((a,b)\)
FIG. 1. (Color online) Creation and detection of macro-entanglement by combining a single-photon Fock state $|1\rangle$ and a coherent state $|\sqrt{n}\alpha\rangle$ on a 50:50 beamsplitter.

The potential of the state (3) for precision phase measurement. We now focus on the robustness of entanglement with respect to loss and phase noise.

IV. ROBUSTNESS WITH RESPECT TO TRANSMISSION LOSS

In general, entanglement is seen to be increasingly fragile to transmission loss as its size increases. Coherent state entanglement $|\alpha\rangle_A|\alpha\rangle_B + |\alpha\rangle_A\langle\alpha|_B$ [17] provides a good example. If the mode $B$ is subject to loss (modeled by a beamsplitter with transmission coefficient $\eta_t$), the amount of entanglement, measured by the negativity (see [18,19] for definition), decreases exponentially, $N = \frac{1}{N(t)}e^{-2(1-\eta_t)|\alpha|^2}$, with size $|\alpha|^2$ and loss $1-\eta_t$ [20]. In comparison, the state (3) exhibits a surprising robustness. Under the assumption that mode $B$ undergoes loss, $|\psi_{out}\rangle$ becomes a statistical mixture of $\frac{1}{2}\sum_{\alpha}D_{\alpha}(\langle\alpha|_B|0\rangle_B - |\alpha\rangle_0D_{\alpha}(\langle\alpha|_B|0\rangle_B$ and $D_{\alpha}(\langle\alpha|_B|0\rangle_B$ with weights $\frac{1}{\sqrt{N}}$ and $\frac{1}{\sqrt{N}}$, respectively. After applying local displacements $D_{\alpha}(-\alpha)$ and $D_{\beta}(\sqrt{\eta}\alpha)$ to modes $A$ and $B$, one finds that the negativity of the resulting state is given by $N = \frac{1}{N(t)}e^{-2(1-\eta_t)|\alpha|^2}$. This makes the sensitivity of entanglement at a beamsplitter, as mentioned before. Indeed, loss, modeled by a beamsplitter, can be seen as an interaction process entangling the nonclassical states and the environment. However, the displacement is a classical operation that does not promote the entanglement of a given quantum system with its environment when it is amplified $[D_{\alpha}(\alpha) \rightarrow D_{\alpha}(\sqrt{\eta}\alpha)D_{\alpha}(\sqrt{1-\eta}\alpha)]$. The robustness of the state (3) makes it well suited for storage in an atomic medium. Entanglement between two ensembles, each containing a macroscopic number of atoms, has been successfully created by mapping a single-photon entanglement into two atomic ensembles [21,22]. The storage of the displaced single-photon entanglement $|\psi_{out}\rangle$ would lead to a similar entanglement in terms of the number of ebits [23], but it would contain a macroscopic number of excited atoms.

V. ROBUSTNESS WITH RESPECT TO COUPLING INEfficIENCY

The starting point in our scheme is the creation of a single photon. It is thus natural to ask how the resulting macro-entanglement degrades when the single photon is subject to loss (i.e., loss before the 50:50 beamsplitter). For comparison, consider, for example, micro-macro entanglement obtained by amplifying one photon of an entangled pair [6] with an optimal universal cloner [24]. Such entanglement can be revealed even if the amplification is followed by arbitrarily large loss [25]. Nonetheless, the state becomes separable as soon as the overall coupling efficiency $\eta_t$ before the cloner is lower than $\frac{1}{\sqrt{n}}$, $n$ being the average number of photons in the macro component [25]. On the other hand, one can show,
following the lines presented in the previous paragraph, that the negativity of the displaced single-photon entanglement scales like $N = -\frac{1}{2} \left[ 1 - \eta_c - \sqrt{1 - 2(1 - \eta_c)\eta_c} \right] \approx \frac{\eta_c}{2} + O(\eta_c^2) \geq 0$, where $\eta_c$ stands for the coupling efficiency of the input single photon. This robustness confers to the proposed scheme a great practical advantage over the one based on the universal cloner.

