Controlled creation of spin domains in spin-1 Bose-Einstein condensates by phase separation

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A method of controlled creation of spin domains in spin-1 antiferromagnetic Bose-Einstein condensates is demonstrated. The method exploits the phenomenon of phase separation of spin components in an external potential. By using an appropriate time-dependent potential, a composition of spin domains can be created, as demonstrated in the particular cases of a double well and a periodic potential. In contrast to other methods, which rely on spatially inhomogeneous magnetic fields, here the domain structure is completely determined by the optical fields, which makes the method versatile and reconfigurable. It allows for creation of domains of various sizes, with the spatial resolution limited by the spin healing length only.

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I. INTRODUCTION

Bose-Einstein condensates with spin degrees of freedom [1] attracted in recent years great interest due to the unique possibility of exploring fundamental concepts of quantum mechanics in a remarkably controllable and tunable environment. The ability to generate spin squeezing and entanglement [2] makes spinor Bose gases promising candidates for applications such as quantum simulators [3], in quantum information [4], and for precise measurements [5]. Moreover, spinor condensates were successfully used to recreate many of the phenomena of condensed-matter physics in experiments displaying an unprecedented level of control over the quantum system. In particular, spin domains [6–8], spin mixing [9], and spin vortices [10] were predicted and observed.

The ability to create spin domains is a crucial component of applications including data storage and spin-based logic implementation [11]. To date, domain structures in Bose-Einstein condensates were created in a controllable way using inhomogeneous magnetic fields [6]. In principle, these can be induced by magnetic coils, electronic chips, or permanent magnets [12]. However, both the spatial resolution of this method and the ability to produce arbitrarily shaped, reconfigurable spin-domain structures is severely limited.

In recent papers [13,14], we considered the possibility of phase separation in the ground state of spinor condensates. We demonstrated that this phenomenon can take place in both antiferromagnetic and ferromagnetic condensates in external potentials. In this paper, we exploit the phenomenon of phase separation to propose a method of controlled creation of metastable spin-domain states with a chosen spatial spin structure. It consists of applying an appropriately designed optical potential, which is subsequently slowly relaxed toward the desired final trapping potential. In contrast to the other method, relying on magnetic field gradients to separate spin components, our method uses more easily reconfigurable optical laser fields to shape the spatial structure. Importantly, these structures are not pinned by local extrema of the magnetic field. An additional advantage is the high spatial resolution, which we demonstrate to be generally limited only by the spin healing length of the condensate.

The paper is organized as follows. Section II reviews the mean-field model of a spin-1 condensate in a homogeneous magnetic field. In Sec. III we present numerical simulations of the spin-domain creation process. In Sec. IV we discuss practical limitations of the proposed method using both analytical and numerical approach. Section V concludes the paper.

II. MODEL

We consider a spin-1 Bose-Einstein condensate in a homogeneous magnetic field pointing along the $z$ axis. Although in general our scheme could be applied to any condensate exhibiting phase separation, we focus on the case of a $^{23}$Na condensate, which has properties that make our method especially effective, such as the very weak dipolar interactions, stability against spin-flip transitions to the other hyperfine manifold, and the relatively long lifetime [7]. We apply the mean-field approximation, which describes dilute Bose-Einstein condensates at zero temperature. In spinor condensates, ground states can substantially deviate from the mean-field solutions even at zero temperature due to large quantum fluctuations [15]. However, it was shown that the introduction of a weak magnetic field restores the validity of the mean-field model [16], due to bosonic enhancement, which favors polarization parallel to the magnetic field.

We start with the Hamiltonian $H = H_S + H_A$, 

$$H = \sum_{j=-0,+,1} \int d\mathbf{r} \psi_j^* \left( -\frac{\hbar^2}{2m} \nabla^2 + \frac{c_0}{2} n + V(\mathbf{r}) \right) \psi_j + H_A,$$  

(1)

where $\psi_{-0,+,1}$ are the wave functions of atoms in magnetic sublevels $m_\pi = -1,0,1$, $m$ is the atomic mass, $V(\mathbf{r})$ is an external potential, and $n = \sum n_j = \sum |\psi_j|^2$ is the total atom density. The asymmetric (spin-dependent) part of the Hamiltonian is given by 

$$H_A = \int d\mathbf{r} \left( \sum_{j=-0,+,1} E_j n_j + \frac{c_2}{2} |\mathbf{F}|^2 \right)$$  

(2)

where $E_j$ is the Zeeman energy shift for state $\psi_j$ and the spin density is 

$$\mathbf{F} = (F_x,F_y,F_z) = (\psi_+\hat{F}_x\psi_1,\psi_1\hat{F}_y\psi_+\psi_1\hat{F}_y\psi_1,\psi_1\hat{F}_z\psi_+\psi_1\hat{F}_z\psi_1),$$  

(3)

where $\hat{F}_{x,y,z}$ are the spin-1 matrices [17] and $\psi = (\psi_+,\psi_0,\psi_-)$. The spin-independent and spin-dependent
interaction coefficients are given by
\begin{equation}
 c_0 = 4\pi\hbar^2(2a_0 + a_0)/3m \\
c_2 = 4\pi\hbar^2(a_2 - a_0)/3m,
\end{equation}
where \(a_s\) is the \(s\)-wave scattering length for colliding atoms with total spin \(S\). The total number of atoms and the total magnetization in the direction of the magnetic field
\begin{equation}
 N = \int n d\mathbf{r},
\end{equation}
\begin{equation}
 M = \int F_z d\mathbf{r} = \int (n_+ - n_-) d\mathbf{r}
\end{equation}
are conserved quantities. A given magnetization can be prepared, for example, by applying a microwave pulse \([18]\).

In the above, we have neglected the dipole-dipole interactions, which would break the conservation of the magnetization. The Zeeman energy shift for each of the components, \(E_j\), can be calculated using the Breit-Rabi formula \([19]\)
\begin{equation}
 E_\pm = -\frac{1}{8} E_{\text{HFS}} (1 + 4 \sqrt{\bar{\alpha} + \alpha^2}) \pm g_I \mu_B B,
\end{equation}
\begin{equation}
 E_0 = -\frac{1}{8} E_{\text{HFS}} (1 + 4 \sqrt{\bar{\alpha} + \alpha^2}),
\end{equation}
where \(E_{\text{HFS}}\) is the hyperfine energy splitting at zero magnetic field, \(\bar{\alpha} = (g_J - g_I) \mu_B B / E_{\text{HFS}},\) where \(\mu_B\) is the Bohr magneton, \(g_I\) and \(g_J\) are the nuclear and electronic \(g\) factors, and \(B\) is the magnetic field strength. The linear part of the Zeeman effect gives rise to an overall shift of the energy, and so we can remove it with the transformation
\begin{equation}
 H \rightarrow H + (N + M)E_+/2 + (N - M)E_-/2.
\end{equation}
This transformation is equivalent to the removal of the Larmor precession of the spin vector around the \(z\) axis \([20]\). We thus consider only the effects of the quadratic Zeeman shift. For sufficiently weak magnetic field we can approximate it by
\begin{equation}
 \delta E = (E_+ + E_- - 2E_0)/2 \approx \alpha^2 E_{\text{HFS}}/16,
\end{equation}
which is positive for \(^{87}\text{Rb}\) and \(^{23}\text{Na}\) condensates.

The Hamiltonian \((1)\) gives rise to the Gross-Pitaevskii equations describing the mean-field dynamics of the system
\begin{equation}
 i\hbar \frac{\partial \psi_\pm}{\partial t} = [\mathcal{L} + c_2(n_+ + n_0 - n_\pm)] \psi_\pm + c_2 \psi_0^* \psi_\pm^*,
\end{equation}
\begin{equation}
 i\hbar \frac{\partial \psi_0}{\partial t} = [\mathcal{L} - \delta E + c_2(n_+ + n_-)] \psi_0 + 2c_2 \psi_+ \psi_- \psi_0^*,
\end{equation}
where \(\mathcal{L}\) is given by \(\mathcal{L} = -\hbar^2 \nabla^2 / 2m + c_0 n + V(r)\).

By comparing the kinetic energy with the interaction energy, we can determine the healing length \(\xi = 2\pi\hbar / \sqrt{2mc_0}\) and the spin healing length \(\xi_s = 2\pi\hbar / \sqrt{2mc_2}\). These quantities give the length scales of spatial variations in the condensate profile induced by the spin-independent or spin-dependent interactions, respectively. Analogously, we define the magnetic healing length as \(\xi_B = 2\pi\hbar / \sqrt{2m\delta E}\).