VI. ROBUSTNESS WITH RESPECT TO PHASE INSTABILITIES

Another decoherence process for path entanglement is associated with the relative phase fluctuations, due to, e.g., vibrations and thermal fluctuations. If the two optical paths corresponding to $A$ and $B$ acquire a phase difference $\varphi$, the displaced single-photon entanglement becomes $|\psi_{\text{out}}\rangle = \frac{1}{\sqrt{2}} \left[ D_\alpha(\sigma_1) \rho |a\rangle_B - |\psi_{\text{in}}\rangle A D_\alpha(\sigma_1) |1\rangle_B \right]$. Furthermore, if $\varphi$ varies from trial to trial, the state $|\psi_{\text{out}}\rangle \langle \psi_{\text{out}}|$ has to be averaged over $\varphi$ with the probability distribution $p(\varphi)$ associated with the phase noise. The question of the sensibility of the displaced single-photon entanglement with respect to phase instability thus reduces to a measure of the entanglement associated with the phase noise. The negativity of the displaced single-photon entanglement can easily be obtained numerically by projecting $|\bar{\psi}\rangle$ out

$\begin{align*}
N &= \frac{1}{2} \left[ 1 - \eta_c - \sqrt{1 - 2(1 - \eta_c)\eta_c} \right] \approx \frac{\eta_c}{2} + O(\eta_c^2) \geq 0,
\end{align*}$

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VII. REVEALING DISPLACED SINGLE-PHOTON ENTANGLEMENT

So far, we have discussed the properties of the state (1). We now present a simple way to reveal its entanglement. The basic idea is to displace each of the electromagnetic fields describing the modes $A$ and $B$ by $-\alpha$. Such a displacement in the phase space can be easily performed by mixing the mode to be displaced with an auxiliary strong coherent field (labeled as the local oscillator in the following) on a highly unbalanced beamsplitter [28], in a manner similar to homodyne measurements. Since $D_\alpha(-\alpha) = D\alpha(-\alpha)^{-1}$, the modes $A$ and $B$ ideally end up in the state (1), which can be revealed by tomography using single-photon detectors. Note that a similar approach was proposed in [29] to reveal entanglement in the scenario where a macroscopic state is created by phase-variant cloning of an entangled photon pair. The authors proposed locally undoing the cloning before doing the measurement.

The approach developed in Ref. [30] does not require a full tomography after the local displacements. It gives a lower bound on the entanglement between the modes $A$ and $B$ from the estimation of the entanglement contained in the two-qubit subspace $\{|00\rangle, |01\rangle, |11\rangle, |10\rangle\}$. More precisely, the concurrence $C$ of the detected fields is bounded by $C \geq \max\{0, V(p_{01} + p_{10}) - 2 - \sqrt{p_{01} p_{11}}\}$ [30], where $V$ is the visibility of the interference obtained by recombining the modes $A$ and $B$ on a 50:50 beamsplitter, and the coefficients $p_{mn}$ are the probabilities of detecting $m$ photons in $A$ and $n$ in $B$. This tomographic approach for characterizing single-photon entanglement is attractive in practice and already triggered highly successful experiments, demonstrating, e.g., heralded entanglement between atomic ensembles [21,22,30–32].

Note that the statistical fluctuations in the phase of local oscillators that are used to perform the displacements limit the precision of the measurement process. Under the assumption that the two local displacements $D_\alpha(-\alpha), D\alpha(-\alpha)$ are performed with a common local oscillator, the measured state is of the form $\int d\varphi \hat{p}(\varphi) \rho_{\text{local}} \hat{D}_\alpha(-\alpha) \rho_{\text{local}} D\alpha(-\alpha)^{*} \hat{D}_\alpha(-\alpha) \rho_{\text{local}} D\alpha(-\alpha)^{*}$, where $\hat{p}(\varphi)$ stands for the phase noise distribution and $\rho_{\text{local}} = |\bar{\psi}\rangle \langle \bar{\psi}|$. Let $V$ be the visibility of the interference that characterizes the phase stability of the local oscillator. One can show that for small imperfections $\epsilon = (1 - V) \ll 1$, the concurrence is bounded by $C \geq \max\{0, 1 - 10(1 - V)|\varphi|^2\}$. The necessary precision of the measurement thus scales as $\frac{1}{\sqrt{10}} = \frac{1}{\eta_{\text{local}}}$. This result strengthens the idea that precise measurements are generally essential for revealing quantum properties of macro systems.

VIII. PROPOSED EXPERIMENT

We now address the question of the experimental feasibility in detail. For concreteness, we focus on a realization of the
single-photon source from a pair source based on spontaneous parametric downconversion, the detection of one photon heralding the production of its twin. Filtering techniques must be used so that the mode of the heralded photon can be made indistinguishable from that of the coherent state produced. (In Ref. [33], we show how one can take mode mismatches into account.) Let $\eta_1$ be the coupling efficiency of the single photon, and let $\eta_2$ be the global detection efficiency, including the transmission from the 50:50 beamsplitter to the detector, as before. For small heralding efficiency, and if the parametric process is weakly pumped so that the success probability for the emission of one photon pair is $\eta_c = \eta |\alpha|^2 - 2(\eta + 3\eta)\eta_2 |\alpha|^2$. Here $\eta = \eta_1 \eta_2$ and $\epsilon = 1 - V$, where $V$ is the interferometric visibility that characterizes the phase stability of $A$, $B$ and the local oscillator (we assume that the modes have independent phase fluctuations with the same variance). To know the value of the visibility that can be obtained in practice, we built a balanced Mach-Zehnder interferometer (see Ref. [33]).

Using an active stabilization, we measured a visibility of $V = 99.996 \pm 0.001\%$. Assuming a coupling efficiency $\eta_1 = 50\%$ and a detection efficiency $\eta_2 = 60\%$, the concurrence remains positive ($C \approx 0.01$) for $|\alpha| = 28$. This translates into entanglement populated by more than $(2 |\alpha|^2 + 1) = 1500$ photons.

### IX. CONCLUSION

We have presented a scheme for creating and revealing macroscopic entanglement with a single photon, coherent states, and linear optical elements. The simplicity of our proposal is conceptually remarkable. On the one hand, it highlights the idea that although quantum systems are difficult to maintain and observe at macro scales, they can easily be created. On the other hand, it naturally raises the following question: is the resulting state really macroscopic? We have shown through experimental results that the entangled state that could be obtained with currently available technologies would involve a large enough number of photons to be seen with the naked eye [34]. This makes our approach satisfactory if macroscopicity is a notion related to size. We also mentioned that the components of the entangled state can easily be distinguished with a mere avalanche photodiode if one looks at the variance of the photon number distribution. This pleases those who believe that macro entangled states need to have components that can be easily distinguished.

Although our study showed that the resulting state features an unexpected robustness against loss, we have shown that it is also more and more fragile under phase disturbance when its size increases. Our approach is thus also satisfactory if macroscopic means sensitive to decoherence, and it highlights the complexity of possible interactions between a given quantum system and its surroundings. We have also seen that the precision of the measurement that is required to reveal the quantum nature of the produced state increases with its size. This also makes our scheme satisfactory if macroscopicity is related to the requirement on the measurement precision. Note that there are many other candidates for macroscopicity measures [35]. Testing each of them is work for the future.

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[7] In the presence of loss before and after the phase covariant cloner, we do not know when the amplified state becomes separable.
[18] Negativity $N$ [19] conveniently quantifies entanglement. It is defined by $N = \sum_{\lambda i} |\lambda|$ , where $\lambda$ are the eigenvalues of the partially transposed density matrix. According to this definition, the negativity of a two-qubit state is bounded by 1/2.
[20] Consider transmission efficiency $\epsilon$ such that $\eta|\alpha|^2 \gg 1$. Furthermore, consider that the mod $B$ undergoes loss. The coherent state entanglement becomes $|\alpha|, |\sqrt{\eta}|\alpha|, |\sqrt{\eta}a|, |\sqrt{\eta}a| + | - |a| - \eta|a|, | - \eta|a|, | - \eta|a| / \sqrt{\eta}a|$, where $\epsilon$ stands for the lost mode. Tracing $\epsilon$ out, one gets

\[ \frac{1}{2} (|\alpha|, |\sqrt{\eta}|\alpha|, |\sqrt{\eta}a|, |\sqrt{\eta}a| - |a| - \eta|a| + | - |a| - \sqrt{\eta}a| - \eta|a| - \sqrt{\eta}a| + | - |a| - \sqrt{\eta}a| - \eta|a|, |\sqrt{\eta}a|, |\sqrt{\eta}a|) \]

The smallest eigenvalue associated with its partial transpose is $-e^{-2|\eta|a|^2}$.

[23] Note that if multimode memories are used, the number of ebits could easily be increased by combining time-bin qudits with a bright coherent state on a beamsplitter.