In spin-1 condensates created to date, the \(c_0\) and \(c_2\) scattering lengths have similar magnitudes. The spin-dependent interaction coefficient \(c_2\) is then much smaller than its spin-independent counterpart \(c_0\). For example, this ratio is about 1 : 30 in a \(^{23}\text{Na}\) condensate and 1 : 220 in a \(^{87}\text{Rb}\) condensate far from Feshbach resonances \([21]\). Consequently, changing the total density \(n\) requires much more energy than changing the relative populations of spin states \(n_j\). In our considerations we will assume that the total atom density profile \(n(r,t)\) is close to the Thomas-Fermi profile for a given potential \(V(r,t)\).

\section{III. Spin-Domain Creation}

In Refs. \([13,14]\) we demonstrated that in the presence of a homogeneous magnetic field, a spin-imbalanced (magnetized) antiferromagnetic condensate is subject to phase separation in the ground state resulting in separate spin domains. The spin domains align according to a simple rule: The magnetized domains reside in the regions with the lowest density, while the unmagnetized domains fill the areas of high density. In this way, the nonlinear spin energy of the antiferromagnetic condensate is minimized for a given total magnetization. In this work, we will show that one can use this property to shape the spatial distribution of spin with an appropriate time-varying external potential.

We consider a quasi-one-dimensional (1D) sodium condensate trapped in an elongated harmonic potential described by the 1D version of Eq. \((8)\) \([13]\) with rescaled interaction coefficients \(c_0^{1D}, c_2^{1D} = (m \omega_\perp) / (2\pi \hbar c_0, c_2),\) where \(\omega_\perp\) is the transverse trapping frequency. The Fermi radius of the transverse trapping potential is smaller than the spin healing length, and the nonlinear energy scale is much smaller than the transverse trap energy scale, which allows us to reduce the problem to one spatial dimension \([21,22]\). To create a double-well potential, we add a Gaussian potential barrier that can be realized, for example, by an additional blue-detuned

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig1}
\caption{(Color online) (a) Ground state of a sodium condensate with fixed magnetization \(M = 0.4\), number of atoms \(N = 8.4 \times 10^3\), trap frequencies \(\omega_\rho = 2\pi \times 1000\text{Hz}\) and \(\omega_z = 2\pi \times 2.5\text{Hz},\) and the magnetic field strength \(B = 120\text{mG}.\) The \(n_+, n_0,\) and \(n_-\) components are depicted by dash-dotted, dashed, and dotted lines, respectively, and the solid lines show the total density. (b) The result of a slow introduction of a Gaussian-shaped barrier with \(A = 3.3 \times 10^{-30}\text{J}\) and \(w = 90\mu\text{m}\) during the time \(t = 5\text{s}.\)}
\end{figure}
CONTROLLED CREATION OF SPIN DOMAINS IN SPIN-1

FIG. 2. (Color online) (a) Ground state of the double-well potential with parameters as in Fig. 1. (b) For comparison, we show the ground state in the absence of the Gaussian barrier. (c) Metastable spin-domain structure obtained starting from (a), after the removal of the Gaussian barrier within \( t = 5 \) s. (d)–(f) Same as (a)–(c), but for a weaker magnetic field \( B = 45 \) mG and magnetization \( M = 0.8 \).

In the absence of the barrier, and under certain experimental conditions [13], the ground state is characterized by spatial separation of the \( m_f = 0 \) and \( m_f = \pm 1 \) atoms (or \( m_f = -1 \), depending on the magnetization) [see Fig. 1(a)]. After switching the barrier on, the condensate splits into an effective longitudinal double-well potential into two symmetric parts. We note that the ground state now has the structure corresponding to Fig. 2(a). However, the result of a slow increase of the barrier on the time scale of several seconds results in a different state, where inner \( m_f = +1 \) domains are absent on both sides of the barrier. The difference between Fig. 1(b) and Fig. 2(a) is a result of the strong spin immiscibility [7] of the \( m_f = 0 \), +1 atoms that suppresses the tunneling of \( m_f = +1 \) atoms through the \( m_f = 0 \) atoms. The final state is thus a metastable state, but practically stable on the relevant experimental time scales.

An even more intriguing effect can occur when the condensate is prepared in the ground state of the double-well potential (with the barrier on) [Fig. 2(a)] and after gradually lowering the potential barrier. The condensate, instead of evolving toward the single-well ground state [Fig. 2(b)] creates another kind of a metastable state, with five alternating spin domains of \( m_f = 0 \) and \( m_f = \pm 1 \) atoms shown in Fig. 2(c). Again, we checked that these spin domains are perfectly stable on the time scales as long as several seconds due to suppressed tunneling. In principle, by applying several potential barriers with certain parameters, it is possible to create a given number of stable spin domains of various sizes.

We note that the above spin-domain creation scenario is possible only for domains which consist of immiscible atoms. The phase separation of \( m_f = 0 \) and \( m_f = \pm 1 \) atoms is a feature of antiferromagnetic spin-1 condensates in a relatively strong magnetic field [13]. In the weaker field regime, the spin-imbalanced antiferromagnetic condensate generally consists of \( m_f = +1 \) and \( m_f = -1 \) atoms, which are miscible. As a result, the spin components do not separate, and the slow process always results in a state close to the ground state of the system, as shown in Figs. 2(d)–2(f). The finer peak structure in Fig. 2(f) results from the fact that the removal of the barrier is not perfectly adiabatic. The small density peaks and dips can be interpreted as dark-bright soliton pairs [23]. It was demonstrated that solitons can be created in condensates during nonadiabatic phase transitions [24].

Furthermore, we consider a condensate placed in a periodic optical potential

\[
V(z) = \frac{1}{2}m_0 \omega_z^2 z^2 + A(t)e^{-z^2/2w^2}.
\]

(9)

with \( d \) being the lattice period. We simulate the dynamics of the condensate with periodic boundary conditions, which can be experimentally realized, for example, with a ring-shaped trapping geometry [25]. Analogously as in the previous case, we start with a condensate with the optical lattice switched on. The density and spin pattern is shown in Fig. 3(a). By gradually decreasing the optical lattice strength, we arrive at a metastable state depicted in Fig. 3(b), composed of multiple \( m_f = 0 \) and \( m_f = \pm 1 \) spin domains. This state is also stable on time scales of the order of seconds. We note that by modifying the lattice potential through introducing higher-order Fourier components, we are also able to adjust the size of individual domains.

IV. LIMITATIONS OF THE METHOD

We now discuss some limitations of the above method of domain creation. First, we need to consider the assumptions
FIG. 3. (Color online) (a) Ground state in a periodic potential with $d = 140 \mu m$ and $\Lambda = 3.3 \times 10^{-30}$ J. (b) The resulting domain structure after switching the potential off during $t = 5$s.

FIG. 4. (Color online) Same as in Fig. 2(c), but (a) for $\omega_z = 2\pi \times 10$ Hz and $N = 2100$ and (b) for $\omega_z = 2\pi \times 20$ Hz and $N = 1050$.

made when deriving the model Hamiltonian. In particular, we neglected the dipole-dipole interactions, which could break the magnetization conservation and possibly influence the spatial patterns. The effects of dipolar interactions have been observed in chromium and rubidium condensates [26,27]. In the presence of strong dipolar interactions, we could expect complicated spin relaxation dynamics analogous to the Einstein-de Haas effect [28]. However, the effect of the tunneling is clearly visible when comparing Fig. 2(c) with Figs. 4(a) and 4(b). With the decreasing distance between the domains the state becomes unstable [see Fig. 4(a)]. In the case of the tightest trap [Fig. 4(b)] the middle domain dissolves completely and the single-well ground state is obtained.

The tunneling rate can be estimated analytically along the lines of Ref. [7], where the case of a spatially varying magnetic field potential was considered. Here, we apply a similar method to the case of a spatially varying external potential. First, we assume that the condensate separates into two components, one of them being $m_f = 0$ and the other either $m_f = +1$ or $m_f = -1$, and neglect the influence of the third component. The potential energy for the $m_f = i$ atoms is given by

$$V_i(z) = V(z) + g_i n_i + g_{ij} n_j,$$

where $j$ is the other present component. The interaction constants in an $F = 1$ condensate are $g_0 = c_0$ and $g_{\pm 1} = g_{0\pm 1} = c_0 + c_2$. We estimate the tunneling rate of $m_f = +1$ atoms from the middle domain in a structure similar to the one shown in Fig. 2(c) toward the edges of the condensate. The chemical potential of atoms in the middle domain is $\mu_{+1} = V(z) + g_1 n_{+1}(z) = \text{const}$. We assume that the density of $m_f = 0$ component is negligible. Analogously, in the neighboring domain the $m_f = 0$ atoms have chemical potential equal to $\mu_0 = V(z) + g_0 n_0(z)$. The $m_f = +1$ atoms that tunnel through this domain feel the potential $V_{+1}(z) = V(z) + g_0 n_0(z)$. 