[26] In practice, the phase stability is usually characterized by the classical interferometric visibility $\bar{V}$ that would be obtained by recombining modes $A$ and $B$ on a 50:50 beamsplitter [27]. Using the relation between the visibility and the phase fluctuation $V = \sqrt{1 - \delta \varphi^2}$, we obtain $\bar{V}_{\text{out}}(V, |\alpha|^2) \geq V (1 + 4|\alpha|^2(1 - V^2)).$


[28] Let $|\gamma\rangle$ and $r$ be the local oscillator and the reflectivity of the beamsplitter used to perform one of the local displacements. In the limit $|\gamma| \to \infty$ and $r \to 0$ such that $\gamma r = \text{const} = -\alpha$, any input state $\hat{\rho}$ evolves as $\hat{D}(-\alpha)\hat{\rho}\hat{D}^\dagger(-\alpha)$, that is, it is displaced in the phase space by $-\alpha$; see M. G. A. Paris, Phys. Lett. A 217, 78 (1996) for detailed proof. For small but nonzero reflectivity, $\gamma$ can always be tuned to fulfill $\gamma r = -\alpha$, so that the corresponding operation simply corresponds to a displacement $\hat{D}(-\alpha)$ followed by loss modeled with a beamsplitter with amplitude transmission $r$.


Supplemental Material - Proposal for Exploring Macroscopic Entanglement with a Single Photon and Coherent States

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I. MODE MISMATCH

In order to create the desired macroscopic entanglement, the A and B modes that are combined into a beamsplitter need to be indistinguishable. If they do not overlap, the output of the beam-splitter is described by

\[
\frac{1}{\sqrt{2}} (|\alpha\rangle_{A_c} |\alpha\rangle_{B_c} \otimes (|1\rangle_{A_s} |0\rangle_{B_s} - |0\rangle_{A_s} |1\rangle_{B_s})) ,
\]

where \(A_c, A_s, B_c, B_s\) label two different modes at the location A (B) that is, the output state is a tensor product of two entangled modes \(A_c, B_c\) (sharing a single photon) with two modes \(A_s, B_s\) filled with coherent states. In this case, the observed concurrence is no longer an entanglement measure of a macro state.

In principle, the indistinguishability can be arbitrary close to unity. The spectral overlap can be ensured using a single narrowband filter. The use of single mode fibers guarantees the transverse spatial overlap. The polarization can be precisely tuned using polarization controllers. Lastly, the temporal overlap is achieved by carefully adjusting the optical path lengths.

The following analysis details the procedure to follow to evaluate the mode overlap and to take the potential mode mismatch into account.

A. Bounding the mode mismatch

Consider the case where the two modes impinging the 50:50 beam-splitter partially overlap, i.e. the bosonic operators \(a\) and \(b\) are replaced by

\[
\sqrt{\mathcal{F}_1}a + \sqrt{1-\mathcal{F}_1}a_s \quad (2)
\]
\[
\sqrt{\mathcal{F}_2}b + \sqrt{1-\mathcal{F}_2}b_c \quad (3)
\]

where \(\mathcal{F}_1, \mathcal{F}_2\) defines the overlap. Two approaches can be used to bound \(\mathcal{F}_1, \mathcal{F}_2\).

The first one is based on an Hong-Ou-Mandel (HOM) interference [1]. When two photodetectors (non-photon number resolving detectors) monitor the output of the beam-splitter as a function of the delay between the two inputs, the visibility of the resulting interference (between a single photon and a coherent state that partially overlap) is given by

\[
V_{\text{HOM}} = \mathcal{F}_1 \mathcal{F}_2 \frac{\eta^2 |\alpha|^2 e^{\delta(\alpha)}}{(\eta |\alpha|^2 - 1)(\eta |\alpha|^2 + \eta - 1)} \quad (4)
\]

where \(\eta\) stands for the detection efficiency. For small \(|\alpha|^2\) \((< 1.4)\), this gives a lower bound on the mode overlap

\[
\mathcal{F}_1 \mathcal{F}_2 > V_{\text{HOM}} |\alpha|^2 - 1 |\alpha|^{-2} > V_{\text{HOM}} . \quad (5)
\]

Under the assumption that the mode associated to the coherent state is unchanged if it is attenuated, one can access a lower bound on the mode overlap from the measurement of the visibility of the HOM interference obtained by combining the single photon with the faint coherent state.