Their density profile can be calculated in the Wentzel-Kramers-Brillouin (WKB) approximation

\[ \frac{dn_{+1}(z)}{dz} = \frac{2\sqrt{2m}}{\hbar} \sqrt{V_{+1}(z) - \mu + 1} \]

\[ = -\frac{2\sqrt{2m}}{\hbar} \sqrt{\mu_0 - \mu + 1 + n_0(z)(g_{01} - g_0)}, \]

(12)

where the decay of the density means that \( dn_{+1}/dz > 0 \) on the right side of the middle domain and \( dn_{+1}/dz < 0 \) on the left side. In the simplest approximation, \( n_0(z) \) does not vary significantly and the decay is exponential. Taking into account the pressure balance at the domain boundary at \( z = z_0, \ g_0n_0(z_0)^2 = g_1n_{+1}(z_0)^2 \), we can estimate the decay as \( n \sim \exp(-\alpha z) \) with \( \alpha = 2\sqrt{mc_0n_0(z_0)}/\hbar = 2^{3/2}\pi\xi^{-1} \), where \( \xi \) is the spin healing length. We can now write down the formula for the tunneling rate in the metastable state

\[ \frac{dN}{dt} = \gamma e^{-\alpha z}, \]

(13)

where \( \Delta z \) is the distance between the domains and \( \gamma \) is the attempt rate \( \gamma = 2\sqrt{2m}n_0(z_0)(\nu_s) \), with \( \nu_s \) being the speed of sound averaged over the transverse density profile [7]. This indicates that the tunneling is greatly suppressed when the distance between the domains is larger than the spin healing length, which determines the spin-domain boundary width through the coefficient \( \alpha \). In Table I we show the ratio of the calculated number of atoms that tunnel through to the total number of atoms in the middle domain. In the case of the tightest trap, the tunneling over the time of the evolution would exceed the number of available atoms, which explains the absence of a metastable state.

The second limitation of the method is related to the minimal size of a domain. For illustration, in Fig. 5 we present the condensate ground state in the double-well potential as in Fig. 2(a), but created with a very narrow Gaussian barrier. In this case the inner domains of \( m_f = +1 \) atoms are absent, and consequently the slow decrease of the barrier height leads to the ground state [Fig. 2(b)] instead of the metastable state [Fig. 2(c)]. The reason for the absence of the domains is the very steep slope of the density profile close to the barrier. According to Ref. [14], in the local density approximation (LDA), the spin state at a given point is determined by the value of the atom density at this point. The \( m_f = +1 \) atoms can reside only in a very narrow region on the density slope where it is smaller than a certain value. If this region is smaller than twice the spin healing length, the domain cannot form. The approximate size of this region can be estimated as twice the distance between the point of maximum density \( n_{max} \) and half the maximum density,

\[ \Delta z \approx \frac{2\sqrt{2\ln 2}}{\sqrt{\ln \frac{\Delta}{n_{0max}}}}. \]

We confirmed through systematic numerical simulations with varying barrier size that with a good accuracy the inner domains appear when \( \Delta z \) becomes larger than \( 2\xi \).

We also note that the preparation of spin domains depends on the quality of the initial states. The fluctuations of atom numbers in potential wells can influence the final patterns, which would manifest in fluctuating sizes of the domains. However, the accurate preparation of initial states is not crucial in our scheme, which relies on the phase separation phenomenon. The only requirements for the phase separation are the appropriate values of interaction constants and a sufficiently strong magnetic field.

V. CONCLUSION

In conclusion, we demonstrated a method of controlled creation of metastable spin domains in an antiferromagnetic condensate in a homogeneous magnetic field. The method exploits the phase separation of spin components in an external potential. In contrast to other methods, which rely on the spatially varying magnetic fields, the domain structure is here completely determined by optical fields, which makes this method more versatile and reconfigurable. Additionally, the method allows for creation of domains of various sizes, with spatial resolution limited by the spin healing length only.

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TABLE I. The ratio of the number of atoms that can tunnel through the barrier according to Eq. (13) within \( t = 5 \) s to the number of available atoms in the middle \( m_f = +1 \) domain.

<table>
<thead>
<tr>
<th>Trap frequency</th>
<th>( \frac{N_{tunnel}}{N_{total}} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.5 Hz</td>
<td>( 10^{-12} )</td>
</tr>
<tr>
<td>10 Hz</td>
<td>0.05</td>
</tr>
<tr>
<td>20 Hz</td>
<td>2</td>
</tr>
</tbody>
</table>

FIG. 5. (Color online) Ground state of the double-well potential for parameters as in Fig. 2(a) but with \( w = 9.4 \mu m \).