The second approach allows one to access the mode overlap without the need to decrease the intensity of the coherent state. It is based on the remark that the variance of the photon number associated to the displaced coherent state \(D(\alpha) |1\rangle\) is 3 times larger than the one for the coherent state \(|\alpha\rangle\) (and thus the one for the state for which the displacement does not overlap with the single photon \(|\alpha\rangle |1\rangle\)). More precisely, the state that is obtained by tracing out one of the two modes of the macro entangled state is given by \(\rho_a = \frac{1}{2} D_a(\alpha) |1\rangle|1\rangle D^\dagger_a(\alpha) + \frac{1}{2} |\alpha\rangle|\alpha\rangle\) if the input modes are perfectly indistinguishable. The decomposition of this state in the photon number basis is characterized by a variance \(\text{Var}(\bar{n}) = 2 |\alpha|^2 + \frac{1}{2}\). In the case where the two input modes do not overlap, the corresponding variance falls down to \(\text{Var}(\bar{n}) = |\alpha|^2 + \frac{1}{2}\). One shows that in the general case, the variance is given by

\[
\text{Var}(\bar{n}) = |\alpha|^2 (\mathcal{F}_1 \mathcal{F}_2 + 1) + \frac{1}{4}. \quad (6)
\]

By measuring the ratio between the variances with and without the single photon input on the beamsplitter, one has a direct access to the mode overlap

\[
\frac{\text{Var}_1(\bar{n})}{\text{Var}_0(\bar{n})} = \frac{|\alpha|^2 (\mathcal{F}_1 \mathcal{F}_2 + 1) + \frac{1}{4}}{|\alpha|^2} \approx \mathcal{F}_1 \mathcal{F}_2 + 1 \quad (7)
\]

where the last equality holds for \(|\alpha|^2 \gg 1\).

B. Evaluating the size of the macro entanglement with imperfect mode matching

We now show how one can use the knowledge on the mode overlap to evaluate the size of the created macro
state, i.e. its effective mean photon number. From the definition of input modes given in Eqs. (2)-(3), the output state is given by

\[
\left[ \sqrt{F_1} D_\alpha(\sqrt{F_2}) D_b(\sqrt{F_2}) \left( |0\rangle_A |1\rangle_B - |1\rangle_A |0\rangle_B \right) \right] \\
\otimes |0\rangle_{A_s} |0\rangle_{B_s} + \\
\sqrt{1-F_1} \left[ \alpha \sqrt{F_2} |A\rangle_A \alpha \sqrt{F_2} |B\rangle_B \right] \\
\otimes \left( |0\rangle_{A_s} |1\rangle_{B_s} - |1\rangle_{A_s} |0\rangle_{B_s} \right) \\
\otimes \left[ \alpha \sqrt{1-F_2} |A\rangle_A \alpha \sqrt{1-F_2} |B\rangle_B \right].
\] (8)

The number of photons contributing to the macro entangled component is thus given by \(2F_2|\alpha|^2 + 1\) and can be bound either from

\[
\text{size} \geq 2V_{HOM} |\alpha|^2 + 1 \quad (9)
\]

or by

\[
\text{size} \geq 2|\alpha|^2 \left( \frac{\text{Var}_1(\hat{n})}{\text{Var}_0(\hat{n})} - 1 \right) + \frac{1}{2} \quad (10)
\]

depending on the way the overlap is estimated.

### C. Witnessing the macro entanglement with imperfect mode matching

One sees from Eq. (8) that the state resulting from mode mismatch is a superposition between the macro component with relative amplitude \(\sqrt{F_1}\) and an undesired component with relative amplitude \(\sqrt{1-F_1}\) where entanglement (between mode \(A_s-B_s\)) involves a single photon. The latter may lead one to wrongly witness the presence of entanglement at the macro level.

We here propose two methods to confidently estimate the entanglement of the macro component. The first one is based on the assumption that the modes \(A_s\) and \(B_s\) can only add noise to the measurements. The observed concurrence would thus increase by filtering them out. Let us take a closer look on the concurrence obtained from the filtered state, i.e. from

\[
\sqrt{F_1} D_\alpha(\hat{a}) D_\alpha(\hat{a}) \left[ \frac{1}{\sqrt{2}} (|1\rangle_0 - |0\rangle_1)_{a,b} \otimes |0\rangle_{a_s} |0\rangle_{b_s} + \\
\left. \sqrt{1-F_1} |\hat{a}\rangle_a |\hat{a}\rangle_b \otimes \frac{1}{\sqrt{2}} (|1\rangle_0 - |0\rangle_1)_{a_s} |b_s\right] \right|_{\Psi_{st}} \right|_{\Psi_s} (11)
\]

with \(\hat{a} = \alpha \sqrt{F_2}\). Using non resolving photon number detector (modeled e.g. by the operator \(1-(1-\eta) a^\dagger a + a^\dagger a\)) for the modes located at \(A\), the coherence between the two terms in Eq. (11) do not play any role, and the measurement results are identical to the one obtained from a state that is a mixture of \(|\Psi_M\rangle\) with probability \(F_1\) and \(|\Psi_s\rangle\) with probability \(1-F_1\). Furthermore, one can show that the concurrence of this mixture is always lower than or equal to \(F_1 C_{\langle \Psi_{st} \rangle} + (1-F_1) C_{\langle \Psi_s \rangle}\) where \(C_{\langle \Psi_{st} \rangle}\) is the concurrence that would be obtained from \(|\Psi_{st}\rangle\). Using the assumption that \(C_{\langle \Psi_s \rangle}\) is smaller than or equal to that measured without amplification \(\alpha = 0\), we obtained \(C_{\langle \Psi_{st} \rangle} \geq \frac{1}{2} C_{\langle \Psi_s \rangle}\) (where \(C\) and \(C_{\alpha=0}\) are the concurrences measured with and without displacement). Finally, using \(F_1 \geq F_1F_2\), one obtains the following inequality for the concurrence of the modes \(a\) and \(b\)

\[
C_{\langle \Psi_{st} \rangle} \geq C - (1-F_1F_2)C_{\alpha=0}. \quad (12)
\]

Therefore, by measuring the overlap between the modes that are combined into a 50:50 beamsplitter to form the desired macro entanglement, the concurrence with the mode \(B\) filled with a coherent state \((C)\) and with vacuum \((C_{\alpha=0})\), the entanglement of the macro component can be bounded using the inequality (12).

Another way to deal with this problem is to use homodyne detections to witness the entanglement from a Bell-like test [2]. If the local oscillators that are required for homodyning are obtained by picking the coherent state (used for the displacement) off with variables beamsplitters, the homodyne detections acts as filters and the contributions \(A_s-A_c\) and \(B_s-B_c\) are detected only. The modes \(A_s\) and \(B_s\) have thus to be traced out and if entanglement is observed, it necessarily comes from the desired component

\[
D_\alpha^{\dagger}(\alpha \sqrt{F_2}) D_\alpha(\alpha \sqrt{F_2}) \left( |0\rangle_A |1\rangle_B + |1\rangle_A |0\rangle_B \right). \quad (13)
\]

### II. PHASE INSTABILITIES

We have seen in the main text that entanglement involving a large number of photons could be observed with the proposed scheme provided that the relative lengths of several optical paths can be interferometrically stabilized. We here show a possible experimental realization and demonstrate with preliminary experimental results how a good enough stabilization could be obtained to measure tunable entanglement involving from one to a thousand photons.

#### A. Possible experimental realization

For concreteness, we focus on a realization using a source of photon pairs, similarly to what was reported in Ref. [3] and [4]. In these works, a continuous wave laser is used to pump a PPKTP waveguide such that photon
pairs are created through spontaneous parametric down-conversion. One photon from a pair is detected to herald the creation of its twin. Both photons are filtered to produce a heralded single photon with a spectral width of 40 MHz. Note that this width is determined by the filtering cavities, and can thus be set to other values that can either be smaller or larger. The coherent state with matching spectral width can be prepared from a broadband laser diode that is filtered with the same cavity used to filter the single photon (here, broadband means that the spectrum of the laser diode is larger than the width of the filtering cavity). Spatial overlap of the single photon and the coherent state is ensured by coupling into single-mode optical fibres. Finally, the polarizations of all beams can be easily prepared to the same state with high-extinction polarizers. We note that the coherence time of the proposed source is of the order of a few nanoseconds, which is greater than the resolution time of usual single photon detectors. This allows one to post-select a single temporal mode.

Given this single photon source and this coherent state, one can use the setup shown on Fig. 1 to prepare and measure the macro entanglement. The local oscillator in state $|\sqrt{2}\alpha/R\rangle$ (where $R$ is defined below) is incident on beamsplitter $BS_1$, whose transmittance is $T^2 = 1$ and whose reflection is $R^2 = 1 - T^2 = 0$. From there, it can recombine with itself either at beamsplitter $BS_2$ or at $BS_3$, whose coefficients are also $T^2$ and $R^2$. This yields two balanced Mach-Zehnder interferometers such that both paths have the same intensity when interfering after $BS_2$ and $BS_3$, i.e. when one path is reflected at $BS_1$ and transmitted $BS_2$ (or $BS_3$), and the other path is transmitted at $BS_1$ and reflected at $BS_2$ (or $BS_3$). Stabilization of these interferometers can be performed by multiplexing another laser in mode $A_{in}$, whose intensity measured into modes $A_2$ and $A_3$ allows to probe and stabilize the relative phase of the interferometers. Finally, there is a third balanced interferometer formed between the $50/50$ beamsplitter $BS_4$ and the $50/50$ beamsplitter used to probe the coherence between the modes obtained after the displacements performed at $BS_2$ and $BS_3$ (this is part the “detection” box in Fig. 1). This interferometer can also be probed with another laser injected in a mode that is different from all the other ones.

**B. Measurement of the interference visibility within a balanced Mach-Zehnder**

Globally, what we obtain is three balanced Mach-Zehnder interferometers that require to be extremely well phase stabilized in order to create and measure the macroscopically populated entangle state. In particular, the size of the state depends on this phase stability, as detailed in the main text. To estimate this phase stability, we built a balanced Mach-Zehnder interferometer with bulk optics (see Fig. 2a). One arm contains a mirror that can be moved using a piezo actuator. The interferometer’s phase is probed using an external cavity laser diode (the locking laser), and it is locked to the side of an interference fringe with a feedback signal on the piezo. The locking laser is widely tunable around 1560 nm. Thus, by tuning its wavelength while is it locked, we can continuously tune the phase of the interferometer with a large precision. The quality of the interference can be probed using another laser (the probe laser) whose wavelength is fixed at 1560 nm and that propagates into different optical modes to avoid an overlap with the locking laser. On Fig. 2b, we see the optical power $P_{min}$ (in dBm) leaking through the interferometer when the phase is set to yield destructive interference for the probe laser. The system is locked to this minimum and is kept stable for 10 minutes. After this, the wavelength of the locking laser is tuned to yield constructive interference with power $P_{max}$ for 8 minutes. The extinction ratio is $P_{max} - P_{min} = 47.2$ dB, corresponding to a visibility of $99.996 \pm 0.001\%$. To obtain this near-perfect visibility, the path imbalance of the interferometer was reduced to less than 200 µm to mitigate the effect of non-zero spectral width of the probe laser ($\approx 1$ GHz). Also, both modes of the interferometer are polarized before being coupled into a single-mode optical fiber. The limit is potentially due to a combination of imperfect mode-matching, unequal loss from both arms, acoustic vibrations and electrical noise in the feedback signal. The measured visibility allows us to estimate the number of photons contained in the macro-macro entangled state $\approx 1500$. Hence, macro-macro entanglement with a thousand photons is within experimental reach with our approach, and states with $10^5$ photons are foreseeable. This would require, in particular, that the coherent state to be stabilized to within 100 MHz or less, and to control the relative loss in both arms to within $10^{-3}$.  

**FIG. 1: Proposed setup to prepare and measure the macro entangled state. The single photon is combined with part of the local oscillator at the 50/50 beamsplitter $BS_1$. In parallel, the coherent state with amplitude $|\sqrt{2}\alpha/R\rangle$ is split among the different modes using $BS_1$. It is subsequently used to displace the single photon at $BS_4$, and later to displace the macro-entangled state back into a single-photon entangled state using the $BS_2$ and $BS_3$ beamsplitters. This arrangement yields two balanced Mach-Zehnder interferometers that need to be phase stabilized.**
FIG. 2: (a) Setup to measure the interference visibility. A balanced Mach-Zehnder interferometer made with two 50/50 beam-splitters (BS) is locked using a tunable “locking laser”. The locking detector’s signal is feed into a locking circuit driving a piezo controlling the length of one of the arms. By tuning its wavelength, the phase of the interferometer can be tuned continuously. The quality of the interference can be probed using a “probe laser” whose wavelength is fixed. (b) Measured power (in dBm) at the “probe detector” as function of time. The interferometer’s phase was first locked to yield destructive interference for the probe laser for 10 minutes, and then was locked to constructive interference for 8 minutes. The measured interference visibility is 99.996 ± 0.001%.